PROCEEDINGS

OF THE

NATIONAL INSTITUTE OF SCIENCES OF INDIA

JANUARY, 1947 — 1, 8

No. 1]	VOL XIII — X, 1	[Pp 1–63

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PUBLISHED BY
THE NATIONAL INSTITUTE OF SCIENCES OF INDIA
1 PARK STREET, CALCUTTA

Price Rs 3-2-0

P 46

ON THE STRUCTURE OF HYDRATED CELLULOSE OBTAINED FROM RAW JUTE FIBRE

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(*Received August 15, read August 30, 1946.*)

ABSTRACT

X ray investigation of the structure of raw jute fibre first treated with NaOH solutions of different concentrations, both under tension and without tension, washed in water and then dried in free air for several days reveals that when tension is applied to the fibre during the treatment even a 30% NaOH solution converts only a part of the native cellulose into hydrated cellulose, but if no tension is applied the cellulose is wholly converted into hydrated cellulose. The structure of this hydrated cellulose is slightly different from that obtained by previous workers from raw cotton or ramie fibre. The dimensions of the unit cell of this hydrated cellulose are $a = 8{\cdot}8$ A.U., $b = 10{\cdot}3$ A.U., $c = 9{\cdot}5$ A.U., and $\beta = 57° 54'$. This treatment is found to make the fibre softer and to diminish its thermal conductivity, so that some of its physical properties are almost the same as those of coarse wool.

INTRODUCTION

It is well known that when cotton or ramie fibre is treated with NaOH solution, washed in water and dried in free air the native cellulose in the fibre is either partially or wholly converted into hydrated cellulose, the percentage of native cellulose present after the treatment depending upon the strength of the solution used and its temperature. The question has been investigated thoroughly by Sisson and Saner (1941). They have shown that raw cotton fibre is completely converted into hydrated cellulose by the action of 18% NaOH solution at the room temperature, no tension being applied to the fibre during the treatment, but above 65°C. the native cellulose in the fibre is only partially converted into hydrated cellulose, even by 50% NaOH solution. Bleached cotton was found to give the same results as raw cotton fibre. The analytical composition of the hydrated cellulose has been observed to be the same as that of native cellulose, but the crystal structure of the former is different from that of the latter. The dimensions of the unit cell of dried hydrated cellulose as determined by Andress (1929) are: $a = 8{\cdot}1$ A.U., $b = 10{\cdot}3$ A.U., $c = 9{\cdot}1$ A.U. and $\beta = 62°$. It is, however, not known whether raw jute fibre, which contains about 10% to 13% of lignin besides native cellulose, behaves in the same way as cotton fibre when treated with NaOH solution. The present investigation was therefore undertaken to study the crystal structure of the product obtained by treating raw jute fibre with NaOH solutions of different concentrations both under tension and without applying any tension, washing it with water and drying. The values of thermal conductivity of the hydrated cellulose obtained by treatment with 30% NaOH solution and of the raw jute fibre have also been determined.

EXPERIMENTAL

The treatments which small bundles of 'white top' raw jute fibre had undergone before their structures were analysed with the help of X-rays are enumerated below.—

(*a*) The small bundle was kept immersed in 18% NaOH solution for half an hour without applying any tension and then washed in tap-water and dried in free air for more than a week.

(*b*) A second group of fibres was slightly stretched with the help of weights and the fibres were treated as in (*a*), (*c*) The process (*a*) was repeated in the case of a third bundle, using 30% NaOH solution

(*d*) The process (*b*) was repeated, using a fourth bundle and 30% NaOH solution

(*e*) The product obtained by treatment (*c*) was again kept immersed in 1% NaOH solution for a few hours, washed in water and dried in free air for a few days

(*f*) A second sample of the product mentioned above was kept immersed in water at about 65°C for a few hours and dried in free air for a few days

(*g*) A third sample of the same product was dried at about 106°C for three hours in an electrically heated chamber

X ray pattern of the product after each of the treatments mentioned above was photographed by exposing a group of about a dozen strands selected from the sample to unfiltered Cu radiation from a Hadding tube The strands were held parallel and close to each other with their lengths vertical in a special holder The width of the bundle was wholly covered by the cross section of the incident X-ray beam A slit system consisting of a cylindrical bore about 0 5 mm in diameter and 4 cm in length along the axis of a lead rod was used An exposure of about 8 to 10 hours was necessary for obtaining a good photograph

As the fibre subjected to treatment (*c*) mentioned above resembled coarse wool the thermal conductivities of this treated fibre and of the original raw jute fibre were also measured[1] using an apparatus used previously by Niyogi and Basu Mallik (1942) and modified recently by Bhattacharyya, P K , of this laboratory The results obtained in all these investigations are discussed in the following section

RESULTS AND DISCUSSION

The X-ray diffraction pattern of the fibre obtained after treatment (*c*) is reproduced in Fig 2, Plate I, while that for the original raw jute fibre is shown in Fig 1 The spacings of the planes giving reflections in the equatorial layer line are given in column 4, Table 1, these planes being marked A_1, A_2 and A_3 respectively starting from the innermost one If these are identified with (101), $(10\bar{1})$ and (002) planes respectively, as has been done by previous workers in the case of hydrated cellulose obtained from cotton or ramie, the dimensions of the unit cell given in the same column are arrived at For the reflections in the other layer lines the relation

$$\frac{4\sin^2\theta}{\lambda^2} = 0\ 018h^2+0\ 01544l^2-0\ 0177hl+0\ 00943k^2$$

is found to be satisfied The spacings of A_1, A_2 and A_3 and the dimensions of the unit cell observed in the case of dry hydrated cellulose by Andress (1929) are given in column 3, Table I It can be seen from Fig 2 that practically the

TABLE I

	Water cellulose Sakurada and Hutino	Hydrated cellulose dried (Andress)	Hydrated cellulose from raw jute (present authors)
A_1	8 98 A U	7 32 A U	7 96 A U
A_2	4 41 ,,	4 45 ,,	4 42 ,,
A_3	3 95 ,,	4 03 ,,	4 03 ,,
a	10 03 ,,	8 14 ,,	8 8 ,,
b	10 3 ,,	10 3 ,	10 3 ,,
c	9 98 ,,	9 14 ,,	9 5 ,,
β	52°	62°	57° 54′

---

[1] The authors' thanks are due to Mr S K Mukherjee for carrying out these measurements

whole of native cellulose in jute fibre is converted into hydrated cellulose by treatment (*c*) in which 30% NaOH solution is used and no tension is applied to the fibre The results given in Table I further show that this hydrated cellulose when dried in free air has a structure different from that found by Andress (1929) in the case of dry hydrated cellulose obtained from other sources The structure is also different from that of water cellulose obtained by Sakurada and Hutino (1936) by treating ramie with 18 5% NaOH solution, washing it in water and without allowing the product to dry, as can be seen from column 2, Table I They pointed out that in the case of the hydrated cellulose obtained by them some water molecules penetrated inside the lattice while the sodium atoms were removed by washing the treated fibre in water, and consequently, the unit cell was larger in the moist state than in the dry state The moist hydrated cellulose which was called by them 'water cellulose' showed a (101) spacing of 8 98 A U, but when it was allowed to dry in free air for three days this spacing was reduced to 7 66 A U and when dried at 105°C for about three hours the same spacing was further reduced to 7 32 A U It is, however, observed in the present investigation that the (101) spacing in the hydrated cellulose obtained from jute fibre by treatment (*c*) and dried in free air for more than a week is 7 96 A U which is greater than 7 66 A U observed by Sakurada and Hutino in the case of hydrated cellulose obtained from ramie and dried in free air When the hydrated cellulose obtained in the present investigation is dried at about 106°C in an electrically heated chamber, it is found to be partly converted into native cellulose and the spacing of the (101) plane of the remaining hydrated cellulose changes to 7 42 A U The pattern obtained after this treatment is shown in Fig 8, Plate I The presence of (101) and ($10\bar{1}$) reflections due to native cellulose is clearly seen between the (101) and ($10\bar{1}$) reflections of hydrated cellulose and the widening of the (002) reflection indicates the presence of (002) reflection from native cellulose corresponding to a spacing of 3 92 A U superposed on that due to hydrated cellulose Further treatments (*e*) and (*f*) do not alter the structure of the hydrated cellulose obtained by treatment (*c*) as can be seen from the corresponding patterns shown in Figs 6 and 7 It has also been found that ageing for three months does not alter the structure (Fig 9)

When tension is applied to the raw jute fibre during treatment with 30% NaOH solution washed in water and dried in free air, the major portion of native cellulose is converted into hydrated cellulose having the structure given in column 4, Table I, but part of the native cellulose remains unchanged as can be seen from the pattern reproduced in Fig 3, Plate I The proportion of such unchanged cellulose observed after treatments (*a*) and (*b*) (with 18 5% NaOH solution) is still larger as can be seen from patterns shown in Figs 4 and 5 Hence the behaviour of raw jute fibre is different from that observed by Sisson and Saner (1941) in the case of cotton fibre

The thermal conductivity K of the hydrated cellulose obtained from jute fibre in the present investigation is given in Table II

TABLE II

Substance	K in $\frac{\text{B T U in}}{\text{ft}^2\ \text{hour}\ ^\circ\text{F}}$
Hydrated cellulose from jute	0 24
Raw jute	0 28
Pure wool	0 24

It can be seen that the thermal conductivity of hydrated cellulose is smaller than that of raw jute fibre and is the same as that of pure wool This hydrated cellulose is much softer than raw jute fibre Hence it is quite suitable for being used as a cheap substitute for coarse wool in making warm fabrics

ACKNOWLEDGMENTS

The work was done under a scheme drawn up by Prof M N Saha, F R S, and financed by the Indian Central Jute Committee. The authors are indebted to Prof M. N Saha for kindly providing facilities for the work in the Palit Laboratory of the Physics Department and for his kind interest in the work, and to the Indian Central Jute Committee for the financial help

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EXPLANATION OF PLATE I

Fig 1 Raw High Top jute fibre
Fig 2 Fibre subjected to treatment (c), (30% NaOH without tension)
Fig 3 Fibre subjected to treatment (d), (30% NaOH with tension)
Fig 4 Fibre subjected to treatment (a), (18% NaOH without tension)
Fig 5 Fibre subjected to treatment (b), (18% NaOH with tension)
Fig 6 Product of treatment (c) subjected to treatment (e), (washed in 1% NaOH solution)
Fig 7 Product of treatment (c) subjected to treatment (f), (steeped in water at 65°C)
Fig 8 Product of treatment (c) dried at 106°C for three hours
Fig 9 Product of treatment (c) dried in free air for three months

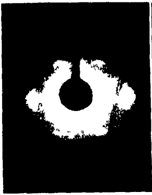

FIG 1

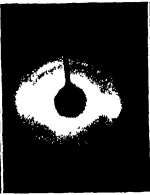

FIG 2

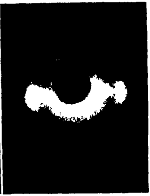

FIG 3

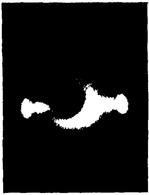

FIG 4

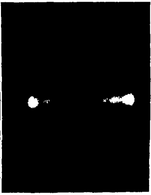

FIG 5

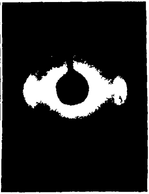

FIG 6

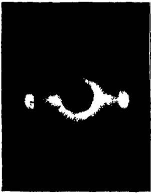

FIG 7.

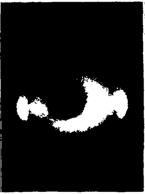

FIG 8

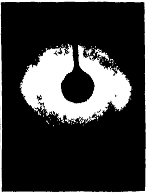

FIG 9.

SOME NEWLY OBSERVED LINKS IN THE NITROGEN CYCLE

By GILBERT J FOWLER, *D Sc* , *F R I C* , *F R San I* , *F N I*

(*Received September 10* , *read October 18, 1946*)

It is now nearly seventeen years since the present writer had the honour of delivering a second Presidential Address to the Indian Chemical Society and chose as his subject the field of work with which he was most familiar, viz 'Recent Researches on the Biochemistry of the Nitrogen Cycle' Following that address he was called upon some years afterwards to deliver a course of lectures as Sakraj Ray Reader in Natural Science in the Patna University and chose the same subject Finally in 1934 he collected such knowledge as he possessed in the book entitled 'An Introduction to the Biochemistry of Nitrogen Conservation' This volume was based on material concerned with the same subject extracted from a former publication, by then out of print, bearing the more extended title of 'An Introduction to Bacteriological and Enzyme Chemistry'

The decade following the publication of the later volume in 1934 was mainly preoccupied with the second world war and in consequence there was during most of the time an almost complete cessation of new public works construction

Nevertheless, perhaps indeed on account of this, it has been possible to devote closer attention to the actual operations for the disposal or purification of waste organic matter whether by means of sewage 'works' or sewage 'farm' As a result new phenomena have been studied and in some cases quite new view points have developed, from which older knowledge can be usefully criticised

In the absence of such research work those who were closely occupied with the daily duties of works or farm were content to base their operations on the simple sequence with which most would be familiar, viz the production of ammonia or amino compounds in the sewage tank by the bacterial decomposition of protein; the oxidation of ammonia to nitrate, and the recovery or fixation of nitrogen from the air by the activity on the farm of leguminous plants By the application of such knowledge it was possible to run a works or a farm with reasonable success There still remained the question of *sludge*, the 'slimy deposit' left in the settling tanks This was 'disposed of' in various ways or 'digested' with production of methane to be used for power, leaving behind the same residue as would be found at the bottom of a bog and with the same lack of agricultural usefulness

Besides liquid nitrogenous waste or sewage, i e 'water after it has been used', is the dry or semi-dry refuse arising from the 'conservancy methods of towns, this material being either burnt in destructors, with resultant piles of useless dust, or disposed of in 'controlled heaps' not infrequently becoming a paradise for rats

It has been well said that if the same attention were to be given to sludge and other kindred waste material as has been given to coal, which, after all, is only an advanced stage of sludge, results of even greater value might be expected The subject, as a valued correspondent put it after a talk with Sir Robert Robertson, affords not merely a field but a 'perfect prairie' of research

Clearly, to vary the old proverb, if a chain is to be strong every link must be able to stand the strain Thus for the economic conversion of waste organic matter into food for man and animal there must not be preventable loss at any point

Pondering upon the many possibilities of new knowledge the writer was led to reconsider some important observations of his earlier years connected with the function of nitrate in the sequence of changes involved in the complete conversion of putrescible organic waste matter into inoffensive plant food This and no less, it may be said, is the true objective of the sanitary chemist and engineer

Consequently, the writer put together his thoughts on the subject in a tentative paper entitled 'The Function of Nitrate in Sewage Purification' and sent a few copies to his fellow workers in the sewage field for their criticism A considerable file of interesting and important comments resulted and the present paper is based on the original draft amplified by still more recent observations which have been made in this and other allied aspects of the cycle

The paper may thus be conveniently divided into two parts, viz —

I The Function of Nitrate in Sewage Purification
II Other Links in the Life Cycle

PART I

The Function of Nitrate in Sewage Purification

Early in the writer's experience in the field of Sewage Purification, viz as far back as 1901, he published a paper before the Royal Institute of Public Health entitled 'Some Points in the Management of Septic Tanks and Bacterial Contact-beds'

The paper was accorded a good deal of attention at the time and the main conclusions were embodied in the Annual Report of the Manchester Corporation for the year ending March, 1901 The following extracts have a special bearing on the subject now under consideration —

> The thorough drainage of a bacteria bed is of the first importance in securing a good effluent If the water cannot get out, the air cannot get in, and the lower parts of the bed rapidly become putrid and the nitrates decrease, perhaps are quite absent Here it must be emphasised that when the nitrates decrease and simultaneously there will always, as a rule, be an increase of nitrites, the bed must be rested
>
> On examining the material of a contact bed in active condition, every piece of it will be found coated over with a slimy growth If this is removed it is found to be a stiff jelly which after a little drying can be cut with a knife
>
> If placed in a tube containing air, and connected with a manometer, the jelly will rapidly absorb all the oxygen with production of carbon dioxide This action will sometimes produce a vacuum equal to several inches of mercury This experiment shows that there is little need to force air into a bed
>
> For the successful working of bacteria beds, the following methods of procedure will be calculated to give the best results The bed must be worked very slowly at first in order to allow it to settle down and the bacterial growths to form
>
> The burden should not be increased till analysis reveals the presence of surplus oxygen, either dissolved or in the form of nitrates, in the effluent

As a result of the careful adherence to this procedure in the days of early enthusiasm effluents of exceptionally high nitrate content were obtained so that it was possible to adopt a policy of mixing highly nitrified effluents with those less efficiently purified giving a mixture which was non-putrefactive and which was accepted by the statutory authorities

There can be no doubt that when a contact-bed is worked strictly according to the rules laid down in the early days of intensive study, high nitrification results, and in the case of secondary bed 10*a* (at the Davyhulme Works) an almost indefinite prolongation of the life of the bed was achieved This bed was stopped on the 22nd February, 1932, after having been in operation for 27 years During the last two years in which the bed was in operation the nitric nitrogen content in the effluent was maintained steadily at an average of 0 82 parts per 100,000 From the corresponding figures for ammoniacal nitrogen in the influent and the effluent, viz 1 81 and 1 09 respectively, there would not appear to be much loss of nitrogen during the passage of the influent through the contact bed In this somewhat remarkable case it is evident that the bacterial jelly with its adsorbed organic colloids was continually converted under steadily maintained aerobic conditions into easily drainable *humus* Some of this no doubt escaped as fine particles in the effluent and the remainder formed part of the active body of the contact bed

It is at any rate clear that the presence of well oxidised humus is favourable to active nitrification

The relation between the presence of humus and the nitrification of organic matter would seem to have been first emphasised by Adeney in his classical researches on the conditions of oxidation of sewage matter

Adeney concluded that oxidation of organic matter proceeds in two well defined stages which may be briefly described as the *carbon* oxidation stage and the *nitrogen* oxidation stage

While confirming the conclusions of earlier workers that the nitrous organism cannot carry oxidation beyond the stage of nitrite whereas the nitric organism only oxidises nitrites to nitrates, he added the further important conclusion that the presence of peaty *humus* matter appears to preserve the vitality of the nitric organisms during the earlier stages of the fermentation process and to establish the conditions whereby it is possible for the nitric organisms to thrive simultaneously with the nitrous

The somewhat curious fact of the apparent necessity of the presence of humus, if sewage matter is to be fully nitrified, is left unexplained and undiscussed The importance in publications of scientific research of accurately recording facts, even if not completely understood, is once more emphasised in these observations of Adeney

A very important paper has since been published entitled 'Some Further Considerations on the Oxidation of Sewage' by F R O'Shaughnessy and S J Roberts (*J Soc Chem Ind*, Vol 57, p 281, 1938) in which the somewhat indefinite reference of Adeney's is shown to be of great significance

O'Shaughnessy and Roberts state that under conditions such as may obtain in practice carbon and nitrogen oxidation may proceed simultaneously but the oxidation of ammonia is dependent not upon the absence of carbon but upon the presence of *humus* solids

It is evident from this observation that the sludge produced after nitrification has set in, i e what may be termed a 'nitrifying sludge', is of a quite different character from that obtained when aeration is continued merely to the 'clarification' or even 'carbon oxidation' stage The earlier partially oxidised product may be described as *floc*, the other as *humus*

There is, however, a still further stage to be considered In describing the biochemical changes taking place in the contact bed it was pointed out that there is an alternation of active nitrification while the bed is emptying and draining and of *denitrification* when it is standing full The denitrification process is at the same time one of carbon oxidation, and cellulose and hemi-cellulose derivatives are then converted into *humus* much as they are in the gardener's leaf mould pit

To limit the aeration in an activated sludge tank merely to what may be termed the flocculation stage is to fail to utilise the reserve of oxygen in the nitrate producible from the ammonia still present and to leave a sludge disposal problem of increased difficulty, *floc* being much less easily drainable than *humus*, and since there is no reserve of nitrate in the interstitial water the sludge is liable to offensive decomposition if allowed to accumulate under anaerobic conditions

Moreover, this denitrification process can be utilised for the final purification of semi-purified effluent by a mere mixing together either in a final denitrification tank or actually in the stream receiving the effluent

As already noted, this method of final purification was actually advocated by the Manchester chemists in the days of the contact bed and was accepted as adequate by the supervising authorities The writer would urge that close study be given to this cycle of nitrification and denitrification changes in the light of all the experience which has accumulated since those early days

In the old and often quoted experiments of Scott-Moncrieff a high degree of nitrification was obtained by trickling an effluent with high ammoniacal nitrogen content over a series of superimposed trays containing filter media The course of nitrification was considerably interfered with if the sequence of the trays was altered The assumption was that the activity of the nitrifying organisms was inhibited

Recent experience would point to a different explanation Researches published by the Water Pollution Research Board in their Annual Report for the year ended June 30th, 1938, show that when two percolating filters are operated in series great efficiency results if the action is from time to time reversed and No 2 filter becomes No 1 filter and *vice versa*

It would thus appear more likely that denitrification takes place resulting in more complete oxidation of the organic matter as a whole but less *apparent* yield of nitrate

That such alternate nitrate formation and reduction takes place even in a compost heap is clear from the early experiments of Rege (*J Ind Inst Sc*, Vol 8A, Part XIII, 1925) which laid the foundation of the technique followed in the production of what is known as 'activated compost'

Rege found that by aerating a mixture of sulphate of ammonia, calcium carbonate and grass powder with activated sludge the ammonia at first was rapidly converted intro nitrate, while on further aeration the nitrate disappeared with, however, no loss of total nitrogen From this point repeated additions of ammonium sulphate and grass powder were made, when it was found that disappearance of nitrate from the solution took place more and more rapidly with successive additions Finally, the additions of ammonium sulphate and grass powder were made simultaneously, and at this stage the whole process of nitrate formation and disappearance took place within 24 hours

Experiments (unpublished) carried out a few years ago at the Indian Institute of Science showed that if finely powdered cellulose matter was added to the aeration tanks in an activated sludge plant (i e in presence of ample aeration by means of diffused air) all the soluble nitrogen was removed from solution and was built into the resulting sludge which was very easily drainable and was of the nature of *humus*

A study of recently published discussions on closed percolating filters leads to the conclusion that improved conditions for nitrification due to rise of temperature is the most important factor in the apparent increased output of a closed filter

The old observations mentioned earlier of the measurable vacuum caused by the 'breathing' of the bacterial jelly on the medium of the contact bed would seem to render unnecessary the forced aeration of the closed filter Provided the natural air supply is not impeded it is doubtful whether any artificial increase over the natural draught will seriously accelerate the rate of oxidation of the black film frequently present on the under surface of the slimy coating on the filter medium On the other hand, the infiltration of a solution of nitrate would facilitate the oxidation of the black film with simultaneous reduction of nitrate

No doubt in absence of adequate aeration *nitrites* may be formed with consequent loss of nitrogen either in the free state or as nitrous oxide which gas was actually found by Letts to be present in a contact bed under certain conditions It must be remembered that every *percolating* filter is really intermittent in its action Unless the influent is actually run through in a single mass as in a mechanical water works filter there is an interval between the arms of a rotary distributor or the return journey of the travelling distributor on a rectangular filter which must produce conditions alternating between nitrification and denitrification

Careful study of the conditions of compost making shows the necessity for adequate aeration if loss of nitrogen is not to take place Howard, indeed, advocates that compost heaps should be provided with aerating chimneys and ample underdrainage

The periodic 'turnings' of a compost heap again are likely to produce conditions of alternate nitrification and denitrification, although simultaneously carbon oxidation will go on If there is no actual loss of nitrogen in this case, and careful frequent analyses seem to negative this possibility, then the ammoniacal nitrogen must eventually be 'demineralised' and be 'built in' to the *humus* which finally results, passing no doubt during the process through the bodies of many living organisms characteristic of the special conditions obtaining in any given case

Attempts have been made by various workers in the past to purify sewage merely by the addition of nitrate of soda in quantity sufficient to supply adequate oxygen through denitrification It is doubtful whether the conditions thus arranged are really the most suitable for efficient purification The following passage from the Annual Report of the Rivers Department of the Manchester Corporation for the year ending March 27th, 1901, p 73, is not without significance —

> 'It was found that when a primary filtrate was allowed to stand overnight in contact with air, or when it was aerated by shaking every quarter of an hour during two hours, a distinct improvement resulted, though almost invariably at the expense of the nitrate present
>
> 'The economical bacterial purification of sewage on a large scale depends essentially upon the addition of oxygen *in presence of the requisite bacteria*[1], consequently, although aeration alone is incapable of appreciably purifying sewage (see City Surveyor's Report, 1897, page 29), yet when aerobic bacteria are introduced by addition of a volume of well-nitrified filtrate, it is probable that aeration may play a much more important part '

Here possibly may be seen an adumbration of the Activated Sludge process

Finally, reference may be made to the remarkable results obtained by Lockett at Mogden by what he terms the 'complete process' In this technique Lockett relies upon the frequent complete conversion of ammoniacal into nitrate nitrogen This is effected by recirculating a large proportion of the highly nitrified effluent together with a high proportion of nitrifying sludge Lockett describes the sludge so obtained as settling very rapidly and being easily filtered and has stated in a letter that it contains more than 8% of nitrogen It might be assumed that this high percentage of nitrogen is primarily due to what may be termed the 'fractionating' of the sludge by the removal in presettlement tanks of a large proportion of the mineral matter which would otherwise form part of the final product In view of other observations, however, it is possible that this high percentage of nitrogen content is due to the 'demineralisation' of ammoniacal nitrogen already referred to in connection with Rege's researches on compost In any event Lockett's results entirely confirm the conclusions of workers in other spheres, notably Howard who writes from the point of view of an agriculturalist, that aerobic conditions are necessary if true *humus* is to be obtained and, it might be added, if the maximum conservation of nitrogen is also to be effected in the various techniques employed in the utilisation of habitation waste

The present writer's conclusion is that the true conditions for the efficient and economical purification of sewage consist in the building up of an initial mass of highly active nitrifying sludge and mixing this with the sewage under conditions of *adequate aeration* A state of things is thus set up in which the nitrate present is alternately reduced and re-formed from fresh additions of ammoniacal nitrogen, the humus which is simultaneously produced showing a high percentage of nitrogen due to the building of ammoniacal nitrogen into the bodies of living organisms and possibly also to the formation of complex organic molecules by the combination of the nitrogen with the carbohydrate residue present

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[1] Italics absent in original reference

In this way the nitric nitrogen present virtually acts as a catalyst and once the correct conditions are established a considerable reduction in the power consumption required for aeration should be achieved A hint of such a result is given in his Report for 1937 by the Chief Sewage Chemist of Shanghai He speaks of having more or less by chance obtained a sludge in all respects satisfactory with a *low power consumption* This means that it is the *quality* of the sludge, quite as much as the quantity of air employed, which has to be studied and properly understood

The present writer would urge the importance of renewed careful and exact research with the object of obtaining further knowledge on these lines Such researches may result in very valuable additions to the underlying theory of the activated sludge process and make ultimately for greater economy and efficiency without loss of the agricultural value of the end products

PART II

Other Links in the Life Cycle

The foregoing pages are thus chiefly concerned with the two end products of sewage purification, viz *nitrate* and *humus*, and their relation to the actual technical processes employed in the 'sewage *works*'

Essentially, the same changes go on in the 'sewage *farm*' when nitrogenous waste matter, whether liquid in the form of sullage or sewage, or semi-solid in the shape of various forms of organic manure, is brought into contact with the soil, there to provide food for the growing plant

Here, however, other factors enter into the cycle Through the interaction of oxidisable matter with the soil particles, purely physico chemical factors have greater scope Forms of life other than specific and well-known types of bacteria play their part, and finally the conditions of life of the growing plant affect the changes occurring in the soil complex in which it grows

Just as knowledge gained on the sewage works may be applied in the running of the sewage farm, so observations made on the 'sewage farm' may find useful application in the operation of the 'sewage works'

The 'Newly Observed Links' may, therefore, be considered under the following categories The active agencies comprised in these categories contribute, in collaboration with the two end products (i e nitrates and humus) so far considered, to the life of the growing plant and so to the life of men and animals —

1 Physico-chemical factors
2 Bacteria concerned with proteolysis, nitrification and denitrification changes, and immobilization or translocation of nitrogen
3 Biological factors concerned with nitrogen fixation Bacteria, free living and symbiotic Other living agencies
4 The Mycorrhizal association
5 Protozoa
6 Animal and man

1 Physico-Chemical Factors

It is to Dr N R Dhar and his co-workers that we owe the most recent and continuous research on this subject While it is commonly believed that the processes of ammonification, nitrification and nitrogen fixation are mainly the result of bacterial activity, Dhar has emphasised the importance of purely physico-chemical factors, sunlight being the source of energy The researches of Dr Dhar and his colleagues up to 1937 are collected and summarised in a valuable paper in the *Proceedings of the National Institute of Sciences of India* (Vol III, No 2, pp 75–131) under the title of 'Nitrogen Transformations in the Soil'

Thus to take the simpler stage of *nitrification* of ammonium salts, experiments in vitro when ammonium salts were mixed with sterilised and unsterilised soil, exposed to sunlight and kept in the dark, a much greater percentage of ammonia was oxidised to nitrite and nitrate in the case of the mixtures exposed to sunlight whether the soil was sterilised or unsterilised Similarly dilute sodium or potassium *nitrite* solutions are readily oxidised to nitrate when exposed to light

Dr Dhar, therefore, considers that the bacterial factor in nitrification has been somewhat over emphasised especially in tropical countries

This point of view finds confirmation in the old experiments of Cavendish who brought about oxidation of ammonia on surfaces in 1777 There is also the well-known technical method of producing nitric acid by the passage of ammonia gas over heated platinum gauze, as well as the experiments by Warburg confirmed by Norris and Ranganathan showing the effect of animal charcoal in accelerating the ordinary bacterial nitrification process

While these purely physico-chemical factors, especially the effect of tropical sunlight, may be freely admitted and deserve further study, especially in connection with production of nitre in so-called nitre beds, yet it is difficult to believe that sunlight has much influence in the nitrification process as effected in the various filtration processes in the sewage works where it is often highly efficient, and where it must occur in the inner depths of the filter bed entirely cut off from sunlight The same conditions are present in the activated sludge tank and also in the compost heap, although to a less extent, since there is in both cases a periodical change of surface at longer or shorter intervals It would be of interest to find out by experiment whether any different results were obtained when these processes were operated under conditions differing only in the presence and absence of sunlight

Ammonification

Dhar and his colleagues found that substances like egg white, gelatine and blood serum, yielded ammonia when exposed to air in presence of sunlight the amount being greatly increased in presence of solid surfaces like TiO_2, ZnO, SiO_2, etc , the best results being obtained with TiO_2

It was found that these reactions were autocatalytic inasmuch as when the pH increased through the increase of ammonia the rate of oxidation increased proportionately Thus it would appear that the formation of ammonia in the soil is a surface reaction and chiefly an oxidative process, taking place on the soil surface with liberation of energy

Similarly experiment indicated that nitrite could be converted to nitrate by direct aeration especially in presence of inductors like ferrous hydroxide, sodium sulphite, etc , where another physico-chemical action is involved, viz *induced* oxidation

Dhar, therefore, concludes that the processes of ammonification and nitrification can be photochemical rather than bacterial especially in tropical countries where the number of bacteria is small being mostly killed by the high temperature of the soil during the summer months

That oxidation changes in the soil, resulting in the quicker availability of organic manures, can be accelerated by purely *chemical* agencies has been shown by the work of C R Harihara Iyer and V Subrahmanyan who have studied the effect of small quantities of manganese and iron salts on the fertilising activity of the soil during the life-time of the crop, thus compensating in some measure for the losses of plant food ingredients during periods of fallow, which in tropical countries are greater than in temperate zone climates Their main findings have been confirmed by others Thus Vyas has reported that manganese and iron oxides could be used to counteract the toxic principles left in the soil by 'jowar' and thus increase the yields of the succeeding crop

These inorganic chemicals could be applied directly to the soil or be incorporated with the manure prior to application to the soil

The researches of the aforesaid workers agree in showing that the purely oxidative changes concerned with the conversion of organic matter into end products of humus and nitrate are at any rate capable of being considerably affected by non-living agencies both chemical and physical

For the practical application of these results experiments are called for which would determine the economic factors involved Of these one of the most obvious is the cost in labour or mechanism incurred by adequate exposure to sunlight of the surface both of soil and of compost heap

It would be of interest to compare the results from two sections of an activated sludge plant comparable in all respects save that one was exposed to sunlight and the other 'blacked out' In both sections there would be an equivalent exposure of fresh surfaces only one of which would be exposed to sunlight

The same principle might be applied in the case of compost heaps though here the change of surface could hardly be complete

In view of recent unpublished experiments by Pillai and Subrahmanyan on the effect of dilution of sewage in presence of oxygen and the observed streaming effect of dissolved oxygen, e g in the rusting of strips of suspended iron, it would be useful to compare the changes taking place in sterilised and unsterilised material not only in presence and absence of sunlight but also in presence or absence of air Does sunlight influence anaerobic changes ?

Nitrogen Fixation

Purely physico-chemical factors play a considerable rôle in the more complicated changes concerned with the important 'link' of nitrogen fixation, important since without it there would be permanent loss of nitrogen from the cycle of life

Dhar brings evidence to show that the energy available by direct oxidation of organic matter in the soil as described in the foregoing paragraphs can be utilised to bring about the oxidation of other organic matter by induction or by catalysis, as well as by bacterial action Thus the addition of molasses to soil in presence of sunlight brings about an appreciable increase in the nitrogen content of the soil under conditions of adequate exposure to air and sun

The same arguments therefore in regard to the economics involved, i e the labour or mechanism needed for continual exposure of fresh surfaces to sunlight holds also in the derived reactions concerned with nitrogen fixation

It is of interest in this connection to note that the activity of worms in chewing up raw organic debris such as leaves, etc must result in making a product which is more readily oxidisable and so any system of manuring which affects the worm population such, e g as an excess of 'mineral' manures, will tend to retard nitrogen fixation

Trace Elements particularly boron and zinc may play some part in these physico chemical effects, as well as in the ammonification effects already referred to. Although the quantities of these may be infinitesimally small, a few parts per million in the soil and less in the plant, they are found to be essential although larger amounts may be toxic In a recent address Dr W G. Ogg, the new Director of Rothamsted, divides trace elements in Agriculture into four groups —

- (i) Those necessary for normal healthy plant growth Among these are comprised Boron, Manganese, Copper and Zinc
- (ii) Those which are toxic to plants, e g excess of Manganese
- (iii) Those necessary for animals if not for plants, viz Cobalt and Iodine.

(iv) Those poisonous to animals but not to plants Among these may be classed Molybdenum and Selenium

For research on such traces quantitative spectrography has been found valuable The necessary apparatus is hardly likely to be found in the normal sewage works laboratory

Besides the necessary presence of these minute quantities of trace elements, the absence from the soil, or it may be from the fertiliser, of certain minerals may have serious effects Thus a deficiency of calcium may be detrimental to the rape plant or may cause manganese toxicity to develop in cauliflowers Potash deficiency may adversely affect barley and mangold plant Magnesium in defect is shown by leaf disease in tomatoes and pears

Clearly in a prepared sludge for the application to a given crop all these deficiencies can be suitably adjusted

2 *Bacteria concerned with proteolysis, nitrification and denitrification changes, and immobilisation or translocation of nitrogen*

Although it may be admitted that the three first of the above-mentioned natural processes occurring in the normal life-cycle can be brought about by physical or physico chemical agencies, yet there is abundant evidence that under normal circumstances of the sewage works or sewage farm the chief agencies are specific bacteria Accounts of their nature and mode of action will be found in text-books dealing with the bacteriology of agriculture and sewage purification

Reference has been made in Part I to the immobilisation of soluble nitrogen in the compost heap and in the activated sludge tank by aeration of soluble ammoniacal salts in presence of hemi cellulose material This reaction is of considerable practical importance in connection with the utilisation of the effluent from the activated sludge tank for the irrigation of crops since it is frequently the case that an excess of nitrogen is prejudicial to the crop at certain stages of its growth Scientific control of the amount and character of the nitrogenous content both of effluent and sludge is therefore needed if the nutritional requirements of the crop are to be properly met

Humus and Plant Nutrition

The extensive and world-wide observations which are being made by Sir Albert Howard and his numerous fellow-workers and which have been published in sundry volumes and in the issues of the *Compost News Letter*, now appearing as *Soil and Health* make the old time crude efforts to dispose of the sludge on the land for crop production seem rather elementary In those days N P K percentages were almost the sole criterion of value and short-time results and ease of application made 'artificials' popular and the 'law of return' was ignored Many years ago I remember while we were washing up the old contact beds at Davyhulme there was a demand for almost any kind of dried sludge, or humus forming material, to bring back fertility to the eroded and wind-swept soils of Canada

In the last issue of *Soil and Health* there is the first of a series of papers by J E R McDonagh, F R C S, on the 'Nature of Health and Disease in Plants' in which particular attention is drawn to the *Rôle of the Sap Protein in Health or Life*

In a recent address on *Food and Phylogeny* Dr C S Haynes, F R S, classifies nutrients as (*a*) sources of cellular energy, and (*b*) sources of specific chemical molecules required for growth which a particular organism is unable to synthesise for itself

Nutritional requirements are the reflection of basic biological problems of biosynthetic capacities of different organisms.

It can hardly be expected that such complex requirements can be met by mere addition of elementary chemicals like sulphate of ammonia any more than any kind of protein molecule will serve as human food More than a decade ago it was shown by Rose that only certain amino acids were of nutritional value Recently these observations have been extended and comparatively small differences in molecular structure of various isomeric amino acids have been shown greatly to affect their susceptibility to enzyme action and consequently their nutritional value Similarly the detailed structure of complicated anti-malarial drugs affects their toxic properties

It is evident that careful preliminary preparation of putrescible sludge (i) by final oxidation by direct aeration or by composting in presence of air, (ii) by elutriation, (iii) or by other means which may be revealed by research, will convert the one-time 'slimy deposit' into valuable plant or even animal food It has been already hinted that there may be a future before sludge chemistry comparable to the coal-tar industry Unlike coal which has to be taken as nature left it, sludge can be modified in composition and properties while it is in the course of production Already in the U S A household implements have been devised for eliminating household kitchen waste by disintegration and discharge into the sewer to be treated along with other sewage solids and recovered with the remaining sludge If there is a large proportion of hemi cellulose residues present it may have the effect of immobilising a certain amount of soluble nitrogen and withdrawing it from the effluent into the sludge Such a process is important when the effluent has to be used for irrigation of crops as these may find too much nitrogen detrimental at certain stages of growth In the absence of refuse cellulose material it may indeed pay to add some suitable source of hemi-cellulose such as chopped grass for the specific purpose of immobilising ammoniacal nitrogen Preliminary experiment at Bangalore has shown this to be possible

Briefly it may be stated that anaerobic action transfers nitrogen from the sludge to the effluent, aerobic action in presence of cellulosic material transfers nitrogen from the effluent to the sludge

3 *Biological factors concerned with Nitrogen fixation Bacteria, free living and symbiotic Other living agencies*

An excellent compendium of information on the 'Fixation of Atmospheric Nitrogen in Living Forms' has been published by T R Bhaskaran and S C Pillai in *The Indian Journal of Agricultural Science*, Vol XII, Part I, February 1942

They confirm the general thesis that the amount of nitrogen fixed is proportional to the energy developed by carbon oxidation in a given time

In their summary they set out no less than 50 items Only the more striking and recently observed of these can be here mentioned

Azotobacter and *Clostridia* are the important non-symbiotic organisms which fix nitrogen in the soil The azotobacter is typical of the aerobes and the clostridia of anaerobes Azotobacter uses carbohydrates, salts of organic acids and alcohols as energy sources Soil humus has been found to exert a stimulatory influence on the organism for nitrogen fixation Vitamin B_1 and phytonucleic acid stimulate growth and nitrogen fixation Certain minerals in optimum concentration are necessary for the growth of azotobacter and among these calcium (replaceable by strontium) and molybdenum (replaceable by vanadium) are specific for nitrogen fixation, manganese and uranium accelerate nitrogen fixation Iron plays no specific rôle in the process

Light is not without effect on the activity of this organism, thus, in a measure, confirming the conclusions of Dhar, but yellow light is better than blue

Hydroxylamine would appear to be the first intermediate product formed during nitrogen fixation

Bhaskaran and Subrahmanyan have reported that the fixation of nitrogen by the mixed flora of the soil follows a different course from that of azotobacter alone in artificial media In the latter case fixation only proceeds so long as the sugar lasts in the medium With a mixed flora only a small quantity is fixed in presence of sugar while the major part, amounting to over two-thirds of the total quantity fixed, is fixed in the later stages They have further shown that the products of decomposition of sugar are utilized in this subsequent fixation

These products of decomposition of sugar consist of simple organic acids and alcohols It is likely that the energy resulting from their oxidation is utilised for nitrogen fixation by purely physico-chemical reactions in accordance with Dhar's observations

Other agencies

Long ago Jamieson of Aberdeen contended that nitrogen fixation took place primarily through the agency of the leaf hairs which produce albumin from the nitrogen of the air According to Bhaskaran and Pillai several workers have reported from time to time that different parts of higher plants exhibit the power of fixing atmospheric nitrogen either by themselves or by their association with the bacteria present in them but evidence so far obtained is still inadequate to draw any definite conclusion regarding the relative importance of these as nitrogen fixers

It may well be that the extent of nitrogen fixation in any given case depends on the conditions obtaining in each case, viz the presence of symbiotic agencies whether plant or micro organism or on the availability or otherwise of nitrogen from other sources

The activities of protozoa in the field of nitrogen fixation will be referred to when considering their specific functions in other portions of the cycle

4 *The Mycorrhizal Association*

The mycorrhizal association may be defined as the mechanism by which living fungous threads (mycelium) invade the cells of the young roots and are gradually digested by these

This important link in the nitrogen cycle has received detailed attention in the writings of Sir Albert Howard and his school I well remember the visit that Sir Martin Forster (Dr Forster as he then was) and I paid to Pusa in the early nineteen twenties and the fascinating examples then shown us by Howard illustrating the importance of root development and of how a well developed root system was virtually a mirror image of the plant above ground This lesson I have since striven to impress upon sundry mahlis who prefer to souse a plant with water rather than to do a little careful digging in order to maintain a reasonable amount of root aeration Fully to expose the root system involves, as Howard showed us, very careful washing away of the surrounding soil

At that time no mention was made of the mycorrhizal association the significance of which was not fully understood, although the association of fungus mycelium with root cells had been observed, and the term mycorrhiza given to the mycelium, as early as 1829

The careful researches of Dr Rayner on the mycorrhizal association in relation to conifers at Wareham in Dorsetshire where small additions of properly made compost had produced spectacular results, led Howard in 1937 to consider the possibility of the phenomenon being general and of its having some special function in connection with the nutrition of the plant on the roots of which it was observed

Careful observations were then made by Dr Rayner and Dr Ida Levisohn and others of the root systems of many plants for evidences of the mycorrhizal association It was found that plants manured with artificials or grown on derelict land showed poor development .Dr Rogers of East Malling in Kent devised an observation chamber for root studies He arranged a vertical darkened glass window on the side of a deep pit in an orchard In this it was possible with the assistance of a conveniently arranged low-power microscope to observe some of the soil fungi actually at work (A photo of one such observation is given in Howard's recent book, p 35)

The universally beneficial effect of organic manure whether in the form of compost or other commonly employed humus-forming material is thus seen to be due to the support given to the necessary fungous mycelium

These careful microscopic observations show that in the invaded cell the mycelium exhibits a regular sequence of changes from invasion to the clumping of the hyphae around the cell nuclei, digestion and disintegration of their granular contents and the final disappearance of the products from the cells In this way the digestion products of the proteins of the fungus pass into the cell sap and thence into the green leaves

The mycorrhizal association has been found to be a very widespread phenomenon The following important crops are all mycorrhiza formers wheat, rice, tea, coffee, cacoa, sugarcane, cotton, sisal, maize, cocoanuts, bananas, citrus fruits, grapes, apples, pears and peaches

Some singular exceptions occur, viz tomato and cabbage The beneficial effect of organic manure is nevertheless clearly observable in these cases and it appears likely that the protein requirement supplied by the mycorrhizal association is in these cases derived from the dead bodies of the bacteria present in the organic debris

While it might be supposed that legumes would be sufficiently supplied with nitrogenous nutriment through the intervention of their root nodules it appears that they also need the assistance of the mycorrhizal association if they are to retain the power to produce seed

The relation between the intake of protein and the observed power of disease resistance is explained by J E R McDonagh by the character of the protein digestion products supplied to the cell sap as already referred to in section 2

The ultimate consequences of these new observations and conclusions are very far reaching In order to maintain the necessary supply of organic manure, 'mixed farming' is essential, i e proper rotation of crops and the intervention of livestock There must indeed be a definite ratio between the number of livestock and the crop acreage

5 *Protozoa*

It has been contended that the ordinary British mind prefers action to meditation with the result that experience is gained through the encountering of practical difficulties which might have been avoided by the expenditure of more time and thought on preliminary investigation A further consequence of this method of procedure is that important work of an apparently recondite character is overlooked and its true significance is only appreciated when its bearing is seen on more practical issues In few instances is this feature more strikingly exhibited than in the history of the *activated sludge process* of sewage purification

I must admit my full share in this apparent blindness to what are now fairly obvious clues to a true scientific theory of the process In preparing my earlier book on 'Bacteriological and Enzyme Chemistry' I must have become aware of Munro's experiments on the acceleration of the nitrification process by a technique of *Activation* identical with that used in the building up of activated sludge Yet, unless subconsciously, it had no direct influence on the course of the early research work on which the process was based, although it was fully recognised later Of possibly even more importance has been the failure to recognise until quite recently the fundamental significance not only in the activated sludge process but in the nitrogen cycle generally of the activity of *protozoa*

That the presence of protozoa in sewage and effluents was long ago recognised is clear from a discussion held at a meeting of the Royal Sanitary Institute in 1909 on the effect of biological conditions on the quality of effluents. Here the discussion centred round the question of how far merely chemical figures of analysis really were sufficient to define the effect of an effluent on the stream into which it flowed It was contended that an effluent adequately purified so far as the chemical figures indicated might yet start up various living growths—fungoid, algal or protozoan—which in turn might adversely affect the amenities of the stream Among the protozoa then under observation was the vorticellid known as *Carchesium Lachmanni* and Mr Glover, chemist in charge, under my direction, at the Withington works of the Manchester Corporation, made some excellent *camera lucida* drawings of the development of this organism and established the fact that fission of one head and formation of two complete organisms took place within three quarters of an hour These drawings were reproduced in the Annual Report of the Rivers Committee and were copied by my former good friend the late Dr Calmette in one of the admirable reports he was then compiling for the advancement of sewage purification technique in France This remarkable rapidity of reproduction was of great significance in relation to the possible function of this or kindred organisms in nature It was to be emphasised that the growth did not occur in effluents which would be classified by chemical standards as unpurified but in discharges which were actually in process of nitrification Thus an abundant growth of *carchesium* was afterwards noticed in the effluent channel from a final or 'secondary' contact bed at the Davyhulme works, this bed by the way being operated on the continuous flow system All these observations can now be seen to have a close bearing on the function of protozoa both in the operation of continuous filters and of the activated sludge process Unfortunately again the findings of distinguished workers in other fields tended unduly to influence the conclusions of many of the earlier workers on the activated sludge process, and so it is only in quite recent years that the true function of protozoa has been recognised and fully investigated The early history of the Activated Sludge process is admirably set out in the remarkable book by A J Martin (*The Activated Sludge Process* by Arthur J Martin—London, Macdonald and Evans, 8 John St , Bedford Row, W C 1, 1927) which becomes of greater historical value as the years go on

Protozoa of differing species were described by various observers but were thought to be incidental to the process rather than essential The study of the experimental activated sludge plant at the Indian Institute of Science by Swaminathan, done largely under my supervision, was influenced by the results of Russell and Hutchinson on the apparently favourable effect on plant growth, notably of tomatoes, by the elimination of the protozoal population by the old gardeners' recipe of heating the soil This was confirmed by Fairbrother and Renshaw who used methylene blue as a partial sterilisation agent Swaminathan's observations did not indicate any marked effect favourable or otherwise on the process from the application of this treatment to the activated sludge as distinct from soil, although it was confirmed that the effect of partial sterilisation was to increase the number of bacteria in the sludge as moderate heating had increased the number of bacteria in the soil

Apart from protozoa it was suggested by Dr Bartow, one of the most distinguished of the early scientific investigators of the activated sludge process (Dr Bartow later was elected President of the American Chemical Society), that the red worms *Aelosoma Lemprichii*, often found in decaying organic matter, played some useful part in the purification process This idea was however soon abandoned The ordinary 'blood worm' the larvae of *chironomus* is unfortunately well known as a parasite on the useful forms of life in the activated sludge tank

As a consequence of all these observations, incomplete as we now know them to be, most workers including myself were of opinion that the activated sludge process was mainly dependent on bacterial activity, associated as in the case of the well-known M_7 with a certain proportion of organic iron compounds as a precipitating agent

I looked upon the process as one of intensive bacterial oxidation, acitification being included under this general category Unfortunately among bacteria could be classed those higher thread like species such as *sphaerotilus* and *leptomitus* characteristic of polluted but partially aerated streams Such growths have been classed as a form of activated sludge, although they have nothing in common with the true product and have little or no clarifying still less purifying effect

Apart from bacterial activity it was also recognised that physical factors, notably the mechanical flocculation of colloidal particles, played their part

Martin's book was published in 1927 At that time the remarkable researches of Cramer of Milwaukee had not been published (1931) and the important detailed working out of the subject by Pillai and Subrahmanyan, and their collaborators had not been undertaken and the detailed work of Gurbaxani only recently successfully submitted as a Ph D thesis is still awaiting publication Dr Gurbaxani is now pursuing his studies under the leading sewage purification specialists in the U S A

Although Cramer s work finds mention in my book published in 1935 it even then appeared more as supporting an interesting theory than as of basic and fundamental importance not only to the theory of the activated sludge process but to the operation of the nitrogen cycle in general Consequently, although a brief mention of it appears in my book, the importance of the remarkable details described in the original paper was not fully realised partly perhaps because the methods of investigation were somewhat unusual In the light of the tabulated experimental results given in Gurbaxani's thesis their importance becomes obvious Cramer employed an original method for preserving an aerobic atmosphere Instead of bubbling air through the liquid under investigation he used a small quantity of sodium chlorate in order to operate under more easily controlled conditions

Cramer draws attention to the fact that while the activated sludge process may give fairly satisfactory results without the formation of nitrates yet adequate oxygen is essential showing the process to depend on living agents Purely mechanical flocculation of colloidal matter either by mechanical stirring or by injection of nitrogen or CO_2 does not deal with impurities in solution

Free access of air was assured by employing dishes of only 4 3 cm in depth with a surface area of 23 sq cm Under these conditions heat sterilised sewage if left alone became septic. The addition of sodium chlorate' to the extent of 0 3% prevented the sewage from becoming septic but did not produce clarification Further addition of 1 c c of raw sewage produced clarification Sewage bacteria in separate culture did not clarify nor did yeasts or the enzymic solution obtained by crushing activated sludge with sand and filtering

It was found that if activated sludge was heated to 60°C for 30 minutes all protozoa were killed but many bacteria remained alive Inoculation with this sludge did not produce clarification in sterile sewage which contained 0 3% of sodium chlorate Further addition of a drop of water containing a *single protozoan* produced clarification in a week

If a small amount of sludge from this clarified sample which contained many individuals of one type of protozoan only was added to sewage that had been first sterilised and then inoculated with bacteria and yeasts only and to which 0 3% of sodium chlorate had been added, clarification resulted in 48 hours or less

In all the experiments in which air was allowed access to the surface of the liquid the neck of the bottle was closed with a sterile cotton wool plug to prevent contamination When this plug was removed from a bottle containing sterile sewage and 0 3% of sodium chlorate the sewage did not clarify Microscopic examination showed that it contained bacteria and yeasts but no protozoa

Further experiments by Cramer himself, for details of which the original paper may be consulted, showed that if the sludge is heated to 50°C for 5 minutes all protozoa are killed except vorticella—an observation of great interest in view of the obviously resistant character of this organism as shown by the Bangalore researches and also in view of its mode of sustenance which can be actually seen to consist in the ingestion of bacteria and of foecal organic matter It was observed that when the protozoa die they rapidly disintegrate and become sludge particles It was believed, although the observation needs confirmation, that during the process of disintegration bacteria could actually be seen emerging from within the protozoa If true this phenomenon partly explains the increase of bacterial population following partial sterilisation

Cramer draws the practical conclusion that clarification depends on (1) Aerobic bacterial life, (2) Live protozoa, (3) Oxygen in solution, and consequently that the activated sludge process can best be controlled microscopically and that a stock of normally active sludge should always be kept on hand for inoculation in case of deterioration from any incidental cause of the main bulk in circulation Cases of temporary stoppage of efficiency through a flush of trade waste, or interruption of aeration, might be met in this way

Cramer expresses the opinion that the protozoa arrive in the sewage through storm water, infiltration water, or kitchen waste

The work was performed in the Research Laboratory of the Sewerage Commission of the City of Milwaukee under the immediate direction of Dr J A Wilson Some portion was done in R Cramer's own laboratory Dr Wilson considers that clarification is proportional to the relation between organic dispersed matter and the amount and vigour of the protozoa

Bangalore Researches

It was in 1938 that I was concerned with the putting into operation of an up-to-date diffused air activated sludge plant for the Military authorities at Cossipore, Calcutta It was designed to deal with the sewage from a population of from 8,000 to 10,000 Having seen that the installation was functioning in mechanical order I placed Mr S C Pillai of the Department of Biochemistry at the Indian Institute of Science in immediate supervision until the plant could be handed over to the authorities In the absence of local facilities for detailed chemical analysis it was fortunate that through the courtesy of the resident medical officer Pillai was able to follow the building up of the sludge by regular microscopical observations It was then that he confirmed the statement of Cramer and his colleagues that efficient clarification was coincident with the appearance of protozoal life particularly *vorticellids* Following these observations he undertook, in collaboration with the Professor of Biochemistry, Dr V Subrahmanyan, and with the detailed assistance particularly of Dr M Gurbaxani, a detailed study of the function of protozoa, with reference not only to the Activated Sludge process but also to the nitrogen cycle throughout nature He was indebted to the late Prof B L Bhatia of Lahore for specific identifications and to Dr B R Seshachar of Central College, Bangalore, for assistance in the photo-micrographic work The general results of these researches are available in recent literature although many valuable details still await publication The following conclusions may be held as established

Flocculation of Colloids

The special function of protozoa in flocculating sewage colloids is clearly evident Amongst the active species *vorticellids* are predominant and among these *epistylis* is specially efficient being twice as effective as the ordinary type of *vorticellids* Observations of the sludge from the installation at the Madura Mills, Tuticorin, where the tanks are operated with sea water show that protozoa are active even under these conditions, indeed a species has been isolated and identified as *Zoothamnium* for the first time reported in India perhaps on account of its habitat in sea water.

Oxidation effects

Of even greater interest than the flocculating effect of protozoal activity is the demonstrated conversion of crude foecal emulsion to the stage certainly of the formation of *nitrite* if not completely to *nitrate* There would seem to be a process of digestion of protein matter akin to the cellular activity of growing plants in digesting the threads of mycorrhizal mycelium and in the case of protozoa of the further oxidation by means of an *oxidase* of the amino compounds so produced Here then it would appear that we have a direct conversion of human waste matter into plant and even animal food since the masses of epistylis have been found to be readily consumed by rats and poultry in much the same manner as yeast Indeed many years ago Buswell and Lang considered that the purification of sewage by microscopic communities is entirely similar to the disposal of garbage by feeding it to hogs

In connection with the remarkable oxidation activity of protozoa and their need for abundant air supply interesting reference is made to an observation of H M Vernon in 1897 that protozoa have the largest respiratory coefficient of all invertebrates, one case being cited where this function was forty times that of a frog

Elimination of Pathogenic Bacteria

A further advantage of this function of the protozoa is that in the course of it, pathogenic bacteria have been found to be completely eliminated This was shown in early days by the empirical observations of Col Stewart, then Director of Public Health in Bengal, and is now confirmed in detail by Gurbaxani with specific cultures both of protozoa, viz *epistylis*, and of *B Typhosus*, *B dysenteriae* and *V cholera* The sanitary importance of these findings is obvious

Sensitiveness to pH conditions

On the other hand, experiment showed that the protozoa were sensitive to changes in *p*H, sudden flushes of acidity proving to be destructive The bearing of this result on the regulation of trade effluents is important

Sources of Protozoa

The Bangalore workers agree with Cramer that the protozoa do not derive directly from human excreta but from soil In dry soil the *vorticellids* exist in the form of cysts, becoming active under conditions of waterlogging They are naturally to be found in all kinds of stagnant and polluted waters The interesting fact is recalled that the very first protozoan described by Leeuwenhoek, the inventor of the microscope, was a species of *vorticella* which he had seen in standing rainwater in 1675

From these observations together with the laboratory experiments it would appear likely that nitrification changes under natural conditions are assisted by the specific activity of certain forms of protozoa

Confirmation of Bangalore Researches

The findings of the Bangalore workers have been confirmed by investigators in widely separated centres Thus Reynoldson has noted the activity of *vorticellids* in the percolating filters at Huddersfield Hardin of Stanford University, California, records a clear cut case of the flocculation of bacteria through a protozoan *Oikomonas termo* Watson of the Wellcome Bureau of Scientific Research reports a similar activity of a soil ciliate *Balantiophorus minutus*

It is of particular interest that in an important report from Manchester by Messrs Wishart, Jepson and Klein on the Dewatering of Sludge, the abundance or otherwise of *vorticella* revealed by microscopic observation, is taken as an indication of the 'condition' of the sludge Since the sewage reaching the Davyhulme works contains almost every type of trade waste it may be concluded that provided these do not unduly affect the pH of their environment the protozoa continue to thrive and perform their function in producing flocculation Careful study is required in order to ascertain the economic limit of air supply, as between the production of fully active protozoa resulting in a sludge of high purifying power, and excessive aeration resulting in the 'burning' out of the protozoa

Protozoa and Nitrogen Fixation

Besides the flocculation of organic colloidal matter and its further oxidation to harmless end products there is evidence, e g through the work of Cutler and Bal that *nitrogen fixation* is facilitated by the symbiosis of certain protozoa with the normal nitrogen fixing organisms of the type of *Azoto bacter chroococcum*

6 *Animals and Man*

Apart from their laboratory studies in connection with the experimental activated sludge plant at the Indian Institute of Science the Bangalore workers have been deputed by the Imperial Council of Agricultural Research to undertake a continuous investigation of *Sewage Farming* under a Scheme of Research approved by the Council Valuable reports have been published by Messrs S C Pillai, R Rajagopalan and V Subrahmanyan The researches thus reported have been mainly concerned with the local sewage farms at Bangalore, with the municipal sewage farm at Madura, and with the treatment of mixtures of sewage and trade waste as instanced by the municipal sewage farm at Ahmedabad

In the course of this work many opportunities naturally were afforded to recognise the numerous links which hold together the chain of living activity from man through animal and plant back to man again These observations have been admirably summarised in a paper by Pillai and Subrahmanyan in the issue of *Science and Culture* for May 1946 Thus they point out that at Madura where some two million gallons of sewage is treated daily on 100 acres of underdrained land, the sewage as it filters through the land and travels down to the effluent channels undergoes rapid oxidation and emerges as a fairly clear effluent supporting micro-organisms including numerous types of protozoa as well as higher forms of life such as worms including earthworms, a variety of insect larvae including those of *chironomus*, gastropods, crabs, frogs, fish (some eight varieties), tortoises and water snakes Some of the visible forms attract birds of prey and many of the fish are consumed for human consumption Since the sewage irrigated areas are intensely farmed for grass, vegetables and fruit trees there is thus at the Madura Sewage Farm an unbroken life cycle

The function of the protozoan link in transforming the organic matter of sewage has been described in the last section It would be of interest if the roots of the various crops were examined for the mycorrhizal association

The various constituents of *plankton*, including protozoa, as food for fish need careful investigation from the important point of view of increased production of fish for human consumption The authors suggest that a stage intermediate between protozoa and fish might be cultivated, e g in special fish tanks from which the fully purified effluent might be used for the irrigation of crops

All these researches have their final bearing on the fundamental issue of the production of adequate and health giving food for man Increasing evidence is forthcoming to show that of even more importance than quantity is the quality of food supplied The researches of McCarrison and his fellow-workers now many years ago showed, e.g that although by plant breeding it might be possible to produce double the yield, e.g of the rice plant, yet unless the crop was grown under suitable conditions a double ration was required in order to provide an equally sustaining meal. The researches on the mycorrhizal association, and on the effects on crop production of the use of organic manure in the form of compost, have indicated that the critical issue is the synthesis of suitable proteins If these are not supplied the nutritive and disease resisting properties of the crops are depreciated, as well as the health and sustenance of the men and animals consuming them

Summary and Conclusions

In the foregoing review an attempt has been made to collect together the outstanding contributions which have been made during the past decade to our knowledge of what is now more than ever seen to be the most vital subject of scientific enquiry, viz the basic economics of food production By this is meant the full utilisation of all available material for the maintenance of the cycle, plant—animal—man

It has been found during this decade of intensive fundamental research, i e research concerned with what is often termed 'academic' as contrasted with 'technological' objectives that, as so often occurs in the history of scientific research, what had seemed simple sequences or reactions really involved many until then unknown or unobserved 'links' the provision or understanding of which was necessary for the full control of the system under investigation The work of Dixon, Brereton Baker, Bone and others on the effect of traces of moisture on combustion phenomena may be cited in illustration In the present review the end product of the 'Nitrogen cycle', viz nitrate, has been seen to function as a *catalyst* through alternate reduction in presence of carbonaceous waste material and reoxidation in presence of adequate oxygen, in addition to its direct utilisation as plant food In addition to the commonly accepted agents in the breaking down and mineralisation of nitrogenous waste material, viz various specific bacteria, sundry purely *chemical* or *physico-chemical agencies* have been shown to play an important part Reference in particular has been made to the researches of Dhar and his co-workers on the effect of sunlight on the sequence of fundamental changes resulting in the conversion of nitrogenous organic materials into nitrate The interesting influence of *trace* elements has been noted

Among living agents other than bacterial, special reference has been made to the *mycorrhizal association* the far reaching importance of which has been dealt with in detail in the writings of Sir Albert Howard, Lady Eve Balfour and their worldwide associates The nature and importance of this 'link' between plant and soil has only thus recently been recognised

Another living 'link', the study of which has been intensively pursued of recent years, with very important results, is that of the activity of *Protozoa* Not only has their useful activity been observed throughout the natural operations of agriculture but they have been shown to be the essential agent in the economic functioning of the activated sludge process of sewage purification These organisms, particularly certain species of *vorticellids*, have been shown to be capable of causing flocculation of colloids, and consequent *clarification* of the sewage, but also of digesting protein matter, including, as a most important corollary, *pathogenic bacteria*, and finally of developing an *oxidase* activity manifested in nitrification Eventually these protozoa may themselves become food for fish which in turn increase the food supply of man

The researches discussed in the foregoing review have introduced a new viewpoint into the control of crop production and of the operations of sewage purification, biological factors attaining much greater prominence

Once more it has been shown that Nature's storehouse of wonders is inexhaustible and that if it is carefully and honestly investigated, and not greedily rifled, we may look forward to an increasing supply of food for the starving millions of the world in place of the wind swept areas of erosion which in our ignorance we have so far created

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ON THE SIMULATION OF BACKGROUND COLOURS BY THE DESERT LOCUST, *SCHISTOCERCA GREGARIA* (FORSKÅL) [ORTHOPTERA, ACRIDIDAE] EXPERIMENTS WITH PAINTED BOXES

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(*Received February 15, 1946*)

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I Introduction

The known types of colour variations in the Desert Locust, *Schistocerca gregaria* (Forskål) are (i) The phase variations (Uvarov, 1923, 1928, and others) In overtone the *gregaria* adults are deep pink when immature and yellowish when mature, as was shown long ago by Kunckel d'Herculais (1892) who was, however, unaware of phase differences, the *solitaria* adults are usually bluish grey *Gregaria* hoppers have a black pattern, while *solitaria* ones are usually green (Johnston, 1926, and others) (ii) In the *solitaria* phase, again, two colour types occur, thus (Roonwal, 1945*a*, 1946*a*) The majority (about 91%) of the adults are blue grey and the remainder fawn, similarly, the majority of the hoppers are green and a small percentage fawn Other types of variations, *e g*, regarding eye stripes, etc, are also known (Roonwal, 1936 *et seq*)

Adaptation to background colours in the field has often been reported in Acridid grasshoppers (*vide* Cott, 1940) Rao (1937, pp 24 25) has observed partial adaptation in the Desert Locust In recent years Faure (1932) and Hertz and Imms (1937) have studied this phenomenon experimentally by rearing locusts of the genera *Locusta* and *Locustana* in variously coloured boxes Here I shall present the results of some preliminary experiments, carried out several years ago on the Baluchistan Coast, on rearing hoppers of the Desert Locust in uniformly coloured boxes [1]

II Experiments

Hoppers were reared singly in small, rectangular, wooden boxes (12 × 9 5 × 8 5 cm) having the inner walls and floor painted with ordinary commercial oil paints in the following different colours Lemon yellow, prussian blue, mahogany, black, dark green, signal red and white, unpainted boxes were used for a pale ochre background. The boxes were closed on all sides except the top which was of unpainted, grey-coloured wire-gauze They were kept in the bright, diffuse daylight of the verandah and were not exposed to direct sunlight For food, the hoppers were supplied twice or thrice daily with fresh twigs of *Heliotropium undulatum* Vohl (= *ramosissimum* Sib) (Baluchi name '*marrand*'), Nat Ord Boraginaceae This shrub grows commonly in the sandy areas on the Baluchistan Coast, and is the favourite food-plant of the Desert Locust (phase *solitaria*) in nature, in both the hopper and adult stages Green (*solitaria*) first stage hoppers were utilised The colour of the hoppers was noted at the start of each experiment and, subsequently, at intervals of 2-3 days Out of 64 experiments which were started, 19 reached a sufficiently advanced stage for reliable deductions to be made, in the remainder, the hoppers died in the first two stages The following experiments were performed for each type of background, the results of the successful experiments being summarised in Table 1 —

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[1] For a brief report of these experiments *vide* Roonwal, 1937, pp 148 149

1 *Lemon yellow* --Nine experiments were started In five the hoppers died in the first stage, and in two in the second In the remaining two, one hopper reached the fourth stage and the other the adult

2 *Prussian blue* —Nine experiments were started In six the hoppers died in the first stage, and in one in the second In the remaining two, the adult stage was reached

3 *Mahogany* —Nine experiments were started In five the hoppers died in the first stage, and in two in the second In the remaining two, one hopper reached the third stage and the other the adult

4 *Black* —Seven experiments were started In four the hoppers died in the first stage In the remaining three, one reached the fourth stage and two the adult

5 *Dark green* —Eight experiments were started In five the hoppers died in the first stage, and in one in the second In the remaining two, one reached the third stage and the other the adult

6 *Signal red* —Nine experiments were started In seven the hoppers died in the first stage In the remaining two, the adult stage was reached

7 *White* —Ten experiments were started In six the hoppers died in the first stage In the remaining four, two hoppers reached the fourth stage, one the fifth and one the adult

8 *Pale ochre* —Three experiments were started In one the hopper died in the first stage, in the second the hopper reached the fourth stage, and in the third the adult

III DISCUSSION AND CONCLUSIONS

In S Africa, Faure (1932), who reared hoppers of *Locusta migratoria migratorioides* (R and F) and *Locustana pardalina* (Walk) in boxes painted in different colours on the inside, found 'good' or 'fair' resemblance on white, black, grey, yellow and brown backgrounds, and no clear resemblance on green, pink and blue backgrounds He also noticed that green hoppers were produced not as a result of green background but only under high humidity with an abundance of fresh and succulent food Hertz and Imms (1937), working on *Locusta migratoria migratorioides* in England, confirmed the dependence of green colour in hoppers on the presence of a moist atmosphere They further elucidated the phenomenon of partial colour adaptation in terms of wave-length of light reflected from the coloured background They found that, except upon a black background, no complete colour adaptation was observed, but the effects of different backgrounds were clearly defined—the background only influences the amount and proportion of the orange-yellow and black produced Yellow reflected rays (5500–6000A) stimulate the production of orange and yellow, their absence, and the presence, instead, of rays shorter than 5000A, produce colourless, pale grey or dark grey hoppers

TABLE I

Results of experiments on rearing Desert Locust hoppers in painted boxes

Inside colour of rearing box	Resulting colour of insects	
	Old hoppers (III–V stages)	Adults
1 Lemon yellow	Bright yellowish green	Light green
2 Prussian blue	Fawn or green base with black markings	Brownish or pinkish grey
3 Mahogany	Ditto	Ditto
4 Black	Dark green or dirty fawn base with black markings	Smoky brown or ash coloured
5 Dark green	Bright green	Light green
6 Signal red	Dirty green or orange	Grey with violet tinge, specially on hind-legs
7 White	Greenish white	Whitish fawn
8 Pale ochre (unpainted wood)	Pale green with or without yellowish tinge	Dull brown

The results of the present experiments on the Desert Locust showed (Table 1) that among hoppers the background colours which were more or less simulated were lemon yellow, black (?), dark green and white, while others, *viz.*, prussian blue and signal red, were not, the results for mahogany and pale ochre were indefinite. The fawn hoppers, as observed on black, prussian blue and mahogany backgrounds, would seem to represent the fawn colour type (Roonwal, 1945*a*, 1946*a*) produced independently of the background in the *solitaria* phase. Among adults the results were less clear, but some simulation was observable on black, dark green and white backgrounds.

IV. SUMMARY

1. To study adaptation to background colours in the Desert Locust, hoppers were reared singly in boxes painted uniformly in different colours on the inside. Nineteen hoppers reached a sufficiently advanced stage to permit of some deductions being made.

2. **Among hoppers, more or less marked colour adaptation was observed on lemon yellow, black (?), dark green and white backgrounds, no adaptation was observed on prussian blue and signal red. Among adults, the results were less clear, but some degree of adaptation was observable on black, dark green and white backgrounds.**

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ON TWO PHASE CONFIGURATION OF SMALL MASSES

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(Communicated by Dr D S Kothari, M Sc , Ph D)

(*Received September 14 , read November 23, 1946*)

ABSTRACT

The paper consists of two parts In Part I physical characteristics of small masses (2○, 1○, 02○, 01○) are calculated for arbitrary values of L/M on the theory of Milne of the two phase configuration of the generalised standard model The calculations do afford some insight as to the conditions in the interior of such small masses In Part II we have studied these masses under conditions of incipient degeneracy

INTRODUCTION

The theory of stellar masses composed of completely degenerate electron gas has been worked out in complete by Chandrasekhar (1939), who isolated the existence of a limiting mass $M_3 = 5\ 75\mu_e^{-2}$ ○ For masses exceeding the limit M_3 the theory predicts no lower limit to the radius nor upper bound to the density Kothari (1936) has incorporated in the usual white dwarf the theory of pressure ionisation and has predicted a maximum radius for a cold body The present paper deals with an investigation of the physical characteristics of masses lying in the region of stellar and proper planetary masses In Part I we have worked out the properties of small masses on the theory of Milne (1932) of two phase configuration of the generalised standard model Milne's theory will, however, be a very crude approximation for such masses since the transition region between the degenerate core and the perfect gas region may be quite extensive At present we shall, however, be satisfied with our approximate calculations, which have been performed for masses, viz 2⊙, 1○, 02○ and 01○ Part II deals with these model stars under conditions of incipient degeneracy We have used Morse's (1940) calculations for the opacity coefficient

§2 NUMERICAL CALCULATIONS

The various physical properties given in the present paper are calculated from Milne's formulae given in Kothari's (1932) paper Since the masses considered here are all less than the critical mass of Milne's theory the configurations are of collapsed type The tables given below may be considered as an extension of the table given in Kothari's (1932) paper for the model white-dwarf (5⊙)

TABLE I

$M = 2\odot$ $L/M = 10^{-2}$

$C^{-\frac{1}{2}} \equiv \omega_3$	$1-\beta_1 = \frac{k_1 L}{4\pi c G M}$	$\frac{a}{\xi_0}$ $\frac{\text{Core rad}}{\text{Total rad}}$	R_1 Total rad cm	ρ_c Central density gm /cm ^{3}	T' Interfacial temperature	T_c Central temp neglecting conduction	T_c * Central temp taking account of conduction
∞	0	1	1 38 10^9	2 22 10^5	0	5 10 10^7	8 15 10^6
20 18	2 42 10^{-5}	0 942	1 47 10^9	2 22 10^5	7 10 10^6	5 10 10^7	8 71 10^6
8 07	1 52 10^{-4}	0 83	1 48 10^9	2 14 10^5	2 41 10^7	5 10 10^7	2 50 10^7
4 04	6 04 10^{-4}	0 61	1 73 10^9	1 92 10^5	6 06 10^7	6 40 10^7	6 08 10^7
2 119	2 18 10^{-3}	0 20	2 90 10^9	1 47 10^5	1 43 10^8	1 43 10^8	1 43 10^8
2 018	2 39 10^{-3}	0	3 78 10^9	9 54 10^4	1 52 10^8	1 52 10^8	1 52 10^8

TABLE II

$M = 1\odot$ $L/M = 10^{-2}$

$C^{-\frac{1}{2}} \equiv \omega_3$	$1-\beta_1 = \frac{k_1 L}{4\pi c G M}$	$\frac{a}{\xi_0}$ $\frac{\text{Core rad}}{\text{Total rad}}$	R_1 Total rad cm	ρ_c Central density gm /cm ^{3}	T' Interfacial temperature	T_c Central temp neglecting conduction	T_c Central temp taking account of conduction
∞	0	1	1 74 10^9	5 54 10^4	0	3 46 10^7	8 45 10^6
20 18	6 05 10^{-6}	0 942	1 86 10^9	5 54 10^4	2 82 10^6	3 46 10^7	8 45 10^6
8 07	3 80 10^{-5}	0 83	1 86 10^9	5 38 10^4	9 58 10^6	3 46 10^7	1 14 10^7
4 04	1 51 10^{-4}	0 61	2 17 10^9	4 79 10^4	2 41 10^7	3 50 10^7	2 46 10^7
2 119	5 50 10^{-4}	0 20	3 65 10^9	3 80 10^4	5 69 10^7	5 70 10^7	5 70 10^7
2 018	6 05 10^{-4}	0	4 76 10^9	2 40 10^4	6 03 10^7	6 03 10^7	6 03 10^7

* In the calculation of T_c (with conduction) we have taken account of both radiative and conductive opacity

The preceding tables for masses $2\odot$ and $1\odot$ show how the physical quantities vary as we pass from a configuration which is no core to one which is all core for a fixed value of $L/M = 10\ ^{2}$ ergs /gm The central density even in a completely degenerate state is not very high

§3 Having enumerated the various physical quantities for arbitrary values of $\frac{k_1 L}{4\pi c G M}$, we shall now estimate a reasonable value for k_1, the opacity in the gaseous envelope for given L and M The opacity expression for a Russell mixture of elements undiluted with hydrogen is given by

$$k_1 = 7\ 34 \quad 10^{25} \frac{\rho}{T^{7/2}} \ \bar{g}/t \qquad (1)$$

where $\bar{g}/t$ is the guillotine factor Let the opacity without the correction factor be given by

$$k_1 = \alpha \ \frac{\rho}{T^{7/2}} \qquad (2)$$

where

$$\alpha = 7\ 34 \quad 10^{25}$$

The expressions for the interfacial temperature and density are respectively,

$$T'' = 8\ 48 \quad 10^9 \left(\frac{k_1 L}{4\pi c G M}\right)^{2/3} \tag{3}$$

$$\rho' = D\mu \quad \left(\frac{k_1 L}{4\pi c G M}\right) \tag{4}$$

where $D = \dfrac{(R/m_H)^4}{\frac{1}{3}\, aK^3} = 1\ 89 \quad 10^7$

From equations (2), (3), and (4) we have,

$$T' = \left(\frac{D\alpha}{4\pi c G}\right)^{2/7} \quad \left(\frac{L}{M}\right)^{2/7} \tag{5}$$

$$\rho' = \frac{(R/m_H)^{3/2}}{K^{3/2}} \quad \left(\frac{D\alpha}{4\pi c G}\right)^{3/7} \quad \left(\frac{L}{M}\right)^{3/7} \quad \mu \tag{6}$$

$$k_1 = \frac{(R/m_H)^{3/2}}{K^{3/2}} \quad \left(\frac{4\pi c G}{D}\right)^{4/7} \quad \left(\frac{M}{L}\right)^{4/7} \quad \alpha^{3/7} \tag{7}$$

The method adopted for determining α is as follows For given L/M and $\mu = 2\ 1$ and $\alpha = 7\ 34 \quad 10^{25}$, as a first approximation T' and ρ' are calculated from (5) and (6) Knowing T'' and ρ', $\log_{10} t/g$ is known from Morse's table by graphical interpolation The new value of $\alpha \equiv \alpha_1 = \dfrac{\alpha}{t/g}$ is used to redetermine T'' and ρ' We continue this process of successive approximation till we get coincident values of T' and ρ' for two successive approximations In practice it was found that generally a third approximation was quite sufficient The results of such calculations are given below

For $\dfrac{L}{M} = 10^{-2}$ ergs/gm

$$\begin{aligned} T' &= 9\ 63 \quad 10^6 \text{ degrees} \\ \rho' &= 7\ 18 \quad 10^2 \text{ gm /cm}^3 \\ k_1 &= 95\ 43 \\ 1-\beta_1 &= 3\ 80 \quad 10^{-5} \end{aligned}$$

§4 Calculations for $M = 2\odot$

$\dfrac{\text{Core radius}}{\text{Total radius}} \approx\ 94$

Central temperature $= 8\ 71 \quad 10^6$ degrees
Central density $= 2\ 22 \quad 10^5$ gm /cm ^{3}
Radius $= 1\ 47 \quad 10^9$ cm
Effective temperature $= 7125$ degrees

§5 Calculations for $M = 1\odot$

$\dfrac{\text{Core radius}}{\text{Total radius}} \approx \cdot 83$

Central temperature $= 1\ 14 \quad 10^7$ degrees
Central density $= 5\ 38 \quad 10^4$ gm /cm ^{3}
Radius $= 1\ 86 \quad 10^9$ cm
Effective temperature $= 5330$ degrees

§6 We have given before the properties of masses $2\odot$ and $1\odot$ for $L/M = 10^{-2}$ We shall now do the same for masses $0 2\odot$ and $0 1\odot$ for arbitrary values of L/M $(10^{-4}-10^{-8})$

TABLE III

$M = 0 2\odot$

$C^{-\frac{1}{3}} = \omega_3$	$1-\beta_1 = \frac{k_1 L}{4\pi cGM}$	$\frac{a}{\xi_0}$ $\frac{\text{Core rad}}{\text{Total rad}}$	R_1 Total rad cm	ρ_c Central density gm /cm ^{3}	T' Interfacial temperature	T_c Central temp with conduction $L/M = 10^{-4}$	$L/M = 10^{-5}$	$L/M = 10^{-6}$	$L/M = 10^{-7}$
∞	0	1	2 97 10^{9}	2 22 10^{3}	0	7 24 10^{5}	3 71 10^{5}	3 39 10^{5}	3 31 10^{5}
20 18	2 42 10^{-7}	0 942	3 18 10^{9}	2 22 10^{3}	3 29 10^{5}	7 24 10^{5}	3 71 10^{5}	3 39 10^{5}	3 31 10^{5}
8 07	1 52 10^{-6}	0 83	3 18 10^{9}	2 14 10^{3}	1 12 10^{6}	1 23 10^{6}	1 15 10^{6}	1 12 10^{6}	1 12 10^{6}
4 04	6 04 10^{-6}	0 61	3 72 10^{9}	1 92 10^{3}	2 81 10^{6}	3 02 10^{6}	2 82 10^{6}	2 82 10^{6}	2 82 10^{6}
2 119	2 18 10^{-5}	0 20	6 24 10^{9}	1 47 10^{3}	3 57 10^{6}	6 62 10^{6}	6 62 10^{6}	6 62 10^{6}	6 62 10^{6}
2 018	2 39 10^{-5}	0	8 15 10^{9}	9 54 10^{2}	7 06 10^{6}	7 06 10^{6}	7 06 10^{6}	7 06 10^{6}	7 06 10^{6}

TABLE IV

$M = 0 1\odot$

$C^{-\frac{1}{3}} = \omega_3$	$1-\beta_1 = \frac{k_1 L}{4\pi cGM}$	$\frac{a}{\xi_0}$ $\frac{\text{Core rad}}{\text{Total rad}}$	R_1 Total rad cm	ρ_c Central density gm /cm ^{3}	T' Interfacial temperature	T_c Central temp with conduction $L/M = 10^{-5}$	$L/M = 10^{-6}$	$L/M = 10^{-7}$	$L/M = 10^{-8}$
∞	0	1	3 75 10^{9}	5 54 10^{2}	0	2 04 10^{5}	1 38 10^{5}	1 31 10^{5}	Same as for $\frac{L}{M} = 10^{-7}$
20 18	6 05 10^{-8}	0 942	4 00 10^{9}	5 54 10^{2}	1 31 10^{5}	2 04 10^{5}	1 38 10^{5}	1 31 10^{5}	
8 07	3 80 10^{-7}	0 83	4 01 10^{9}	5 38 10^{2}	4 45 10^{5}	4 68 10^{5}	4 46 10^{5}	4 46 10^{5}	
4 04	1 51 10^{-6}	0 61	4 68 10^{9}	4 79 10^{2}	1 12 10^{6}	1 12 10^{6}	1 12 10^{6}	1 12 10^{6}	
2 119	5 50 10^{-6}	0 20	7 85 10^{9}	3 80 10^{2}	2 64 10^{6}	2 64 10^{6}	2 64 10^{6}	2 64 10^{6}	
2 018	6 05 10^{-6}	0	1 03 10^{10}	2 40 10^{2}	2 82 10^{6}	2 82 10^{6}	2 82 10^{6}	2 82 10^{6}	

§7 The interfacial temperature and density are respectively given by,

Also

$$\left.\begin{aligned} T' &= 4\ 61\quad 10^{7}\left(\frac{L}{M}\right)^{2/7} \\ \rho' &= 1\ 59\quad 10^{4}\left(\frac{L}{M}\right)^{3/7} \\ k_1 &= 10\ 1\left(\frac{M}{L}\right)^{4/7} \\ 1-\beta_1 &= 4{\cdot}01\quad 10^{-4}\left(\frac{L}{M}\right)^{3/7} \end{aligned}\right\} \quad \cdots \qquad (8)$$

The above expressions are not very accurate since in their deduction we have used ordinary Kramer's formula for the non degenerate opacity, i e $k_1=4\ 23\quad 10^{28}\,\frac{\rho'}{T'^{7/2}}$ instead of the more accurate formula given by Morse

Having thus estimated a reasonable value for $(1-\beta_1)$ we can now enumerate reasonable values for the physical quantities for the masses $M = 02\bigcirc$ and $M = 01\bigcirc$

TABLE V

$M = 02\bigcirc$

L/M	$(1-\beta_1)$	$\frac{a}{\xi_0}$ $\frac{\text{Core rad}}{\text{Total rad}}$	R Total radius cm	T' Interfacial temperature	ρ' Interfacial density gm/cm ^{3}	k_1 Non degenerate opacity	T_c Central temperature	ρ_c Central density gm/cm ^{3}	T Effective temperature
10^{-4}	7 74 10^{-6}	0 57	3 8 10^9	3 32 10^6	3 07 10^2	1 95 10^3	3 0 10^6	1 9 10^3	788
10^{-5}	2 88 10^{-6}	0 75	3 4 10^9	1 72 10^6	1 14 10^2	7 27 10^3	2 0 10^6	2 1 10^3	468
10^{-6}	1 07 10^{-6}	0 87	3 2 10^9	8 90 10^5	4 26 10^1	2 72 10^4	1 2 10^6	2 2 10^3	273
10^{-7}	4 01 10^{-7}	0 95	3 2 10^9	4 61 10^5	1 59 10^1	1 01 10^5	3 4 10^6	2 2 10^3	153

TABLE VI

$M = 01\bigcirc$

L/M	$(1-\beta_1)$	$\frac{a}{\xi_0}$ $\frac{\text{Core rad}}{\text{Total rad}}$	R Total radius cm	T' Interfacial temperature	ρ Interfacial density gm/cm ^{3}	k_1 Non degenerate opacity	T_c Central temperature	ρ_c Central density gm/cm ^{3}	T_e Effective temperature
10^{-5}	2 88 10^{-6}	0 44	5 2 10^9	1 72 10^6	1 14 10^2	7 27 10^3	2 0 10^6	4 2 10^3	318
10^{-6}	1 07 10^{-6}	0 68	4 6 10^9	8 90 10^5	4 26 10^1	2 72 10^4	1 1 10^6	4 8 10^3	190
10^{-7}	4 01 10^{-7}	0 82	4 0 10^9	4 61 10^5	1 59 10^1	1 01 10^5	1 5 10^6	5 4 10^3	115
10^{-8}	1 49 10^{-7}	0 9	4 0 10^9	2 39 10^5	5 92	3 75 10^5	1 3 10^6	5 5 10^3	64

§8 PART II STARS AT THEIR MAXIMUM CENTRAL TEMPERATURE AND LUMINOSITY

We have seen in Part I that the configuration in which degeneracy is just setting in has the maximum central temperature The luminosity of small masses decreases very rapidly with the decrease of mass It would be worthwhile to calculate the luminosity when the central temperature is maximum The luminosity L is given by,

$$L = \frac{4\pi cGM(1-\beta_0)}{k_c\ \alpha} \qquad (9)$$

where $1-\beta_0$ is given by the quartic equation,

$$1-\beta_0 = 6\ 00\quad 10^{-2}\left(\frac{M}{\bigcirc}\right)^2\left(\frac{\mu}{2\ 1}\right)^4\cdot\beta_0^4, \qquad (10)$$

In our calculations we have assumed the star to be composed of Russell mixture of elements diluted with 35% hydrogen and have taken μ to be equal to unity This is admittedly a crude assumption For opacity we have used the expression given by Morse The guillotine factor is known from Morse's table corresponding to the mean condition $T' = \frac{2}{3} T_c$ The results of such calculations are summarised in the table below

TABLE VII

Mass $M/\odot$	$\text{Log}_{10} \frac{R}{R\odot}$	T_c Central temperature	ρ_c Central density	$\text{Log}_{10} k_c$	T_e Effective temperature	$\text{Log}_{10} \frac{L}{L\odot}$ (Calculated)	$\text{Log}_{10} \frac{L}{L\odot}$ (Observed)
25	−0 76	7 46 10^7	3 68 10^3	0 76	6360	−1 35	−1 77 Krüger 60
2	−0 73	2 13 10^7	2 36 10^3	1 12	4217	−1 99	−1 96 0_2 Eri C
1	−0 63	8 44 10^6	5 89 10^2	2 12	1253	−3 90	
02	−0 39	9 87 10^5	2 36 10^1	4 0	97	−7 9	
01	−0 29	3 92 10^5	5 89	5 1	27	−9 9	

The calculations given in the table are based on the assumption that the central temperature reaches maximum when the core just vanishes, i e for $w_3 = 0$ As a matter of fact the maximum will reach somewhere in the region of partial degeneracy We shall discuss this point in another paper, dealing with partially degenerate stars Prof Russell (1944) has performed a similar calculation based on Eddington's idea that high maxima of surface and central temperature should occur for values of the radius about 3 or 4 times that of the degenerate state

It is clear from the above table that even under the most favourable condition it would be impossible to observe stars of mass less than 05○ by their own light The opacity as calculated from Morse's accurate expression is so large for these small masses that the radiative flux of heat is very small for them

It is a pleasure to acknowledge my indebtedness to Prof D S Kothari for his help during the course of the present investigation Thanks are also due to Prof K S Krishnan for the kind interest he has taken in this work

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A TABLE OF VALUES OF $N_3(t)$

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(Communicated by Dr D S Kothari)

(*Received September 23, read November 22, 1946*)

1 In 1940, I listed numbers[1] less than 20,000, which cannot be represented by Ramanujan's ternary quadratic form

$$x^2+y^2+10\,z^2$$

In this connection, I computed a table giving the non negative solutions of the equation

$$x^2+y^2=t, \tag{1}$$

with $x \leqslant y$ The following is a specimen of the entries in this table The third column gives $n_2(t)$—the number of non negative solutions of (1), the solutions (a, b) and (b, a) being considered different when a and b are unequal The last column gives the value of $N_2(t)$ defined by the relation

$$N_2(t)=\sum_{i=0}^{t} n_2(i) \tag{2}$$

t	(x, y)	$n_2(t)$	$N_2(t)$
6961	(20, 81)	2	5551
6962	(59, 59)	1	5552
6964	(58, 60)	2	5554
6970	(9, 83)(27, 79)(47, 69)(57, 61)	8	5562
6976	(24, 80)	2	5564

2 The present table has been calculated at the suggestion of Dr Kothari It gives the values of $n_3(t)$—the number of non-negative solutions of the equation

$$x^2+y^2+z^2=t \tag{3}$$

for values of t up to 10000 Since

$$n_3(t)=0 \text{ when } t \equiv 7 \pmod 8, \tag{4}$$

I have not thought it necessary to give in the table, the value of $n_3(t)$ when t is of the form $8m+7$ To find the value of $n_3(t)$ in the other cases, we express t in the form $8m+n$, where $0 \leqslant n \leqslant 6$ The values of m are given in the first column of the table and those of n in the top row The last column gives the value of $N_3(8m+7)$ where

$$N_3(t)=\sum_{i=0}^{t} n_3(i) \tag{5}$$

The value of $N_3(t)$ for other values of t, is easily found with the help of the table

Evidently

$$n_3(t) = \sum_{i=0}^{[\sqrt{t}]} n_2(t-i^2) \tag{6}$$

By considering z in (3) as not less than either of the integers x or y, the work of computation was considerably shortened While (6) would have required 100 entries when $t = 10000$, the shortened table required only 43

3 To ensure correctness several checks were applied Thus the values of $N_3(r^2)$ were computed from (5) and independently by using the formula

$$N_3(t) = \sum_{i=0}^{[\sqrt{t}]} N_2(t-i^2), \tag{7}$$

and were found to tally The fact that

$$n_3(t) = 0 \text{ if and only if } t \text{ is of the form } 4^k(8a+b) \tag{8}$$

was also of great use The results have finally been checked against the class-number table[2] computed by me in 1936, it being well-known that $n_3(8i+3)$ is the number of classes of the Gaussian binary quadratic form

$$ax^2+2bxy+cy^2$$

with a negative determinant

$$b^2-ac = -(8i+3)$$

These checks and the extreme care I have taken in the work of calculation make me sure that the entries in the table can be fully relied upon

4 It may be noticed that the number $c(t)$ of lattice points on the circle

$$x^2+y^2 = t, \quad t>0, \tag{A}$$

is given by the formula

$$c(t) = 4\{n_2(t)-n_1(t)\} \tag{9}$$

The total number of lattice points inside or on the circle (A) is given by

$$C(t) = 4\{N_2(t)-[\sqrt{t}]\}-3 \tag{10}$$

Similarly, $s(t)$—the number of lattice points on the sphere

$$x^2+y^2+z^2 = t, \quad t > 0, \tag{B}$$

is given by

$$s(t) = 8n_3(t)-12n_2(t)+6n_1(t) \tag{11}$$

and the total number of lattice points inside or on the sphere (B) is given by the formula

$$S(t) = 8N_3(t)-12N_2(t)+6[\sqrt{t}]+5 \tag{12}$$

In the above formulae

$$n_1(t) = 1 \text{ or } 0,$$

according as t is or is not a square and $[x]$ denotes as usual the greatest integer in x

5 The following short table gives the values of $N_2(r^2)$, $N_3(r^2)$, $C(r^2)$, $S(r^2)$ and $V(r)$ where $V(r) = [\frac{4}{3}\pi r^3+0\ 5]$ for values of r up to 100

It is well-known that for large r

$$S(r^2) \sim V(r)$$

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[2] ———(1942) On the Class Numbers of Binary Quadratic Forms *Revista A*, **3**, 283–99

r	N_2	C	N_3	S	V
0	1	1	1	1	0
1	3	5	4	7	4
2	6	13	11	33	34
3	11	29	29	123	113
4	17	49	54	257	268
5	26	81	99	515	524
6	35	113	163	925	905
7	45	149	239	1419	1437
8	58	197	344	2109	2145
9	73	253	486	3071	3054
10	90	317	648	4169	4189
11	106	377	847	5575	5575
12	123	441	1069	7153	7238
13	146	529	1355	9171	9203
14	168	613	1680	11513	11494
15	193	709	2046	14147	14137
16	216	797	2446	17077	17157
17	243	901	2911	20479	20580
18	271	1009	3443	24405	24429
19	302	1129	4022	28671	28731
20	335	1257	4662	33401	33510
21	365	1373	5395	38911	38792
22	402	1517	6145	44473	44602
23	437	1653	6998	50883	50965
24	473	1793	7913	57777	57906
25	516	1961	8913	65267	65450
26	557	2121	10006	73525	73622
27	600	2289	11194	82519	82448
28	642	2453	12437	91965	91952
29	687	2629	13751	1 01943	1 02160
30	736	2821	15216	1 13081	1 13097
31	782	3001	16710	1 24487	1 24788
32	835	3209	18361	1 37065	1 37258
33	886	3409	20123	1 50555	1 50533
34	941	3625	21950	1 64517	1 64636
35	999	3853	23919	1 79579	1 79594
36	1050	4053	25956	1 95269	1 95432
37	1111	4293	28150	2 12095	2 12175
38	1167	4513	30415	2 29549	2 29847
39	1234	4777	32876	2 48439	2 48475
40	1297	5025	35385	2 67761	2 68083
41	1357	5261	38049	2 88359	2 88696
42	1424	5525	40876	3 10177	3 10339
43	1491	5789	43801	3 32779	3 33038
44	1564	6077	46892	3 56637	3 56818
45	1636	6361	50159	3 81915	3 81704
46	1703	6625	53469	4 07597	4 07720
47	1778	6921	56950	4 34551	4 34893
48	1852	7213	60589	4 62781	4 63247
49	1931	7525	64430	4 92567	4 92807

r	N_2	C	N_3	S	V
50	2012	7845	68393	5 23305	5 23599
51	2095	8173	72540	5 55491	5 55647
52	2177	8497	76828	5 88817	5 88977
53	2256	8809	81274	6 23443	6 23615
54	2341	9145	85940	6 59757	6 59584
55	2425	9477	90659	6 96507	6 96910
56	2518	9845	95649	7 35317	7 35619
57	2605	10189	1 00836	7 75775	7 75735
58	2698	10557	1 06075	8 16577	8 17283
59	2788	10913	1 11647	8 60079	8 60290
60	2883	11289	1 17290	9 04089	9 04779
61	2982	11681	1 23261	9 50675	9 50776
62	3078	12061	1 29300	9 97841	9 98306
63	3177	12453	1 35634	10 47331	10 47394
64	3278	12853	1 42108	10 97917	10 98066
65	3384	13273	1 48733	11 49651	11 50347
66	3485	13673	1 55736	12 04469	12 04260
67	3586	14073	1 62741	12 59303	12 59833
68	3695	14505	1 70044	13 16425	13 17090
69	3807	14949	1 77691	13 76263	13 76055
70	3914	15373	1 85366	14 36385	14 36755
71	4025	15813	1 93328	14 98755	14 99214
72	4133	16241	2 01453	15 62465	15 63458
73	4256	16729	2 09923	16 28755	16 29511
74	4373	17193	2 18683	16 97437	16 97398
75	4492	17665	2 27564	17 67063	17 67146
76	4608	18125	2 36687	18 38661	18 38778
77	4729	18605	2 46075	19 12319	19 12321
78	4856	19109	2 55655	19 87441	19 87799
79	4974	19577	2 65498	20 64775	20 65237
80	5101	20081	2 75546	21 43641	21 44661
81	5230	20593	2 85992	22 25667	22 26095
82	5358	21101	2 96547	23 08577	23 09565
83	5491	21629	3 07560	23 95091	23 95096
84	5618	22133	3 18640	24 82213	24 82713
85	5761	22701	3 30041	25 71711	25 72441
86	5891	23217	3 41762	26 63925	26 64305
87	6030	23769	3 53772	27 58343	27 58331
88	6167	24313	3 65937	28 54025	28 54543
89	6301	24845	3 78340	29 51647	29 52967
90	6452	25445	3 91312	30 53617	30 53628
91	6591	25997	4 04393	31 56603	31 56551
92	6734	26565	4 17660	32 61029	32 61761
93	6880	27145	4 31430	33 69443	33 69283
94	7027	27729	4 45291	34 78573	34 79142
95	7182	28345	4 59559	35 90935	35 91364
96	7326	28917	4 74053	37 05093	37 05973
97	7479	29525	4 88877	38 21855	38 22996
98	7636	30149	5 04096	39 41720	39 42456
99	7789	30757	5 19679	40 64563	40 64379
100	7955	31417	5 35339	41 87857	41 88790

6 Table of values of $n_3(t)$, $t = 8m+n$

m \ n	0	1	2	3	4	5	6	
0	1	3	3	1	3	6	3	20
1	3	6	6	3	1	6	6	51
2	3	9	6	3	6	6	3	87
3	3	9	12	4	0	12	6	133
4	3	6	9	6	6	6	9	178
5	6	15	6	3	3	12	6	229
6	1	9	15	6	6	12	12	290
7	6	6	6	9	0	12	12	341
8	3	18	12	3	9	12	6	404
9	6	9	18	7	3	12	6	465
10	6	15	9	9	6	12	15	537
11	3	21	18	6	0	6	12	603
12	3	9	15	9	9	24	6	678
13	12	12	12	9	4	12	18	757
14	0	15	12	6	12	18	9	829
15	6	12	18	6	0	24	18	913
16	3	18	12	15	6	6	21	994
17	9	15	12	9	6	12	6	1063
18	6	18	27	7	6	24	15	1166
19	9	21	12	12	0	12	12	1244
20	6	24	15	3	15	12	15	1334
21	6	15	24	15	3	24	18	1439
22	3	6	15	15	12	18	18	1526
23	6	30	18	6	0	24	6	1616
24	1	9	33	12	9	18	15	1713
25	15	18	12	12	6	18	30	1824
26	6	30	12	9	12	12	9	1914
27	12	12	18	12	0	30	18	2016
28	6	24	15	15	6	18	30	2130
29	6	21	30	6	9	18	12	2232
30	0	21	18	13	12	24	18	2338
31	12	18	24	21	0	6	24	2443
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431	12	153	84	36	0	48	66	110449
432	12	45	60	36	30	144	48	110824
433	69	60	42	57	19	48	114	111233
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436	18	102	36	69	33	48	117	112385
437	30	54	60	33	36	102	24	112724
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486	13	57	132	72	18	66	72	131938
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492	18	36	72	48	42	90	48	134323
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494	0	90	72	36	54	36	33	135061
495	54	78	90	24	0	108	90	135505
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546	12	78	84	60	18	108	120	156786
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614	9	114	144	30	60	72	33	186506
615	36	72	66	45	0	114	90	186929
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617	39	69	54	48	36	156	36	187868
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640	6	147	48	78	36	84	150	198357
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653	30	150	60	15	33	126	42	204357
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657	75	48	84	36	18	126	78	206232
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659	15	63	138	45	0	78	36	207078
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666	18	60	168	60	30	96	120	210397
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676	27	153	72	102	24	30	171	215128
677	36	111	108	39	36	96	42	215596
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682	42	48	57	102	24	66	69	217882
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688	15	60	69	69	57	78	108	220759
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694	15	57	93	72	42	54	78	223646
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703	66	102	96	84	0	66	66	227876
704	3	114	165	42	57	42	57	228356
705	48	63	120	48	12	174	90	228911
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708	24	60	123	30	30	162	102	230447
709	54	60	48	105	36	54	108	230912
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713	48	84	108	24	45	72	66	232742
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716	9	162	72	60	57	108	60	234227
717	48	57	132	66	12	144	90	234776
718	0	84	99	72	36	78	180	235325
719	24	90	60	36	0	48	36	235619
720	18	84	84	36	42	120	93	236096
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725	30	171	54	30	39	96	78	238607
726	13	66	156	36	24	120	120	239142
727	90	48	42	69	0	72	96	239559
728	6	135	84	45	36	120	60	240045
729	42	42	168	36	33	108	72	240546
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736	0	60	72	78	42	72	108	244038
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745	60	90	48	72	36	60	126	248445
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747	75	42	108	72	0	192	54	249468
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752	12	90	72	66	60	72	45	251889
753	36	99	168	54	12	108	90	252456
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757	78	93	66	78	36	96	84	254373
758	9	114	111	45	78	102	54	254886
759	36	39	192	91	0	72	54	255370
760	0	90	69	72	54	60	144	255859
761	33	177	72	45	21	138	78	256423
762	30	36	75	48	54	162	120	256948
763	72	72	60	90	0	96	120	257458
764	12	105	120	30	78	60	48	257911
765	60	69	84	48	33	132	90	258427
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767	30	129	90	60	0	84	42	259342
768	3	90	168	45	30	180	90	259948
769	69	60	60	84	51	36	126	260434
770	12	126	60	33	54	96	87	260902
771	30	60	156	66	0	114	144	261472
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774	15	42	132	48	30	108	60	262861
775	84	186	48	51	0	72	150	263452
776	9	144	96	45	63	60	90	263959
777	36	51	72	75	12	192	72	264469
778	36	90	108	54	66	72	132	265027
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780	24	63	111	66	48	90	75	265888
781	42	90	102	72	18	54	210	266476
782	0	117	60	30	48	120	36	266887
783	72	84	132	36	0	174	96	267481
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786	18	114	84	120	36	84	102	269029
787	81	54	66	129	0	66	96	269521
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789	24	54	132	48	27	120	78	270592
790	24	126	60	63	60	42	159	271126
791	24	99	72	54	0	60	60	271495
792	9	45	93	60	42	174	72	271990
793	72	144	60	84	25	72	120	272567
794	12	111	183	48	78	60	57	273116
795	48	81	78	54	0	96	126	273599
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801	63	84	174	72	18	120	66	276653
802	33	108	57	105	24	66	117	277163
803	42	177	144	27	0	90	54	277697
804	24	36	183	60	45	108	72	278225
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806	6	123	84	51	66	96	78	279272
807	42	54	84	90	0	144	90	279776
808	24	72	60	60	42	54	138	280226
809	21	135	144	60	45	60	84	280775
810	30	87	96	36	30	156	96	281306
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812	24	102	87	42	84	144	51	282326
813	30	54	150	48	21	210	72	282911
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825	60	72	78	36	24	114	180	289052
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838	27	126	84	84	24	102	84	295836
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886	15	144	84	114	48	42	207	321412
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910	0	147	36	75	54	72	135	334331
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912	12	69	90	105	51	192	66	335468
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953	33	156	84	30	39	144	60	358131
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955	84	168	54	87	0	48	120	359220
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959	24	87	102	63	0	78	54	361368
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1146	24	126	132	117	24	144	84	471107
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1162	42	93	57	150	42	96	123	480899
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1188	48	78	213	48	27	168	96	496923
1189	96	120	78	84	48	60	126	497535
1190	12	195	114	36	60	150	72	498174
1191	42	90	156	72	0	186	120	498840
1192	24	102	48	165	66	108	84	499437
1193	51	156	156	39	36	120	84	500079
1194	18	66	246	84	54	78	153	500778
1195	72	126	48	54	0	120	144	501342
1196	12	228	72	60	93	60	81	501948
1197	90	54	132	96	24	168	90	502602
1198	0	96	105	69	48	120	180	503220
1199	30	126	156	48	0	72	60	503712

$m \setminus n$	0	1	2	3	4	5	6	
1200	15	75	135	108	51	144	102	504342
1201	63	150	102	132	24	66	192	505071
1202	33	138	60	57	72	228	54	505713
1203	90	72	138	54	0	168	162	506397
1204	36	90	87	132	60	78	114	506994
1205	30	144	84	33	69	96	84	507534
1206	15	87	159	84	36	138	132	508185
1207	108	114	84	114	0	60	96	508761
1208	0	198	192	36	57	96	66	509406
1209	36	78	174	105	24	150	54	510027
1210	42	132	72	54	102	72	252	510753
1211	30	159	96	60	0	144	114	511356
1212	18	63	180	84	51	234	135	512121
1213	84	108	72	78	42	108	126	512739
1214	0	78	120	72	84	60	42	513195
1215	78	141	132	72	0	174	162	513954
1216	9	120	60	96	30	66	168	514503
1217	51	132	102	39	66	120	66	515079
1218	24	78	180	76	30	246	144	515857
1219	42	66	108	60	0	84	96	516313
1220	24	174	72	48	117	144	90	516982
1221	48	81	132	78	18	120	168	517627
1222	30	90	81	144	48	78	108	518206
1223	42	180	132	33	0	168	72	518833
1224	21	60	156	60	60	102	120	519412
1225	99	168	54	111	24	96	210	520174
1226	18	198	96	63	78	66	96	520789
1227	48	75	150	153	0	120	90	521425
1228	51	126	114	84	48	66	150	522064
1229	30	159	120	48	57	78	78	522634
1230	0	138	156	36	54	192	117	523327
1231	48	126	84	135	0	54	222	523996
1232	12	93	84	63	60	144	63	524515
1233	93	54	234	48	21	246	72	525283
1234	21	165	75	144	78	60	84	525910
1235	48	174	138	51	0	84	84	526489
1236	18	108	204	90	33	108	132	527182
1237	120	102	66	90	45	96	150	527851
1238	15	132	84	45	60	168	108	528463
1239	48	60	120	72	0	138	150	529051
1240	12	90	81	75	78	72	168	529627
1241	42	231	96	69	60	72	54	530251
1242	48	42	93	84	36	204	60	530818
1243	84	132	144	96	0	78	210	531562
1244	18	138	216	48	66	114	54	532216
1245	48	96	138	78	36	108	156	532876
1246	0	90	72	135	48	72	255	533548
1247	30	114	90	60	0	228	48	534118
1248	0	102	99	78	42	162	96	534697
1249	63	78	138	120	54	60	102	535312
1250	27							

PROCEEDINGS

OF THE

NATIONAL INSTITUTE OF SCIENCES OF INDIA

FEBRUARY, 1947

No. 2] VOL XIII [Pp 65–104

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PUBLISHED BY

THE NATIONAL INSTITUTE OF SCIENCES OF INDIA

1 PARK STREET, CALCUTTA

Price Rs 1-14-0

P 47

ON COPULATION AND INSEMINATION IN THE COCKROACH *PERIPLANETA AMERICANA* (LINN)

By P D. Gupta, *Lecturer in Zoology, University of Lucknow*

(Communicated by Prof K N Bahl, F N I)

(*Received November 27, 1946*)

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Introduction

The structure and development of male and female genitalia in different groups of Orthopteroid Insects have been studied by a number of workers Observations on copulation have also been recorded in several instances, and it is now established that a spermatophore is formed within the genital organs of a male in the various families of Orthoptera, and that insemination is accomplished by transference of a spermatophore from the male to the female Our knowledge regarding copulation and insemination in cockroaches is meagre A spermatophore has no doubt been observed in *Blatta orientalis* by Zabinski (1933), and Qadri (1938), but little is known of the exact function of the different parts of the male and female genitalia during the process of copulation Snodgrass (1937) writes 'Considering how intimate some of the cockroaches are with us, it is disconcerting to find how little we know of their private lives ' The following account incorporates the observations on the various aspects of copulation and insemination in *Periplaneta americana*, and also deals with the exact working and function of the component parts of the male and female genitalia during the act of copulation

Material and Technique

Large number of nymphs of cockroaches were reared in the laboratory and a complete record of the date and time of their final moult was maintained The adults were removed from the common stock soon after their emergence and males were segregated from the females For observing copulation, the male and female cockroaches were brought together in large glass dishes covered with gauze In some cases several females and one or two males and in others several males and a few females were kept in the same dish In the event of their not copulating, and also when it was not possible to watch the process, care was taken to separate the males from the females

Ordinary methods of killing the copulating individuals, with interlocking of their genitalia intact, did not prove successful However, hot water was found very useful for killing such insects. The copulating pair was led into a beaker or a tall glass dish and sufficient quantity of hot water at about 70°C was poured over them This resulted in quick and simultaneous death of the pair with all their parts in the copulating position In case immediate dissection was not required, a small incision on the sides of the body of the animals was made and the pair was preserved in 70% alcohol to which a few drops of glycerine were added After watching progressive stages of copulation, it was found advisable to kill the animals about 45 minutes after the beginning of copulation, since that is the time when they are completely engrossed in the act and do not easily separate

OBSERVATIONS ON COPULATION

It has been observed that under laboratory conditions, cockroaches copulate only during night from March to September On an average the process of conjugation lasts an hour and a half but under disturbed conditions it may get prolonged to two hours or even more In many cases the males were found excited after six days of their last ecdysis and a few did even copulate with females of the same age In no case either a male or a female copulated earlier than six days, although, both Zabinski and Qadri are of opinion that at least the females are ready to copulate just after their final moult Both the males and females copulate several times during their lifetime, the male is capable of copulating at intervals of six to seven days but the female less frequently since the latter, after copulation, gets busy depositing cocoons The female cockroach becomes once more ready for copulation only when its store of sperms in the spermatheca is exhausted during cocoon formation A few of these were observed to copulate even as early as three to four hours after the cocoon laying was over

Unlike some other insects there is no evidence to show any kind of courtship in *P americana* and female in particular gives no indication of its desirability for the action An excited male, whose abdomen becomes extended, genitalia partly extruded, and the cerci stretched out, bustles about in search of a female and runs after other cockroaches touching their bodies by means of its antennae On approaching a female, it attempts to insert its abdomen beneath that of the female, and also tries to catch hold of the female genitalia This is done usually from behind, sometimes from one side, while Wille's remark, 'the male brings the end of its abdomen close to the head of the female,' indicates that it tries to do so from in front An unwilling female avoids the male and quietly walks away leaving the latter alone on the spot Rarely does a male find itself readily acceptable to the female, and in this connection Qadri s statement, the young males avoid the females and flee away if the latter approach them', gives a wrong impression that the female rather than the male is initially excited A male in search of a companion may, in the heat of the moment, insert its abdomen beneath that of another male but on finding it one of the same sex, it soon withdraws from there At times, after a few minutes attempt, it is able to catch hold of a part of the genitalia of a female but in case the latter is unwilling a tussle ensues and the two separate

A willing female permits the male to insert its genitalia within her vestibulum The two soon become joined by their posterior ends and remain connected together in a tail-to tail position till insemination is normally completed After their mutual union is established, the male becomes perfectly passive while the female takes an active part and moves about dragging the male behind her in search of some dark cosy place Having secured one, the pair remains stationary and they do not indicate any movement for some time After about an hour of their union they tap each other by their hind or fore legs, apparently indicating the climax of the operation It has been ascertained by dissecting a number of copulating pairs, forcibly separated during the process, that the spermatophore is not discharged from the male within at least an hour from the beginning of copulation This has also been made sure that even after the deposition of a spermatophore, the pair remains connected together for about fifteen minutes after which the male withdraws its genitalia and the two separate The female remains motionless for some time while the male retires from the spot

SEX ATTRACTION

Observations on the mating habits of *P americana* show that there is no sex-attraction in cockroaches, these appear to be guided mainly by instinct Wille has demonstrated in *Blatella germanica* that the secretion of two pairs of glands, dorsally situated at the ends of sixth and seventh tergites of adult males, serves to attract the females at the time of mating He writes When a male encounters a female, it raises the wings at right angles to the body and exposes the depression of its back to the female, who being soon attracted to them, first explores them with her palpi and then proceeds to lick them with her mouth parts ' Rau (1924), Zabinski, and Qadri in their observation on *B orientalis* agree with Wille on this point and further record that the male attracts the female on its back and extends the abdomen below her to secure a hold on the ovipositors I, however, failed to observe in *P americana* any attempt on the part of the male to attract a female on its back, nor have I found the female exploring and licking the glands on the back of the male Only a pair of dorsal glands,[1] equally developed in both the sexes, occurs in nymphs as well as in adults of *B orientalis* and *P americana* and I regard these as odour producing glands as suggested by Hasse (1889) and Oettinger (1906) I therefore feel sure that these have nothing to do with sex attraction, there being no such phenomenon in these insects

INTER RELATIONSHIP OF GENITALIA [2] AND OTHER PARTS OF MALE AND FEMALE DURING THEIR UNION

A male cockroach, prior to getting an actual hold on the female pulls down the gynovalvular portion of the seventh sternite of the female by the tip of its protruded titillator This action of the male releases the ovipositors of the female and their free distal ends are then pushed up by the male genitalia to widen out the entrance of the female genital pouch (gynatrium), into which the male inserts its genitalia In firmly united specimens, the distal portion of the ninth sternite of the male comes to lie above the gynovalvular portion of the seventh sternite of the female and the styles of the male press hard against the notches of the seventh sternite of the female The cerci of the female lie beneath those of the male, the two resting in a cross wise manner on either side The wings of the male usually come to lie beneath those of the female The epiprocts of the male stretch beneath those of the female, press against the paraprocts of the latter and also get bent upon themselves

The right phallomere [3] of the male genitalia occupies almost a median dorsal position within the gynatrium and is situated between the posterior gynapophyses and the ejaculatory duct, the former taking no part in the actual process of copulation The distal extremity of the serrate lobe of the phallomere reaches the bases of the anterior valvulae of the ovipositor The curved spine of the right limb of serrate lobe entangles the left valvula while the two prong like processes of its left limb hold the right valvula The right and left anterior valvulae come to lie above the left [4] and ventral [5] phallomeres respectively These valvulae extend back to the opposing lobes which open to allow the entry of the former into the space enclosed between themselves The opposing lobes soon tighten their hold to grasp firmly these valvulae between their lips It will thus be seen that the anterior valvulae are the only parts of the female genitalia that are held tightly by those of the male

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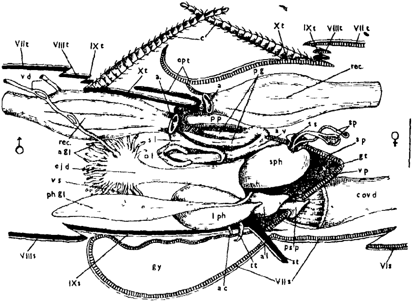
[1] Dorsal glands were described in *Periplaneta* by Minchin (1889 1890)

[2] The terminology has been adopted from Crampton (1925) and Snodgrass (1937)

[3], [4], & [5] The right, left and ventral phallomeres are equivalent to the *Right Dorsal, left* (combined dorsal and ventral), and *Right Ventral penis valves* respectively (Qadri, 1938, 1940)



TEXT FIG 1 Sectional view (Schematic) of the tail ends of a copulating pair of *P americana* showing the interlocking arrangement of their genitalia

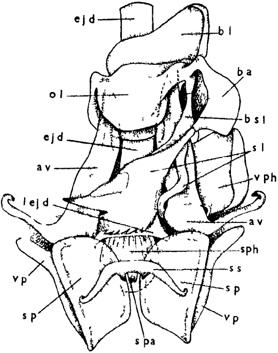
a, anus, *a c*, acutolobus, *a gl*, accessory glands, *a l*, asperate lobe, *a v*, anterior valvula, *c*, cerci, *c ov d*, common oviduct, *ej d*, ejaculatory duct, *ep t*, epiproct, *gt*, gynatrium, *gv*, gynovalvula, *l ph*, left phallomere, *o l*, opposing lobes, *p g*, posterior gonapophysis, *ph gl*, phallic gland, *p p*, paraproct, *ps p*, pseudopenis, *rec*, rectum, *s l*, serrate lobe, *sp*, spermatheca, *s p*, spermathecal plate, *sph*, spermatophore, *s s*, spermathecal sclerite, *st*, style, *tt*, titillator, *v p*, vulvular plate, *v s*, vesicula seminalis, *VI s*, *VII s*, *VIII s*, *IX s*, sixth, seventh, eighth and ninth sternites, *VII t*, *VIII t*, *IX t*, *X t*, seventh, eighth, ninth and tenth tergites

during the act of copulation and are released only after the completion of the act The remaining sclerites of the right phallomere are only subsidiary and mainly provide attachment to the muscles

The other parts of the male and female genitalia lie just under the right phallomere of the male and the ovipositors of the female The vulvular plates which ordinarily lie flat on the seventh sternite become vertically slanting and female gonopore situated between the vulvular plates consequently points obliquely backwards The lateral spermathecal plates, which in the natural condition superpose the perivulvulars get likewise tilted and thus widen out the vaginal opening The arms of the median spermathecal sclerite are pulled anteroventrally and all these adjustments cause the spermathecal papilla face vertically downward

The ventral phallomere, on which the terminal opening of the ejaculatory duct is present, shifts to the right making room for the expansion of the lower lip of the ejaculatory duct It does not hold any part of the female genitalia while Qadri writes 'In *B orientalis* it holds the anterior ovipositor valves tightly' The ejaculatory duct becomes fully dilated and its membranous upper lip as well as the thick lower lip, both extend up to the entrance of the vagina The spermatophore is expelled out by the muscular contraction of the wall of the ejaculatory duct and it is directly attached on to the ventrally projecting spermathecal papilla

The left phallomere which moves at an angle of about 30° from its original position projects towards the right from beneath the ejaculatory duct The expanded tip of the pseudopenis enters the female gonopore where it rotates about 90° on its own axis getting an anchorage on the perivulvular selerites The asperate lobe lies just above the pseudopenis, a little to the right side, while the opening of the phallic gland is situated immediately adjacent to the opening of the ejaculatory duct The titillator, which in the very beginning of the process brought about the opening of the female gynatrium to allow the entrance of the male genitalia into it, comes to lie ventrally in a slanting position The acutolobus is situated above and slightly to the left of titillator and its curved spine presses against a depression on the endogynal plate in the bothecal membrane The other scerlites are likewise tilted and mainly provide attachment to the muscles.



TEXT FIG 2 Dorsal view (semi diagrammatic) of the male and female genitalia during copulation after the removal of the posterior gonapophyses

a v, anterior valvula, *b a*, basarcus, *b l*, basilamina, *b s l*, basi serrate lobe, *ej d.*, ejaculatory duct, *l ej d*, lip of the ejaculatory duct, *o l*, opposing lobe, *s l*, serrate lobe, *s p*, spermathecal plate, *s pa*, spermathecal papilla, *sph*, spermatophore; *s s*, spermathecal sclerite, *v p*, vulvular plate, *v ph*, ventral phallomere

Rau's, Zabinski's, and Qadri's descriptions of the copulatory process are inadequate and for want of clear and properly labelled sketches an understanding of the inter-relationship of the parts of male and female genitalia is hardly possible. I agree with Zabinski's observations that the removal of the long hooked process (titillator) from the genitalia of a male disables the same to copulate with a female because such a male will not be able even to open out the female gynatrium (vestibulum) Likewise, a normal male will be unable to retain hold on a female cockroach from whose genitalia the anterior valvulae have been removed, because this is the only part of a female that is well within the grasp of the male during the act of copulation

Qadri (1938) writes *The right ventral penis valve* which is probably incorrectly referred to as the penis by Zabinski and others, *holds the anterior ovipositor valves tightly*, while the right dorsal penis valve lies between the anterior valves, and *appears* to be the main clasping organ The ejaculatory duct enters from the side and its membranous lip is applied to the bases of the anterior valves where the spermatophore is attached (Italics are mine) There appears to be some confusion in the use of the terms left and right dorsal and ventral penis valves in the genitalia of male (Qadri 1940) My observations on *P americana* are not in agreement with those of Qadri in many respects The right ventral penis valve, which is equivalent to the ventral phallomere, is plane and simple part incapable of holding anything, still Qadri has assigned to it the function of holding the anterior ovipositor valves which are actually grasped by the right phallomere equivalent to the right dorsal penis valve mentioned by him The details regarding the functions of the other component parts of the genitalia have not been described by him The membranous lip of the ejaculatory duct lies below the ovipositor valves and is not applied to the bases of these as pointed out by Qadri and neither is the spermatophore attached to the bases of the anterior valves

FATE OF THE SPERMATOPHORE

As already mentioned the spermatophore is expelled out of the ejaculatory duct and it is attached on to the spermathecal papilla of the female where it has been found sticking up to about 21 hours after the copulation In a freshly mated female the outer wall of the spermatophore is soft and it is loosely attached to the spermathecal papilla but within about two hours the wall of the spermatophore sets in and hardens so that the spermatophore becomes firmly fixed on to the papilla After about 18 hours the spermatophore attachment becomes loose and 3 to 4 hours later it is no more found within the genital chamber of the female The ultimate fate of the empty spermatophore is obscure possibly it drops out and is eaten up by the female as is recorded in some other Orthopteroid insects (Gerhardt 1913 1914)

Different views have been expressed regarding the place of attachment of the spermatophore and its fate Zabinski says Copulation in *B orientalis* results in the attachment of a spermatophore on the papilla of the female containing the spermathecal orifice the spermatophore is carried by the female for two or three days and is then rejected Qadri wrongly criticises Zabinski regarding the initial place of attachment of the spermatophore and writes In just mated female, the spermatophore is *far* from the spermathecal aperture and lies between the bases of the ovipositor valves In a male dissected eight hours after mating, the spermatophore was *shifted* from the ovipositor valves to the interior of the genital cavity in the *vicinity* of the spermathecal aperture' (Italics are mine) My observations on *P americana* are in agreement with those of Zabinski since I also find that a spermatophore is attached from the very beginning to the spermathecal papilla It appears that while Qadri was handling a just mated female, the soft, freshly laid spermatophore which was then loosely attached to the spermathecal papilla got displaced from its original position during dissection and remained attached to the ovipositors which lie just above and press against the posterior part of the spermatophore It is otherwise, not possible to explain the deposition of a spermatophore far away from the spermathecal aperture and also its shifting from the ovipositor valves to the interior of genital cavity in the vicinity of the spermathecal aperture as the time advances

ACKNOWLEDGMENTS

The observations incorporated herein have been made in the Zoological Laboratory of the University of Lucknow The work was carried out under the kind guidance of Professor K N Bahl to whom I am indebted for the correction of the manuscript I am grateful to Dr M L Bhatia of the University for his helpful criticism and valuable suggestions I am thankful to the University of Lucknow for the award of a Research Fellowship

SUMMARY

Copulation in cockroaches usually takes place during night from March to September and lasts about an hour and a half There is no courtship A male becomes sexually excited when fully formed spermatophore is present in its ejaculatory duct and moves about in search of a female while the latter behaves indifferently and it does not try to mount the back of the male as mentioned by previous workers There is no glandular secretion to affect sex-attraction Copulation takes place only six days after the final moult in both the male and female The males copulate several times at intesvals of about seven days while the females less frequently and they can do so even a few hours after laying a cocoon

During copulation the male and female remain joined together in a tail-to-tail position The titillator of the male genitalia forces open the female gynatrium thus allowing the entry of the former into the latter The pseudopenis actually enters the female gonopore and anchors the vulvular plates, the right phallomere works as the main clasping organ since its opposing lobes and the serrate lobe hold the ventral valvulae and their bases The spermatophore is expelled out and is directly attached on to the ventrally projecting spermathecal papilla

The secretion of the phallic gland is poured over the spermatophore during its attachment to the spermathecal papilla and hardens to form the outer wall of the former The spermatophore remains attached to the papilla for about twenty-one hours during which the spermatic fluid within the spermatophore passes into the spermatheca

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PHOTOCHEMICAL STUDIES IN SOLS AND GELS PART 1

THE OXIDATION OF GLYCERINE BY METHYLENE BLUE IN LIGHT OF DIFFERENT FREQUENCIES WITH COLLOIDAL ZINC OXIDE AS THE PHOTO-SENSITISER IN MEDIA OF THIXOTROPIC ALUMINIUM HYDROXIDE SOL AND GEL

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(Received August 10, read August 30, 1946)

Photochemical reactions in liquid as well as in gaseous phases have been studied in considerable detail but in comparison, very few reactions have been studied in a sol or a gel phase As examples, we may cite the decomposition of silver halides in gelatine studied by a number of workers, notably by Eggart and Noddack in wavelengths 365, 406 and 436 $\mu\mu$ and the decomposition of AgCl (sensitised by Ag) on printing out paper studied by Weigert in wavelength 436 $\mu\mu$ But no worker has yet made any comparative study of the kinetics of any reactions in both sol and gel phases It would, therefore, be very interesting if methods could be devised by means of which photochemical reactions could be studied in a sol phase and the results compared with those studied in the gel phase Chemical processes in nature mostly take place in sol and gel phases It is hoped that similar laboratory investigations in sol and gel phases may throw light on the mechanism of such chemical processes in nature With this idea in mind, we started our work using some transparent and colourless thixotropic sols and gels as solvent media The advantage of using the thixotropic gels is that they liquefy on shaking which set again on standing for some time

In the present investigation we have studied the oxidation of glycerine by methylene blue with colloidal zinc oxide as the photosensitiser in light of frequencies, 366, 406 and 436 $\mu\mu$ in media of thixotropic aluminium hydroxide sol and gel

The action of zinc oxide in photosensitising the decomposition of various organic compounds and a large number of inorganic reactions has long been known Eibner was the first to observe that several inorganic and organic coloured substances (e g , prussian blue, lead chromate, etc) are reduced in the light by zinc oxide in the presence of a depolariser such as glycerine or sugar Winther studied the fluorescence of zinc oxide and some of its photosensitising action For both processes, the near ultraviolet spectral region was found to be effective Winther suggested that the zinc oxide remains chemically unchanged during these reactions and supported the view by experiments, which indicate that actually the size and shape of single zinc oxide particles remain unchanged even on prolonged illumination in presence of glycerine and lead carbonate Tammann observed that, in presence of solid zinc oxide, silver is deposited from a solution of silver nitrate on illumination and zinc goes into solution Tammann suggested that light accelerates the ionic exchange according to the equation, $ZnO+2AgNO_3 = Ag_2O+Zn(NO_3)_2$ Kohlshutter and d'Almendra demonstrated that metallic silver and not its oxide is the product of light action In this connection the investigations carried out by Baur and his collaborators are outstanding Baur and Perret and Perret studied the photosensitising action of zinc oxide by exposing to sunlight a suspension of zinc oxide or other insoluble inorganic substances in silver nitrate solutions The result indicates that the photosensitised decomposition of silver nitrate solution is due to a specific action of zinc oxide and not to the presence in general of solid particles in the solution. It was further observed that on short illumination, not only metallic silver, but also silver oxide and peroxide are deposited on zinc oxide surfaces and that an equivalent amount of this latter goes into solution The photochemical reaction was also found to be followed by evolution of gas, identified as pure oxygen Perret suggested that the integral process taking place on illumination is expressed by the following equation

$$2AgNO_3+ZnO(+light) = 2Ag+Zn(NO_3)_2+1/2\ O_2$$

and demonstrated that, actually, the total amount of oxygen formed (free and bound in silver oxides) is given roughly by the relation

$$1/4\ O_2 = Ag$$

Baur developed in a series of publications a novel theory of photosensitisation to explain the action of zinc oxide or uranyl salts in promoting various photochemical processes The sensitiser is thought to become on absorption of light a polarised molecule comparable to the two electrodes of an electrolytic cell Perret (loc cit) observed that solid zinc oxide particles exert a specific photosensitising action also on solutions of mercuric chloride and the reaction proceeds according to the equation

$$2HgCl_2+ZnO = Hg_2Cl_2+ZnCl_2+1/2\ O_2$$

The reaction is greatly accelerated by dextrose and by sucrose, when the reaction is of zero order The third reaction of zinc oxide, studied by Perret, was its action in light on methylene blue Methylene blue has long been known to be light-sensitive Lasareff studied the kinetics of reduction of methylene blue solutions in solid gelatine He found that the process was reversible and that, under the influence of free oxygen, the methylene blue leuco base was oxidised in the dark to the dye-stuff again According to Perret (loc cit) the photochemical reaction of methylene blue is noticeably accelerated by the presence of zinc oxide

In presence of glycerine or glucose, the reaction has been schematically described as follows

$$ZnO\begin{matrix}\oplus\\ \ominus\end{matrix}\begin{cases}\text{glucose or glycerine}\rightarrow\text{oxidation products}\\ \text{methylene blue}\rightarrow\text{leuco-dye}\end{cases}$$

Baur and Neuweiler have shown that by exposing aqueous suspensions of zinc oxide in contact with air to sunlight hydrogen peroxide is formed They represented the reaction as follows

$$ZnO\begin{matrix}\oplus\\ \ominus\end{matrix}\begin{cases}OH^-\rightarrow 1/4\ O_2+1/2\ H_2O\\ H^+ +1/2\ O_2\rightarrow 1/2\ H_2O_2\end{cases}$$

References may also be made to the work done by Bohi, Fukhushima, McMorris and Dickinson, Goodeve, Narasimhachari and Qureshi and lastly by Dhar and Bhattacharyya

None of these workers, however, have studied the *kinetics* of the reactions sensitised by zinc oxide, because such studies present considerable difficulties on account of the heterogeneity of the reacting substances

After a number of trials we have succeeded in getting a suitable protective colloid,—thixotropic aluminium hydroxide sol,—which keeps zinc oxide in a colloidal state for nearly 10–12 hours during which each experiment recorded in this paper was completed

Section A—deals with the reaction in aluminium hydroxide sol as medium

Section B—deals with the reaction in aluminium hydroxide gel as medium.

EXPERIMENTAL.

The source of light was a mercury arc lamp whose strength of current and voltage were maintained constant by means of a regulating resistance Parallel beams of light were obtained by means of quartz cylindrical lenses of different focal lengths Monochromatic radiations at 366 and 406 and 436 $\mu\mu$ were obtained by using as filter a dilute solution of copper sulphate in combination with the corresponding Schott and Gen monochromatic filter The reaction cell was made of corex glass and was circular with a thickness of 1 cm and having a capacity of 2 5 c c The cell was placed inside a double jacketed metal box with a window in front The temperature was kept constant by passing, with the aid of a circulating pump, water from a thermostat through the annular space of the box It is essential for accurate work that no extraneous light should enter the reaction cell and the reaction was carried out in a perfectly dark room, the only light entering the reaction cell being that obtained from the lamp through the window

Reagents

Merck's extra pure zinc oxide, bi distilled glycerine and methylene blue supplied by B D H were used throughout For making thixotropic aluminium hydroxide sol, extra pure aluminium sulphate, lead acetate and barium acetate supplied by B D H were used For making solutions bi-distilled water was used

Preparation of thixotropic aluminium hydroxide sol

Thixotropic aluminium hydroxide sol was prepared by slow hydrolysis of aluminium diacetate following the method of Crum This information being not generally accessible, the method is given in detail below Aluminium acetate was first prepared by mixing together strong solutions of tersulphate of alumina and of acetate of lead They were poured slowly together into a beaker surrounded with ice cold water To the filtrate was added H_2S to precipitate lead sulphate which remained in solution and next barium acetate to throw down sulphuric acid

The filtrate, thus obtained free from lead and H_2SO_4, speedily became turbid on heating and a heavy deposit of white crystalline powder was formed This basic diacetate of aluminium was washed several times with distilled water and then dissolved in 200 times its weight of boiling bi distilled water The solution was then maintained at about 90°C for 15 days when complete hydrolysis occurred The liquid was then boiled in a wide beaker for nearly 12 hours with constant addition of fresh water to retain the same volume so that most of the acetic acid had been volatilised and then the aluminium hydroxide sol was put to dialysis for nearly a week until the sol became neutral The dialysed sol was concentrated, filtered and used

Estimation of the Aluminium hydroxide sol

The concentration of aluminium hydroxide in the sol was estimated by dissolving $Al(OH)_3$ in HNO_3 and precipitating $Al(OH)_3$ by ammonia The investigated sol contained 11 65 $\times 10^{-2}$ Mols of Al_2O_3 per litre

Preparation of colloidal zinc oxide

It is not possible to prepare colloidal zinc oxide in water as a medium After many trials we have found that thixotropic aluminium hydroxide sol acts as a protective colloid in keeping zinc oxide in a colloidal state for more than 12 hours The colloidal zinc oxide was prepared daily, before use, by triturating a weighed quantity of zinc oxide with $Al(OH)_3$ sol in a glass mortar for 10 minutes and then diluting it either with bi-distilled water or with further quantity of $Al(OH)_3$ sol

To study the reaction in sol-phase dilution was made by adding water whereas for studying in the solid gel phase dilution was made by adding $Al(OH)_3$ sol. For solid phase reaction Merck's extra pure K_2SO_4 was used for setting the sol to a jelly.

Measurement of the velocity of reaction

Spectrophotometric method was adopted for the estimation of methylene blue at any moment For this purpose 'Spectrophotometer Assembly' supplied by the Gaertner and Co was used Spectrophotometer readings (θ) were taken with the Corex cell filled with mixtures of different concentrations of methylene blue and colloidal zinc oxide in the green region (546 $\mu\mu$) where the leuco-dye has got no absorption—the blank corex cell being filled up with $Al(OH)_3$ sol of the same concentration as in the dye solution The calibration curve was obtained by plotting log tan θ against the concentration of methylene blue at a particular sol concentration (Fig 1)

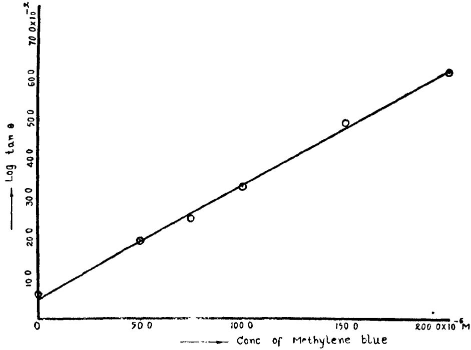

FIG 1

As colloidal zinc oxide has got appreciable general absorption in the visible, separate curves for absorption at 546 $\mu\mu$ were obtained corresponding to different concentration of colloidal zinc oxide in the mixture

TABLE I

Concentration of colloidal zinc oxide	Concentration of methylene blue	Spectrophotometer readings with mixtures of dye and colloidal zinc oxide (θ)	Spectrophotometer readings With water in both the cells (θ')	log tan θ
$\times 10^4$M	$\times 10^4$M			
6 15	2 0	76 8	45 0	0 6208
,,	1 5	72 0	,,	0 4882
,,	1 0	64 9	,,	0 3294
,,	0 75	60 6	,,	0 2491
,,	0 5	57 0	,,	0 1875
,,		49 0	,,	0 0608

The cell containing the reaction mixture was made absolutely airtight by covering the stopper with paraffin wax as the leuco-methylene blue is easily oxidised by air There is an induction period in this reaction which was eliminated mostly by passing pure and dry N_2 gas through the reaction mixture just after mixing the ingredients and partly by exposing the reaction mixture to the whole light of the mercury-arc for 5 minutes

It was observed that methylene blue does not get reduced when exposed to light of frequencies 366, 406 and 436 $\mu\mu$ either alone or in presence of glycerine or in presence of aluminium hydroxide sol No dark reaction was observed when a mixture of colloidal zinc oxide, methylene blue and glycerine was kept in the dark for more than 24 hours

Determination of pH

The *p*H of the reaction mixture was determined potentiometrically by using a glass electrode

Measurement of Intensity

The intensity of radiation absorbed by the reaction mixture was measured by means of a 'Weston's Photronic cell' and a sensitive galvanometer The photronic cell was calibrated by means of a standard lamp (12V, 4 0 W) standardised by means of a Moll thermopile and a Hefner lamp The intensity of absorbed radiation was measured by noting the deflections when the light passed through (*a*) thixotropic aluminium hydroxide sol of the same concentration as was used in the reaction mixture, (*b*) the reaction mixture The difference in deflections in the two cases gave the intensity of radiation absorbed by the reaction mixture. It is to be pointed out that aluminium hydroxide sol or gel has got no absorption in 366, 406 and 436 $\mu\mu$

As mentioned before, light absorbed by methylene blue is not effective in the photochemical reduction by glycerine and hence the photo oxidation of glycerine by methylene blue is due to the light absorbed by colloidal zinc oxide alone

The amount of light absorbed by colloidal zinc oxide alone can be calculated by the formula for mixtures I_{abs} for colloidal zinc oxide =

$$I_0\left(1-e^{-\epsilon_1 c_1 d-\epsilon_2 c_2 d}\right)\times\frac{\epsilon_1 c_1}{\epsilon_1 c_1+\epsilon_2 c_2} \qquad (\alpha)$$

where ϵ_1 = molecular extinction coefficient of colloidal zinc oxide
ϵ_2 = molecular extinction coefficient of methylene blue
c_1 = conc of colloidal zinc oxide in gm mols/litre
c_2 = conc of methylene blue in gm mols/litre
d = thickness of the reaction cell in cm
I_0 = intensity of incident radiation

The relation (α) can be roughly taken as

$$I_{abs} = I\frac{\epsilon_1 c_1}{\epsilon_1 c_1+\epsilon_1 c_2} \qquad . \quad .. \quad .. \quad .. \quad (\beta)$$

where I = intensity of radiation absorbed by the reaction mixture

The molecular extinction coefficients ϵ_1 and ϵ_2 of colloidal zinc oxide and methylene blue were determined experimentally by intensity measurements in the following way. The deflections in the galvanometer were noted, first of all, with the solvent alone (e g, aluminium hydroxide sol in the case of colloidal zinc oxide or water in the case of methylene blue) and secondly with colloidal zinc oxide or methylene blue of known concentrations The molecular extinction coefficients ϵ_1 and ϵ_2 were then calculated according to the equation,

$$\epsilon = \frac{1}{c\,d} \log_e \frac{I_0}{I_t}$$

where c and d have their usual significance and I_t = intensity of transmitted light

The values of ϵ_1 and ϵ_2 for different wavelengths (λ) e g , 366, 406 and 436 $\mu\mu$ are recorded in Table II

TABLE II

$\lambda(\mu\mu)$	366	406	436
ϵ_1	633 4	478 6	220 8
ϵ_2	2500 0	1710 0	1733 0

The extinction coefficients of colloidal zinc oxide as recorded above cannot be taken as perfectly accurate as the necessary corrections for the scattered light have not been made As the concentrations of colloidal zinc oxide had been taken throughout this investigation so low, that the colloid had a pale white opalescence, the errors due to scattered light might have been very small

The reaction was studied at 25°C The experimental data are recorded in Tables III to XII The reaction was found to be zero-molecular with respect to methylene blue In the following tables,

$$\frac{\Delta x}{\Delta t} = \text{zero molecular velocity constant}$$

= No of gm mols of methylene blue transformed per litre per minute.

In the tables, T = temperature, I_{abs} = No of quanta absorbed by colloidal zinc oxide per c c per sec ,

a, b, A and B are the concentrations of methylene blue, glycerine, colloidal zinc oxide and aluminium hydroxide sol respectively, in gm mols/litre

Section A

Determination of the order of the reaction

TABLE III

λ = 436 $\mu\mu$ a = 15 0 × 10^{-6}M b = 20 0 × 10^{-2}M
A = 6 15 × 10^{-4}M B = 5 96 × 10^{-3}M I_{abs} = 160 × 10^{13}
Temperature (T) = 25°C pH = 6 92

	Time (minutes)	Spectro reading (θ)	log tan θ	C 10^6 Methylene blue	$\frac{\Delta x}{\Delta t}\,10^7$	
(1)	$t+0$	68 4	0 4024	12 57	19 7	from (1) and (2)
(2)	$t+10$	65 6	0 3433	10 6	19 8	from (1) and (3)
(3)	$t+20$	62 8	0 2891	8 6	19 5	from (1) and (4)
(4)	$t+30$	60 0	0 2386	6 8	19 0	from (1) and (5)
(5)	$t+40$	57 2	0 1908	5 0		
					19 5	(Mean)

Effect of varying the concentration of methylene blue

TABLE IV

$T = 25°C$ $pH = 6\ 92$

λ ($\mu\mu$)	$a\ 10^5$ (Mol)	$b\ 10^2$ (Mol)	$A\ 10^4$ (Mol)	$B\ 10^3$ (Mol)	$I_{abs}\ 10^{-13}$	$\frac{\Delta x}{\Delta t}\ 10^7$	$\frac{\Delta x}{\Delta t}\ 10^{21}/I_{abs}$	γ
366	20 0	10 0	6·15	5 96	40	32 0	8 0	0 08
,,	15·0	,,	,,	,,	,,	29 5	7 4	0 07
,,	10 0	,,	,,	,,	,,	32 7	8 2	0 08
,,	7 5	,,	,,	,,	,,	32 0	8 0	0 08
,,	5 0	,,	,,	,,	,,	31 0	7 8	0 08
406	20 0	10 0	12 3	11 92	20	2 9	1 4	0 014
,,	15 0	,,	,,	,,	,,	2 6	1 3	0 013
,,	10 0	,,	,,	,,	,,	2 6	1 3	0 013
,,	7 5	,,	,,	,,	,,	2 6	1 3	0 013
,,	5 0	,,	,,	,,	,,	2·6	1 3	0 013
436	20 0	20·0	6 15	5 96	160	19 3	1 2	0 012
,,	15 0	,,	,,	,,	,,	19 5	1 2	0 012
,,	10 0	,,	,,	,,	,,	17 8	1 1	0 01
,,	7 5	,,	,,	,,	,,	17 8	1 1	0 01
,,	5 0	,,	,,	,,	,,	17 8	1 1	0 01

Effect of varying the concentration of glycerine

TABLE V

$T = 25°C$ $pH = 6\ 92$

λ ($\mu\mu$)	$a\ 10^5$ (Mol)	$b\ 10^2$ (Mol)	$A\ 10^4$ (Mol)	$B\ 10^3$ (Mol)	$I_{abs}\ 10^{-13}$	$\frac{\Delta x}{\Delta t}\ 10^7$	$\frac{\Delta x}{\Delta t}\ 10^{21}/I_{abs}$	γ
366	10 0	20 0	6 15	5 96	34	26 7	7 8	0 08
,,	,,	8 0	,,	,,	,,	26 7	7 8	0 08
,,	,,	4 0	,,	,,	,,	23 6	6 9	0 07
,,	,,	2 0	,,	,,	,,	18 6	5 5	0 06
,,	,,	1 0	,,	,,	,,	12 8	3 8	0 04
406	15 0	20 0	12 3	11 92	20	2 6	1 3	0 013
,,	,,	10 0	,,	,,	,	2 6	1 3	0 013
,,	,,	8 0	,,	,,	,,	2 6	1 3	0·013
,,	,,	4·0	,,	,,	,,	2 3	1 2	0 012
,,	,,	2·0	,,	,,	,,	2 1	1 1	0 011
,,	,,	1·0	,,	,,	,,	1 7	0 9	0 009
436	10 0	20 0	6 15	5 96	160	17 8	1 1	0 01
,,	,,	10 0	,,	,,	,,	18 0	1 1	0 01
,,	,,	8 0	,,	,,		18 0	1 1	0 01
,,	,,	5 0	,,	,,	,,	14 5	0 9	0 009
,,	,,	2 5	,,	,,	,,	9 8	0 6	0·006

Effect of varying the concentration of colloidal zinc oxide

TABLE VI

$T = 25°C$ $pH = 6\ 92$

λ ($\mu\mu$)	$a\ 10^5$ (Mol)	$b\ 10^2$ (Mol)	$A\ 10^4$ (Mol)	$B\ 10^3$ (Mol)	$I_{abs}\ 10^{-13}$	$\frac{\Delta x}{\Delta t}\ 10^7$	$\frac{\Delta x}{\Delta t}\ 10^{21}/I_{abs}$	γ
366	10 0	10·0	12 3	11 92	50	36 0	7 2	0 07
,,	,,	,,	6 15	5 96	40	32 7	8 0	0·08
,,	,,	,,	3 08	2 98	20	15 7	7 5	0 075
406	15 0	10·0	12 3	11 92	20	2 6	1 3	0 013
,,	,,	,,	6 15	5 96	10	1 4	1 3	0 013
436	20 0	10 0	12 3	11 92	166	20 6	1 2	0 01
,,	,,	,,	6 15	5 96	123	14 0	1 1	0 01
,,	,,	,,	3 08	2 98	69	6 95	1 0	0 01

Effect of varying the intensity of incident and hence of absorbed radiation

TABLE VII

$T = 25°C$ $pH = 6·92$

λ (μμ)	$a\ 10^5$ (Mol)	$b\ 10^3$ (Mol)	$A\ 10^4$ (Mol)	$B\ 10^3$ (Mol)	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^7$	$\frac{\Delta x}{\Delta t}\ 10^{21}/I_{abs}$	γ
366	10 0	10 0	6 15	5 96	68	53 0	7 8	0 08
,,	,,	,,	,,	,,	40	32 7	8 0	0 08
,,	,,	,,	,,	,,	34	26 7	7 8	0 08
406	15 0	10 0	12 3	11 92	24	3 2	1 3	0 013
,,	,,	,,	,,	,,	12	1 6	1 3	0 013
436	10 0	10 0	6 15	5 96	160	17 8	1 1	0 01
,,	,,	,,	,,	,,	123	14 0	1 1	0 01
,,	,,	,,	,,	,,	81	9 0	1 1	0 01

Effect of varying the concentration of aluminium hydroxide sol

TABLE VIII

$T = 25°C$ $pH = 6\ 92$

λ (μμ)	$a\ 10^5$ (Mol)	$b\ 10^3$ (Mol)	$A\ 10^4$ (Mol)	$B\ 10^3$ (Mol)	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^7$	$\frac{\Delta x}{\Delta t}\ 10^{21}/I_{abs}$	γ
366	10 0	8 0	6 15	5 96	34	26 7	7 8	0 08
,,	,,	,,	,,	47 6	,,	26 7	7 8	0 08

Section B

Effect of varying the concentration of K_2SO_4

TABLE IX

$T = 25°C$ $pH = 6\ 92$

λ (μμ)	$[K_2SO_4]\ 10^4$ (Mol)	$a\ 10^5$ (Mol)	$b\ 10^3$ (Mol)	$A\ 10^4$ (Mol)	$B\ 10^3$ (Mol)	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^7$
366		16 0	8 0	12 3	47 6	40	29 8
,,	2 0	,,	,,	,,	,,	,,	21 5
*,,	3 0	,,	,,	,,	,,	,,	14 2

* The reaction mixture set to a firm jelly within 5 minutes when the concentration of added $K_2SO_4 = 3\ 0 \times 10^{-4}M$

Effect of varying the concentration of methylene blue

TABLE X

$T = 25°C$ $pH = 6\ 92$ $K_2SO_4 = 3\ 0 \times 10^{-4}M$

λ (μμ)	$a\ 10^5$ (Mol)	$b\ 10^3$ (Mol)	$A\ 10^4$ (Mol)	$B\ 10^3$ (Mol)	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^7$	$\frac{\Delta x}{\Delta t}\ 10^{21}/I_{abs}$	γ
366	16 0	8 0	12 3	47 6	40 1	14 2	3 5	0·035
,,	12 0	,,	,,	,,	,,	14 2	3 5	0 035
,,	8 0	,,	,,	,,	,,	13 7	3 4	0 034
,,	4 0	,,	,,	,,	,,	14 0	3 5	0 035

Effect of varying the concentration of glycerine.

TABLE XI

$T = 25°C$ $pH = 6\ 92$ $K_2SO_4 = 3{\cdot}0 \times 10^{-4}M$

λ ($\mu\mu$)	$a\ 10^5$ (Mol)	$b\ 10^2$ (Mol)	$A\ 10^4$ (Mol)	$B\ 10^3$ (Mol)	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^7$	$\frac{\Delta x}{\Delta t}.\ 10^{21}/I_{abs.}$	γ
366	16 0	8 0	12 3	47 6	40 1	14 2	3 5	0 035
,	,,	4 0	,,	,,		12 1	3 0	0 030
,,	,,	2 0	,,	,,	,,	7 4	1 8	0 018

Effect of varying the intensity

TABLE XII

$T = 25°C$ $pH - 6\ 92$ $K_2SO_4 - 3\ 0 \times 10^{-4}M$

λ ($\mu\mu$)	$a\ 10^5$ (Mol)	$b\ 10^2$ (Mol)	$A\ 10^4$ (Mol)	$B\ 10^3$ (Mol)	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^7$	$\frac{\Delta x}{\Delta t}\ 10^{21}/I_{abs}$	γ
366	16 0	8 0	12 3	47 6	40 1	14 2	3 5	0 035
,,	,	,,	,,	,,	62 1	21 1	3 4	0 034

DISCUSSION

The reaction has got the following common characteristics in both the sol and gel states —

(1) The reaction is zero-molecular with respect to methylene blue
(2) There is a slight induction period in the sol phase whereas in the gel phase it is appreciable
(3) The velocity constant is independent of the initial concentration of methylene blue
(4) The velocity constant is independent of the initial concentration of glycerine when [glycerine] $\geqq$ 0 08M Below 0 08M, the velocity constant diminishes with decreasing concentration of glycerine In fact, $1\Big/\frac{\Delta x}{\Delta t}$ plotted against $\frac{1}{C_{glycerine}}$ gives a straight line (Fig 2)
(5) The velocity constant increases with increase in the concentration of colloidal zinc oxide, but $\frac{\Delta x}{\Delta t}\Big/I_{abs}$ remains practically the same With the maximum concentration, the light is, more or less, completely absorbed
(6) For the same concentration of glycerine, the velocity constant is directly proportional to the intensity of radiation absorbed by colloidal zinc oxide In fact, $\frac{\Delta x}{\Delta t}\Big/I_{abs.}$ is constant for a particular wavelength, which increases with increase in the quanta absorbed.
(7) The quantum efficiency is much less than unity
(8) The photo-active range of ZnO and consequently its absorption extends up to the mercury violet, i e, 436 $\mu\mu$

Winther has studied the light absorption of ZnO in the ultraviolet by its effect in reducing the fluorescence of mercurous chloride with which it is intimately mixed.

He gives a curve for ZnO with a maximum at 360 $\mu\mu$ and a minimum at about 300 $\mu\mu$ Goodeve from a study of the diffuse reflecting power of the solid ZnO has obtained

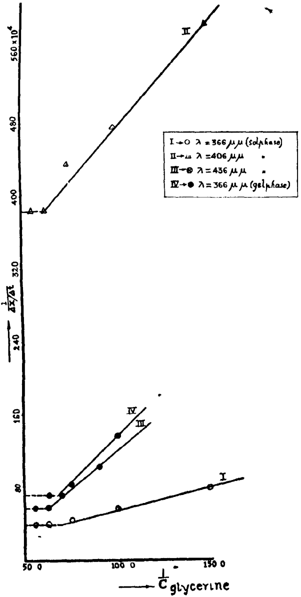

FIG 2

the absorption spectra of ZnO He has observed that there is a sharp fall in the reflecting power of the powder at 385 $\mu\mu$ and the curve becomes flat again at a reflecting power of about 2% and remains unchanged as the wavelength is decreased.

The rapid fall in the reflecting power indicates the entry of a strong absorption band with a fairly sharp threshold at the wavelength The light of wavelength below this threshold, reflected from the powder, undoubtedly comes from the front surface of the particles—this accounting for the flatness of the curve From these observations Goodeve has concluded that zinc oxide absorbs in the near ultraviolet, which is exactly the region in which the oxide is found to be photoactive

The results of the present authors show that the photoactive range and consequently its absorption extends even up to the mercury violet, i e 436 $\mu\mu$ From a recent investigation on the formation of H_2O_2 from water in presence of solid ZnO as a sensitiser, Narasimhachari and Qureshi have drawn similar conclusions

The following mechanism will explain all these characteristic features —

The colloidal zinc oxide surface is completely covered with a unimolecular layer of the dye-molecules even at very low concentration of the dye It is remarkable that the absorption of radiation by the dye-molecules directly does not lead to the photoreduction by reaction with glycerine But a dye-molecule can be brought into activated state by receiving energy from the elementary spaces of the colloidal zinc oxide surface which is also excited by the absorption of radiation

The low quantum efficiency may be due to two reasons —

(*a*) Only a small fraction of the radiation absorbed by the colloidal ZnO is available for the activation of the dye molecules adsorbed on the surface of the colloidal ZnO, or,

(*b*) the velocity of reaction between activated molecules and the reductant adsorbed on the surface of ZnO is so slow that most of the former reverts spontaneously to the normal state It is difficult to decide between these two possibilities

The velocity is given by

$$\frac{\Delta x}{\Delta t} = K' \frac{I}{N h \nu} C^R_s \qquad \text{(i)}$$

where C^R_s is the surface concentration of the reductant According to Langmuir

$$C^R_s = \frac{K_1 C^R_B}{K_3 + K_2 C^R_B} \qquad \text{(ii)}$$

where C^R_B is the concentration of reductant in solution

When C^R_B is very large, $K_3 + K_2 C^R_B$ may be taken equal to $K_2 C^R_B$

or
$$\frac{\Delta x}{\Delta t} = K' \frac{I}{N h \nu} \frac{K_1}{K_2} \quad . \qquad \text{(iii)}$$

That is, the velocity constant is independent of reductant concentration when the latter is high as has been experimentally found to be the case

At a lower concentration of glycerine, K_3 cannot be neglected in comparison to $K_2 C^R_B$

Hence
$$1\Big/\frac{\Delta x}{\Delta t} = K_4 \frac{K_3 + K_1 C^R_B}{K_2 C^R_B}$$

$$= \frac{K_5}{C^R_B} + K_6 \quad .. \quad . \quad .. \quad .. \quad .. \qquad \text{(iv)}$$

That is, at low concentration of reductant, $1\Big/\frac{\Delta x}{\Delta t}$ plotted against $1/C_{\text{glycerine}}$ should give a straight line This has been found experimentally.

The velocity of reaction has been found to be proportional to the intensity of absorbed radiation as is demanded by equation (i)

The rate of reaction and consequently the quantum efficiency is greater in medium of thixotropic aluminium hydroxide sol than that in aluminium hydroxide gel This is not in agreement with the observations made by the authors in their work on the photo reduction of ferric chloride by mandelic acid in media of thorium phosphate and thorium molybdate sols gels (Part II of the series) where they have found equal rate of reaction in both sol and gel states of thorium phosphate and thorium molybdate as media

This discrepancy may be explained by the observations made by the authors that the number of colloidal zinc oxide particles diminishes with the addition of coagulating agent, e g , K_2SO_4 with the consequent diminution in the rate of reaction The particles of zinc oxide become bigger with additions of coagulating agent with the result that the specific reaction surface becomes reduced The following table will clearly show that the approximate number of particles observed by means of an ultramicroscope with a very narrow slit opening is much smaller in the gel phase than in the sol phase, the concentration of colloidal zinc oxide being the same

TABLE XIII

Concentration of Colloidal Zinc Oxide = $6\ 1 \times 10^{-4}$M
,, Aluminium hydroxide = $47\ 6 \times 10^{-3}$M
,, Glycerine = $8\ 0 \times 10^{-2}$M

$[K_2SO_4]$	—0 0M	*$3\ 2 \times 10^{-4}$M
n	—30	5

where n = approximate no of particles instantaneously visible in an ultramicroscope with a very small slit opening

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PHOTOCHEMICAL STUDIES IN SOLS AND GELS PART II

THE REDUCTION OF FERRIC CHLORIDE BY MANDELIC ACID IN LIGHT OF DIFFERENT FREQUENCIES IN MEDIA OF THIXOTROPIC THORIUM PHOSPHATE AND THORIUM MOLYBDATE SOLS AND GELS

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(*Received August 10, read August 30, 1946*)

In the present investigation we have used thixotropic thorium phosphate and thorium molybdate sols and gels as solvent media as they are transparent and perfectly colourless and have studied a very simple reaction—the photoreduction of ferric chloride by mandelic acid in light of wavelengths 366 and 436$\mu\mu$ The kinetics of this reaction were studied, in details, in the aqueous phase by a number of workers, notably by Eder, Jodin, Lemoine, Ross, Winther and Oxolt-Howe, Kornfeld and Mencke, Ghosh and Purakayastha, Purakayastha, Allmand and Young, Kornfeld, Talpade and Bavdekar

In aqueous phase and with mandelic acid as a reductant, Ghosh and Purakayastha (*loc cit*) have shown that (i) the reaction is zero-molecular with respect to ferric chloride, (ii) the zero-molecular velocity constant increases with increasing concentration of the reductant and in fact, $\dfrac{1}{\Delta x/\Delta t}$ plotted against $\dfrac{1}{(\text{Reductant})}$ gives a straight line, (iii) the velocity constant is directly proportional to the intensity of absorbed radiation, (iv) the velocity constant varies very slightly with increasing concentration of hydrochloric acid, (v) the quantum efficiency was found to be 1 06, 1 18 and 1 36 in wavelengths 488, 448 and 390$\mu\mu$ respectively

The reaction has been studied in media of (1) thorium phosphate and thorium molybdate sols, (2) thorium phosphate and thorium molybdate gels, and (3) water, and a comparative study of the photoprocess has been made under these conditions

The reaction may be expressed by the equation,

$$2FeCl_3+C_6H_5CH(OH)COOH = 2FeCl_2+2HCl+CO_2+C_6H_5CHO$$

Section A—deals with the photoreduction of ferric chloride by mandelic acid in unpolarised light of wavelengths 366 and 436$\mu\mu$

Section B—deals with the photoreduction of ferric chloride by mandelic acid in polarised light of wavelength 436$\mu\mu$

Experimental

The experimental arrangement was the same as was described by the authors in Part I of this series with the following alteration —

(*a*) The reaction cell was 4 cm × 4 cm × 1 cm thick and made of plane glass plates fused into one another with a stopper at the top for measurements in unpolarised light

(*b*) For measurements in polarised light, a circular corex glass cell of 1 cm thickness and 2 5 c c capacity was used

Reagents

Kahlbaum's extra-pure ferric chloride, Merck's extra-pure mandelic acid, thorium nitrate, potassium phosphate (KH_2PO_4), potassium iodide, hydrochloric acid and sodium thiosulphate and sodium molybdate supplied by B D H were used For making solutions bi-distilled water was used

Preparation of thorium phosphate sol and gel

Thorium phosphate sol and gel were prepared according to the method of Prakash and Dhar by mixing 0 25 c c of a solution of potassium phosphate (22 0%) with 5 c c of a solution of thorium nitrate (48 14 g /litre), making the total volume in all cases to 6 c c The mixture on shaking for 3 minutes and allowing to stand for 5 minutes gave a transparent, colourless and viscous sol which set to a firm gel after 4 hours

Preparation of thorium molybdate sol and gel

Thorium molybdate sol and gel were prepared according to the method of Prakash and Dhar (*loc cit*) modified to some extent by Mata Prasad *et al* 5 c c of a solution of thorium nitrate (48 14 g /litre) were taken in one test tube and in another test tube were taken 1 5 c c of a solution of sodium molybdate (3%), 1 5 c c water and 2 c c of HCl (0 1N) The solution contained in one test tube was poured into the other when a white precipitate was formed, which on shaking for 5 minutes, dissolved giving a transparent and colourless sol, which set to a firm gel after about 1 hour

To study the reaction in the gel phase, the reactants were mixed in the corresponding sol and allowed to stand in the dark until the reaction mixture set to a firm and transparent gel The set reaction mixture was then exposed to monochromatic light

To study the reaction in the sol phase, the reactants were mixed in the corresponding sol and exposed just after mixing to monochromatic light The reaction was stopped within 60 minutes during which the sol did not set to a gel In order to prevent the hydrolysis of ferric chloride, a certain amount of hydrochloric acid was added to the solution of ferric chloride When the reaction mixtures were made in thorium molybdate sol or gel, a large number of air bubbles were always found entrapped in the viscous liquid Since these bubbles did not disappear spontaneously, they were removed before pipetting, by the application of vacuum A clear reaction mixture was then obtained

Measurement of the velocity of reaction

Thorium phosphate as well as thorium molybdate gels, though thixotropic, liquefy to a very viscous liquid on shaking vigorously and so it was found very difficult to pipette out the exposed reaction mixture at definite intervals For this reason 2 c c of the reaction mixture were exposed in the reaction cell each time and the whole amount was taken out in a stoppered conical flask after a definite period and the ferric chloride was estimated iodometrically by titration, in an atmosphere of CO_2, with standard thiosulphate solution by means of a micro-burette. The initial concentration of ferric chloride was determined in the same way

The *p*H of the reaction mixture was determined potentiometrically by using glass electrode The *p*H of different mixtures were kept constant by required quantities of HCl or KOH The *p*H of the reaction mixture was varied by adding HCl

Measurement of intensity

The intensity of radiation absorbed by the reaction mixture was measured in the same way as was described in Part I of the series The intensity of absorbed radiation was measured by noting the deflections when the light passed through (*a*) pure solvent, *i e* water or pure sol or pure gel, and (*b*) the reaction mixture The difference in deflections in the two cases gave the intensity of radiation absorbed by the reaction mixture It is to be pointed out here that thorium phosphate and thorium molybdate sols and gels have got no absorption in 366 or 436$\mu\mu$ There is no difference in the absorption of the sol before and after gelation

It was found that the intense yellow colour produced by mixing mandelic acid with ferric chloride solution in water became pale yellow in media of thorium phosphate as well as thorium molybdate sols and gels The extinction coefficients of ferric chloride were measured in different media by means of intensity measurements, keeping the concentration of mandelic acid greater than that of ferric chloride and the ratio $\frac{(\text{Mandelic acid})}{(\text{FeCl}_3)}$ constant The extinction coefficients at different wavelengths and in different media were found in the following way the deflections in the galvanometer were noted, first of all, with the solvent alone and secondly with ferric chloride and mandelic acid mixtures of known concentrations The molecular extinction coefficients of ferric chloride were then calculated according to the equation,

$$\epsilon = \frac{1}{c\,d} \log_{10} \frac{I_0}{I_t}$$

where ϵ = molecular extinction coefficient,

c = concentration of ferric chloride in gm mol per litre,

d = thickness of the reaction cell in cm ,

and I_0, I_t are the incident and transmitted radiations measured The extinction coefficients of ferric chloride were also measured in presence of varying quantities of thorium nitrate The results are tabulated in tables I and II

TABLE I

(λ)	*phase*	ϵ
366	Aqueous	1629 0
,,	Thorium phosphate sol	106 2
,,	Thorium phosphate gel	106 2
,,	Thorium molybdate sol	217 6
,,	Thorium molybdate gel	217 6
436	Aqueous	610 3
,,	Thorium phosphate sol	47 6
,,	Thorium phosphate gel	46 2
,,	Thorium molybdate sol	52 1
,,	Thorium molybdate gel	52 1

TABLE II

$\lambda = 366\mu\mu$

Concentration of ferric chloride = 1 94 × 10^{-4}M

,, mandelic acid = 2 24 × 10^{-4}M

(Thorium nitrate) 10^4	0	2 3	6 9	13 8	20 3	33 2
ϵ	1629 0	1173 0	739·0	565 7	438 5	366 1

From table II we can see that even a small amount of thorium nitrate lowers the extinction coefficient to a great extent

The reactions which do not take place in the dark were carried out at 25°C The experimental data, are recorded in tables III to XV The reaction was found to be zero-molecular with respect to ferric chloride In the given tables, in media of thorium phosphate sol and gel, $\frac{\Delta x}{\Delta t}$ = zero-molecular velocity constant = changes in concentration of ferric chloride in 2 c c of reaction mixture per minute in terms of c c of 0 0029N thiosulphate and in media of thorium molybdate sol and gel, $\frac{\Delta x}{\Delta t}$ = changes in concentration of ferric chloride in 2 c c of reaction mixture per minute in terms of c c of 0 0017N thiosulphate

In the tables, θ = temperature, I_{abs} = number of quanta absorbed per c c. per sec a = Initial concentration of ferric chloride in gm mol per litre and b = Initial concentration of mandelic acid in gm mol per litre, γ = Quantum efficiency

SECTION A

Determination of the order of the reaction

TABLE III

Medium = Thorium phosphate gel, $\lambda = 436\mu\mu$, $\theta = 25°C$, $a = 6\ 23 \times 10^{-3}M$, $b = 7\ 4 \times 10^{-3}M$, $I_{abs} = 300\ 7 \times 10^{13}$, $pH = 1\ 74$

Time	0 0029N—thio for 2 c c reaction mixture	$\frac{\Delta x}{\Delta t}\ 10^3$
(I) 0	4 30	7 67 from (I) and (II)
(II) 60	3 84	7 50 from (II) and (III)
(III) 120	3 39	
		7 59 (mean)

TABLE IV

Medium = Thorium molybdate gel $\lambda = 366\mu\mu$ $\theta = 25°C$, $a = 6\ 1 \times 10^{-3}M$, $b = 7\ 4 \times 10^{-3}M$, $I_{abs} = 107\ 0 \times 10^{13}$ $pH = 1\ 42$

Time	0 0017N—thio for 2 c c reaction mixture	$\frac{\Delta x}{\Delta t}\ 10^{13}$
(I) 0	7 30	9 3 from (I) and (II)
(II) 30	7 02	9 0 from (II) and (III)
(III) 60	6 75	
(IV) 90	6 47	9 3 from (III) and (IV)
		9 2 (mean)

Effect of varying the concentration of ferric chloride

TABLE V

Medium = Thorium phosphate gel, $\theta = 25°C$ $pH = 1\ 74$, Thiosulphate = 0 0029N

λ ($\mu\mu$)	$a\ 10^3$ (mol)	$b\ 10^3$ (mol)	$I_{abs}\ 10^{-13}$	$\frac{\Delta x}{\Delta t}\ 10^8$	$\frac{\Delta x}{\Delta t}\ 10^{16}/I_{abs}$	γ
366	9 33	7 4	128 5	5 2	4 0	0 06
,,	6 22	,,	120 9	4 7	3 9	0 06
	3 11		76 3	3 0	3 9	0 06
436	12 44	7 4	386 8	9 0	2 3	0 03
,,	9 33	,,	355 0	8 4	2 4	0 04
,	6 22	,,	300 7	7 6	2 5	0 04
,,	3 11	,,	206 2	4 6	2 2	0 03

TABLE VI

Medium = Thorium molybdate gel, $\theta = 25°C$, $pH = 1\ 42$, Thiosulphate = 0 0017N

λ $(\mu\mu)$	$a\ 10^3$ (mol)	$b\ 10^3$ (mol)	$I_{abs}\ 10^{-13}$	$\frac{\Delta x}{\Delta t}\ 10^3$	$\frac{\Delta x}{\Delta t}\ 10^{18}/I_{abs}$	γ
366	6 1	7 4	107 0	9 2	8 6	0 07
,,	3 05	,,	90 2	7 8	8 6	0 07
,,	1 53	,,	54 1	4 6	8 4	0·07
436	6 1	7 4	153 5	11 0	7 2	0 06
,	3 05	,,	138 1	10 0	7 2	0 06
,,	1 53	,,	97 0	7 0	7 2	0 06

Effect of varying the concentration of mandelic acid

TABLE VII

Medium = Thorium phosphate gel, $\theta = 25°C$, $pH = 1\ 74$, Thiosulphate = 0 0029N

λ $(\mu\mu)$	$a\ 10^3$ (mol)	$b\ 10^3$ (mol)	$I_{abs}\ 10^{-13}$	$\frac{\Delta x}{\Delta t}\ 10^3$	$\frac{\Delta x}{\Delta t}\ 10^{18}/I_{abs}$	γ
366	6 22	14 8	122 2	5 3	4 4	0 07
,,	,,	7 4	120 9	4 7	3 9	0 06
,	,,	3 7	110 1	3 7	3 4	0 05
436	6 23	14 8	343 7	9 5	2 8	0 04
,,	,,	11 1	339 3	9 0	2 6	0 04
,,	,,	7 4	300 7	7 6	2 5	0 04
,,	,,	3 7	231 9	4 0	1 7	0 03

TABLE VIII

Medium = Thorium molybdate gel, $\theta = 25°C$, $pH = 1\ 42$, Thiosulphate = 0 0017N

λ $(\mu\mu)$	$a\ 10^3$ (mol)	$b\ 10^3$ (mol)	$I_{abs}\ 10^{-13}$	$\frac{\Delta x}{\Delta t}\ 10^3$	$\frac{\Delta x}{\Delta t}\ 10^{18}/I_{abs}$	γ
366	6 1	7 4	107 0	9 2	8 6	0 07
,,	,,	3 7	,,	8 6	8 0	0 07
,,	,,	1 85	,,	7 3	6 8	0 06
436	3 05	7 4	138 0	10 0	7 2	0 06
,,	,,	3 7	110 0	7 9	7 1	0 06
,,	,,	1 85	49 0	3 5	7 1	0 06
,,	1 53	5 55	77 0	5 5	7 1	0 06
,,	,,	3 7	48 2	3 3	7 0	0 06
,,	,,	1 85	28 0	2 0	7 0	0 06

Effect of varying the intensity of absorbed radiation

TABLE IX

Medium = Thorium phosphate gel, $\theta = 25°C$, $pH = 1\ 74$, Thiosulphate = 0 0029N

λ $(\mu\mu)$	$a\ 10^3$ (mol)	$b\ 10^3$ (mol)	$I_{abs}\ 10^{-13}$	$\frac{\Delta x}{\Delta t}\ 10^3$	$\frac{\Delta x}{\Delta t}\ 10^{18}/I_{abs}$	γ
366	6 22	7 4	120 9	4 7	3 9	0 06
,,	,,	,,	76 3	3 0	3 9	0 06
436	6 22	7 4	300 7	7 6	2 5	0 04
,,	,,	,,	186 1	5 0	2 7	0 04

TABLE X

Medium – Thorium molybdate gel, $\theta = 25°C$, $pH = 1.42$, Thiosulphate = 0.0017N

λ ($\mu\mu$)	$a\ 10^3$ (mol)	$b\ 10^3$ (mol)	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^8$	$\frac{\Delta x}{\Delta t}\ 10^{18}/I_{abs}$	γ
366	6.1	3.7	240.1	19.0	7.9	0.07
	,,	,,	100.0	8.0	8.0	0.07
436	1.55	5.55	56.0	4.0	7.1	0.06
,,	,,	,,	36.0	2.6	7.2	0.06
,,	,,	3.7	36.0	2.6	7.2	0.06
,,	,,	,,	18.6	1.33	7.1	0.06

Effect of varying pH

TABLE XI

Medium - Thorium phosphate gel, $\theta = 25°C$, Thiosulphate = 0.0029N

λ ($\mu\mu$)	$a\ 10^3$ (mol)	$b\ 10^3$ (mol)	pH	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^8$	$\frac{\Delta x}{\Delta t}\ 10^{18}/I_{abs}$	γ
366	6.22	7.4	1.74	120.9	4.7	3.9	0.06
,,	,,	,,	1.43	,,	4.7	3.9	0.06
436	6.22	7.4	1.74	300.7	7.6	2.5	0.04
,,	,,	,,	1.43	,,	7.5	2.5	0.04

TABLE XII

Medium = Thorium molybdate gel, θ 25°C, Thiosulphate = 0.0017N

λ ($\mu\mu$)	$a\ 10^3$ (mol)	$b\ 10^3$ (mol)	pH	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^8$	$\frac{\Delta x}{\Delta t}\ 10^{18}/I_{abs}$	γ
366	6.1	3.7	1.80	118.2	9.7	8.2	0.07
,,	,,	,,	1.48	,,	9.5	8.0	0.07
436	1.55	5.55	1.42	56.0	4.0	7.1	0.06
,,	,,	,,	1.18	56.0	4.0	7.1	0.06

Effect of the nature of medium.

TABLE XIII

$\theta = 25°C$, $pH = 1.74$, $a\ 10^3 = 6.22M$, $b\ 10^3 = 7.4M$, Thiosulphate = 0.0029N

λ ($\mu\mu$)	Medium	$I_{abs}\ 10^{-18}$	$\frac{\Delta x}{\Delta t}\ 10^8$	$\frac{\Delta x}{\Delta t}\ 10^{18}/I_{abs}$	γ
366	Aqueous	131.0	9.5	7.3	0.11
,,	Thorium phosphate sol	120.9	4.7	3.9	0.06
,,	Thorium phosphate gel	120.9	4.7	3.9	0.06
436	Aqueous	595.7	25.7	4.3	0.06
,,	Thorium phosphate sol	300.7	7.6	2.5	0.04
,,	Thorium phosphate gel	300.7	7.6	2.5	0.04

TABLE XIV

$\theta = 25°C$, $pH = 1·42$, $a\,10^3 = 6·1M$, Thiosulphate = 0·0017N

λ ($\mu\mu$)	$b\,10^3$ (mol.)	Medium	$I_{abs}\,10^{-18}$	$\frac{\Delta x}{\Delta t}\,10^3$	$\frac{\Delta x}{\Delta t}\,10^{18}/I_{abs}$	γ
366	3·7	Aqueous	142·6	15·2	10·7	0·09
,,	,,	Thorium molybdate sol	100·0	8·0	8·0	0·07
,,	,,	Thorium molybdate gel	100·0	8·0	8·0	0·07
436	7·4	Aqueous	394·6	29·0	7·3	0·09
,,	,,	Thorium molybdate sol	335·0	24·0	7·2	0·06
,,	,,	Thorium molybdate gel	335·0	24·0	7·2	0·06

SECTION B

Ghosh and his collaborators have made an extensive study of a large number of chemical reactions on the surface of certain inorganic micro-heterogeneous photocatalysts under the influence of light in various states of polarisation. In their experiments, they have observed in certain cases a differential reaction velocity with *d*- and *l*- circularly polarised light of equal amplitudes.

It appeared interesting to investigate the effect on the photoreduction of ferric chloride by mandelic acid in thixotropic thorium molybdate gel as a solvent medium.

Experimental

The apparatus and the experimental procedure were the same as in Section A with the following alterations :—

The polarising apparatus was placed between the ultraviolet filter and the reaction cell. The polarising apparatus consists of a nicol prism and a glass Rhomb. For plane polarised light, the nicol prism was used and for the circularly polarised light, the nicol prism and the Rhomb were used in conjunction. The description and working principle of the polarising apparatus have been discussed by Ghosh, Banerjee and Mukherjee.

The experimental results are recorded in table XV.

TABLE XV

$\lambda = 436\mu\mu$, $\theta = 25°C$, $a\,10^3 = 6·1M$, $b\,10^3 = 7·4M$, $pH = 1·42$

Nature of light	$I_{abs}\,10^{-18}$	$\frac{\Delta x}{\Delta t}\,10^3$	$\frac{\Delta x}{\Delta t}.10^{18}/I_{abs}$
Unpolarised	334·9	24·0	7·2
Plane polarised—			
(*a*) Axis of vibration—vertical	27·9	2·0	7·2
(*b*) Axis of vibration—horizontal	27·9	2·1	7·5
Circularly polarised—			
(*a*) *d*-circularly	21·0	1·5	7·1
(*b*) *l* circularly	21·0	1·5	7·1

DISCUSSION.

The reaction has the following similar characteristics in both thorium phosphate and thorium molybdate gels as solvent media :—

(1) The reaction is zero-molecular with respect to ferric chloride.

(2) The zero-molecular velocity constant increases with increasing concentration of mandelic acid, the intensity of absorbed radiation being also

increased In fact, $\frac{\Delta x}{\Delta t} 10^{18}/I_{abs}$ increases slightly with increasing concentration of mandelic acid

(3) The velocity constant increases with increasing concentration of ferric chloride In fact, $\frac{\Delta x}{\Delta t} 10^{18}/I_{abs}$ remains always constant.

(4) The velocity constant is directly proportional to the intensity of absorbed radiation In fact, $\frac{\Delta x}{\Delta t} 10^{18}/I_{abs}$ remains always constant for a particular wavelength but increases with increase in the magnitude of the quanta absorbed

(5) The velocity constant is practically independent of pH

(6) The rate of reaction remains the same in media of both sol and gel but it is greater in aqueous media In thorium phosphate sol and gel as media, $\frac{\Delta x}{\Delta t} 10^{18}/I_{abs}$ remains the same but in water it is much greater In thorium molybdate sol and gel as media, $\frac{\Delta x}{\Delta t} 10^{18}/I_{abs}$ has got the same value as in aqueous medium

(7) The velocity constant remains the same in polarised light having axes of vibration vertical as well as horizontal

(8) The velocity constant remains the same in both *d* and 1- circularly polarised light of equal amplitudes

(9) The quantum efficiency is much less than unity

On the observations made by Kistler that the dielectric constants of thixotropic sols remain the same before and after gelation and also on the observations made by Heyman that there is no change in volume of transparent thixotropic sols after gelation so that the average distance between the constituent particles does not alter, we can explain the reason for the same rate of reaction in media of thixotropic sol, before and after gelation, by assuming that the activated ferric ion deactivates to the same extent in media of both sol and gel

The same rate of reaction in media of thixotropic sol, before and after gelation, means that the reaction can proceed equally fast whether the water of the medium is *free* or *bound*

The quicker reaction in water may be due to a complex formation between mandelic acid and ferric chloride as is evidenced by the deep yellow colour of the mixture

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A THEOREM IN ANALYTIC NUMBER THEORY

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(Communicated by Sir S S Bhatnagar, F R S)

(*Received November 8, read November 22, 1946*)

§ 1 Let p denote a prime such that p^t is a factor of n, while p^{t+1} is not, we then say that 'n contains the prime p to the power t' Using a recent theorem of Selberg (*Skr Norske Vid Akad* , Oslo, I, No 5, 49 pp , 1942) I have proved the

Theorem 1 *Let k and r be given positive integers Then 'almost all' positive integers contain at least r different primes $\equiv -1$ (mod k), each to an odd power*

From this and well-known congruence properties of Ramanujan's function $\tau(n)$ we derive, without difficulty,

Theorem 2 *Let $\theta_1, \theta_2, \ldots, \theta_6$ be arbitrary positive integers, then the congruence*

$$\tau(n) \equiv 0 \pmod{2^{\theta_1} 3^{\theta_2} 5^{\theta_3} 7^{\theta_4} 23^{\theta_5} 691^{\theta_6}}$$

is true for almost all n

In Theorems 1 and 2, the words 'almost all' carry the usual sense in the analytic theory of numbers The proof of Theorem 1 is based on the case $r = 1$, which is well-known (see Hardy's *Ramanujan*, Cambridge, 1940, p 168) In the general case I have not been able to accomplish the proof without using the rather difficult results of Selberg

This note contains the proof for the case $r = 2$ Let $\sigma(x)$ denote the number of positive integers n not exceeding x, such that every prime factor of n which is $\equiv -1$ (mod k) is contained in n to an even power Then we have (Hardy's *Ramanujan*, p 168)

$$\sigma(x) = O\left(\frac{x}{\log^c x}\right) \tag{1}$$

where $0 < c < 1$ and $c = c(k)$

It is easy to see that the sum

$$S(x) = \sum \sigma\left(\frac{x}{p}\right) \tag{2}$$

where p is subject to (3) and (4) below

$$p \text{ prime}, \; p \leq x \tag{3}$$

$$p \equiv -1 \pmod{k} \tag{4}$$

represents the number of numbers n not exceeding x and such that n contains exactly one prime factor $\equiv -1$ (mod k) to an odd power [such n, may, naturally, contain any number of primes $\equiv -1$ (mod k) to an even power].

We split $S(x)$ into 2 parts, thus

$$S(x) = S_1(x) + S_2(x) \tag{5}$$

Here

(6) $$S_1(x) = \sum \sigma\left(\frac{x}{p}\right)$$

where p is subject to

α $$p \text{ prime}, p \equiv -1 \pmod{k}$$

β $$p \leqq \frac{x}{e^{\sqrt{\log x}}},$$

and

(7) $$S_2(x) = \sum \sigma\left(\frac{x}{p}\right)$$

where p is subject to

γ $$p \text{ prime}, p \equiv -1 \pmod{k}$$

δ $$\frac{x}{e^{\sqrt{\log x}}} < p \leqq x.$$

We now estimate $S_1(x)$ From (1) we have

$$S_1(x) = O\left(\frac{x}{\log^c\left(e^{\sqrt{\log x}}\right)}\right) \sum_{p \leqq x} \frac{1}{p}$$

where p runs through primes Hence

(8) $$S_1(x) = O\left(\frac{x \log \log x}{\log^{\frac{c}{2}}(x)}\right) = O(x)$$

Again using the crude inequality $\sigma(x) \leqq x$ to estimate $S_2(x)$ we get ·

(9) $$S_2(x) = O\Big(\sum_{y \leqq p \leqq x} \frac{x}{p}\Big)$$

where

(10) $$y = \frac{x}{e^{\sqrt{\log x}}}$$

and p runs through primes in (9)

To estimate (9) we use the classical result (in Prime Number Theory)

(11) $$\sum_{p \leqq x} \frac{1}{p} = \log \log x + B + O\left(\frac{1}{\log x}\right)$$

where p runs through primes From (10) and (11) we get

$$\sum_{y < p \leqq x} \frac{1}{p} = -\log\left(\frac{\log y}{\log x}\right) + O\left(\frac{1}{\log y}\right)$$

$$= -\log\left(1 - \frac{1}{\sqrt{\log x}}\right) + O\left(\frac{1}{\log x}\right)$$

(12) $$= O\left(\frac{1}{\sqrt{\log x}}\right)$$

From (9) and (12) we get

$$S_2(x) = O\left(\frac{x}{\sqrt{\log x}}\right) \tag{13}$$

From (5), (8), (13) we have finally

$$S(x) = O(x) \tag{14}$$

From the definition of $S(x)$ it now follows that almost all positive integers $n \leq x$ have the property that n contains *at least two* prime factors $\equiv -1 \pmod{k}$ to an odd power

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Walfisz, A (1938) 'Zur additiven Zahlentheorie' *Travaux de L'Institut Mathematique de Tbilissi*, 5, 145–152

ON NUMBERS WHICH CAN BE EXPRESSED AS A SUM OF TWO SQUARES

By R P BAMBAH *and* S CHOWLA

(Communicated by Sir S S Bhatnagar, F R S)

(*Received November 8, read November 22, 1946*)

§1 Denote by $b_1\ (=1),\ b_2,\ b_3,\ b_4,$ the numbers, arranged in ascending order of magnitude, which can be expressed as a sum of two squares (of integers) It is known that (Landau 1)

$$\sum_{b_n \leqq x} 1 \sim \frac{Cx}{\sqrt{\log x}}$$

where C is a positive constant In connection with the theory of lattice points in a circle it is known that

$$\sum_{\substack{u,\, v \\ u^2+v^2 \leqq x \\ u,\, v \gtreqless 0}} 1 = \pi x + P(x)$$

where

(1) $$P(x) = O(x^{\frac{27}{82}})\,;$$

and it has been conjectured that, for every positive ϵ,

(2) $$P(x) = O(x^{\frac{1}{4}+\epsilon})$$

We are concerned in this paper with the problem of the magnitude of the difference $b_{n+1} - b_n$ More precisely we wish to seek a function $f(x)$ such that between

$$x \text{ and } x + f(x)$$

there is at least one number expressible as a sum of 2 squares for all large x

From (1) it follows at once that

$$f(x) = O(x^{\frac{27}{82}}),$$

if (2) is true we would get

$$f(x) = O(x^{\frac{1}{4}+\epsilon})$$

for every positive ϵ

We prove in this paper by a simple argument that (see the more precise result at the end of §2)

(3) $$f(x) = O(x^{\frac{1}{4}}).$$

It has been conjectured that if $(a, b) = 1$ there is (see Chowla 2) at least one prime $\equiv a \pmod{b}$ between x and $x+x^{\epsilon}$ when $x > x_0 = x_0(\epsilon, a, b)$ If this is true then taking $a = 1$, $b = 4$, it would follow that

$$f(x) = x^{\epsilon}$$

for any fixed positive ϵ This shows that (3) is still very far from the probable truth It would also be of interest to know whether (3) can be improved by elementary arguments

We have to acknowledge here that the results (3) was found some years ago* by Dr T Vijayaraghavan by an argument not quite as simple as the one we give This paper has its roots in this letter of Dr Vijayaraghavan

§2. In this section all letters denote positive real numbers

Let ϵ be an arbitrary positive number and $x > x_0(\epsilon)$ Let $[g]$ denote the greatest integer contained in g Write

(4) $$t = [\sqrt{x}] = \sqrt{x} - \theta$$

where

(5) $$0 \leqq \theta < 1$$

Let (here x_1, x_2 are not necessarily integers)

$$x_1^2 + t^2 = x$$

$$x_2^2 + t^2 = x + 2\sqrt{2+\epsilon}\, x^{\frac{1}{4}}$$

Then

$$x_2^2 - x_1^2 = 2\sqrt{2+\epsilon}\, x^{\frac{1}{4}}$$

(6) $$x_2 - x_1 = \frac{2\sqrt{2+\epsilon}\, x^{\frac{1}{4}}}{x_1 + x_2}$$

now

$$x_1 = \sqrt{x - t^2} = \sqrt{x - (\sqrt{x} - \theta)^2}$$

$$= \sqrt{2\,\theta\sqrt{x} - \theta^2} < \sqrt{2}\sqrt[4]{x}$$

similarly for $x > x_0(\epsilon)$,

$$x_2 = \sqrt{x + 2\,(\sqrt{2+\epsilon})\, x_v^{\frac{1}{4}} - (\sqrt{x} - \theta)^2}$$

$$\leqq \sqrt{2 + \frac{\epsilon}{10}}\, x^{\frac{1}{4}}.$$

Hence, for $x > x_0(\epsilon)$,

(7) $$x_1 + x_2 < 2\sqrt{2 + \frac{\epsilon}{10}}\, x^{\frac{1}{4}}$$

From (6) and (7),

$$x_2 - x_1 > \sqrt{\frac{2+\epsilon}{2 + \frac{\epsilon}{10}}} > 1$$

Hence there exists an *integer* x_3 between x_1 and x_2

---

* In a letter addressed to one of us (S C), but unfortunately mislaid

Hence

$$t^2+x_1^2<t^2+x_3^2<t^2+x_2^2$$

so that

$$x<t^2+x_3^2<x+2\sqrt{2+\epsilon}\,x^{\frac{1}{4}}$$

(where t and x_3 are integers). Thus we have the

Theorem Let ϵ denote an arbitrary positive number Then there exists between x and $x+2\sqrt{2+\epsilon}\,x^{\frac{1}{4}}$ an integer which can be expressed as a sum of two squares (of integers) for all $x>x_0(\epsilon)$

§3 REFERENCES

Chowla, S (1935) 'On the representation of a large number as a sum of light almost equal cubes' *Quart Journ of Maths* (Oxford), **6**, 146–148

Landau (Handbuch), Band 2

11 MAY 1948

PROCEEDINGS

OF THE

NATIONAL INSTITUTE OF SCIENCES OF INDIA

MARCH, 1947

No. 3] VOL. XIII [Pp. 105–139

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PUBLISHED BY
THE NATIONAL INSTITUTE OF SCIENCES OF INDIA
UNIVERSITY BUILDINGS, DELHI

Price Rs 1 14 0

P 48

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ON *PISIONE COMPLEXA*, N SP FROM THE SANDY BEACH, MADRAS[1]

By K H Alikunhi, *M Sc*

(*From the University Zoological Research Laboratory, Madras*)

(Communicated by Dr S L Hora, F N I)

(*Received October 21, read November 23, 1946*)

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Introductory and Historical

Collections from coarse sand near low water level of the Madras beach revealed the abundant occurrence therein of a Pisionid which proved to be an undescribed species Two other Pisionids, *Pisionidens indica* (Aiyar and Alikunhi, 1940, 1943) and *Praegeria gopalai* (Alikunhi, 1941) have been previously recorded from the same area

Hartman (1939) dealing with the family Pisionidae, considers that since the type of the genus *Praegeria*—*P remota* Southern—resembles the type of the genus *Pisione*—*P oerstedi* Grube—the retention of the genus *Praegeria* is unnecessary *Praegeria remota* Southern becomes only a valid species of *Pisione*—*P remota* (Southern) Hartman further finds that *Pisione germanica* described by Augener (1924) from the North Sea is identical with *Pisione remota* (Southern) from Ireland Augener (*loc cit*) has already shown that *Pisione contracta* Ehlers, from Peru, is identical with *P oerstedi* Grube Hartman therefore concludes that 'two valid species are known to occur, *Pisione oerstedi* Grube from Peru, Ceylon and New Zealand, and *Pisione remota* (Southern) from Ireland and the North Sea' To this he adds a new genus *Pisionella* which differs from *Pisione* in the possession of a median cirriform antenna at the anterior margin of the prostomium, three pairs of cirriform cirri, and the longitudinal series of spinelets on the setae tips and in the absence of the acicular setae from the buccal segment Aiyar and Alikunhi (1940) created a third genus for an entirely different form under the name *Pisionella*, but owing to Hartman's genus having priority, have since changed it to *Pisionidens* (Aiyar and Alikunhi, 1943).

Hartman's paper was not accessible to me when my account of *Praegeria gopalai* was published in 1941 Since then I have had occasion to go through his paper and I am now convinced that it is unnecessary to retain the genus *Praegeria* *P gopalai* thus becomes *Pisione gopalai* (Alikunhi), and it forms the third valid species

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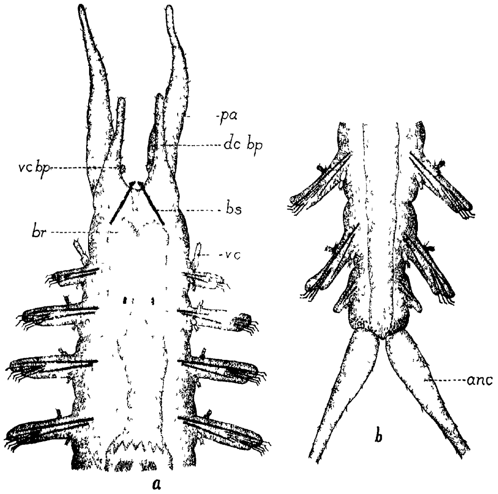
[1] Thesis, in part, accepted for the Degree, Master of Science, of the University of Madras

Linlithgow Library.

of the genus The present form also belongs to the genus *Pisione* and it possesses all the peculiar features characterising the family Pisionidae as illustrated in the accounts of *Pisionidens indica* and *Pisione gopalai* However, it shows a number of features peculiar to itself and in the following pages I propose to describe it as a new species under the name *Pisione complexa*

EXTERNAL CHARACTERS

The worms are comparatively large and measure about 10 to 25 mm in length in the mature condition The number of segments varies from 70 to 100 or even



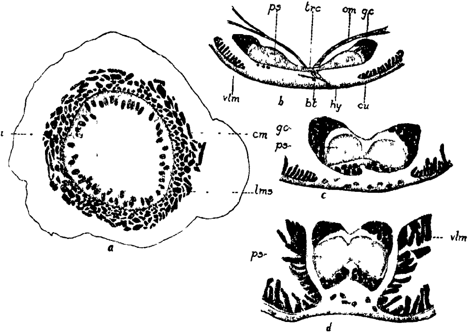
TEXT FIG 1 *Pisione complexa*, sp nov

(*a*) Anterior end of the worm showing the cephalic appendages, body slightly contracted, drawn from specimen in spirit ×120
(*b*) Posterior end of the worm, from life ×200

anc, anal cirrus, *br*, brain, *bs*, buccal spine, *dc bp*, dorsal cirrus of buccal parapodium, *pa*, palp, *vc*, ventral cirrus, *vc bp*, ventral cirrus of buccal parapodium more. The cephalic appendages are similar to those of *P. gopalai*. The ventral cirri of the first pair of parapodia are only slightly elongated (Fig. 1*a*). The buccal spines are devoid of any serrations at the tip and are shorter than those of *P. remota*. A pair of eyes are present attached to the brain. Parapodia are comparatively smaller than those of *P. gopalai*. The setigerous lobe is bifid at the tip where there are two papillae, the larger one of which becomes broader and foliaceous towards the posterior region. The setigerous support is identical with the same in *P. gopalai* and consists of two acicula and five setae in each foot, excepting in the first and the last four or five, in each of which there are only four setae. The body tapers considerably towards the posterior extremity. The anal segment is quite simple and resembles that of *P. remota* (Fig. 1*b*). It is a button-shaped structure to the posterior extremity of which is attached a pair of long anal cirri swollen at the base and gradually tapering to the tip. At the posterior extremity of the anal segment are aggregated a few coiled hypodermal glands the secretions of which serve for adhesion, like the secretion from the pygidial glands of *P. gopalai*.

BODY-WALL

Transverse sections are usually circular in outline but may be slightly compressed dorso-ventrally (Figs. 5 and 8). The epidermal layer is very thin, and the nuclei stain lightly. The circular muscle layer is inconspicuous and can be made out only



TEXT-FIG. 2. *Pisione complexa*, sp. nov.

(*a*) Transverse section of the stomach showing the musculature. ×267
(*b*) Nerve area—anterior region. ×400
(*c*) Nerve area—stomach region. ×400
(*d*) Nerve area—middle region. ×400

bt, basement tissue, ***cm***, circular muscle, ***cu***, cuticle, ***epi***, epithelium, ***gc***, ganglion cells, ***hy***, hypoderm, ***lms***, longitudinal muscle, ***om***, oblique muscle, ***ps***, punctated substance, ***trc***, transverse connective, ***vlm***, ventral longitudinal muscle

at the sides The longitudinal muscle bands form an almost complete wall to the body cavity They are better developed than in the other species and in the ventral bands the folded edges usually come very near each other, thereby approaching the condition in *Pisionidens indica* (Aiyar and Alikunhi, 1940) The nerve area is located between the folded inner edges of the ventral longitudinal muscle bands (Fig 2*b*–*d*) The coelomic membrane lining the body cavity is extremely thin Epidermal glands are mostly confined to the sides of the body

Coelomic corpuscles, oval, circular or disc-like, float freely in the coelomic fluid in the anterior segments

ALIMENTARY CANAL

The various regions of the alimentary canal are similar to those described in *P gopalai*, except for some minor differences such as the greater development, generally, of the musculature, and especially the presence of a layer of circular muscles internal to the longitudinal muscles of the stomach and intestine, and the larger size of the cells constituting the inner epithelium of the stomach (Fig 2*a*)

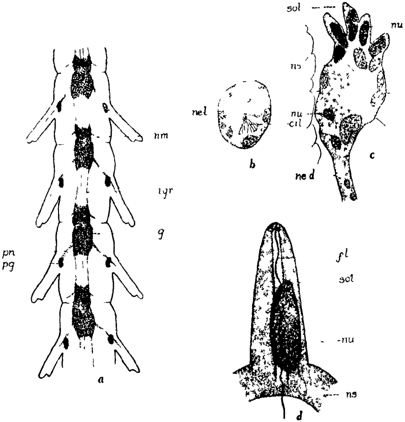
NERVOUS SYSTEM

The posterior lobes of the brain are short and extend only into the anterior part of the third setigerous segment (Fig 1*a*) The anteriorly directed dorsal cirri of the buccal parapodia are supplied by a pair of slender nerves which arise from the anterior part of the oesophageal commissures The origin of these nerves from the oesophageal commissures is evidence that these structures are modified parapodial cirri The commissures fuse together in the second setigerous segment, to form the ventral nerve cord In the anterior segments the two cords stand rather apart and at the ganglionic swellings there are distinct connectives in between (Fig 2*b*) In the front region the ventral longitudinal muscles being poorly developed, the nerve area is very broad. In these segments a part of the ganglionic swelling lies on the inner aspect of the muscle band, on each side (Fig 2*b*) Externally to the nerve cord occur the basement tissue, the thin hypoderm and the cuticle It is difficult to make out the circular muscles in this area The oblique muscles are powerfully developed and are inserted into the basement tissue below the nerve cords At the level of the stomach the nerve cords come closer together and the punctated substance in each half assumes a semi-circular outline (Fig 2*c*) The ventral longitudinal muscles are better developed here and in consequence the nerve area is deeper and narrower than in front This condition is maintained in the hinder regions of the body also

The nerve cord has lost the primitive condition of being in contact with the hypoderm In the absence of the circular muscle coat in the nerve area *P complexa* resembles the Glyceridae (*Glysera capitata*) wherein 'the great external circular muscular layer ceases before reaching the nerve area, so that externally the latter has only the hypoderm and the specially developed cuticle' (McIntosh, 1877) But in *G capitata* the ventral longitudinal muscles form an arch over the nerves, thereby differing from the condition in *P complexa*

As in *P gopalai*, the nature of the ganglionic swelling in the ventral nerve chain is peculiar Examined in the living condition, under the microscope, each segment shows a big ganglionic enlargement, the major part of which lying in the segment to which it belongs, while a small part extends into the segment in front This is clearly seen in whole mounts (Fig 3*a*) A pair of podial nerves arise from the main portion of the ganglionic swelling and each of these enlarges into a prominent ganglion situated at the base of the parapodial lobe and then proceeds further into the latter

A second pair of very slender nerves originate from the anterior portion of the ventral ganglia and innervate the muscles of the body-wall



TEXT FIG 3 *Pisione complexa*, sp nov

(*a*) Diagrammatic representation of four segments of the worm, from a whole mount stained in Delafield's haematoxylin, showing the ventral ganglia and their disposition
(*b*) Transverse section of the nephridial swelling × 1200
(*c*) Longitudinal section of the nephridial swelling and solenocytes × 1200
(*d*) Reconstructed diagram of a solenocyte

cil, cilium, *fl*, flagellum, *g*, ganglion, *igr*, interganglionic region, *ne d*, nephridial duct, *ne l*, nephridial lumen, *nm*, nerve to muscle, *ns*, nephridial swelling, *nu*, nucleus, *pg*, parapodial ganglion, *pn*, parapodial nerve, *sol*, solenocyte

EXCRETORY SYSTEM

Excretory organs in the form of paired nephridia are present in all the segments excepting the anterior five The nephridium ends internally in the form of a swelling projecting into the body cavity from the posterior corner of the segment In general structure it shows close resemblance to that of *P gopalai* The first two or three pairs of nephridia are larger with a larger number of solenocytes The nephridial swelling has a spacious cavity which is richly ciliated and is almost circular in cross-section (Fig 3*b*) The cells forming the nephridial swelling are large, though their boundaries cannot be clearly made out The nuclei stain but lightly

The solenocytes are situated on the anterior face of the nephridial swelling and have a crowded appearance (Fig 3c) Each solenocyte has a broad base and tapers gradually to the tip There is no distinction into a cell-body and a flagellum-carrying tube (Fig 3d) There is a narrow lumen and a long flagellum, attached to the wall at the distal extremity, works rapidly down the lumen The nucleus is elongated and has a baso-lateral position (Fig 3c) The nephridial duct at its commencement describes one or rarely two spirals before piercing the septum to open at the base of the ventral cirrus

Nothing is known of the nephridia of *Pisione oerstedi* and *Pisionella hancocki* In *Pisionidens indica* and *Pisione gopalai* the nephridial system has been shown to be very much like that described by Goodrich (1900) for a number of phyllodocids In *Pisione remota* (Southern) which also occur in the Madras beach—an account of the nephridia and the reproductive organs of which will form the subject matter of a separate communication—the nephridia are similar to those in other pisionids As the foregoing account shows, in *Pisione complexa* also there is that phyllodocid type of nephridium There is the same nephridial swelling from which solenocytes arise Only the solenocytes lack a flagellum-carrying tube distinct from the cell-body—a condition which is mentioned for the first time It will thus be seen that the affinities of the Pisionidae are altogether with the Phyllodocids, Nephthyds and the Glyderids, in the structure of their excretory organs

The nephridia undergo modifications in the genital segments and these will be described in detail along with the reproductive organs

REPRODUCTIVE SYSTEM

Male

Testis

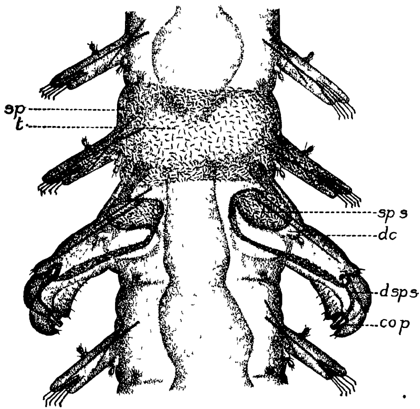
The males are usually smaller than the females The reproductive organs are highly localised and are invariably found to be developed after the 30th segment In the mature male the testes are developed in a varying number of separate segments The smallest mature specimen examined was one with 38 setigerous segments and this had a single pair of testes situated in the 32nd segment In a specimen with 55 setigerous segments testes were developed in segments 32 and 44, while another, 18 mm long and with 98 segments, had 6 pairs of testes occurring in segments 32, 44, 56, 61, 71 and 77 respectively In yet another with 100 setigerous segments only four pairs of testes were developed and these occurred in segments 35, 38, 58 and 82 respectively The arrangement therefore is not regular

The worms probably mature when they develop about 38 to 40 segments and then there is only a single pair of testes, situated in one of the posterior segments Growth in length continues by the addition of fresh segments at the posterior end and along with this at varying intervals fresh testes groups are also developed This, therefore, accounts for the occurrence of individuals with varying number of testes, sperm-sacs and copulatory organs in various stages of development The maximum number of testes developed depends upon the length of the worm, but of the numerous specimens examined none has been found to possess more than six pairs of testes

Each testis is invariably confined to a single segment which always precedes the one carrying the sperm-sacs (Fig 4) It originates as paired cell proliferations attached to the septum and has a thin outer limiting membrane, which is clearly visible in transverse sections In the later stages of maturity the testis becomes smaller and less conspicuous owing to the liberation of sperms

Sperm-sacs

Each testis is invariably followed in the next segment by a pair of sperm-sacs which become associated with a pair of copulatory organs (Fig 4) The nephridia in the testis-bearing segments are inconspicuous and a pair of genital funnels of about 4 to 5 times the size of the unmodified nephridial swelling becomes associated



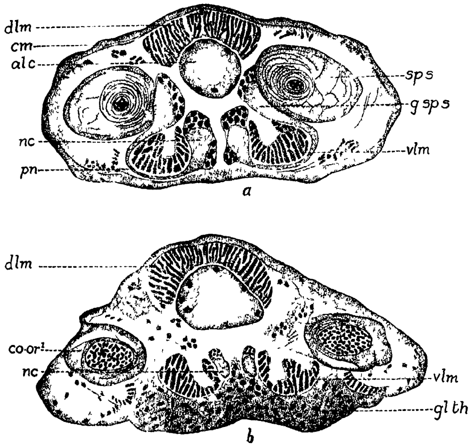
TEXT FIG 4 *Pisione complexa*, sp nov

Genital segments of a mature male showing the distribution of the reproductive organs × 200

co p, copulatory process, *dc*, dorsal cirrus, *d sp s*, duct of sperm sac, *sp*, sperm, *sp s*, sperm sac, *t*, testis

with them When fully formed these funnels are situated close to the body-wall, on the dorso-lateral aspect of the reduced nephridial swelling (Figs 4 and 6) They are deep spoon-shaped structures with ciliated margins The solenocytes of the nephridia of these segments get shorter and inconspicuous and can be distinguished under high magnification only by the characteristic downward lashing movement of their flagella The nephridial lumen is narrow and the genital funnel opens into it at the point where it pierces the septum There is no loop or coiling of the nephridial duct before its piercing the septum and in this respect also the nephridia of the testis-bearing segments differ from those of other segments Behind the septum, the nephridial duct runs down to a short distance and then sharply bends upwards and forwards to get enlarged into a narrow thin-walled sac (Figs 4 and 6) Proceeding further, it narrows and bends sideways and backwards and running almost straight down, enters the copulatory organ It is interesting to note that this second descending portion corresponds to the highly spacious, muscular, second dilatation in the sperm-sac of *P. gopalai* Therefore, in this form even though there is a well developed sheath of circular muscles surrounding the hind portion of the sperm-sac, the nephridial duct is not distended into a second saccular portion In living specimens the surrounding muscles are not very conspicuous The liberated sperms are thickly packed inside the saccular portion The interior of the sperm sac is powerfully ciliated The external duct on entering the copulatory organ enlarges imperceptibly, forms a loop about the middle of its course and is continued to its external opening situated at the tip of a papilla (Figs 4 and 6)

The parapodial nerves are stout and at the outer edge of the ventral longitudinal muscles they turn inwards and enlarge into a pair of ganglia which come to be situated close to the ventral wall of the sperm-sacs (Fig 5*a*) From each of these ganglia a stout nerve—corresponding to the nerve to the parapodial lobe—is given



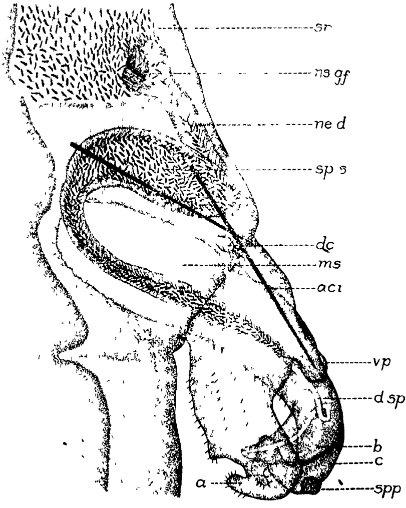
TEXT FIG 5 *Pisione complexa*, sp nov

(*a*) Transverse section of the sperm-sac segment of the male showing the nerves arising from the ventral nerve cord going to the sperm-sacs × 540

(*b*) Transverse section of the posterior part of the sperm sac segment showing the glandular pad on the ventral side × 540

al c, alimentary canal, *co or*,¹ proximal part of copulatory organ, *dlm*, dorsal longitudinal muscle, *gl th*, glandular thickening, *g sp s*, ganglion to sperm-sac, *n c*, nerve cord (Other letters as in previous figures) off which proceeds to the tip of the copulatory organ along the dorsal aspect of the sperm-sac The presence of this nerve throws light on the homology of the copulatory organ discussed in a subsequent section

In the sperm-sac bearing segment the longitudinal muscles are very much reduced The dorsal and ventral bands are strictly confined to the dorsal and ventral sides respectively (Fig 5*a*) The alimentary canal is pushed dorsad and is in the form of a narrow tube The available coelomic space is thereby increased and it is mainly occupied by the sperm-sacs Towards the hinder part of this segment a fairly thick glandular pad or thickening is formed ventrally (Fig 5*b*) The cells



TEXT FIG 6 *Pisione complexa*, sp nov

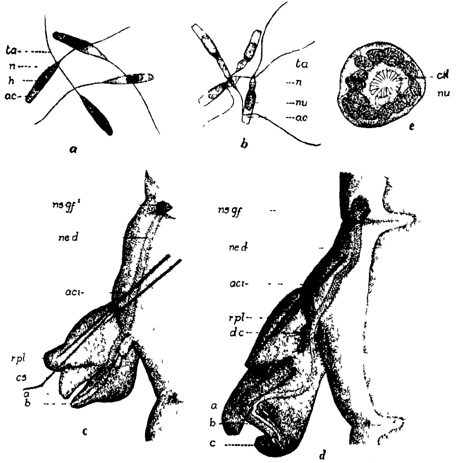
Magnified drawing of sperm sac and copulatory organ of one side ×400.

a b c, processes of copulatory apparatus; *aci*, aciculum; *ms*, muscular sheath; *ns gf*, nephridial swelling and genital funnel, *spp*, spinous papilla, *vp*, vestigeal papilla (Other letters as in previous figures.)

forming this thickening have darkly staining nuclei of various shapes It is probable that this glandular pad in *P complexa* serves at least to some extent the function of the ventral suckers in *Pisionidens indica* and are probably epidermal glands specially developed for purposes of adhesion during copulation

Copulatory Organs

The number of copulatory organs varies from 1 to 6 pairs, or even more depending upon the number of pairs of sperm-sacs and testes developed Each



TEXT FIG 7 *Pisione complexa*, sp nov

(*a*) Sperms from the male—fresh preparation ×1200
(*b*) Sperms from the male—from a smear stained in Delafield's haematoxylin ×1200
(*c*) Copulatory organ—early stage ×267
(*d*) Copulatory organ—later stage ×267
(*e*) Transverse section of the nephridial duct in the genital segment of a developing male ×1200

ac, acrosome, *cs*, compound seta, *h*, head, *n*, neck, *ns gf*,[1] nephridial swelling and rudiment of genital funnel, *r pl*, reduced parapodium, *ta*, tail (Other letters as in previous figures) copulatory organ is an elongated, non retractile structure, about twice the size of the normal parapodium In the segment carrying the copulatory organs the dorsal cirrus of the parapodium remains unmodified The main lobe of the parapodium has undergone extreme reduction and is represented only by a minute vestigial papilla (Fig 6) Its setigerous support consists of only two acicula, compound as well as simple setae being absent From the ventral aspect of the base of the vestigial papilla representing the parapodial lobe, is given off a broad foliacious structure ending in a curved papilla provided with palpocils (*a* in Fig 6) As will be shown later, this papilla represents a highly modified ventral cirrus, and probably forms part of the copulatory apparatus Such a structure is entirely lacking in *P gopalai* From the ventral aspect of this papilla arises a finger-like process which divides distally into two The ventral one of these processes is elongated and carries at its end a minute, retractile, spinous pad just behind the tip (*c* in Fig 6) This spinous structure corresponds to the muscular papilla so conspicuous in the copulatory organs of *P gopalai* The efferent duct of the sperm-sac runs through the dorsal process to open to the outside at the tip of a conical papilla (*b* in Fig 6)

Sperms

Sperms when liberated pass into the coelomic space of the testis segment They do not usually spread into the neighbouring segments To the naked eye the testis and sperm-sac bearing segments appear white The sperms are non-motile Each sperm is slender and elongated and measures about 40 to 45 microns in length, including the flagellum The acrosome is prominent but blunt and measures about 4 μ (Fig 7*a*) The nuclear portion is oval in outline, more refractile than the acrosome, and measures about 3 4 microns The neck portion following the head is longer, about 5 microns, narrow and tapering to the posterior extremity whence commences the long flagellum measuring about 28 to 30 microns

Higher magnification reveals that at the commencement of the neck there is a minute granule, probably representing the centriole

DEVELOPMENT OF SPERM-SACS AND COPULATORY ORGANS

The formation and differentiation of the adult sperm sacs and copulatory organs have been followed by examining a number of developing male specimens, in various stages of maturity Figures 7*c* to 7*e* illustrate some of the stages In the segment that is destined to develop the sperm-sacs in the adult, the parapodium is very much reduced An unmodified ventral cirrus is absent while the dorsal cirrus remains unchanged Two acicula and one compound seta support the reduced parapodial lobe From the ventral aspect of this lobe arises a conspicuous structure which is sub divided into two, the dorsal one of which, judging from its position, seems to correspond to the ventral cirrus of the unmodified parapodium (*a* in Fig 7*c*) The rudiments of the genital funnels are present in association with the nephridia, in the form of a few large cells Cilia are not yet developed The nephridial duct is considerably thicker than in other segments and running down almost straight, enters the base of the newly formed ventral structure and proceeding forwards opens at the tip of the ventral of the two processes into which it is divided (*b*, Fig 7*c*) Testis is developed in the preceding segment but is small and does not fill the com partment At this stage there is no trace of the sperm-sacs in the segment carrying the developing copulatory organs

In another specimen, slightly more advanced in development, the setigerous lobe in the segment carrying the developing copulatory organ was smaller and had lost the compound seta which was present in the previous stage The dorsal of the two processes—the modified ventral cirrus—mentioned in the previous stage is elongated (*a*, Fig 7*d*) The ventral process, now very much enlarged in size, splits into two by developing a broad, slightly curved structure ventrally (*c*, Fig 7*d*)

The papilla carrying the efferent duct of the nephridium has further elongated and is bent at right angles (*b*, Fig 7*d*) The rudiments of the genital funnels are larger and have developed a central lumen with a crown of vibratile cilia This lumen has attained communication with the nephridial duct as in the fully mature specimen The nephridial swelling is very much reduced The nephridial duct has a straight course in the body and there are no saccular expansions in its course Immediately behind the septum the nephridial duct is prominent and broad It has a thick wall, the constituent cells being highly protoplasmic The nuclei are large and closely situated and form an almost complete ring surrounding the central ciliated lumen (Fig 7*e*) In the region behind the duct gradually becomes narrow and thin-walled On entering the copulatory organ it has assumed a similar course as in the mature specimen The testis in the preceding segment has undergone further development and cell division and occupies the major portion of the segmental chamber

In the next stage the developing copulatory organ assumes the adult condition The nephridial duct behind the septum makes a bend and enlarges into a saccular portion as in the adult Behind this saccular portion a thick muscular covering is developed around the nephridial duct The testis is fully developed and a few sperms have been liberated into the body cavity The essential parts of the adult sperm sac have now been differentiated, and in the next stage which is the fully ripe condition, more sperms are liberated from the testis and are carried down the nephridial duct to be stored in the sperm-sac It is, therefore, quite clear that the adult sperm sac is formed by the differentiation of the simple straight nephridial duct The latter thus becomes the sperm-duct and in so functioning as a passage for the genital elements to the exterior, has undergone some transformatory changes, and in this feature the Pisionids are more specialised than most of the polychaetes

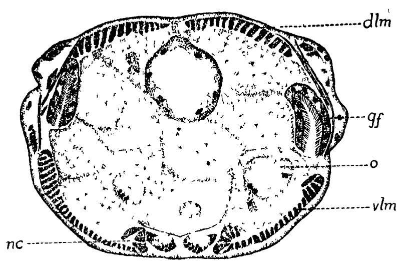
A comparison of the structure of the copulatory organ with that of *P gopalai* is instructive As has been described, the entire structure in *P gopalai* is comparatively simple, there being developed only a single papilla, carrying the efferent duct of the sperm sac, besides the reduced parapodial lobe which later gets completely suppressed leaving no trace behind In the corresponding stage in *P complexa* we get two additional processes making the structure more complicated Again, a compound seta is never present in the reduced parapodium of the segment carrying the copulatory organs in *P gopalai*, at least when the worm is attaining sexual maturity for the first time (see below)

The structure of the developing copulatory organ suggests its probable origin In the segment which is destined to become the genital segment the parapodium consists of a dorsal cirrus, a highly reduced main lobe, and the developing copulatory organ The ventral cirrus has undergone considerable modification Normally, in the non-genital segments the nephridial opening is situated below the ventral cirrus Now, judging from the position of the developing copulatory apparatus in relation to the parapodium, it seems probable that the finger-like process immediately ventral to the reduced main lobe of the parapodium corresponds to the ventral cirrus (*a* in Figs 6, 7*c* and 7*d*) The body-wall below this modified ventral cirrus, and probably partly from its base, undergoes a projection, lengthens out and splits into two processes, the dorsal one of which carries to its end the nephridial duct, while the ventral one undergoes further differentiation, becomes flattened and develops a pad of spinous processes We must, therefore, regard the copulatory organ proper as being formed almost entirely *de novo* from the side of the body-wall This, together with the adjoining highly modified ventral cirrus may be said to constitute the copulatory apparatus

Female

The ovaries are confined to the second-half of the body and usually 6 to 20 pairs of them are developed They are highly localised and definitely paired in origin.

Each pair is situated within the confines of a single segment Transverse sections reveal the presence of an extremely thin membrane covering the ovary In the ripe individual the segmental chamber is fully occupied by ova and distinction between the right and the left ovary is lost With the expansion of the ovarian mass the great longitudinal muscle bands are extremely reduced The hypodermal layer is very much attenuated, and the alimentary canal is considerably narrowed and pushed very much dorsad (Fig 8)



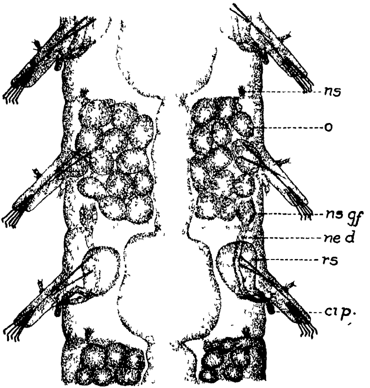
TEXT FIG 8 *Pisione complexa*, sp nov

Transverse section through the posterior region of an ova bearing segment of a ripe female × 540

gf, genital funnel, *o*, ovum

The reduction in the thickness of musculature and its partial atrophy is a common feature in the sexually mature forms of various polychaetes and such changes have been noted in considerable detail in several families (Fage and Legendre, 1927) In the present case, however, the musculature only undergoes reduction without any actual destruction Fage and Legendre (*loc cit*) and Caullery and Mesnil (1898) have given detailed accounts of the modifications undergone by the digestive tube in the genital segments of a number of polychaetes In certain forms in the sexually ripe individuals the alimentary canal is so pressed by the gonads that the inner sides of the intestine are applied one against the other, without leaving any space in between, with the result that the digestive tube is no longer functional and the animal cannot nourish itself during this period In such forms as *Eulalia punctifera* when the segments are relieved of the genital elements the alimentary canal assumes its normal condition and becomes functional again But there are others in which hystolysis takes place in the contracted region of the alimentary canal, which, in consequence, gets disintegrated and the animal does not survive oviposition Instances in point are met with in certain Phyllodocids, Glycerids, Cirratulids, etc In *P complexa* even though the digestive tube in the genital segments is greatly attenuated, there is no fusion of the intestinal walls and it is probable that the organ carries on its usual function, at least in a restricted sense, throughout the sexual period The worm survives oviposition and resembles *Eulalia punctifera* in that the digestive tube assumes its normal dimensions when the genital elements are shed

Each ovarian segment is invariably followed by another in which ova are never developed but in which a pair of receptacula seminis are situated (Fig 9) The ovaries and the receptacula seminis, in other words, have an alternating arrangement In a worm with 60 setigerous segments 8 pairs of ovaries were developed in segments 41, 43, 45, 47, 49, 51, 53 and 55 respectively, while in the intervening segments were situated the corresponding pairs of receptacula seminis The number and position of the receptacula seminis, therefore, depend upon the number and position of the ovarian groups In this localisation of a certain number of segments for the ovaries and an equal number solely for the receptacula seminis, *P complexa* differs from *P remota* but closely resembles *Pisionidens indica*, even though in the latter each ovarian group extends through more than one segment

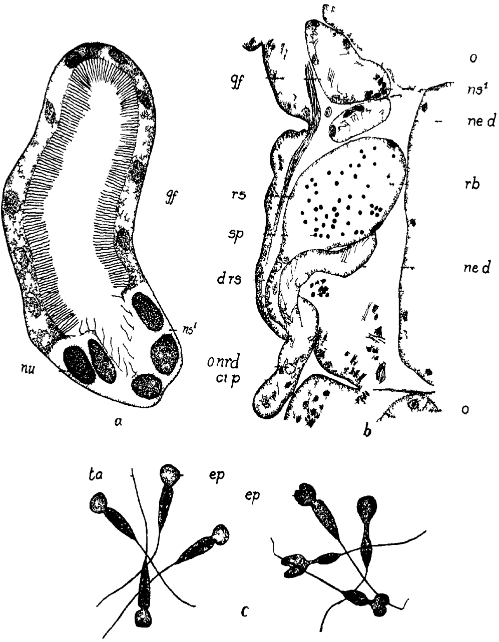


TEXT FIG 9 *Pisione complexa*, sp nov

Genital segments of a ripe female showing the arrangement of the various structures × 200

ci p, cirriform process, rs, receptaculum seminis

The parapodia of the segments in which the receptacula seminis are situated, are unmodified In ripe individuals from the ventral aspect of the base of the parapodium of the particular segment, arises an elongated papilla with the nephridio-receptacular aperture (see below) at its base (Figs 9 and 10*b*) This process is larger than either of the parapodial cirri It corresponds, in the female, to the highly developed copulatory organ proper of the male Such a structure is absent in *P gopalai* In *Pisionella hancocki* Hartman (1939) figures such a cirriform structure attached to the ventral basal aspect of the parapodial lobe



TEXT FIG 10 *Pisione complexa* sp nov

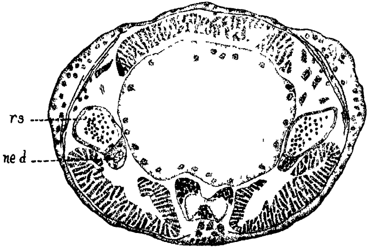
(*a*) Longitudinal section of the nephridium and the associated genital funnel of a ripe female ×1800
(*b*) Longitudinal (horizontal) section—female—showing the opening of the nephridial duct into the distal portion of the receptacular duct Note the cirriform structure at the base of which the duct opens ×600
(*c*) Sperms from the receptaculum seminis of the female ×1800

d rs duct of receptaculum seminis ***ep*** expanded portion ***ns*** ¹ reduced nephridial swelling ***o nrd*** nephridio receptacular opening; ***rb*** refractile bodies

A pair of well-developed genital funnels are present in each of the ova-bearing segments (Fig 9) These funnels in the fully mature condition are about twice the size of those in the males and become closely associated with the nephridia The relative position of the nephridium and the genital funnel is the same as in the male Each funnel is in the form of an inverted spoon with the two edges of it projecting beyond above the highly reduced nephridial swelling (Fig 8) In other words, the funnel is hood-like with the concavity directed ventrally A transverse section passing through the anterior region of the funnel is in the form of an inverted *V* or *U*, while further behind the gap between the limbs of the *U* gets closed up by the nephridial swelling (Figs 8 and 10*a*) The wall of the genital funnel is formed of a single layer of large ciliated cells, the nuclei of which stain lightly In the nephridial portion the nuclei of the solenocytes take deep stain (Fig 10*a* and *b*)

It may be pointed out that nephridia still function in the testis and ova-bearing segments, though in a reduced condition, and the ciliated funnel is only grafted on to the dorso-lateral aspect of the nephridial swelling—a condition which generally resembles the relationship between corresponding organs in the Phyllodocidae and the Glyceridae In this connection the relation between the genital funnel and the nephridium in *Pisionidens indica* may be recalled in that the nephridia in the genital segments do not undergo any reduction in size or difference in structure, but that only a large conspicuous ciliated funnel is grafted on to it for the conduction of the genital elements to the exterior—a condition much the same as in Phyllodocids The reduction in size of the nephridial swelling in the genital segments of *Pisione gopalai* has already been described (Alikunhi, 1941) This reduction in size and the highly inconspicuous nature of the protonephridium when associated with the genital funnel make these composite structures in their fully developed condition, superficially resemble the typical nephromixia of the Hesionids, Spionids, Syllids, etc, but the presence of solenocytes functioning at least in a reduced condition along with the genital funnel is proof that their real structure is on the Phyllodocid plan

After piercing the septum the nephridial duct runs down between the alimentary canal and the receptaculum seminis At this stage it is wide and powerfully ciliated Further down it proceeds along the ventral aspect of the receptaculum seminis and gets narrow (Fig 11) The receptaculum seminis is a closed thin-walled sac,



TEXT FIG 11 *Pisione complexa*, sp nov

Transverse section passing through a pair of receptacula seminis × 300.

usually containing spermatozoa It is not ciliated internally The cells in its wall at the closed end are extremely flattened The sac opens to the exterior, as already mentioned, at the parapodial base on a special process (Figs 9 and 10*b*) The external duct is thick-walled, the constituent cells being large and protoplasmic It is further thickened by the development of a thin layer of circular muscles Some distance in front of the external opening of the duct of the receptaculum seminis, the nephridial duct joins the former, with the result that there is only one common external aperture—the nephridio-receptacular aperture, as in the case of *P gopalai* (Fig 10*b*)

Structure of Sperms within the Receptaculum Seminis

The presence of copulatory organs in the male and receptacula seminis containing sperms, in the female, point to the occurrence of copulation, but I have not been able to observe the process The arrangement of the accessory reproductive structures, however, suggests that the process might be similar to that described in *Pisionidens* But genital papillae or suckers are absent in *Pisione* and adhesion between the copulating individuals may be brought about by the large flattened process of the copulatory organ, with the help of the specially developed glands on the ventral surface of the sperm sac bearing segments, and probably also by the modified ventral cirrus

Mature females are invariably found to have been inseminated The sperms inside the receptacula seminis have undergone metamorphosis and as in *P gopalai*, the new modifications have apparently taken place after their transference to the receptaculum seminis The sperms when pressed out of the receptaculum seminis are seen to be loosely held together by a sticky fluid, probably secreted by the wall of the receptaculum seminis itself There is no formation of spermatophores

Each sperm has developed a broad, more or less circular anterior extremity which is marked out from the succeeding region by a distinct constriction (Fig 10*c*). The expanded portion probably represents the acrosome The nuclear portion that follows the constriction is spindle-shaped, with the tapering neck region behind. The sperm measures up to 28μ, of which 8 microns are made up by the anterior expansion and the head, while the rest, by the flagellum

Certain variations from the structure described above are often met with in some individuals It might be that these variations are progressive stages before the final condition is arrived at, but I have no confirmatory evidence on this point In some of these sperms the broad anterior expansion is found to be partly bisected by a deep median indentation (Fig 10*c*) In some others the constriction that follows the anterior expansion is very much elongated in the form of a connective The median indentation may be absent in some In sections the nuclear portion takes a deep stain and is circular in outline The sperms have undergone a distinct reduction in size as compared to those from the sperm-sacs of the male. How this is effected and how the changes are brought about, I am not now in a position to explain

The changes undergone by the sperms while inside the receptacula seminis, in *P gopalai*, have already been described in detail The sperms being non-motile the remarkable transformations undergone by them after copulation seem to have something to do with ensuring fertilisation by the development of an adhesive structure which would enable them to adhere firmly to the surface of the female gamete at the first contact.

In this connection it may be pointed out that somewhat similar instances of 'metamorphosis' of the spermatozoa in the receptacula seminis have been noted in Molluscs Ikeda (1929) has described an instance of metamorphosis of spermatozoa in the Japanese slug ***Phylomicus bilineatus*** Benson Similar instances have also been observed by the same author in other slugs like *P fruhstobei*, ***Eulimax flavus***, ***Limacella agrestis***, etc. In ***P. bilineatus***, Ikeda says that the mature spermatozoon stored in the receptaculum seminis has no tail, but only a sperm-head and that the metamorphosis takes place in the atrium after copulation During metamorphosis the head and the middle piece get separated from each other by the formation of a slender filament in between, and finally by the breaking of this filament the connection between the two is severed Ikeda is finally inclined to believe, on the evidence of his own experiments, that this metamorphosis of the spermatozoon is probably related to the mechanism of fertilisation rather than to the prevention of self-fertilisation in the hermaphrodite molluscs

In the case of *P complexa* the sexes being separated the prevention of self-fertilisation as a reason for metamorphosis of the spermatozoon does not arise In this case also it is therefore probable that these changes, as suggested before, have something to do with the mechanism of fertilization A detailed and comparative study of the metamorphosis of the spermatozoon in this and other Pisionids is being made in the hope that it would be possible to throw some light on this exceedingly interesting phenomenon

The ova are large, greenish in colour and in the fresh condition have big transparent nuclei Sections show that in the early oocyte the nuclear portion is more or less clear with a circular central darkly staining nucleolus In the fully formed ovum the nuclear portion takes a greyish tinge with iron haematoxylin, is rather opaque, and usually has an inner clear area The nuclear membrane is present The nucleolus has undergone division and portions of it have already been passed into the surrounding cytoplasm The details of these changes being outside the scope of the present paper are not now considered

When mature, the ova are liberated into the body cavity and are attracted towards the wide mouth of the genital funnels by their ciliary action They are then taken into and passed down the enlarged nephridial duct As in the case of *P gopalai*, it is believed that as the ova pass down the nephridial duct the sperms that are stored into the receptaculum seminis are also sent down which would enable them to get fertilized just prior to, or immediately on their being extruded

CHANGES AFTER LIBERATION OF GENITAL ELEMENTS

The account that is given below is based upon observations of the changes occurring in a number of individuals after the shedding of the genital elements These observations have been put together in a connected form for the sake of convenience and continuity, though it should be understood that all these changes have not been observed in a single form Most of the observations have been confirmed from worms directly obtained from the beach at different times

Female

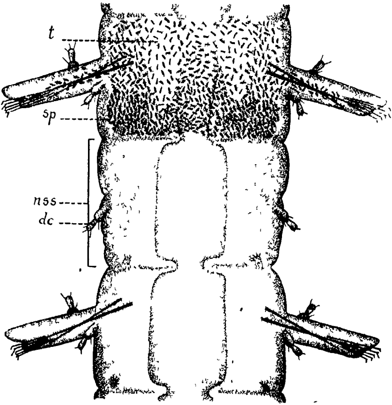
Sexually mature specimens are obtainable throughout the year When ripe females are kept in the laboratory for a week or so the ova are completely shed Breeding takes place continuously and when the genital elements are shed, probably after a short resting period, the individual develops another set of genital elements It is, however, found that in either sex the accessory structures such as the copulatory organs, sperm-sacs, genital funnels and receptacula seminis are gradually lost after the shedding of the genital elements.

With the shedding of the ova the genital funnels begin to get smaller The receptacula seminis, some of them still containing sperms, are pushed nearer the anterior septum and their external ducts show signs of hystolysis. The nephridial duct is unchanged The ovary has returned to its immature condition and is now represented by a small mass of tissue

Prolonged captivity results in further changes The genital funnels disappear, with the result that the nephridial swellings of the respective segments stand out prominently. There is no trace of the receptaculum seminis. The nephridial duct now communicates independently to the exterior The cirrus like structure, developed from the parapodial base is absent The worm now looks just like an immature one, with just a trace of the ovary

Male

A similar series of changes take place in the males also The testis becomes reduced and is often represented by minute groups of cells surrounded by sperms The genital funnels are also very much reduced and portions of the sperm-sac show signs of histolysis Copulatory organs still retain their outer shape, but internally histolysis has commenced At a later stage the genital funnels, sperm-sacs and copulatory organs are all completely shed and the dorsal cirri alone persist in the genital segment (Fig 12) The segment is now nonsetigerous, the acicula also having been cast off



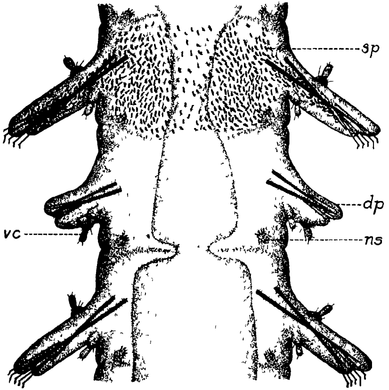
TEXT FIG 12 *Pisione complexa*, sp nov

Middle segments of a male worm showing the genital segments from which sperm sacs and copulatory organs have been dropped off × 200

nss, non setigerous segment

The changes undergone by the nephridial duct are evidently more complicated here than in the female Since in the male genital segment the external opening of the nephridial duct—modified into the genital duct—is situated at the tip of the copulatory organ, the question arises whether the nephridia in these segments function when the sperm-sacs—mere dilatations of the nephridial duct—and the copulatory organs are undergoing profound histolysis which culminate in their entire loss With the disintegration of these structures it is clear that the nephridial duct behind the septum is also lost It is therefore, highly probable that the nephridia in these segments do not function when histolysis is taking place and that the nephridial swelling which now stands alone in the testis bearing segment develops a fresh external duct by the time the ventral cirrus and parapodial lobe are developed in place of the copulatory organs (see below)

At a later stage fresh parapodia begin to develop from either side of the segments in which sperm-sacs and copulatory organs were present (Fig 13) Ventral cirrus is formed Compound as well as simple setae are developed in the newly formed parapodial lobe These parapodia get further elongated and in this condition differ in no way from the parapodia of the non-genital segments The testis is very minute and like the ovary has returned to the undeveloped condition The segment still contains some worn out sperms in the coelom (Fig 13) The ultimate fate of these sperms is not fully known Since the nephridia have no direct internal opening, with the disintegration of the genital funnels and sperm sacs these sperms are effectively prevented from being taken to the exterior In the males, therefore, the accessory organs are lost before the genital elements are completely shed



TEXT FIG 13 *Pisione complexa*, sp nov

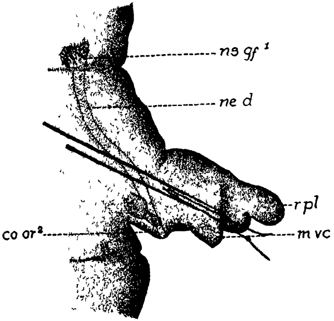
Middle segments of a male that has cast off the sperm sacs and other accessory structures, developing ordinary parapodia Note the condition of the fresh parapodia and the presence of sperms in the preceding segment × 200,
dp, developing parapodium.

Changes of a similar nature taking place in *Pionosyllis neapolitana* have been described by Goodrich (1930), but in the male segments of this hermaphrodite syllid the testes are always in a well developed condition, apparently ready to produce more sperms This is not the case in *P complexa* Whether any change takes place in the sperm-sacs of *Pionosyllis* is not known

It is clear that the reproductive organs are lost and renewed probably several times in the life history of an individual But the time taken for the development of the first set, the interval between its loss and the formation of the next, are points that remain to be elucidated

SECOND GENERATION OF REPRODUCTIVE ORGANS

The second set of reproductive elements is developed in the very same segments which contained the first, since the remnants of the ovary or the testis as the case may be, are present in those segments in an inactive condition In the male, the parapodia of the segment following the testis, though resembling the others, were smaller and their bases were considerably swollen (Fig 14) The ventral cirrus was absent and in its place there was a broad papilla with a swelling at the base On this swelling was situated the external aperture of the nephridium, the course and appearance of which were similar to those of the nephridial duct in the developing copulatory organ (Fig 7c) The absence of the ventral cirrus at this stage and the presence of an enlarging papilla in its place definitely shows that the normal ventral cirrus is being modified during the formation of the copulatory process Also the swelling at the base of the enlarging ventral cirrus supports the conclusion arrived at previously that the actual copulatory process is formed as a prolongation of the



TEXT FIG 14 *Pisione complexa*, sp nov

Regeneration of copulatory organ—early stage, viewed from the ventral aspect ×400

co.or.,² developing copulatory organ; *m vc*, modified ventral cirrus.

body-wall at the base of the ventral cirrus A cluster of cells was attached to the nephridial swelling, evidently forming the rudiments of the genital funnel The parapodial lobe was supported by two acicula, one simple and one compound seta The testis was still minute and undeveloped It is to be observed that in the formation of the copulatory organ during the second generation the well developed parapodium gets gradually shorter and shorter, at the same time casting off its setae until only the two acicula remain In this feature it differs from its first formation when a well developed parapodium was not present in its place The succeeding stages are similar to those in the first formation of the copulatory organ described above It is to be noted that a few sperms of the first generation are still present in the segmental chamber It is possible that these are senile sperms which might be absorbed during the increased metabolism that is taking place as a result of the formation of fresh genital funnels, sperm sacs and copulatory organs It is also suggestive that the retention of sperms in the testis segments till the regeneration of the reproductive organs incidently forms one of the means by which the animal finds material to meet the demands of the increased metabolic activity, and might probably explain what would otherwise seem a waste of energy on the part of the worm in producing more sperms than are actually used during copulation

Might not these phenomena indicate some sort of rejuvenation of the worm accompanying the replacement of the once functional reproductive organs by an entirely new set? Recent researches on rejuvenation in other animals seem to indicate that this is not altogether impossible

SPECIFIC CHARACTERS

Pisione complexa n sp

Worms 10 to 20 mm long, with 70 to 100 or more segments, cephalic region identical with that of *Pisione gopalai*, anal segment semi-circular, without anal glands, one to six or more pairs of testes and a corresponding number of genital funnels, sperm-sacs and copulatory organs in the male, each testis invariably confined to a single segment, sperm-sac with only one saccular expansion, copulatory apparatus of extraordinary complexity, parapodial lobe in the sperm-sac bearing segment highly reduced and supported by two acicula in the adult and by two acicula and one compound seta in the earlier stage, the ventral cirrus modified into a broad papilla, ventral glandular pad on the sperm-sac bearing segment, sperms non-motile and of large size, females having up to 20 pairs of ovaries, enclosed in membraneous coverings, arranged in alternating segments, the intervening segments occupied by the paired receptacula seminis, genital funnels in every ova-bearing segment, parapodia of segments carrying receptacula seminis unmodified, nephridio-receptacular aperture situated on a special cirriform structure ventrally to the parapodium, and sperms within the receptacula seminis also non-motile but with highly expanded anterior extremity

Locality Madras beach, India

ACKNOWLEDGMENTS

The author wishes to express his deep sense of gratitude to Prof R Gopala Aiyar for his helpful criticisms and constant encouragement during the preparation of this paper His thanks are also due to the syndicate of the University of Madras for awarding him a research studentship during the tenure of which this work was carried out

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STUDIES ON THE CYTOLOGY OF YEASTS

II INDUCTION OF POLYPLOIDY AND HETEROCHROMATIN

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(Original received June 14, received after revision November 14, read August 30, 1946)

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INTRODUCTION

The distribution of the chromosomes in a distillery yeast to the two daughter nuclei during mitosis presented some interesting variations (Subramaniam and Ranganathan, 1946b) While, in many, the distribution was equal, each getting four chromosomes, in others it was unequal, owing to chromosome lagging, and nuclei with two, three, five or six chromosomes were formed These diploid, triploid, pentaploid and hexaploid cells appear to disintegrate after a few divisions since they could not be isolated in wort-agar plates

From investigations on animals (White, 1945, Koller, 1943) and plants (Darlington, 1942) it appears that variations in the amount of heterochromatin not only upset the nucleic acid metabolism of the cell but even alter the timings of mitoses and often result in sterility Mutation is suggested to be the cause of these variations Can such an explanation be extended to include the lethal mutations in the distillery yeast ? Do yeast chromosomes carry heterochromatic regions ?

Though in the distillery yeast all except the tetraploid have only a short span of existence, the four original strains having distinct morphological characters isolated immediately after exposure of a brewery yeast to low temperatures for 90 days (Subramaniam and Ranganathan, 1946d) have given rise to 15 different strains having distinct characteristics

A cytogenetical analysis of the above depends, therefore, on an elucidation of what happens in yeasts when the chromosomes are duplicated and whether yeasts have heterochromatin

LITERATURE

A perusal of the literature on the reactions of yeast to biologically active substances indicates that almost similar effects could be produced by a variety of chemicals (Bauch, 1941, 1942, Fabian and McCullough, 1934, Thaysen and Morris, 1943, Levan and Sandwall, 1943, and Thomas, 1945) The conclusions drawn by different investigators appear, however, to be different While Bauch (1941, 1942) claims production of tetraploids and even octoploids, Thaysen and Morris (1943) considered the change to be 'deep seated', while Thomas (1945) suggests a 'plasmagene mutation' Levan and Sandwall (1943) consider that their results do not justify a comparison of the effects of various chemicals on yeast with the colchicine effect on higher plants

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All the above tentative conclusions are, however, uncorroborated by critical cytological investigations Entire dependence on morphological data is often misleading, for, agencies which induce polyploidy also produce gene mutations (Kerkis, 1939, Darlington and La Cour, 1940)

Any comparison between the cell division of yeasts and cell division of higher plants should be based on a clear appreciation of the different intervals at which duplication of cells takes place in different organisms (Darlington, 1937, Darlington and La Cour, 1940) While yeasts divide almost every hour in well-aerated wort, intervals between divisions in higher plants have been known to extend in some cases to a week or more Thus, while yeast can theoretically be converted into a tetraploid by treatment of very short duration, longer treatment is necessary to produce a tetraploid seedling Another important observation on higher plants ignored by workers on yeast is that it has been possible to produce octoploid plants only very rarely by a single treatment (Dermen, 1940)

Our own experience with a particular strain of yeast confirms the above observation After three months of treatment we obtained only a tetraploid (Subramaniam, 1945) Lack of realisation of the importance of the above observation has resulted in different conclusions being drawn from almost identical observations Richards (1938) found that when colchicine is present, a maximum crop was produced in a single growth cycle In such cultures alcohol and other products of fermentation were found to be greater Acceptance of Richards' suggestion that colchicine not only buffers the medium but is also a food, necessarily depends on proof that the cells have not become polyploid A careful perusal of Richards' paper shows that his colchicine cultures differed from the controls (1) in having only a single growth cycle, and (2) in their increased fermentative activity as evidenced by increased output of alcohol and increased utilisation of sugar The difference may be as much due to presence of colchicine as due to duplication of chromosome sets

Though many substances have been known to induce 'colchicine mitosis', few are useful for the production of polyploids Apart from colchicine the largest number of polyploids have been produced by treatment with acenaphthene Even in the selection of a suitable chemical for induction of polyploidy in yeasts several factors have to be taken into consideration Recent experiments have shown an antagonism of ethyl alcohol to colchicine Levan and Ostergren (1943) found that while the threshold value of colchicine for *Allium* was 0 0055%, in the presence of 0 5% alcohol even 0 008% was found to be ineffective The increasing percentages of alcohol produced during fermentation may therefore desensitize the yeast to the action of colchicine

Organisms differ in their sensitivity to different polyploidizing agents The Gramineae differ from the Leguminosae in their sensitivity to acenaphthene (Kostoff, 1938b) While C-mitosis could be induced in wheat by different substances, flax reacts only to colchicine (Simonet and Guinochet, 1939) Ulva, a green alga, reacts only to acenaphthene but not to colchicine (Levan and Levring, 1942)

Thus, if we take the four factors into consideration, viz (1) that yeast cells should be compared to whole plants and not to individual cells of higher organisms, (2) that action of a short duration may bring about a duplication of chromosomes in yeasts, (3) that biologically active substances do not generally produce higher polyploids among treated seedlings by a single treatment, and (4) that the tetraploids and octoploids may have a shorter generation time, then, the observations of various workers show a remarkable coincidence

Results on higher plants indicate that chromosome duplication may produce entirely different types of tetraploids '(1) There may occur an appreciable increase in size of each vegetative cell in the tetraploid individual while the total number of cells making up the plant remains relatively the same as in the diploid form, consequently, the tetraploid plant appears larger than the diploid individual Most of the changes following polyploidy appear to fall into this category (2) An increase in cell volume may follow a doubling of chromosomes, but there may be a decrease in the total number of cells making up the tetraploid plant, therefore, the tetraploid individual will not appear different from the diploid (3) The doubling of the chromosomes may not have any effect on the size of the cells The polyploid individuals remain indistinguishable except probably in sexual and in some obscure physiological behaviour' (Dermen, 1940)

It would be well to remember that volume changes in induced polyploids depend not merely on an increase in the number of chromosomes but also on the occurrence of particular genetic factors in the organism The possibility of different types of gene mutations being induced by different biologically active substances has also to be kept in view

The results of previous workers on yeast, when analysed in the light of the above considerations fall into two separate groups (1) polyploidy with augmentation of size appears to be likely when cultures are treated with camphor and borneol, and (2) polyploidy without any remarkable increase in size in cultures treated with colchicine (Levan and Sandwall, 1943, Thaysen and Morris, 1943, Richards, 1938, Bauch, 1941, 1942)

Though a large number of observations have been recorded on variation in yeasts, *the absence of cytological data has resulted in an absolute lack of criteria for any rational analysis of the changes occurring in yeasts when treated with biologically active substances* Many workers have recorded the tendency of the variants to revert to their original morphological condition A careful perusal of the literature shows that variants themselves are of two types (1) 'saltants' and (2) 'Dauermodifications' (Henrici, 1941) Both these seem to occur in cultures either spontaneously or when exposed to cold or the action of polyploidizing or carcinogenic substances We do not know yet how many of these variations are the result of changes in chromosome number and how many to causes such as deficiency (duplication) and gene mutations (Subramaniam and Ranganathan, 1946a) Even in higher plants the origin of aneuploid individuals and chimeras after colchicine treatment still awaits a rational explanation While some consider that such aberrations are the result of partial instead of total arrest of chromosome division, others have shown origin of such individuals by multipolar divisions of some polyploid nuclei (Dermen, 1940, Kostoff, 1938b)

The unstable nature of some of the mutants and the 'sporting' behaviour of the distillery yeast observed in this laboratory (Subramaniam and Ranganathan, 1946b) led me to a closer study of the problem of chromosomal behaviour in yeast and compare it with that in higher plants and animals One curious observation on the structure of the chromosomes in the distillery yeast appeared to be significant In Carnoy iron haematoxylin as well as in Feulgen slides the chromosomes were 'glistening' The cause of this 'glistening' appears to be due to the chromosomes having a lightly stained core and a chromophile cortex While in the control Sc 9 (Subramaniam, 1946a) such a phenomenon was not observed, in material stained while the cultures were undergoing treatment with acenaphthene a similar staining reaction of the chromosomes was observed

Levan (1945) has reported recently deviations in the staining qualities of chromosomes when treated with inorganic salt solutions Not only could he observe the relational spirals of the two half chromatids clearly, but the heterochromatic regions retained the stain when the other regions of the chromosomes had lost their colour.

To understand the different staining reactions of the chromosome when it becomes duplicated, we have to turn to the results on higher plants and animals for an explanation *Even there the whole matter remains still as a speculation* Pontecorvo (1944) says 'It is implicit in the view expressed here that a heterochromatic segment should arise every time that a minute euchromatic region undergoes repeated duplications in the genotype and the replicas remain adjacent to each other on the chromosome ' Mather (1944), on the other hand, states 'The functional distinction between the two kinds of gene must not, however, be held to imply that one type can never change into the other So far as the effect on the phenotype is concerned it would not seem impossible that the polygene of relatively small effect could become the major gene of relatively large effect if the developmental history of the organism became elaborated in an appropriate way '

Are we seeing in yeast chromosomes a transformation of a part of the euchromatin into heterochromatin ? Is the chromophobic interior of the chromosomes the heterochromatin ?

Increase in the rate of cell division is said to be associated with increased synthesis of nucleic acids by the cell, this again being supposed to be regulated by the action of heterochromatin (Thomas, 1945) The shorter generation time of the major strain of *Torulopsis* produced by Thaysen and Morris (1943) necessarily presupposes such a series of events Since the amount of alcohol produced by any strain depends on its genic make up and since duplication of the genes should in all probability lead to increased alcohol output, it necessarily follows that the cells should produce increased quantities of nucleic acids It is only the ribose nucleic acids which are concerned in cellular syntheses (Caspersson and Schultz, 1940, p 512)

If we extend the concept of the important rôle of nucleoproteins in cellular synthesis to yeasts also (Henneberg, 1916), then the increased concentration of both types of nucleic acids in the major strain of Thaysen and Morris (1943) appears intelligible Production and regulation of nucleic acids is said to be intimately associated with the heterochromatic regions, for, it is stated 'Heterochromatic regions have the capacity (1) to form large amounts of thymonucleic acid (or better perhaps, thymonucleoproteins) in the chromosomes themselves, (2) to form or affect the composition of the nucleoli, (3) to affect the characteristics of neighbouring regions translocated to them in such a way as to change the developmental effects of these regions in somatic cells , and (4) to affect the content of the ribonucleic acids in the egg cytoplasm of *Drosophila*' (Schultz *et al* , 1940, p 521)

Looked at from different angles, the occurrence of heterochromatin in polyploid yeasts appears to be a possibility

MATERIAL AND METHODS

The accidental choice of acenaphthene for experiments on yeast necessitated by non-availability of even colchicine was a happy one, for, its valuable feature appears to be its lack of toxic properties (Levan and Ostergren, 1943) As qualitative investigations should precede quantitative ones, experiments were devised more with an idea of producing polyploids than to gauge the effect of differing concentrations of acenaphthene on the mitotic cycle The workers who discovered that acenaphthene could successfully be used for the induction of polyploidy in plants have stressed the importance of having an undissolved excess of the substance, as a 'saturated solution alone was not sufficient to induce chromosome doubling' (Kostoff, 1938a, p 753)

Yeast could be made either to grow or ferment Therefore an attempt was made to bring as far as possible growing cells under the action of acenaphthene Tubes containing 24-hour cultures were well shaken and the contents poured out Fresh wort was added to the tube and the few cells left in the tube were distributed uniformly throughout the medium by vigorous shaking A loop from the above, which would usually contain not more than 50 cells, was inoculated into the experimental tube containing a few crystals of acenaphthene and about 10 c c of wort Every day most of the material was discarded and the same quantity of wort was added and the crystals of acenaphthene were renewed By this method actively growing cells were exposed to the action of acenaphthene and the above procedure was continued for 90 days Bauch (1942) has stressed the importance of temperature at the time of treatment, but the control of temperature was thought unnecessary in the present instance because of the prolonged treatment Examination of the contents of the tube was carried out every day After the third day, every 24-hour culture would show a layer of small cells at the top

Since vigorous growth has been noticed in higher plants immediately on return to normal environment after treatment with polyploidizing agents (Kostoff, 1938c, Nebel and Ruttle, 1938b, Muntzing and Runquist, 1939), the acenaphthene treated culture was kept in an active condition in wort before isolating the various types of cells by their distinct colony characteristics in wort-agar plates

A description of the immediate cytological effects of acenaphthene on active cells of the brewery strain Sc 9 (N C T C 3,007) having only two chromosomes (Subramaniam, 1946a) was held up pending a preliminary analysis of the cytology of the new types produced (Subramaniam, 1945, Subramaniam and Ranganathan, 1946a) by 90 days' treatment This was just to confirm the suspicion based on a careful perusal of the literature that higher polyploids may not occur even on continuous treatment It was thought that if such a confirmation was available it may enable a correlation of the contradictory results obtained by different workers on the effect of colchicine on yeasts

The observations recorded in this paper, however, are the results of a cytological investigation of the immediate effects of acenaphthene on growing cells of the above two chromosome strain Since few cells are introduced at the beginning, the time of fixation was arbitrarily fixed at six hours in order to get sufficient material for preparing a few slides The contents of the tubes were centrifuged at regular intervals to get a series showing the chromosome stages (Subramaniam, 1946a) Unsolved technical difficulties have prevented a study of the first mitosis under acenaphthene treatment and hence the results recorded are those of later divisions from acenaphthene cultures

Confusion of volutin with chromatin was avoided by choosing cultures of cells with clear cytoplasm showing no granular inclusions (Caspersson and Brandt, 1941) and fixed in Bouin or Carnoy to prevent the mitochondria vitiating the picture The slides were stained in Heidenhain's haematoxylin

OBSERVATIONS

The nucleus of the yeast preparing for division may be observed as a vacuole enclosing a chromophile mass having an irregular shape (Fig 3) The staining of this chromatin mass is uniform and it divides into two and soon after into four as the indentation in one of the chromophile masses in Fig 4 would suggest Or, the nuclear vesicle disappears and the chromatin mass first assumes an irregular vesicular shape with a chromophile rim and a chromophobic interior (Fig 5) before dividing into two very similar bodies having an identical shape and structure (Fig 1) The differentiation into chromophilic and chromophobic regions appears at this stage even in those chromophile bodies which appeared uniformly stained at the time of division of the initial chromatin mass (Figs 3 and 4) Division of the two bodies is not simultaneous as Figs 2 and 7 would indicate One of the bodies divides first (Fig 2) and then the other (Fig 7). This appears to be the typical tetraploid metaphase condition which appears to be succeeded by the typical anaphase illustrated in Fig 12 . Among the large number of cells seen in any field, though four chromosome stages predominate (Figs 7 and 13), cells showing other chromosome numbers are also present The two initial chromophile bodies may give rise by division to four chromosomes, two of which may differ in size (Fig 6) One pair of these chromosomes may differ not only in size but also in structure A pair of vesicular chromosomes and two chromatin grains occur in the cell illustrated in Fig 8 In Fig 10 may be seen three vesicular chromosomes and three granular ones On careful examination of large numbers of cells it appears as if these granular chromosomes are the result of unequal division or breakage of the chromosomes and that they eventually disintegrate The vesicular shape of the chromosomes in cells undergoing treatment with acenaphthene renders it impossible to judge the exact cause for such inequality in size between the two daughter chromosomes Since the mutant isolated from cultures grown under normal conditions after undergoing treatment for 90 days with acenaphthene showed two unequal chromosomes (Subramaniam and Ranganathan, 1946a), the question of fragmentation and translocation of bits of chromosomes have to be seriously considered

The occurrence of triploid anaphases (Fig 16) suggests either disintegration and loss of one chromosome or the non-division of some chromosomes, the later segregation of the six chromosomes into two groups and the reconstitution of two nuclei each with three chromosomes The occurrence of pentaploids and triploids (Figs 9, 10, 11, 14, 15 and 16) suggests the latter possibility Colchicine and acenaphthene though they inhibit spindle formation do not, however, have any effect on the streaming movements of the cytoplasm (Nebel and Ruttle, 1938a) Observations on the control suggest that apart from the spindle, the streaming movements also play an important rôle in the distribution of the daughter chromosomes or the reconstituted nucleus to the bud (see Pictographic summary, Subramaniam, 1946a) Unco ordinated division of the chromosomes into chromatids and streaming movements may explain the curious disposition of the chromosomes in Figs 9, 14 and 15 Thus in Figs 14 and 15 one of the chromophile bodies appears to have divided giving an odd number of chromosomes The possibility that in the majority, the ultimate division of all the bodies may finally be followed by the anaphase illustrated in Fig 12 is worth consideration since pentaploid anaphases have not been noticed in the material While in Figs 9, 14 and 15 five chromosomes are present, in Fig 17 there are six, two of which appear to be considerably bigger than the rest In Fig 18 there are seven bodies, the result probably of non-division of one of the chromosomes, which when completed would probably proceed later to the anaphase shown in Fig 12

The behaviour of the chromosomes in both plants and animals under continued treatment appear to be similar 'The sequence of events is similar to that described by Levan (1938) in *Allium* The chromatid attraction lapses and the division of the centromere takes place, but the two chromatids remain parallel Either a single resting nucleus, which will be tetraploid is produced or several unbalanced nuclei which will degenerate' (Barber and Callan, 1943, p 264, Kostoff, 1938b)

The vagaries of different types of cells from different plants to identical concentration of these chemicals are slowly coming to light It is well known that after treatment with colchicine aberrant forms with unchanged chromosome numbers also occur

The results recorded above of the action of acenaphthene on a brewery strain of yeasts do not appear to be unusual The mutant with two unequal chromosomes (Subramaniam and Ranganathan, 1946a) may be the result of a simple division of the cell shown in Fig 6, or the multipolar division of a cell containing eight unequal chromosomes The occurrence of only a tetraploid even after 90 days' treatment suggests that the efficiency of the chemical as a polyploidizing agent ceases once a duplication of the chromosomes had occurred It is quite likely that several types with unbalanced chromosome numbers may occur in the cultures. But all these various forms may have only a short span of existence since they did not appear in wort-agar plates.

DISCUSSION.

Heteropycnosis of entire chromosomes or parts of chromosomes have been known for a long time In fact, the 'chromomeres' of Wenrich (1916) and the 'prochromosomes' of Rosenberg (1909) seem to belong to this category Chromosomes show different types of heteropycnosis during the various stages and members of the same set may differ from one another even at the same stage The X-chromosome of Acrididae show positive heteropycnosis during the prophase of meiosis and negative heteropycnosis during the early spermatogonial divisions The autosomes of the same set, however, are positively heteropycnotic during the meiotic prophase but show no reversal in the early spermatogonial divisions Even among these autosomes, the 'precocious chromosome' resembles in its staining capacity the X-chromosome itself (White, 1945)

In many plants (Darlington and La Cour, 1940) such a reversibility has been demonstrated Particular regions which show negative heteropycnosis during metaphase at low temperatures appear deeply stained during the resting stages The Y-chromosome of *Drosophila* is completely heteropycnotic and genetical evidence suggests that it does not carry the major genes On this basis chromosomes or chromosomal regions are classified into eu- and heterochromatic regions As in the case of the Y-chromosome, in the autosomes also the heterochromatic regions have been shown to exhibit a different type of genic behaviour Apart from all this Caspersson (1941) has shown that the protein synthesized by the genes are less complex in the heterochromatic regions

The identification of heterochromatic regions in chromosomes does not appear to be an easy affair (Callan, 1942) Though in certain plants the chromosomes show the differential segments when mitosis or meiosis takes place at low temperatures, this method is not of universal application The number of chromocentres in the resting nuclei (Manton, 1935) may not also be a safe guide Darlington and La Cour (1940) found the number of chromocentres to correspond to the heterochromatic segments in *Paris polyphylla* but not in others In *T erectum*, the number of chromocentres were fewer than the heterochromatic segments They conclude that it is not always possible to distinguish the intercalary segments (Kaufmann, 1939) and that the possibility of two neighbouring differential segments appearing as a single chromocentre should be kept in view

Mirsky (1943) suggested that heterochromatin may be characterised as 'that portion of a chromosome which retains its high content of nucleic acid in the interphase when the rest of the chromosome (the so-called euchromatin) loses much of its nucleic acid' (p 28) The realisation of the fact that the detection of heterochromatin being not an easy affair, the failure to locate such regions by cytological methods need not necessarily indicate its absence, has led Darlington to re define it as 'parts of chromosomes which are liable to remain charged with thymonucleotides in the resting stage' (Mather, 1944)

This naturally leads to a consideration of the position of heterochromatin in the chromosomes The supernumerary chromosomes in many plants and animals (White, 1945, Darlington and Thomas, 1941) are almost wholly composed of heterochromatin Even different tissues in the same plant or animal may have different numbers of heterochromatic supernumerary chromosomes The supernumeraries are limited to the germ track in *Sorghum* In *Sciara* the male and female have seven and eight chromosomes in their soma while the cells of the germ line contain in addition a pair of supernumeraries, the 'limited chromosomes'

Even in the same autosome the heterochromatic regions may be limited to the areas around the centromere or they may have in addition such regions at their ends as also minute ones distributed at intervals Translocation of a gene to the heterochromatic segment leads initially to irregularities in reproduction before a mutation to the heterochromatic type (Caspersson and Schultz, 1938). Since in

Drosophila the inert regions contain most of the repeats (Kaufmann, 1939) it has been tentatively suggested that inertness may be the cause of reduplication Since additions or deletions of the heterochromatic segments have only slight phenotypic effects and since heterochromatin carries 'polygenes' it has been surmised that 'by virtue of its complement of polygenes it must play an important part in the fine adjustment of the phenotype to the immediate environment and in the storing of the variability on which will depend the future adaptation and evolution of the organism' (Mather, 1944) In animals *repeats* represent 'an important kind of "raw material" for evolution', for, 'mutations which would be lethal or at any rate lower the viability of the organism if they occurred in a non-repeated region, may in many cases have no such disastrous consequences if they occur in a tetraploid segment' (White, 1945, p 48) Polyploidy appears to have played a major rôle in the evolution of plants and the yeast is perhaps no exception

The specific question therefore is—whether the chromophobic portions of the chromosomes in yeast cells undergoing treatment with acenaphthene represent the heterochromatin? Any duplication of the chromosome sets should make some set of genes more or less superfluous

Before discussing the possibility of the chromophobic portion of the chromosomes being heterochromatin, the question whether such a staining reaction may not be a mere indication of the structure of the chromosome itself has to be considered Chambers (1925) has shown that in favourable material the chromosomes during certain stages possess a cortex which can be optically differentiated from a central core 'This structure is significant in view of the way in which the artificially induced chromatin filaments come to view in the prophase spermatocyte of the grasshopper Granules appear out of the hyaline nuclear material and align themselves in rows As the granules increase and accumulate, their arrangements about a hyaline non-granular core becomes more and more appreciable The definitive chromosome finally results by a shortening of the core and the fusion of the granules into a hyaline cortex' (p 274) Levan (1945) found that treatment with many salts produced clear pictures of the internal structure of the chromosomes

That the lightly stained region in the chromosome in acenaphthene material is not merely a clear picture of the internal structure would be evident owing to the following reasons (1) The control strain shows no such differential staining (2) In actively growing cells of the distillery yeast under normal conditions the chromosomes show such a differential staining (3) In Carnoy or Bouin iron haematoxylin and in Feulgen's nucleal reaction an identical picture is obtained Since the cells under discussion differ from the control in that they are polyploid, the differentially understained region is in all probability the heterochromatin

In rod-shaped chromosomes the heterochromatin occurs either intercalated or as continuation of the euchromatin Not only does the amount of heterochromatin differ in some groups from individual to individual owing to duplications and deletions, but also they show variations in amount in different tissues In *Drosophila melanogaster* the heteropycnotic region which is about one-third the length of the X chromosome at mitosis is represented in the salivary chromosomes by less than one-tenth its length While, in *Drosophila* salivary glands the heterochromatic regions around the centromeres fuse to form the chromocentre, in the Chironomidae no such fusion occurs and intercalary and terminal heterochromatic regions have been observed

The heterochromatin of the autosomes have been known to differ from that of the sex chromosomes and it seems as if 'there is difference between "compact heterochromatin" in which chromomeres still form bands and "loose heterochromatin" in which the regular arrangement of the chromomeres is entirely lost' (White, 1945, p 44) Thus the range of variations observed in the position and distribution of heterochromatin in animals and plants renders it possible to consider the lightly stained interior of the yeast chromosomes as heterochromatin Very little attention seems to have been paid to the location of heterochromatin in granular chromosomes We have an example of such a type in *Drosophila melanogaster* itself The 'dot' chromosome appears in salivary glands as a short strand attached to the chromocentre Often both ends of the 'dot' chromosome may be attached to the chromocentre showing the existence at the ends of heterochromatic segments

The differentially stained region in the yeast chromosomes thus appears in all probability to be the heterochromatin In certain species of Chironomidae, Bauer has observed that 'single bands of large "vesicular" chromomeres occur in the middle of a chromosome These he interprets as heterochromatic segments consisting of only one band, they may also occur at the end of a chromosome' (White, 1945) The resemblance in structure of these heterochromomeres to the chromosomes in the yeast exposed to the action of acenaphthene is rather striking

The peculiar position of the heterochromatin in the yeast chromosomes militates in no way against its identification since Painter and Taylor (1942) describe in the toad discrete granules of heterochromatin entirely removed from the chromosomes and still appearing to function Caspersson and Brandt (1941) suggested that the volutin granules and thymonucleic acid of yeast cells may correspond respectively to the hetero- and euchromatin of animals and plants

The demonstration of a change from eu- to heterochromatin on duplication of chromosomes appears to be of considerable significance Since gradations between polygenes and major genes as well as transformation of one into the other have all been envisaged, definite statements based on observations on yeast would be precarious Only planned experiments on higher plants on the effect of induced polyploidy on eu- and heterochromatin may furnish us with any rational explanation

SUMMARY

1 Lethal mutations observed in a distillery yeast necessitated an elucidation of what happens in yeasts when the chromosomes are duplicated and whether yeasts have heterochromatin

2 A review of the effects of polyploidizing agents on yeasts and higher plants is presented It is suggested that if we take the four factors into consideration, viz (1) that yeast cells should be compared to whole plants and not to individual cells of higher organisms, (2) that action of a short duration may bring about a duplication of chromosomes in yeasts, (3) that biologically active substances generally do not by a single treatment give higher polyploids among treated seedlings, and (4) that the tetraploids and octoploids may have a shorter generation time, then the observations of various workers show a remarkable coincidence

3 The observations and speculations on the problem of heterochromatin are reviewed and the possible occurrence of heterochromatin in polyploid yeasts is indicated

4 Details of the method of treatment of actively growing yeast cells with acenaphthene are given

5 The various chromosome pictures seen during mitosis in cells undergoing treatment with acenaphthene are described The chromosomes have a chromophilic cortex and a chromophobic interior

6 The possibility of a change from eu to heterochromatin on induction of polyploidy is discussed and it is suggested that the chromophobic core of the chromosomes may correspond to the heterochromatin of higher plants and animals

7 It is shown that the peculiar position of the heterochromatin militates in no way against its identification since Caspersson and Brandt suggested a correspondence of volutin granules of yeast to heterochromatin, while Painter and Taylor describe in the toad discrete granules of heterochromatin entirely removed from the chromosomes and still appearing to function.

ACKNOWLEDGMENTS

I am very grateful to Sir J C Ghosh, Kt , D Sc , F N I , and Mr M Sreenivasaya for their active interest and encouragement and to the Council of Scientific and Industrial Research for generous financial assistance

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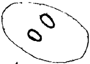
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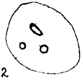
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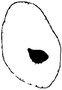
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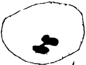
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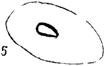
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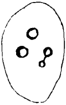
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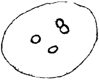
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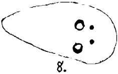
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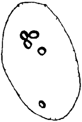
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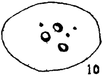
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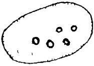
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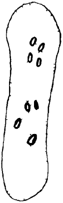
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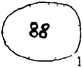
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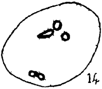
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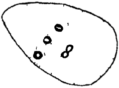
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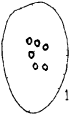
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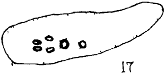
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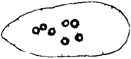
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THE NATIONAL INSTITUTE OF SCIENCES OF INDIA

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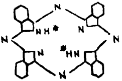
STUDIES ON PHYSICO-CHEMICAL PROPERTIES OF PHTHALOCYANINES

DETERMINATION OF MOLECULAR WEIGHTS OF FREE PHTHALOCYANINE AND SOME OF ITS METALLIC DERIVATIVES

By M V SIBUR, *B Sc*, M S MUTHANNA, *M Sc*, S K BHATTACHARYYA, *D Sc.*, *and*
SIR J C GHOSH, *Kt*, *D Sc*, *F N I*

(*Received August 10, read August 30, 1946*)

Phthalocyanines, since their discovery in 1928, and synthetic preparations in the pure state by Linstead and his school have been materials of such absorbing interest to the chemist as well as to the industrialist that it was considered worthwhile to investigate some of their physico-chemical properties Phthalocyanines combine the complexity of organic structure with the stability of inorganic compounds, resulting in a new chromophore with a splendid resonance and a unique resistance to light, heat, acids and alkalies and to most of the organic solvents The structure of free phthalocyanine, as given in I, has been finally accepted as correct



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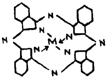


II (*M* = Divalent metal)

The structure consists of four corners with iso-indole nuclei which are bound through nitrogen atoms, so that the middle part consists of a 16 membered ring Within this ring are found two imino-hydrogen atoms (marked with asterisk) which are replaceable by metal This structure was assigned by Robertson and established with certainty with the help of X-ray investigations It has been proved by Linstead *et al* (1936) that the metal is held to the two iso-indole N atoms by primary valencies and is co-ordinated with the other two N atoms to form four chelate rings, this sort of chelation leading to greater stability (Structure II)

Phthalocyanine derivatives with 26 elements are known so far Their solubilities are anomalous Though they exhibit general similarity, there exist distinct differences depending on the constituent metallic atom and they can be classified into three groups —

- (*a*) Derivatives of Na, Ca, Hg, etc , are amorphous powder, insoluble in organic solvents and do not sublime The metal can be removed by dilute acids and some organic solvents
- (*b*) Stable covalent co-ordination compounds, like Cu, Ni, Zn, Pt, etc , derivatives They are stable towards cold concentrated sulphuric acid and hot alkalies and the metal cannot be separated without disrupting the whole molecule They are soluble in high boiling organic solvents, crystallise in monoclinic crystals and sublime at 500–600°C
- (*c*) Labile covalent co-ordinating compounds, like those of Mg, Mn, Sn, etc They cannot be crystallised or sublimed The metal is easily removed by the acids.

Of the various physico chemical properties of phthalocyanines studied so far, reference may be made to their oxidisability studied by Linstead *et al* (1934), their halogenation studied by Linstead *et al* , their catalytic activities studied by Cook and by Tamamusha and Tohonatsu, their absorption spectra studied by Linstead *et al* in a number of organic solvents like chloronaphthalene, bromonaphthalene, pyridine, quinoline, ethyl alcohol, acetone, and in an inorganic solvent like absolute sulphuric acid As regards the determination of molecular weights of phthalocyanines, the only reference available in literature (other than X-ray method) is the determination of the molecular weight of magnesium phthalocyanine in naphthalene as a solvent, by ebullioscopic method studied by Linstead and Lowe, who have found that the experimental value is in good agreement with the theoretical value

The object of the present investigation was to determine (*a*) the molecular weights of free phthalocyanine and its derivatives of copper, lithium, chloro-chloraluminium and silver in sulphuric acid by the cryoscopic method, (*b*) the molecular weight of dilithium phthalocyanine by the ebullioscopic method in absolute ethyl alcohol as the solvent

Sulphuric acid has been found to be very suitable for this purpose as it is a good solvent for the phthalocyanines It has a high dielectric constant—greater than 84 The complications due to inter-ionic forces and ion-association are of considerably smaller magnitude In this connection references may be made to the valuable work done by Hantzsch, Oddo and Scandola, Conant and Werner, and by Hamet *et al* on the determination of molecular weights of a large number of organic and inorganic substances by cryoscopic methods with sulphuric acid as a solvent

Section A deals with the determination of molecular weights of free phthalocyanine, copper phthalocyanine, dilithium phthalocyanine, silver phthalocyanine and chloro-aluminium chloro-phthalocyanine by the cryoscopic method in sulphuric acid

Section B deals with the determination of molecular weight of dilithium phthalocyanine in absolute alcohol by ebullioscopic method

Section A

EXPERIMENTAL

Reagents

Sulphuric acid Sulphuric acid (A R quality) was distilled in an all-pyrex distillation set under a stream of dry CO_2 The acid distilled over without decomposition at 318°C -319°C at atmospheric pressure (690 mm of Hg) The distillate was stored in an air-tight bottle in a desiccator The fuming sulphuric acid was prepared by heating 33% oleum in the distillation flask and absorbing the evolved SO_3 into distilled H_2SO_4 This fuming acid was also preserved in an air-tight bottle inside a desiccator

Free phthalocyanine and its various derivatives were prepared according to the methods of Linstead *et al*

$KHSO_4$ Merck's A R quality $KHSO_4$ was used in this investigation

Apparatus and Experimental Procedure.

For determining the freezing point Beckmann apparatus was used with the following modifications to prevent absorption of moisture (1) The rubber and cork stoppers were replaced by ground joints *A* and *B* as shown in Fig 1, (2) For stirring the following simple arrangement was made A rubber tube is placed over the side tube *C* and another glass tube passed through it A slightly thinner glass rod is introduced into the latter tube and connected to the platinum stirrer by means of a flexible wire as shown in Fig 1 The stirrer was operated by hand There is little friction and the sealing was almost complete. This was further protected by another glass tubing whenever stirrer was not in operation For recording the freezing point a Beckmann thermometer reading to 1/100° was used The thermometer was first of all set at about 10 5°C to approximate to the freezing point of absolute sulphuric acid

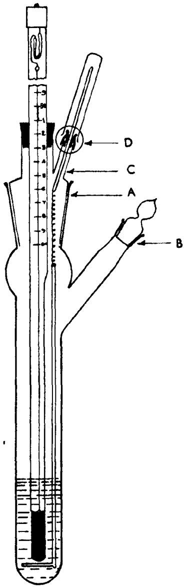


FIG 1

During the cryoscopic measurements, (*a*) the temperature of the cooling was not allowed to exceed 2° to 3°, (*b*) the stirrer was operated as uniformly as possible; (*c*) the thermometer was always tapped before taking a reading

Stock sulphuric acid was prepared by mixing fuming acid with the distilled acid in a ratio calculated to give slightly less than 100% H_2SO_4 and it was stored in an automatic sealing tube of the type used by Oddo and Scandola (Fig 2) This

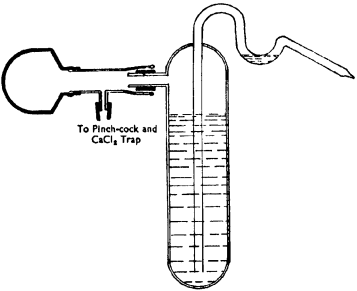


FIG 2

acid was then forced out into the freezing point tube up to a definite mark The freezing point of the acid was next adjusted to the desired point by adding to it small quantities of fuming H_2SO_4 by means of a pipette The total quantity of H_2SO_4 was found out by weighing the freezing point tube before and after addition of acid

There is a marked change in the freezing point of H_2SO_4 on either side of the maximum point which corresponds to absolute H_2SO_4 with freezing point of 10 5°C The freezing point of sulphuric acid of composition, either more or less than 100%, is less than 10 5°C After determining the freezing point (F P) of the solvent the phthalocyanine or its derivative was added, and the lowering of the freezing point produced was measured

Determination of the cryoscopic constants of the solvents used

In the present investigation two types of solvent were used —

(a) Sulphuric acid containing about 0 1% water, the freezing point being 0 11°C less than the maximum

(b) Sulphuric acid containing about 1% water, the freezing point being 4·15°C less than the maximum

Before determining the molecular weights of the phthalocyanines, the cryoscopic constant K was determined for the solvents by observing freezing point depressions of $KHSO_4$ at different dilutions. Taking Vant Hoff's factor $i = 2$, the molecular weight of $KHSO_4$ comes down to 68 Assuming this value of the molecular weight of $KHSO_4$ in solution, the cryoscopic constants K were calculated from the formula

$$K = \frac{M\ \delta t\ W}{\omega},$$

where $M = 68$, δt = observed freezing point depression, W = amount of sulphuric acid in grams, and ω = amount of $KHSO_4$ in grams

Graphs were drawn taking K as ordinates and the concentrations of $KHSO_4$ as abscissae (Fig 3) and from the graphs, K for zero concentration of $KHSO_4$ was extrapolated The results are tabulated in Tables I and II

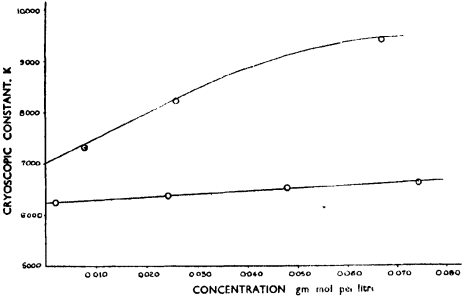


FIG 3

Cryoscopic constant K for sulphuric acid containing about 0 1% water

TABLE I

Volume of acid taken = 20 4 c c W = 37 4 grams

ω (grams)	$C\ 10^5$ (Mols)	F P (Beckmann reading)	δt (°C)	K	K (extrapolated)
		0 220			6200
0·0049	177	0 232	0 012	6230	
0 0670	2420	0 388	0 168	6380	
0 1826	4780	0 559	0 339	6500	
0 2072	7450	0 758	0 538	6610	

Cryoscopic constant for sulphuric acid containing about 1% *water*

TABLE II

Volume of acid taken = 29 8 c c W = 54 4 grams

w (grams)	C 10⁵ (Mol)	F P (Beckmann reading)	δt (°C)	K	K (extrapolated)
		4 279			7000
0 0283	713	4 335	0 056	7320	
0 1663	4104	4 697	0 418	9290	
0 2737	6740	4 977	0 698	9450	

The experimental results on the molecular weights of free phthalocyanine and its various derivatives are recorded in Tables III and IV The molecular weight was calculated from the relation $M = \frac{K w}{\Delta t W}$.

TABLE III

F P of acid used = 0 11° below the maximum F P of absolute sulphuric acid, cryoscopic constant K = 6,200 (extrapolated) M = Molecular weight calculated according to the formula, M_{obs} = Molecular weight found experimentally

Substance	M_{calc}	W gms	w gms	C 10⁵ (Mol)	t (°C)	M_{obs}	i
Free Phthalocyanine							
($C_{32}H_{18}N_8$)	514	33 25	0 1002	1074	0 177	105 5	4 9
do do	514	33 05	0 1098	1176	0 199	102 9	5 0
Copper Phthalocyanine							
($C_{32}H_{16}N_8Cu$)	576	32 65	0 1108	1078	0 170	123 8	4 7
do do	576	32 45	0 1254	1228	0 196	122 3	4 7
Silver Phthalocyanine							
($C_{32}H_{16}N_8Ag$)	619	30 00	0 1578	1549	0 500	64 9	9 5
do do	619	32 90	0 1476	1326	0 452	61 5	10 0
Chloraluminium Chloro phthalocyanine							
($C_{32}H_{15}N_8Cl$ AlCl $2H_2O$)	644	37 25	0 1316	1005	0 543	40 3	16 0

TABLE IV

F P of acid used = 4 15° below the maximum F P of absolute sulphuric acid, cryoscopic constant K = 7,000 (extrapolated)

Substance	M_{calc}	W grams	w grams	C 10⁵ (Mol)	t (°C)	M_{obs}	i
Free Phthalocyanine ($C_{32}H_{18}N_8$)	514	38 7	0 1068	988	0 285	67 8	7 6
do do	514	38 7	0 1137	1076	0 291	70 7	7 3
Copper Phthalocyanine ($C_{32}H_{16}N_8Cu$)	576	36 4	0 0886	778	0 222	76 7	7 5
do do	576	36 5	0 0980	858	0 252	74 6	7 7
Dilithium Phthalocyanine ($C_{32}H_{16}N_8Li_2$)	526	37 6	0 0995	908	0 470	39 4	13 4
do do	526	37 3	0 1176	1108	0 542	40 7	13 0
Silver Phthalocyanine ($C_{32}H_{16}N_8Ag$)	619	38 8	0 1178	902	0 353	60 2	10 3
do do	619	39 0	0 1334	1016	0 381	62 8	9 9
Chloraluminium Chloro phthalocyanine ($C_{32}H_{16}N_8Cl\ AlCl\ H_2O$)	644	36 7	0 1122	874	0 550	39 0	16 5
do do	644	36 8	0 0922	700	0 451	38 7	16 0

Section B

Reagents —Pure dilithium phthalocyanine prepared according to the method described before, and pure and crystallised benzoic acid were used As a solvent absolute alcohol distilled over metallic calcium, and diethyl phthalate was used

Apparatus and Method of Experimental Procedure

For the determination of boiling point Landsberger's method, as modified by Walker and Lumsden, was used The thermometer used in these experiments was graduated to 1/100° When the thermometer registered a constant temperature, the reading was taken as the boiling point of the pure solvent Next the molecular weight of a simple substance, like benzoic acid, was determined in absolute alcohol to test the accuracy of the method

The molecular weight of dilithium phthalocyanine was determined in the same way and the results are recorded in Table V

TABLE V

k' = 1560 0

Substance	θ (°C)	ω gms	θ_1 (°C)	e (°C)	v (c c)	M Obs	M Calc
Benzoic acid	75 50	0 5	76 10	0 60	11 0	118 2	122 0
Dilithium phthalocyanine	75 50	0 5	75 67	0 17	9 25	496 1	525 9
,,	75 50	0 5	75 64	0 14	11 5	484 0	525 9

θ = boiling point of absolute alcohol , θ_1 = boiling point of solution , ω = weight of the solute , e = elevation of the boiling point , v = volume of the solution , M_{obs} = molecular weight found experimentally, and M_{calc} = molecular weight calculated according to the formula

From the above table we can see that the molecular weight, as determined by experiment in the solvent of absolute alcohol, is in good agreement with the theoretical value

DISCUSSION

Section A

The cryoscopic measurement of Hantzsch in sulphuric acid as a solvent with compounds, organic as well as inorganic, containing N atoms not linked to oxygen atoms, led him to the following conclusions —

The nitrogen bearing compounds will dissolve in sulphuric acid with complete salt formation and forming a polyvalent ion depending on the number of N atoms in the compound This is demonstrated by the lowering of the freezing point and the molecular weight corresponding to an increase in the number of ions in the solution

Thus

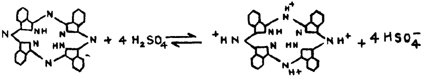
No of N atoms in the compound	1	2	3	4	5
Maximum No of ions	2	3	4	5	6

Accordingly the number of ions formed in sulphuric acid solution can be estimated and thus one can know how many N atoms of the compound take part in salt formation Hantzsch has studied many compounds including basic, neutral and even acidic substances like NH_3, HNO_2, HNO_3, and benzoic acid

The experimental results of Tables III and IV may be explained schematically as outlined below

(1) *Free Phthalocyanine*

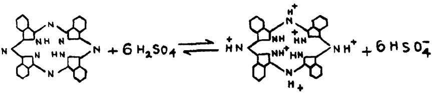
(*a*) With sulphuric acid containing 0 1% water



It follows from the scheme that $n = 5$ which is in good agreement with experimental values If we assume complete dissociation as, we could do logically, at such low concentration in a solvent with so high a dielectric strength, i which is equal to $1+(n-1)\alpha$ becomes $1+(n-1)$, that is, $i = n$

(*b*) With sulphuric acid containing 1 0% water

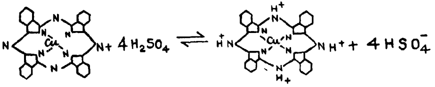
The two central nitrogen atoms (which are not imino), with one pair of electrons, take up two more protons under changed condition of the solvent as



Hence i should be 7 which has been experimentally observed From this it seems evident that the inner two co-ordinating nitrogen atoms also take up protons in the presence of small quantities of water, whose function is difficult to explain at present

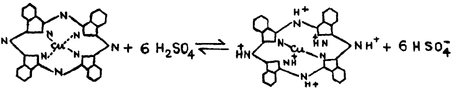
(2) *Copper Phthalocyanine*

(*a*) With 0 1% water in the solvent



Hence *i* should be 5 which has been experimentally observed

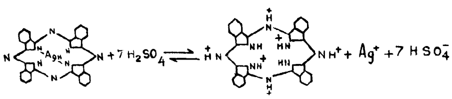
(*b*) With 1 0% water in the solvent



Here also we observe similar results as in the case of free phthalocyanine, that is, *i* = 7.

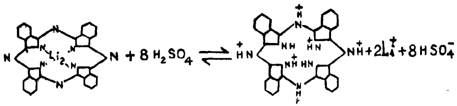
In other cases the same reaction scheme may be postulated with sulphuric acid containing 0 1% as well as 1 0% water

(3) *Silver Phthalocyanine*



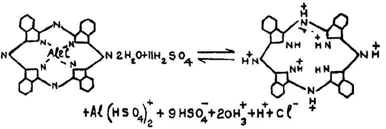
which should give *i* = 9, which has been found to agree fairly well with the experimental value

(4) *Dilithium Phthalocyanine*



Here *i* = 11 whereas the experimental values are in the neighbourhood of 13 This discrepancy may be due to experimental error.

(5) *Chloro-aluminium Chloro phthalocyanine*



Here $i = 15$ which is in fair agreement with the experimental value of 16

Section B

The agreement between the theoretical and the observed value is quite satisfactory and evidently the molecule of dilithium phthalocyanine remains undissociated in absolute alcohol

Our thanks are due to Dr T L Rama Char for carrying out some preliminary experiments

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ON STATIONARY LINE ELEMENTS

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(Communicated by Prof V V Narlikar, F N I)

(*Received September 3 , read October 18, 1946*)

ABSTRACT

A set of necessary and sufficient conditions for a spherically symmetrical line element to be stationary is obtained and a method is given of transforming it to a static form when the conditions are satisfied A new proof of Birkhoff's theorem is given

1 INTRODUCTION

Consider a static line element, $ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$

This may be transformed to a non static form by an arbitrary non singular transformation of co-ordinates Line elements obtained in this way may be called stationary as is the usual practice It is the object of this paper to investigate the necessary and sufficient conditions for a non static line-element to be stationary Only spherically symmetrical line elements have been considered

2 NECESSARY CONDITIONS

A general spherically symmetrical static line-element is given by

$$ds^2 = -A\,dr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + D\,dt^2 + 2C\,dr\,dt, \tag{1}$$

where A, B, C, D are functions of r alone By the successive transformations

$$\bar{r}^2 = B, \quad \bar{t} = t \tag{2 1}$$

and

$$\left.\begin{aligned} \bar{r} &= \rho, \\ \bar{t} &= \int -\frac{C}{D}\, d\rho + k\tau, \end{aligned}\right\} \tag{2 2}$$

we get

$$ds^2 = -\bar{A}\,d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta\, d\phi^2) + \bar{D}\,d\tau^2, \tag{3}$$

where $\bar{A}$ and $\bar{D}$ are functions of ρ alone

Hence there is no loss of generality in taking the general spherically symmetrical line-element in the form (3)

To get the necessary conditions let it be assumed that the line element,

$$ds^2 = -A\,dr^2 - B(d\theta^2 + \sin^2\theta\, d\phi^2) + D\,dt^2 + 2C\,dr\,dt,$$

where A, B, C, D are functions of r and t, is transformable into

$$ds^2 = -\bar{A}\,d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta\, d\phi^2) + \bar{D}\,d\tau^2 \tag{4}$$

where $\bar{A}$ and $\bar{D}$ are functions of ρ alone

The law of transformation of tensors gives—

$$\bar{A} = A\left(\frac{\partial r}{\partial \rho}\right)^2 - D\left(\frac{\partial t}{\partial \rho}\right)^2 - 2C\frac{\partial r}{\partial \rho}\frac{\partial t}{\partial \rho}, \tag{5.1}$$

$$\rho^2 = B, \tag{5.2}$$

$$\bar{D} = D\left(\frac{\partial t}{\partial \tau}\right)^2 - A\left(\frac{\partial r}{\partial \tau}\right)^2 + 2C\frac{\partial r}{\partial \tau}\frac{\partial t}{\partial \tau}, \tag{5.3}$$

$$0 = C\left(\frac{\partial r}{\partial \rho}\frac{\partial t}{\partial \tau} + \frac{\partial r}{\partial \tau}\frac{\partial t}{\partial \rho}\right) + D\frac{\partial t}{\partial \rho}\frac{\partial t}{\partial \tau} - A\frac{\partial r}{\partial \rho}\frac{\partial r}{\partial \tau} \tag{5.4}$$

Differentiating (5.2) with respect to ρ and τ respectively we get

$$1 = \frac{B'}{2B^{1/2}}\frac{\partial r}{\partial \rho} + \frac{B}{2B^{1/2}}\frac{\partial t}{\partial \rho}, \tag{5.5}$$

$$0 = B'\frac{\partial r}{\partial \tau} + B\frac{\partial t}{\partial \tau}, \tag{5.6}$$

a dot representing a differentiation with regard to t and a dash representing a differentiation with regard to r

Solving (5.3) to (5.6) we get

$$\frac{\partial r}{\partial \rho} = M, \quad \frac{\partial t}{\partial \rho} = N, \quad \frac{\partial r}{\partial \tau} = R\bar{F}, \quad \frac{\partial t}{\partial \tau} = S\bar{F}, \tag{6.1}$$

where

$$M = (DB' - CB) \quad 2B^{1/2}(DB'^2 - 2CBB' - AB^2)^{-1} \tag{6.2}$$

$$N = -(AB + CB') \quad 2B^{1/2}(DB'^2 - 2CBB' - AB^2)^{-1} \tag{6.3}$$

$$R = B(DB'^2 - 2CBB' - AB^2)^{-\frac{1}{2}} \tag{6.4}$$

$$S = -B'(DB'^2 - 2CBB' - AB^2)^{-\frac{1}{2}}, \tag{6.5}$$

$$\bar{F} = \bar{D}^{1/2} \tag{6.6}$$

Differentiating $\frac{\partial r}{\partial \rho}$ with respect to τ and $\frac{\partial r}{\partial \tau}$ with respect to ρ and equating we obtain

$$\left[(RM' + MS) - (R'M + RN)\right]\bar{F} - R\frac{d\bar{F}}{d\rho} = 0 \tag{7.1}$$

Similarly, differentiating $\frac{\partial t}{\partial \tau}$ with respect to ρ and $\frac{\partial t}{\partial \rho}$ with respect to τ and equating we obtain

$$\left[(RN' + NS) - (S'M + SN)\right]\bar{F} - S\frac{d\bar{F}}{d\rho} = 0 \tag{7.2}$$

Calculation shows that

$$\begin{aligned}\frac{(RM' + MS) - (R'M + RN)}{R} &= \frac{(RN' + NS) - (S'M + SN)}{S} \\ &= \frac{B^{\frac{1}{2}}}{(mB' - nB)^2}\Big[(-m^2B'' + 2mnB' - n^2B) \\ &\qquad + B'(mm' + nm - 2\dot{n}m) \\ &\qquad + B(n\dot{n} + n'm - 2nm')\Big],\end{aligned} \tag{8.1}$$

where

$$m = (DB'-CB),\ n = (CB'+AB) \tag{8 2}$$

Thus (7 1) and (7 2) reduce to one independent equation From (7 1) we obtain

$$\frac{\frac{d\overline{F}}{d\rho}}{\overline{F}} = \frac{(RM'+MS)-(R'M+RN)}{R} \quad . \tag{9}$$

Since $\frac{d\overline{F}}{d\rho}\Big/\overline{F}$ is by hypothesis a function of ρ alone, it follows from (9) and (5 2) that

$$\frac{(RM'+MS)-(R'M+RN)}{R} = \alpha\ (B^{\frac{1}{2}}), \tag{10 1}$$

where α is arbitrary Thus we have obtained one of the necessary conditions By substituting the values of

$$\frac{\partial r}{\partial \rho},\ \frac{\partial t}{\partial \rho}$$

from (6 1) in (5 1) we obtain

$$\bar{A} = AM^2-2CMN-DN^2 \tag{11}$$

Since by hypothesis $\bar{A}$ is a function of ρ alone we obtain from (11) and (5 2)

$$AM^2-2CMN-DN^2 = \beta(B^{\frac{1}{2}}) \tag{10 2}$$

as the second necessary condition (10 1) and (10 2) is a set of necessary conditions (Vaidya, 1945)

3 SUFFICIENCY OF NECESSARY CONDITIONS

The conditions obtained in the last section will now be shown to be sufficient For the line-element,

$$ds^2 = -Adr^2-B(d\theta^2+\sin^2\theta d\phi^2)+Ddt^2+2Cdrdt,$$

where A, B, C, D are functions of r and t, let it be assumed that

$$\frac{(RM'+MS)-(R'M+RN)}{R} = \alpha(B^{\frac{1}{2}}) \tag{13 1}$$

and,

$$AM^2-2CMN-DN^2 = \beta(B^{\frac{1}{2}}),$$

where M, N, R, S have been defined by (6 2) to (6 6) Define a function $\overline{F}\ (x)$ by

$$\frac{d\overline{F}}{dx}\Big/\overline{F} = \alpha(x) \quad . \tag{14}$$

Next consider a transformation defined by

$$\frac{\partial r}{\partial \rho} = M\ ,\ \frac{\partial t}{\partial \rho} = N\ ,\ \frac{\partial r}{\partial \tau} = R\overline{F}(B^{\frac{1}{2}})\ ,\ \frac{\partial t}{\partial \tau} = S\overline{F}(B^{\frac{1}{2}}) \tag{15}$$

It can be easily verified that

$$\frac{B'}{2B^{\frac{1}{2}}}\ \frac{\partial r}{\partial \rho}+\frac{B}{2B^{\frac{1}{2}}}\ \frac{\partial t}{\partial \rho} = 1, \tag{16 1}$$

$$B'\frac{\partial r}{\partial \tau}+B\frac{\partial t}{\partial \tau} = 0\ , \tag{16 2}$$

these give

$$\rho^2 = B \tag{17}$$

The transformation is consistent as can be verified by differentiating $\frac{\partial r}{\partial \rho} = M$ with respect to τ and $\frac{\partial r}{\partial \tau} = N$ with respect to ρ and equating the two values of $\frac{\partial^2 r}{\partial \rho\, \partial \tau}$ so obtained This equality is ensured by our definition of $\overline{F}$ The law of transformation of tensors gives the transformed line element as

$$ds^2 = -\beta(\rho)d\rho^2 - \rho^2(d\theta^2 + \sin^2\theta d\phi^2) + (\overline{F})^2 d\tau^2, \tag{18}$$

where $\overline{F}$ is given by (14) This is clearly seen to be a static line element Thus we have shown that the conditions are sufficient

4 PROOF OF BIRKHOFF'S THEOREM

Consider a line-element

$$ds^2 = -A\,dr^2 - B(d\theta^2 + \sin^2\theta d\phi^2) + D dt^2, \tag{19}$$

where A, B, D are functions of r and t and which is a solution of Einstein's field equations for empty space Hence we have

$$T_\nu^\mu = 0 \tag{20}$$

T_1^1, T_4^1, T_4^4 equated to zero give (Tolman, 1934*a*)

$$B = BD\left[\frac{1}{2}\frac{BD}{BD^2} + \frac{1}{4}\frac{B^2}{DB^2} + \frac{1}{4}\frac{B'^2}{AB^2} + \frac{1}{2}\frac{B'D'}{ABD} - \frac{1}{B}\right] \tag{21 1}$$

$$B' = \frac{B}{2}\left[\frac{BB'}{B^2} + \frac{AB'}{AB} + \frac{D'B}{DB}\right], \tag{21 2}$$

$$B'' = AB\left[\frac{1}{4}\frac{B'^2}{AB^2} + \frac{1}{2}\frac{B'A'}{BA^2} + \frac{1}{2}\frac{AB}{ABD} + \frac{1}{4}\frac{B^2}{B^2D} + \frac{1}{B}\right] \tag{21 3}$$

The necessary and sufficient conditions become for this line-element,

$$\begin{aligned}\frac{B^{\frac{1}{2}}}{(DB'^2 - AB^2)^2}\big[-2AD\{B''B^2 - 2B'BB' + B\ B'^2\}\\ +B'\{B'^2DD' + BB'(AD - 2AD)\}\\ +B\{BB'(A'D - 2AD') + B^2AA\}\big] = \alpha(B^{\frac{1}{2}}),\end{aligned} \tag{22 1}$$

$$\frac{4ADB}{DB'^2 - AB^2} = \beta(B^{\frac{1}{2}}) \tag{22.2}$$

On substituting the values of B'', B' and B given by equations (21) in the following

$$\frac{\partial}{\partial r}\left[\frac{4ADB}{DB'^2 - AB^2}\right]B - \frac{\partial}{\partial t}\left[\frac{4ADB}{DB'^2 - AB^2}\right]B' = 0, \tag{23}$$

we find that it becomes an identity Hence (22 2) is satisfied On substituting the values of B'', B', B in the left-hand side of (22 1) we find that it reduces to

$$-\frac{B^4}{2B}\left(1-\frac{4ADB}{DB'^2-AB^2}\right) \tag{24}$$

Hence (22 1) is also satisfied

Thus the necessary and sufficient conditions being satisfied all solutions of Einstein's field-equations for empty space which are spherically symmetrical and which are of the form

$$ds^2 = -A\,dr^2-B(d\theta^2+\sin^2\theta d\phi^2)+Ddt^2,$$

are stationary Thus another proof of Birkhoff's theorem has been given (Tolman, 1934*b*, Einstein and Straus, 1946)

5 CONCLUDING REMARKS

In the course of some recent investigation it was necessary for us to verify whether the following line-element is stationary

$$ds^2 = -(A+Br^2)^{-2}(dx^2+dy^2+dz^2)-\frac{(A+Br^2)^2}{(A+Br^2)^2}\,\frac{1}{4AB}\,dt^2, \tag{25}$$

where A and B are arbitrary functions of t The above set of conditions (10) enabled us to show that it is stationary and the transform (3) for this reveals that it represents flat space time The results obtained here should be of use in investigations of spherical distributions where it is necessary to know whether a non static line-element is stationary and if so to what static form it is transformable

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ON THE SUMMABILITY OF THE CONJUGATE SERIES OF A FOURIER SERIES BY LOGARITHMIC MEANS

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(Communicated by Dr. B. N. Prasad.)

(*Received October 10, read October 18, 1946.*)

1. Let

$$\sum_{n=1}^{\infty}(b_n\cos n\theta-a_n\sin n\theta) \tag{1.1}$$

be the conjugate series of the Fourier series associated with a function $f(\theta)$ which is integrable (L) over the interval $(-\pi, \pi)$ and defined outside by periodicity. The 'conjugate' function associated with the series (1.1) is

$$\bar{f}(\theta)=\frac{1}{2\pi}\int_0^{\pi}\{f(\theta+t)-f(\theta-t)\}\cot\frac{1}{2}t\,dt, \tag{1.2}$$

the integral being a Cauchy integral at the origin.

Prasad* proved that if the function $f(\theta)$ is bounded, the conjugate series (1.1) is summable (C, δ) for every positive δ to $\bar{f}(\theta)$ wherever the integral (1.2) exists. This theorem of Prasad was in a certain sense extended by Hardy and Littlewood † in the form of a simple necessary and sufficient condition for the Cesaro summability of the conjugate series corresponding to a bounded function as follows:—

Suppose that $f(t)$ is bounded in the neighbourhood of $t=0$. Then the conjugate series of $f(t)$, for $t=0$, is either summable by Cesaro means of every positive order or summable by no Cesaro mean. A necessary and sufficient condition for the summability is the convergence of the integral (1.2).

Now there are simple bounded functions for which the integral (1.2) is not convergent. For example, let $\theta=0$ and let an odd function $f(t)$ be defined in $(0, \pi)$ by

$$\begin{cases} f(0)=0 & \\ f(t)=\dfrac{1}{t}\tan\dfrac{1}{2}t\sin\left(\log\dfrac{1}{t}\right), & \text{for } 0<t\leq 1, \\ \quad =0, & \text{for } 1<t\leq\pi, \end{cases} \tag{1.3}$$

and by periodicity elsewhere. Then the integral for $\bar{f}(\theta)$ oscillates between 0 and $\frac{2}{\pi}$ in the neighbourhood of $t=0$ and the conjugate series for this function $f(t)$ is not summable (C). This series is, however, summable by Riesz's logarithmic means of order unity as we shall see later.

Definition.—A series Σc_n is said to be summable by Riesz's logarithmic means of order $k>0$, or summable (R, k), to the sum s, provided that

$$R_k(w)=\frac{1}{(\log w)^k}\sum_{n<w}\left(\log\frac{w}{n}\right)^k c_n \tag{1.4}$$

tends to a limit s as $w\to\infty$.

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* Prasad, 4.

† Hardy and Littlewood, 3.

The problem that naturally suggests itself is to find some general theorem, concerning Rieszian summability of the conjugate series of a bounded function, which may be of the same type as the theorem of Hardy and Littlewood given above. Theorem A, which we prove below, covers a class of functions much wider than that of bounded functions and is quite of the type desired. In what follows we use logarithmic integral means which are more general than fractional integral means of a function.

Let us write

$$\psi(t) = \frac{1}{2}\{f(\theta+t)-f(\theta-t)\},\quad g(t) = \frac{1}{\pi}\int_t^{\pi} \psi(t)\cot\frac{1}{2}t\,dt - s,$$

$$g_\alpha(t) = \frac{1}{\Gamma(\alpha)}\int_t^{\pi}\left(\log\frac{u}{t}\right)^{\alpha-1}\frac{g(u)}{u}\,du,\quad \alpha>0,$$

$$g_0(t) = g(t)$$

It is known[*] that

$$g_{\alpha+\beta}(t) = \frac{1}{\Gamma(\beta)}\int_t^{\pi}\left(\log\frac{u}{t}\right)^{\beta-1} g_\alpha(u)\,\frac{du}{u},\quad \beta>0$$

Accordingly we have

$$g_{\alpha+1}(t) = \int_t^{\pi} g_\alpha(u)\,\frac{du}{u},$$

and we define

$$g_\alpha(t) = -t\frac{d}{dt}g_{\alpha+1}(t),\ \text{for } -1\leq\alpha<0,$$

so that

$$g_{-1}(t) = \frac{1}{\pi}\,t\cot\frac{1}{2}t\,\psi(t)$$

Theorem A *Let*

$$(1.5)\qquad \int_0^t |\,g_{\alpha-1}(t)\,|\,dt = O\left\{t\left(\log\frac{1}{t}\right)^{\alpha+1}\right\},\quad \alpha>0,$$

and

$$(1.6)\qquad g_\alpha(t) = o\left\{\left(\log\frac{1}{t}\right)^{\alpha+1}\right\},$$

as $t\to 0$. *Then a necessary and sufficient condition that the conjugate series* (1.2) *be summable* $(R, \alpha+1)$ *for* $t=\theta$ *to the sum* s *is that*

$$(1.7)\qquad g_{\alpha+1}(t) = o\left\{\left(\log\frac{1}{t}\right)^{\alpha+1}\right\},$$

as $t\to 0$

An analogous theorem for Fourier series was given by Wang[†] for integral α. He has also shown[‡] that if the condition (1.7) is satisfied, the conjugate series is summable $(R, \alpha+2)$ and conversely if the conjugate series is summable $(R, \alpha+1)$ to the sum s, then

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[*] Wang, 7

[†] Wang, 6.

[‡] Wang, 8

$$g_{\alpha+2}(t) = o\left\{\left(\log\frac{1}{t}\right)^{\alpha+2}\right\},$$

as $t \to 0$.

The case $\alpha = 0$ of Theorem A gives a simple, elegant result, namely

Theorem B *Let*

$$\int_0^t |\psi(u)|\, du = O\left(t \log\frac{1}{t}\right), \tag{1 8}$$

and

$$g(t) = o\left(\log\frac{1}{t}\right), \tag{1 9}$$

as $t \to 0$ *Then a necessary and sufficient condition that the conjugate series* (1 2) *be summable* $(R, 1)$ *for* $t = \theta$ *to the sum* s *is that*

$$\int_t^\pi \frac{g(u)}{u} du = o\left(\log\frac{1}{t}\right), \tag{1 10}$$

as $t \to 0$

The condition (1 8) is satisfied wherever $\psi(t) = O\left(\log\frac{1}{t}\right)$ and in particular when $f(t)$ is bounded near $t = 0$, and the condition (1 9) holds in particular when $g(t)$ is bounded

A result for Fourier series analogous to the Theorem B was given by Hardy and generalised by Takahasi and by Bosanquet and Offord * Theorem B can also be deduced from a theorem of Bosanquet and Offord

In § 3 we find a necessary and sufficient condition for the summability $(R, \alpha+1)$ of the conjugate series In § 4 we prove Theorem A We show in § 5 that the conjugate series for the function $f(t)$ defined by (1 3) is summable $(R, 1)$ to the sum $\frac{1}{\pi}$ although, as previously remarked, it is not summable (C) In § 6, we construct an example to show that the holding of only one of the conditions (1 8) and (1 9), namely

$$g(t) = o\left(\log\frac{1}{t}\right) \neq o(1)$$

is not sufficient to ensure the summability $(R, 1)$ of the conjugate series

I am much indebted to Dr B N Prasad for his kind interest and advice in the preparation of this paper

2 We shall make use of functions $S_k(t)$ defined by

$$S_k(t) = \int_0^1 \left(\log\frac{1}{u}\right)^k \sin tu\, du = \frac{1}{t}\int_0^t \left(\log\frac{t}{u}\right)^k \sin u\, du,\ k > -1$$

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* Hardy, 2; Takahasi, 5; Bosanquet and Offord, 1

It is known that *

$$\frac{d}{dt}\left\{tS_k(t)\right\} = kS_{k-1}(t),\ k>0, \tag{2 1}$$

$$S_{r+s+1}(t) = \frac{\Gamma(r+s+2)}{\Gamma(r+1)\Gamma(s+1)}\int_0^1 S_s(ut)\left(\log\frac{1}{u}\right)^r du,\ r>-1,\ s>-1 \tag{2 2}$$

We shall also require the following lemma —

Lemma *For* $\alpha>0$,

$$\int_0^\pi g(t)S_\alpha(\omega t)\,dt = \Gamma(\alpha+1)\int_0^\pi g_\alpha(t)S_0(\omega t)\,dt$$

$$= \frac{\Gamma(\alpha+1)}{\omega}\int_0^\pi g_\alpha(t)\frac{1-\cos\omega t}{t}\,dt$$

This has also been shown by Wang †

3 We may make the usual simplifications, supposing that $f(t)$ is odd and that $\theta=0$, $s=0$ so that

$$\psi(t)=f(t) \text{ and } g(t)=\frac{1}{\pi}\int_t^\pi f(t)\cot\frac{1}{2}t\,dt$$

We have

$$s_0(\omega)=\sum_{n<\omega} b_n = \frac{1}{\pi}\int_0^\pi f(t)\cot\frac{1}{2}t(1-\cos\omega t)\,dt+o(1)$$

$$=\left[-(1-\cos\omega t)g(t)\right]_0^\pi+\omega\int_0^\pi g(t)\sin\omega t\,dt+o(1)$$

$$=\omega\int_0^\pi g(t)\sin\omega t\,dt+o(1),$$

as $\omega\to\infty$ And for $k>0$,

$$s_k(\omega)=\sum_{n<\omega}\left(\log\frac{\omega}{n}\right)^k b_n = k\int_1^\omega\left(\log\frac{\omega}{n}\right)^{k-1} s_0(x)\frac{dx}{x}$$

$$=k\int_1^\omega\left(\log\frac{\omega}{x}\right)^{k-1}dx\int_0^\pi g(t)\sin xt\,dt+o\{(\log\omega)^k\}$$

$$=k\int_0^\pi g(t)\,dt\int_1^\omega\left(\log\frac{\omega}{x}\right)^{k-1}\sin xt\,dx+o\{(\log\omega)^k\}$$

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* Wang, 7

† Wang, 8

$$= k\int_0^\pi g(t)\,dt\int_0^\omega \left(\log\frac{\omega}{x}\right)^{k-1}\sin xt\,dx - k\int_0^\pi g(t)\,dt\int_0^1\left(\log\frac{\omega}{x}\right)^{k-1}\sin xt\,dx$$

$$+o\{(\log\omega)^k\}$$

$$= k\omega\int_0^\pi g(t)S_{k-1}(\omega t)\,dt + O\left\{(\log\omega)^{k-1}\int_0^\pi |\,g(t)\,|\,dt\right\} + o\{(\log\omega)^k\}$$

$$= k\omega\int_0^\pi g(t)S_{k-1}(\omega t)\,dt + o\{(\log\omega)^k\},$$

as $\omega\to\infty$, $g(t)$ being integrable (L) in $(0,\pi)$ Hence

$$R_k(\omega) = \frac{1}{(\log\omega)^k}s_k(\omega) = \frac{k\omega}{(\log\omega)^k}\int_0^\pi g(t)S_{k-1}(\omega t)\,dt + o(1)$$

Putting $k = \alpha+1$, we have by the lemma

$$R_{\alpha+1}(\omega) = \frac{(\log\omega)^{\alpha+1}}{(\alpha+1)\omega}\int_0^\pi g(t)S_\alpha(\omega t)\,dt + o(1)$$

$$= \frac{\Gamma(\alpha+2)}{(\log\omega)^{\alpha+1}}\int_0^\pi g_\alpha(t)\frac{1-\cos\omega t}{t}\,dt + o(1)$$

Thus the necessary and sufficient condition that the conjugate series be summable $(R, \alpha+1)$ *at* $t=\theta$ *to the sum* s *is that*

$$\int_0^\pi g_\alpha(t)\frac{1-\cos\omega t}{t}\,dt = o\{(\log\omega)^{\alpha+1}\}, \tag{3 1}$$

as $\omega\to\infty$

4 *Proof of Theorem A*

(i) To prove that the condition is sufficient, we have

$$\int_0^\pi g_\alpha(t)\frac{1-\cos\omega t}{t}\,dt = \int_0^{\lambda/\omega} g_\alpha(t)\frac{1-\cos\omega t}{t}\,dt + \int_{\lambda/\omega}^\pi \frac{g_\alpha(t)}{t}\,dt - \int_{\lambda/\omega}^\pi \frac{g_\alpha(t)}{t}\cos\omega t\,dt$$

$$= I+J-K, \text{ say,} \tag{4 1}$$

where λ is large but fixed Now by (1 6)

$$I = O\left(\omega\int_0^{\lambda/\omega}|\,g_\alpha(t)\,|\,dt\right) = o\left\{\omega\ \frac{\lambda}{\omega}\left(\log\frac{\omega}{\lambda}\right)^{\alpha+1}\right\} = o\{(\log\omega)^{\alpha+1}\}, \tag{4 2}$$

as $\omega\to\infty$ By (1.7)

$$J = o\left\{\left(\log\frac{\omega}{\lambda}\right)^{\alpha+1}\right\} = o\{(\log\omega)^{\alpha+1}\} \tag{4.3}$$

Also $g_\alpha(t)$ being an integral for $\lambda/\omega \leq t \leq \pi$, we have

$$K = \int_{\lambda/\omega}^{\pi} g_\alpha(t)\frac{\cos \omega t}{t}\,dt = \frac{1}{\omega}\left[g_\alpha(t)\frac{\sin \omega t}{t}\right]_{\lambda/\omega}^{\pi} - \frac{1}{\omega}\int_{\lambda/\omega}^{\pi} \sin \omega t\left[\frac{d}{dt}\frac{g_\alpha(t)}{t}\right]dt$$

$$= -\frac{1}{\lambda}\sin \lambda g_\alpha(\lambda/\omega) + \frac{1}{\omega}\int_{\lambda/\omega}^{\pi} g_\alpha(t)\frac{\sin \omega t}{t^2}\,dt + \frac{1}{\omega}\int_{\lambda/\omega}^{\pi} g_{\alpha-1}(t)\frac{\sin \omega t}{t^2}\,dt$$

$$= o\{(\log \omega)^{\alpha+1}\} + K_1 + K_2 \tag{4 4}$$

And by (1 6)

$$K_1 = \frac{1}{\omega}\int_{\lambda/\omega}^{\pi} o\left(\log\frac{1}{t}\right)^{\alpha+1}\frac{dt}{t^2} = \frac{1}{\omega}\left[o\left\{\frac{1}{t}\left(\log\frac{1}{t}\right)^{\alpha+1}\right\}\right]_{\lambda/\omega}^{\pi}$$

$$= o\{(\log \omega)^{\alpha+1}\} \tag{4 5}$$

Also by (1 5),

$$|K_2| \leq \frac{1}{\omega}\int_{\lambda/\omega}^{\pi} |g_{\alpha-1}(t)|\frac{dt}{t^2}$$

$$= \frac{1}{\omega}\left[\frac{1}{t^2}\int_0^t |g_{\alpha-1}(t)|\,dt\right]_{\lambda/\omega}^{\pi} + \frac{2}{\omega}\int_{\lambda/\omega}^{\pi}\left\{\int_0^t |g_{\alpha-1}(t)|\,dt\right\}\frac{dt}{t^3}$$

$$= O\left(\frac{1}{\omega}\right) + \frac{1}{\omega}\,\frac{\omega}{\lambda}\,O\left\{\left(\log\frac{\omega}{\lambda}\right)^{\alpha+1}\right\} + \frac{2}{\omega}\int_{\lambda/\omega}^{\pi} O\left\{\left(\log\frac{1}{t}\right)^{\alpha+1}\right\}\frac{dt}{t^2}$$

$$= o(1) + \frac{1}{\lambda}O\left\{\left(\log\frac{\omega}{\lambda}\right)^{\alpha+1}\right\} + \frac{1}{\omega}O\left\{\frac{\omega}{\lambda}\left(\log\frac{\omega}{\lambda}\right)^{\alpha+1}\right\}$$

$$= o(1) + \frac{1}{\lambda}O\left\{(\log \omega)^{\alpha+1}\right\}, \tag{4 6}$$

as $\omega \to \infty$

Hence combining the results from (4 1) to (4 6), we have

$$\int_0^{\pi} g_\alpha(t)\frac{1-\cos \omega t}{t}\,dt = o\{(\log \omega)^{\alpha+1}\} + \frac{1}{\lambda}O\{\log \omega)^{\alpha+1}\}$$

$$= o\{(\log \omega)^{\alpha+1}\},$$

if $\lambda \to \infty$ after $\omega \to \infty$ This, by (3 1) proves the sufficiency part of Theorem A

(ii) To prove that the condition is necessary, we assume (3 1), that is,

$$\int_0^{\pi} g_\alpha(t)\frac{1-\cos \omega t}{t}\,dt = o\{(\log \omega)^{\alpha+1}\}, \tag{4 7}$$

and deduce (1 7), provided that (1 6) is satisfied Now

$$\int_0^\pi g_\alpha(t)\frac{1-\cos \omega t}{t}\,dt = \Big[-g_{\alpha+1}(t)(1-\cos \omega t)\Big]_0^\pi + \omega\int_0^\pi g_{\alpha+1}(t)\sin \omega t dt$$

$$= o(1)+\omega\int_0^\pi g_{\alpha+1}(t)\sin \omega t dt$$

So we have, by (4 7),

$$\int_0^\pi g_{\alpha+1}(t)\sin \omega t dt = o\left\{\frac{(\log \omega)^{\alpha+1}}{\omega}\right\},$$

as $\omega \to \infty$.

Also after Hardy,* we take

$$g_{\alpha+1}(t) \sim \sum_{n=1}^{\infty} b_n^{(\alpha+1)} \sin nt,$$

for $0 \leq t \leq \pi$, then

$$b_n^{(\alpha+1)} = \frac{1}{\pi}\int_0^\pi g_{\alpha+1}(t)\sin nt\, dt = o\left\{\frac{(\log n)^{\alpha+1}}{n}\right\},$$

as $n \to \infty$ Now

$$\frac{1}{t}\int_0^t g_{\alpha+1}(t)\,dt = \sum_{n=1}^{\infty} b_n^{(\alpha+1)}\frac{1-\cos nt}{nt}$$

$$= \sum_{n<\frac{1}{t}} b_n^{(\alpha+1)}\frac{\sin nt/2}{nt/2}\cdot \sin nt/2 + \sum_{n \geq \frac{1}{t}} b_n^{(\alpha+1)}\frac{1-\cos nt}{nt}$$

$$= \sum_{n<\frac{1}{t}} o\left\{\frac{(\log n)^{\alpha+1}}{n}\right\} + \sum_{n\geq\frac{1}{t}} o\left\{\frac{(\log n)^{\alpha+1}}{n}\right\} O\left(\frac{1}{nt}\right)$$

$$= o\left\{\left(\log \frac{1}{t}\right)^{\alpha+1}\right\}$$

But

$$\frac{1}{t}\int_0^t g_{\alpha+1}(t)dt = g_{\alpha+1}(t)+\frac{1}{t}\int_0^t g_\alpha(t)\,dt$$

Or, by (1 6)

$$o\left\{\left(\log\frac{1}{t}\right)^{\alpha+1}\right\} = g_{\alpha+1}(t)+o\left\{\left(\log\frac{1}{t}\right)^{\alpha+1}\right\}$$

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* Hardy, 2

Hence

$$g_{\alpha+1}(t) = o\left\{\left(\log\frac{1}{t}\right)^{\alpha+1}\right\},$$

as $t \to 0$, which proves the necessary part of the theorem

5. The conjugate series for the function $f(t)$ given by (1 3) is summable $(R, 1)$ to $\frac{1}{\pi}$ For

$$g(t) = \frac{1}{\pi}\int_t^\pi f(t)\cot\frac{1}{2}\,dt - \frac{1}{\pi}$$

$$= \frac{1}{\pi}\int_t^1 \sin\left(\log\frac{1}{t}\right)\frac{dt}{t} - \frac{1}{\pi}$$

$$= -\frac{1}{\pi}\cos\left(\log\frac{1}{t}\right) = o\left(\log\frac{1}{t}\right),$$

as $t \to 0$, Also

$$g_1(t) = \int_t^\pi \frac{g(u)}{u}\,du = o\left(\log\frac{1}{t}\right),$$

and $$f(t) = O(1),$$

as $t \to 0$ Thus $f(t)$ satisfies the conditions of Theorem B with $s = \frac{1}{\pi}$

6. We shall now give an example* of a function $f(t)$ such that

$$g(t) = \frac{1}{\pi}\int_t^\pi \psi(t)\cot\frac{1}{2}t\,dt - s = o\left(\log\frac{1}{t}\right) \neq o(1),$$

but the conjugate series of the Fourier series of $f(t)$ is not summable $(R, 1)$ to s

We take $\theta = o$, $s = o$, $\psi(t) = f(t) =$ an odd function

Example We choose sequences $\{t_n\}$, $\{\lambda_n\}$ and $\{c_n\}$ such that

$$\frac{\pi}{2} = t_0 > t_1 > t_2 > \quad . > t_n \to 0,\ 0 < \lambda_1 < \lambda_2 < \quad . < \lambda_n \to \infty$$

and

$$c_n = \frac{\log\lambda_n}{\sqrt{\log(4n+1)}},$$

and we take

$$\lambda_r = 1\ 5.9\ldots\ (4r+1),\ t_r = \frac{\pi}{2\lambda_r}$$

---

* The corresponding example for Fourier series was given by Wang, 7

We define an odd function $f(t)$ in (o, π) by

$$(6\;1)\qquad \begin{cases} f(0) = 0 \\ f(t) = -c_r\lambda_r \tan\frac{t}{2}\sin\lambda_r t, \text{ for } t_r < t \leq t_{r-1},\ (r = 1, 2, 3, \quad), \\ f(t) = 0, \text{ for } \frac{\pi}{2} < t \leq \pi \end{cases}$$

and by periodicity elsewhere
Then

$$\int_{t_r}^{t_{r-1}} | f(t) | \, dt = c_r\lambda_r \int_{t_r}^{t_{r-1}} \tan\frac{t}{2} \mid \sin\lambda_r t \mid dt$$

$$= c_r\lambda_r \int_{\frac{\pi}{2\lambda_r}}^{\frac{\pi}{2\lambda_{r-1}}} \tan\frac{t}{2} \mid \sin\lambda_r t \mid dt$$

$$= O\left(\frac{c_r}{\lambda_r}\int_{\frac{\pi}{2}}^{\frac{\pi}{2}(4r+1)} u \mid \sin u \mid du\right)$$

$$= O\left\{\frac{c_r}{\lambda_r} r(2r+1)\right\}$$

$$= O\left\{\frac{1}{\lambda_r - 4}\right\},\ r \geq 4$$

Thus

$$\int_0^{\pi} | f(t) | \, dt = \int_{t_1}^{\frac{\pi}{2}} + \int_{t_2}^{t_1} + \int_{t_3}^{t_2} + \sum_{r=4}^{\infty} O\left(\frac{1}{1\;5\;9 \qquad (4r-15)}\right)$$
$$= O(1)$$

so that $f(t)$ is integrable (L) in $(0, \pi)$
Also for $r = 1, 2, 3, \quad$,

$$\int_{t_r}^{t_{r-1}} f(u) \cot\frac{1}{2}u \, du = -c_r\lambda_r \int_{\frac{\pi}{2\lambda_r}}^{\frac{\pi}{2\lambda_{r-1}}} \sin\lambda_r t \, dt = c_r \Big[\cos\lambda_r t \Big]_{\frac{\pi}{2\lambda_r}}^{\frac{\pi}{2\lambda_{r-1}}}$$
$$= 0$$

Hence if $t_r < t \leq t_r - 1$, then

$$g(t) = \frac{1}{\pi}\int_t^{\pi} f(u) \cot\frac{u}{2} \, du$$

$$= \frac{1}{\pi}\int_{t}^{t_{r-1}} f(u)\cot\frac{1}{2}u\,du$$

$$= -\frac{1}{\pi}c_r\cos\lambda_r t$$

And

$$\left|\frac{g(t)}{\log\frac{1}{t}}\right| \leq \frac{c_r}{\log 1/t_{r-1}} = \frac{\log\lambda_r}{\sqrt{\log(4r+1)}}\;\frac{1}{\log\frac{2\lambda_{r-1}}{\pi}} = o(1),$$

as $r\to\infty$ so that $g(t) = o\left(\log\frac{1}{t}\right)$ as $t\to 0$

Now from (3 1), the necessary and sufficient condition that the conjugate series of the Fourier series of $f(t)$ be summable $(R, 1)$ to zero is

$$\int_{0}^{\frac{\pi}{2}} g(t)\frac{1-\cos\omega t}{t}\,dt = o(\log\omega)$$

We shall now show that as ω takes successively the values of the sequence $\{\lambda_r\}$,

$$\frac{1}{\log\lambda_r}\int_{0}^{\frac{\pi}{2}} g(t)\frac{1-\cos\lambda_r t}{t}\,dt$$

is not $o(1)$, but tends to infinity as $r\to\infty$

For

$$\int_{0}^{\frac{\pi}{2}} g(t)\frac{1-\cos\lambda_r t}{t}\,dt = \left\{\int_{0}^{t_r} + \int_{t_r}^{t_{r-1}} + \sum_{k=1}^{r-1}\int_{t_k}^{t_{k-1}}\right\} g(t)\;\frac{1-\cos\lambda_r t}{t}\,dt$$

$$= \int_{0}^{t_r} g(t)\frac{1-\cos\lambda_r t}{t}\,dt - \int_{t_r}^{t_{r-1}} g(t)\frac{\cos\lambda_r t}{t}\,dt$$

$$+\sum_{k=1}^{r}\int_{t_k}^{t_{k-1}}\frac{g(t)}{t}\,dt - \sum_{k=1}^{r-1}\int_{t_k}^{t_{k-1}} g(t)\frac{\cos\lambda_r t}{t}\,dt$$

$$= I+J+K+L,\ \text{say}$$

Now

$$I = \int_{0}^{\frac{\pi}{2\lambda_r}} g(t)\frac{1-\cos\lambda_r t}{t}\,dt = \lambda_r\int_{0}^{\frac{\pi}{2\lambda_r}} o\left(\log\frac{1}{t}\right)dt = o(\log\lambda_r).$$

$$J = -\int_{t_r}^{t_{r-1}} g(t)\frac{\cos\lambda_r t}{t}\,dt = \frac{c_r}{\pi}\int_{\frac{\pi}{2\lambda_r}}^{\frac{\pi}{2\lambda_{r-1}}} \cos^2\lambda_r t\,\frac{dt}{t}$$

$$= \frac{1}{2\pi}c_r \int_{\frac{\pi}{2\lambda_r}}^{\frac{\pi}{2\lambda_{r-1}}} \frac{1+\cos 2\lambda_r t}{t}\,dt$$

$$= \frac{c_r}{2\pi}\log\frac{\lambda_r}{\lambda_{r-1}} + \frac{c_r}{2\pi}\int_{\pi}^{(4r+1)\pi}\frac{\cos u}{u}\,du$$

$$= \frac{1}{2\pi}\frac{\log\lambda_r}{\sqrt{\log(4r+1)}}\log(4r+1) + O(c_r)$$

$$= \frac{1}{2\pi}\log\lambda_r\sqrt{\log(4r+1)} + o(\log\lambda_r)$$

$$K = \sum_{k=1}^{r}\int_{t_k}^{t_{k-1}}\frac{g(t)}{t}\,dt = -\sum_{k=1}^{r}\frac{1}{\pi}c_k\int_{\frac{\pi}{2\lambda_k}}^{\frac{\pi}{2\lambda_{k-1}}}\frac{\cos\lambda_k t}{t}\,dt$$

$$= -\sum_{k=1}^{r}\frac{1}{\pi}c_k\int_{\frac{\pi}{2}}^{(4k+1)\frac{\pi}{2}}\frac{\cos u}{u}\,du$$

$$= -\sum_{k=1}^{r}\frac{1}{\pi}\,\frac{2}{\pi}c_k\int_{\frac{\pi}{2}}^{\epsilon_k}\cos u\,du,\quad \frac{\pi}{2}<\epsilon_k<\frac{\pi}{2}(4k+1),$$

$$= \sum_{k=1}^{r}2c_k(1-\sin\epsilon_k) = \sum_{k=1}^{r}2(1-\sin\epsilon_k)\frac{\log\lambda_k}{\sqrt{\log(4k+1)}},$$

so that K is positive Again

$$L = -\sum_{k=1}^{r-1}\int_{t_k}^{t_{k-1}} g(t)\frac{\cos\lambda_r t}{t}\,dt = \sum\frac{1}{\pi}c_k\int_{t_k}^{t_{k-1}}\frac{\cos\lambda_k t\cos\lambda_r t}{t}\,dt$$

$$= \frac{1}{2\pi}\sum_{k=1}^{r-1}c_k\int_{\frac{\pi}{2\lambda_k}}^{\frac{\pi}{2\lambda_{k-1}}}\frac{\cos(\lambda_r-\lambda_k)t+\cos(\lambda_r+\lambda_k)t}{t}\,dt$$

$$= \sum_{k=1}^{r-1}c_k\lambda_k O\left(\frac{1}{\lambda_r-\lambda_k}\right) = \sum_{k=1}^{r-1}\frac{\log\lambda_k}{\sqrt{\log(4k+1)}}\,O\left(\frac{\lambda_k}{\lambda_r-\lambda_k}\right)$$

$$= r \log \lambda_{r-1} O\left(\frac{\lambda_{r-1}}{\lambda_r - \lambda_{r-1}}\right)$$

$$= O(\log \lambda_{r-1}) = o(\log \lambda_r),$$

as $r \to \infty$

Hence combining our results we have

$$\frac{1}{\log \lambda_r} \int_0^{\frac{\pi}{2}} g(t)\, \frac{1 - \cos \lambda_r t}{t}\, dt \geq \frac{1}{2\pi} \sqrt{\log (4r+1)} + o(1)$$

$$\to \infty, \text{ as } r \to \infty,$$

which proves that the conjugate series for the function $f(t)$ defined by (6 1) is not summable $(R, 1)$ although

$$g(t) = o\left(\log \frac{1}{t}\right)$$

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MODULAR EQUATIONS AS SOLUTIONS OF ALGEBRAIC DIFFERENTIAL EQUATIONS OF THE SIXTH ORDER

By S CHOWLA

(Communicated by Sir S S Bhatnagar, F R S)

(*Read January 17, 1947*)

In a recent paper (reviewed in *Mathematical Reviews*, Vol 7, p 243) I pointed out how identities of the Ramanujan Rademacher-Zuchermann type can be proved by showing that both sides of the identity satisfy the same algebraic differential equation

Let

$$K=\int_0^{\frac{\pi}{2}}\frac{d\phi}{\sqrt{1-k^2\sin^2\phi}},\quad K'=\int_0^{\frac{\pi}{2}}\frac{d\phi}{\sqrt{1-k'^2\sin^2\phi}}$$

$$L=\int_0^{\frac{\pi}{2}}\frac{d\phi}{\sqrt{1-l^2\sin^2\phi}},\quad L'=\int_0^{\frac{\pi}{2}}\frac{d\phi}{\sqrt{1-l'^2\sin^2\phi}}$$

where $$k^2+k'^2=1,\ l^2+l'^2=1\quad \text{If}$$

(1) $$\frac{L'}{L}=n\,\frac{K'}{K}$$

where n is a positive integer, we have

$$\sqrt{kl}+\sqrt{k'l'}=1\quad (n=3)$$

$$\sqrt[4]{kl}+\sqrt[4]{k'l'}=1\quad (n=7)$$

such algebraic relations between k and l are called 'modular equations' The object of this note is to point out that the 'modular equations' are solutions of algebraic differential equations of the sixth order

Write

$$E=\int_0^{\frac{\pi}{2}}\sqrt{1-k^2\sin^2\phi}\,d\phi,\quad E'=\int_0^{\frac{\pi}{2}}\sqrt{1-k'^2\sin^2\phi}\,d\phi$$

$$M=\int_0^{\frac{\pi}{2}}\sqrt{1-l^2\sin^2\phi}\,d\phi,\quad M'=\int_0^{\frac{\pi}{2}}\sqrt{1-l'^2\sin^2\phi}\,d\phi$$

Then we have

$$EK'+KE'-KK' = \frac{\pi}{2} \tag{2}$$

$$LM'+ML'-LL' = \frac{\pi}{2} \tag{3}$$

$$\frac{dK}{dk} = \frac{E-k'^2K}{kk'^2} \tag{4}$$

$$\frac{dE}{dk} = \frac{E-K}{k} \tag{5}$$

$$\frac{dL}{dl} = \frac{M-l'^2L}{ll'^2} \tag{6}$$

$$\frac{dM}{dl} = \frac{M-L}{l} \tag{7}$$

Differentiating (1) 6 times with respect to k and using (2), (3), (4), (5), (6), (7) to eliminate K, K', L, L', E, M from (1) and the 6 equations obtained by differentiation, we get

Theorem If (1) *is true then*

$$f\left(k, l, \frac{dl}{dk}, \frac{d^2l}{dk^2}, \frac{d^3l}{dk^3}, \frac{d^4l}{dk^4}, \frac{d^5l}{dk^5}, \frac{d^6l}{dk^6}\right) = 0$$

where $f(x_1, x_2,$ $, x_8)$ *denotes a polynomial in the* x*'s with integral coefficients depending on* n *alone*

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See *Mathematical Reviews*, Vol 7 1946 page 243

ON SERIES OF THE LAMBERT TYPE WHICH ASSUME IRRATIONAL VALUES FOR RATIONAL VALUES OF THE ARGUMENT

By S CHOWLA

(Communicated by Sir S S Bhatnagar, F R S)

(*Read January 17, 1947*)

Let

$$f(x) = \sum_{1}^{\infty} \frac{x^n}{1-x^n},$$

$$g(x) = \sum_{1}^{\infty} \frac{x^n}{1-x^n} \sin \frac{n\pi}{2},$$

where $|x| < 1$ It is not unlikely that $f(x)$ and $g(x)$ are irrational when x is a rational number different from 0 I am unable to prove anything about $f(x)$, but I can show that $g(x)$ is irrational when x is a rational number of the form $1/t$ where t is a positive integer >5

We have

Lemma 1 *We have*

$$1+4g(x) = \sum_{0}^{\infty} r(n)x^n$$

where $r(n)$ *is the number of representations of* n *as a sum of two squares*

This is well-known

Lemma 2 *Let* ϵ *denote an arbitrary positive number and* m *an arbitrary positive integer Then we can find an integer* x *such that*

(i) $$r(x+t) = 0 \text{ for } 1 < t < m$$

(ii) $$m > (\tfrac{1}{2}-\epsilon)\frac{\log x}{\log\log x}$$

for all $m > m_0(\epsilon)$

Proof Let q_m denote the mth prime $\equiv 3 \pmod 4$ Then the system of congruences

$$x+1 \equiv q_1 \pmod{q_1^2}$$

$$x+2 \equiv q_2 \pmod{q_2^2}$$

$$x+m \equiv q_m \pmod{q_m^2}$$

is soluble, and in fact with

$$q_1^2 q_2^2 \quad q_m^2 < x < 2q_1^2 q_2^2 \cdot \quad q_m^2$$

Now from the extended Prime Number Theorem,

$$q_m \sim 2m \log m$$

whence

$$\log x \sim 2 \sum_{t=1}^{m} \log t \sim 2m \log m$$

$$\log\log x \sim \log m$$

$$\frac{\log x}{\log\log x} \sim 2m$$

so that for any $\epsilon > 0$ and $m > m_0(\epsilon)$ we have

$$m > (\tfrac{1}{2} - \epsilon)\frac{\log x}{\log\log x}.$$

Further $r(x+t) = 0$ for $1 \leqslant t \leqslant m$

since x satisfies the above m congruences

Now it is known that

Lemma 3 *We have*

$$r(n) < 2^{(1+\epsilon)\frac{\log n}{\log\log n}}$$

where $\epsilon > 0$, *for all* $n > n_0(\epsilon)$

Now consider

$$\sum_{g=m+1}^{\infty} \frac{r(x+g)}{t^{x+g}}$$

$$= \sum_{n=x+m+1}^{2x} \frac{r(n)}{t^n} + \sum_{2x+1}^{\infty} \frac{r(n)}{t^n}$$

$$\leq \frac{2^{\frac{(1+\epsilon)\log x}{\log\log x}}}{t^{x+m+1}} + \sum_{2x+1}^{\infty} \frac{n}{t^n}$$

$$\leqq \frac{2^{\frac{(1+\epsilon)\log x}{\log\log x}}}{t^{x+m+1}} + 0\left(\frac{1}{t^{2x}}\right)$$

(1) $$\leqq \frac{t^{\frac{(1+\epsilon)\log 2 \log x}{\log t \log\log x}}}{t^{x+m+1}} + 0\left(\frac{1}{t^{2x}}\right).$$

Let us represent

$$S = \sum_{1}^{\infty} \frac{r(n)}{t^n}$$

as a decimal in the scale of t

Writing

$$S = \sum_{n=1}^{x} + \sum_{x+1}^{x+m} + \sum_{x+m}^{\infty}$$

$$= \sum\nolimits_1 + \sum\nolimits_2 + \sum\nolimits_3$$

On account of $\Sigma_2 = 0$ it would follow that all the decimal places of x from the $(x+1)$th to the $(x+m)$th are zero, had Σ_3 not butted into this part of the decimal representation (in the scale of t) of S But roughly

$$\frac{(1+\epsilon)\log 2\log x}{\log t\log\log x} \tag{2}$$

decimal places to the left of the $(x+m)$th decimal places are affected by Σ_3 on account of (1) Now

$$m > \left(\frac{1}{2}-\epsilon\right)\frac{\log t}{\log\log x} \tag{3}$$

From (2) and (3) it

$$\frac{\log 2}{\log t} < \frac{1}{2}$$

i e $t \geq 5$, S has a block of at least

$$\left(\frac{1}{2}-\frac{\log 2}{\log t}\right)\frac{(1-\epsilon)\log x}{\log\log x}$$

decimal places all equal to 0 Since S has an infinity of decimal places $\neq 0$ it follows that S is irrational —Q E D

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ON THE SIGN OF THE GAUSSIAN SUM

By R. P BAMBAH *and* S CHOWLA

(Communicated by Prof D S Kothari, Ph D , F N I)

(Received December 21, 1946 , read January 1, 1947)

It is a classical result due to van der Corput that

$$\int_a^b e^{2\pi i f(x)}\,dx-\sum_{a\leqslant n\leqslant b} e^{2\pi i f(n)}=\tfrac{9}{4}\theta_1$$

where $f(x)$ is real, $f'(x)$ monotonic and $|f'(x)|\leqslant\frac{1}{2}$ in (a, b), $\theta_1, \theta_2, \theta_3$, denote complex numbers whose absolute value does not exceed 1 Hence

$$\sum_0^{k-1} e^{\frac{\pi i m^2}{2k}}-\int_0^{k-1} e^{\frac{\pi i x^2}{2k}}\,dx=\tfrac{9}{4}\theta_2 \tag{2}$$

Further as pointed out by Estermann it is trivial that for odd k

$$S=\sum_{m=0}^{k-1} e^{\frac{2\pi i m^2}{k}}=1+\frac{2}{(1+i^k)}\sum_{m=1}^{k-1} e^{\frac{\pi i m^2}{2k}} \tag{3}$$

$$\tfrac{1}{2}(1-i)(1+i^k)S=\pm\sqrt{k} \tag{4}$$

Again by the second mean-value theorem (or graphically), for odd $k\geqslant 13$,

$$\int_{k-1}^{\infty} e^{\frac{\pi i x^2}{2k}}\,dx=\sqrt{k}\int_{\frac{(k-1)^2}{k}}^{\infty}\frac{e^{\frac{\pi i t}{2}}\,dt}{2\sqrt{t}}=\frac{k}{2(k-1)}\,\frac{2\sqrt{2}\ 2\theta_3}{\pi} \tag{5}$$

$$=\frac{2\ 84}{3\ 1}\,\frac{k}{(k-1)}\,\theta_4=\theta_5$$

From (2), (3), (5)

$$S\tfrac{1}{2}(1-i)(1+i^k)=\theta_6+(1-i)\left(\sum_0^{k-1} e^{\frac{\pi i m^2}{2k}}-1\right)$$

$$=\theta_6+(\tfrac{9}{4}+1)\sqrt{2}\theta_7+(1-i)\int_0^{k-1} e^{\frac{\pi i x^2}{2k}}\,dx$$

$$=\theta_6+\tfrac{17}{4}\sqrt{2}\theta_8+(1-i)\int^{\infty} e^{\frac{\pi i x^2}{2k}}\,dx \tag{6}$$

Now, using (4),

$$\pm\sqrt{k} = (1-i)\sqrt{k}\int_0^\infty e^{\frac{\pi i t^2}{2}}\,dt + \theta_6 + \frac{17\sqrt{2}}{4}\theta_8$$

$$= \sqrt{k}\int_0^\infty \left(\cos\frac{\pi x^2}{2} + \sin\frac{\pi x^2}{2}\right)dx + \sqrt{k}\,i\int_0^\infty \left(\sin\frac{\pi x^2}{2} - \cos\frac{\pi x^2}{2}\right)dx + \left(1+\frac{17\sqrt{2}}{4}\right)\theta_9 \tag{7}$$

Making $k \to \infty$, it follows that

$$\int_0^\infty \sin\frac{\pi x^2}{2}\,dx = \int_0^\infty \cos\frac{\pi x^2}{2}\,dx, \tag{8}$$

so that (7) becomes

$$\pm\sqrt{k} = 2\sqrt{k}\int_0^\infty \sin\frac{\pi x^2}{2}\,dx + \left(1+\frac{17\sqrt{2}}{4}\right)\theta_9 \tag{9}$$

Now, since (graphically)

$$\int_0^\infty \sin\left(\frac{\pi x^2}{2}\right)dx = \int_0^\infty \frac{\sin\left(\frac{\pi y}{2}\right)}{2\sqrt{y}}\,dy > 0$$

it follows from (9) that (again by making $k \to \infty$)

$$\int_0^\infty \sin\left(\frac{\pi x^2}{2}\right)dx = \frac{1}{2},$$

so that

$$\pm\sqrt{k} = \sqrt{k} + \left(1+\frac{17\sqrt{2}}{4}\right)\theta_9 \tag{10}$$

Hence the + sign holds in (10) if

$$k > \left(1+\frac{17\sqrt{2}}{4}\right)^2, \text{ i e if } k > 49$$

We have thus proved that the + sign holds in (4) whenever k (which is odd) >49 By actual calculation we can show that the + sign also holds when $k < 49$

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ON AN UNSUSPECTED REAL ZERO OF EPSTEIN'S ZETA FUNCTION

By S Chowla

(Communicated by Sir S S Bhatnagar, F R S)

(*Read January 17, 1947*)

Let ($s = \sigma + it$)

$$F(s) = \sum\sum \frac{1}{(x^2+dy^2)^s} \qquad (\sigma > 1).$$

[where the summation is for all integers x, y going from $-\infty$ to $+\infty$ ($x = y = 0$) being excluded)] and its analytical continuations It was for a long time considered likely that the only roots of $F(s) = 0$ for $0 < \sigma < 1$, satisfy $\sigma = \frac{1}{2}$ (the analogue of Riemann's hypothesis) Davenport and Heilbronn proved the existence of complex zeroes of $F(s) = 0$ in the half plane $\sigma > 1$ I prove the surprising result that $F(s) = 0$ has a real zero s with $\frac{1}{2} < s < 1$ for all large $d > d_0$ (d is a positive integer)

Theorem *For all $d > d_0$ we have*

$$F(1-S) = 0$$

where the real number s satisfies

$$\lim_{d \to \infty} (s\sqrt{d}) = \frac{3}{\pi}$$

Proof For real s we have (Deuring)

$$F(s) = \zeta(2s) + \frac{d^{\frac{1}{2}-s}\sqrt{\pi}\,\zeta(2s-1)\Gamma(s-\frac{1}{2})}{\Gamma(s)} + O(e^{-2\sqrt{d}})$$

Proceeding as in my paper in *Quart J of Maths* (Oxford), 1934, it follows that

$$F(1-S) = 0$$

for a real s satisfying

$$s \sim \frac{3}{\pi\sqrt{d}}$$

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ON THE HELIUM CONTENT OF STARS OF LARGE MASSES

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(Communicated by Prof N R Sen, D Sc, Ph.D)

(*Received February 7, read April 4, 1947*)

ABSTRACT

The effect of the assumption of a substantial proportion of helium together with hydrogen in the composition of certain stars of large masses has been worked out for the energy generation law of Bethe It is found that the problem of determining the hydrogen and helium contents in stars of large masses with the observed values of their mass, radius and luminosity, though mathematically soluble, does not in general lead to a physically significant solution The case in which a physically significant solution may be expected is indicated in this paper

1 INTRODUCTION

This paper is a continuation of a previous one* (Burman, 1946) where the internal constitution of stars of large masses with appreciable radiation pressure, was studied on the basis of Bethe's law of energy generation The helium content of these stars was assumed to be negligible, it was then found that the Bethe formula of energy generation does not fit in satisfactorily with the mass-luminosity relation in these stars It seems therefore reasonable to study the effect of helium on the constitution of these stars and it is the result of this study that we deal with in the present communication

The models we consider here are of the convective-radiative type, and have assigned values for the ratio y_c of radiation to gas pressure at the centre A number of *point source* stellar models for different values of y_c have been constructed by Henrich (1942), and we make use of some of them for our present purpose, introducing, however, the assumption that the energy generation in them is governed by Bethe's law We consider a number of models for $y_c = 0\ 01$, and $0\ 10$ and for various values of X and Y, the hydrogen and helium contents respectively, so that y_c, X and Y serve as *three* parameters in these models, whereas in the previous paper we had y_c and X only as the *two* parameters in the models, the helium content there being assumed zero throughout As explained in the previous paper we make use of the approximate power law representation of the exact exponential law of energy generation

2 THE EQUATIONS

The equilibrium equations in the convective core, in the usual notations are

$$\frac{dP}{dr} = -G\frac{M(r)}{r^2}\rho \qquad (1)$$

$$\frac{dM(r)}{dr} = 4\pi r^2\rho \qquad (2)$$

and

$$\frac{dP}{P} = \Gamma_1\frac{d\rho}{\rho} \qquad (3)$$

---

* Referred to hereinafter as the 'previous paper'

where the adiabatic exponent Γ_1 is given by

$$\Gamma_1 = \beta + \frac{(4-3\beta)^2(\gamma-1)}{\beta+12(\gamma-1)(1-\beta)} \tag{4}$$

In the radiative envelope, equation (3) is to be replaced by the equation

$$\frac{d}{dr}\left(\frac{1}{3}aT^4\right) = -\frac{\kappa\rho}{c}\frac{L}{4\pi r^2}, \tag{5}$$

where the luminosity L is assumed constant in the envelope The law of opacity (Henrich, 1942) is taken as in the previous paper as

$$\left.\begin{aligned} \kappa &= \frac{\kappa_0\rho T^{-3.5}}{10\theta} & \theta > \theta_\kappa \\ &= \frac{\kappa_0\rho T^{-3.5}}{10\theta_\kappa} & \theta < \theta_\kappa \end{aligned}\right\} \tag{6}$$

where $\theta = T/T_c$, T_c being the central temperature, and θ_κ, a certain value of θ in the envelope region where the opacity changes

The luminosity equation is

$$L(r) = \int_0^r 4\pi r^2\rho\epsilon\, dr \tag{7}$$

with

$$\epsilon = EX\rho T^n, \tag{8}$$

the coefficient E and the exponent n being suitably chosen

We had in the previous paper chosen different values of E and n for different values of y_c, and this choice was made to suit the temperature conditions in those models

It is now found that the introduction of a not very high concentration of helium and a varying concentration of hydrogen in the models does not materially affect the mean value of n for the corresponding ranges of central temperature We therefore retain in the present calculations the previous values of E and n for the same value of y_c

Introducing the variables

$$r = \alpha\xi,\ \rho = \rho_c\sigma,\ T = T_c\theta$$

we obtain the total luminosity of the model, ignoring the slight generation of energy outside the convective core, as

$$L(\xi_i) = 4\pi EX\left(\frac{5k}{8\pi GH}\right)^{3/2}\frac{1}{\mu}\left(\frac{a}{3}\frac{H}{k}\right)^{1/2}\frac{T_c^{n+3}}{y_c^{1/2}}\ I(\xi_i,\ y_c) \tag{9}$$

where

$$I(\xi_i,\ y_c) = \int_0^{\xi_i}\sigma^2\theta^n\xi^2\, d\xi, \tag{10}$$

ξ_i, being the interface between the convective core and the radiative envelope The method of evaluating the integral $I\ (\xi_i,\ y_c)$ has been shown in the previous paper

For a model with given y_c, we have the following relations (Henrich, 1942) between L, M, R, T_c, ρ_c and μ, the quantities L, M, R, being expressed in solar units

$$\mu^2 M = 4\,420\, y_c^{1/2}\psi_R \tag{11}$$

$$\log \rho_c = \log\left(\frac{1}{3}\frac{\alpha_2^3}{\psi_R}\right) + \log\frac{M}{R^3} + 0.149 \tag{12}$$

$$\log T_c = 6.966 + \log\frac{\alpha_2}{\psi_R} + \log\frac{\mu M}{R} \tag{13}$$

$$\log L = 28.742 + \log\left(\frac{\alpha_2^{0.5} Q_2}{\psi_R^{5.5}}\right) + \log\left(\frac{\mu^{7.5} M^{5.5}}{\kappa_0 R^{0.5}}\right), \tag{14}$$

where α_2, ψ_R, Q_2 are known in terms of y_c alone It is to be noted that the energy generation formula has not been used in the derivation of these relations

The values of L as determined by equations (9) and (14) should agree and this requires that

$$\log\left\{4\pi E X\left(\frac{5k}{8\pi G H}\right)^{3/2}\frac{1}{\mu}\left(\frac{a}{3}\frac{H}{k}\right)^{1/2}\frac{T_c^{\,n+3}}{y_c^{1/2}}\ I(\xi_1,\ y_c)\right\}$$

$$= 28.742 + \log\left(\frac{\alpha_2^{0.5} Q_2}{\psi_R^{5.5}}\right) + \log\left(\frac{\mu^7 M^5}{\kappa_0}\right) + \frac{1}{2}\log T_c - \log\frac{\alpha_2}{\psi_R}$$

$$-3.483 + \log L_\odot \tag{15}$$

where $L_\odot$ denotes the solar luminosity

The opacity coefficient κ_0 and the mean molecular weight μ are given by

$$\kappa_0 = 4.32 \times 10^{25}(1+X)(1-X-Y) \tag{16}$$

and

$$\mu = 2/(1+3X+\tfrac{1}{2}Y) \tag{17}$$

We now explain how the configuration is determined by y_c, X and Y when the energy generation formula is taken into account For given y_c, equation (11) determines the mass when X and Y are assigned Equation (15) now determines T_c and the radius R is then obtained from equation (13) The central density and the luminosity are then given by equations (12) and (14) respectively, so that the configuration is completely determined We saw in the previous paper how y_c and X, i e μ (when $Y = 0$) alone determined the configuration Here, however, we do not put $Y = 0$ and therefore replace μ by the two parameters X and Y The number of equations, however, is just sufficient to determine the configuration when y_c, X and Y are assigned

3 RESULTS OF CALCULATIONS

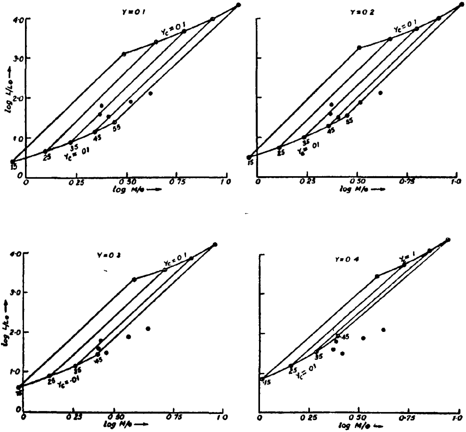
We have constructed a number of models for $y_c = 0.01$ and 0.10 and for different values of X and Y, the hydrogen and helium contents respectively For each value of y_c we take $Y = 0.10$, 0.20, 0.30 and 0.40 and corresponding to each value of Y the values of X taken vary in general from 15 to 45 per cent The stellar parameters L, M, R, T_c and ρ_c are calculated in each case according to the formulae in the last section, and the results are shewn in Table 1 As stated before, the coefficient E and the exponent n in the energy generation formula are taken to be different for different values of y_c and their values for a given y_c are taken to be the same as those in the previous paper We shall now see how it is possible from a study of these results to calculate y_c, X, Y, T_c and ρ_c for a star whose L, M, R values are observationally known

TABLE I

Luminosity, Mass, Radius, Central Temperature and Density of Configurations with assigned y_c (ratio of radiation to gas pressure at the centre) and composition

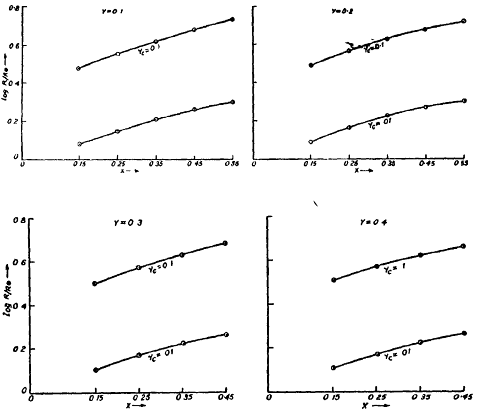
Y	X	y_c	$\log L/L_{\odot}$	$\log M/\odot$	$\log R/R_{\odot}$	$T_c 10^{-6}$	ρ_c
0 10	0 15	0 01	0 391	−0 072	0 083	22 7	47 4
		0 10	3 12	0 493	0 483	32 4	13 9
	0 25	0 01	0 659	0 087	0 153	23 1	42 0
		0 10	3 39	0 652	0 553	33 2	12 4
	0 35	0 01	0 903	0 220	0 211	23 6	38 3
		0 10	3 63	0 786	0 611	33 9	11 3
	0 45	0 01	1 14	0 337	0 261	24 1	35 5
		0 10	3 87	0 902	0 660	34 5	10 5
0 20	0 15	0 01	0 498	−0 043	0 093	22 9	47 3
		0 10	3 23	0 522	0 492	32 8	13 9
	0 25	0 01	0 770	0 110	0 160	23 4	42 3
		0 10	3 50	0 676	0 560	33 5	12 5
	0 35	0 01	1 02	0 241	0 216	23 9	38 8
		0 10	3 75	0 806	0 616	34 3	11 4
	0 45	0 01	1 28	0 355	0 263	24 4	36 4
		0 10	4 01	0 920	0 662	35 1	10 7
0 30	0 15	0 01	0 614	−0 016	0 102	23 2	47 4
		0 10	3 34	0 550	0 500	33 2	14 0
	0 25	0 01	0 895	0 134	0 166	23 7	42 8
		0 10	3 62	0 700	0 566	34 0	12 6
	0 35	0 01	1 17	0 261	0 220	24 3	39 6
		0 10	3 90	0 826	0 619	34 8	11 7
	0 45	0 01	1 45	0 372	0 264	24 9	37 7
		0 10	4 18	0 937	0 663	35 8	11 2
0 40	0 15	0 01	0 845	0 011	0 109	23 4	47 8
		0 10	3 47	0 576	0 509	33 7	14 1
	0 25	0 01	1 21	0 156	0 171	24 0	43 6
		0 10	3 77	0 721	0 570	34 5	12 9
	0 35	0 01	1 57	0 281	0 221	24 7	41 0
		0 10	4 08	0 845	0 620	35 5	12 2
	0 45	0 01	1 97	0 389	0 260	25 6	40 2
		0 10	4 30	0 954	0 659	36 7	11 9

From Table I, we construct a mass-luminosity diagram for each value of Y and the corresponding values of y_c and X as in Figs 1-4 Each diagram consists of two sets of curves $y_c =$ const and $X =$ const We also plot $\log R/R_{\odot}$ against X for each value of Y as in Figs 5-8



FIGS 1–4

The mass-luminosity diagrams for $Y = 0.1$, 0.2, 0.3 and 0.4, with $y_c = 0.01$ and 0.10 in each case The values of X for the various cases are indicated at the bottom of the curves



FIGS 5–8

The variation of radius with X for $Y = 0·1$, $0·2$, $0·3$, $0·4$ and $y_c = 0·01$ and $0·1$

These curves would enable us to calculate in the manner explained below, y_c, X and Y for a model when the values of L, M and R are assigned. The central density and temperature can then be obtained from the equations (12) and (13) respectively, the quantities α_2 and ψ_R for a given y_c being obtained by interpolation from Henrich's (1942) values.

The position of a star in each of the mass-luminosity diagrams would give by interpolation the values of X and y_c corresponding to the different values of Y, that is to say, if we assume a given helium content in the composition of a star of given L, M, R, the position of the representative point of the star in the mass-luminosity diagram corresponding to the assumed value of Y, would give us a value for its hydrogen content X and a value for the central ratio y_c of radiation to gas pressure. A further interpolation in the R–X diagram for the appropriate Y would give for the radius of the star a value which is not necessarily the same as the observed value of the radius. In this manner we obtain different sets of values for y_c, X, Y and R for the same L, M values of a star. Interpolation between these sets of values for a definite value of R (which is the observed value of the radius of the star whose L, M-values we have taken) would now fix the parameters y_c, X and Y for the star considered. The procedure is as follows —

Consider, for example, any one of these sets of values, say y_c', X', Y' and R'. We note that for the given L, M and R' as parameters, only the simultaneous set of values y_c', X' and Y' is possible, as this set is the uniquely determined solution of our equations for the given L, M and R'. This will be true of every one of the (four) sets of y_c, X, Y and R determined by us, corresponding to given L, M-values. This being the situation, the values of y_c, X and Y for some intermediate value of R can be easily obtained by a one-parametric interpolation. The central density and temperature would then be obtained from equations (12) and (13), so that all the parameters of the star are determined by its L, M, R-values. From this point of view we have here examined some stars of Kuiper's table considered in the previous paper and the results are shown in Table II.

TABLE II

Solutions for X, Y, y_c, T_c and ρ_c for some stars whose L, M, R values are known from observation

Star	log $L/L_\odot$	log $M/\odot$	log $R/R_\odot$	X	Y	y_c	$T_c\ 10^{-6}$	ρ_c
α Aur A	2.08	0.62	1.20	no physically significant solutions				
α Aur B	1.90	0.52	0.82					
β Aur A	1.83	0.38	0.43					
ζ C Ma A	1.48	0.41	0.28	0.46	0.20	0.013	26.4	35.0
α C Ma A	1.59	0.37	0.25	0.45	0.45	0.015	24.8	24.2

The first 3 stars correspond to the case $R_{obs} > R_{calc}$ (for $Y = 0$) as may be seen from Table II of the previous paper. The other two correspond to the case $R_{obs} < R_{calc}$ (for $Y = 0$). It is only in the latter case that a physically significant solution is possible.

4 CONCLUSIONS

It will be noticed from Table II of the previous paper that for an assumed zero helium content it is not possible to so choose a hydrogen content as to obtain an agreement in the observed and calculated values of L, M, R of a star in which the effect of radiation pressure is not negligible and in which energy generation takes place according to Bethe's law. We have in this paper attempted to obtain the desired agreement in the calculated and observed values of L, M, R of a star in which energy is generated according to Bethe's law, by introducing a suitable proportion of helium into its composition.

It is found that the effect of introducing helium into the composition of a star of given L, M is to cause a decrease in the calculated value of its radius as compared with its value calculated on the assumption of $Y = 0$. It is therefore not possible by giving suitable values to the helium content (Y) to obtain an agreement between the observed and calculated values of L, M, R in the case of those stars where the observed radius is greater than that calculated on the basis of $Y = 0$. This will be clear from Table II where the desired agreement has been obtained in the case of only two stars for which $R_{obs} < R_{calc}$ (with $Y = 0$). The other stars of the table correspond to the case $R_{obs} > R_{calc}$ (with $Y = 0$) and no agreement has been possible in their cases with any physically significant values of X and Y. It is thus permissible to conclude that Bethe's law will not generally explain the generation of energy in stars of large masses.

This result is of considerable importance in deciding whether a star of large mass with given L, M, R can be supplied with energy according to Bethe's scheme. Our previous paper gives a simple method of calculating the radius of a configuration of given L, M and zero helium content, assuming that energy is generated within it according to Bethe's law. Unless the calculated radius is larger than the given radius of the configuration, we may decide that Bethe's law will not be applicable to this configuration

It has, however, been shewn that so far as accurate observations go Bethe's law satisfactorily accounts for the energy generation in stars whose masses are in the neighbourhood of the solar mass

The author takes pleasure in acknowledging his indebtedness to Professor N R Sen for valuable discussions and criticisms during the preparation of this paper

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ON THE SELF ENERGY OF THE ELECTRONS

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(*Received November, 1946, read March 7, 1947*)

SUMMARY

The self energy of an electron in motion is investigated from quantum electrodynamics In the first article the interaction energy of an electron with electromagnetic field is given and in the second and third articles the evaluation of the dynamic and static self energies is undertaken The divergence of the self energy in the hole theory is logarithmic which is shown to be due to the symmetrisation in the behaviour of the electron with respect to emission and absorption of photon in its initial state

INTRODUCTION

The self-energy of an electron is its total energy in free space when isolated from other particles In the classical theory of Lorentz where the electron is considered to have a finite extension over which the total charge is distributed the self-energy includes besides the energy of its mass also the interaction energy between the different charge elements of the electron integrated over its volume, the interaction taking place via the electromagnetic field produced at each point of the electron by its other parts In the first approximation when the retardation of the electromagnetic field inside the electron is neglected we obtain the usual electrostatic self energy, i e , the energy which is required to keep the different parts of the charges of the electron together and this tends to infinity in the limit for a point charge In the higher approximations the self energy is due to the self-force which arises because of the retardation of the electromagnetic field inside the electron and contains terms the first of which is independent of the structure of the electron and is the usual radiation damping, the second and the higher approximation terms on the other hand can be expressed, particularly for the case of a simple oscillator, as a series in powers of r_0/λ where r_0 is the radius of the electron and λ the wave length of the emitted radiation This part of the self-energy therefore vanishes for a point charge In the quantum theory where the electron is considered to be a point charge the self-energy which has been calculated by Waller (1930) Oppenheimer (1930) and Rosenfeld (1931) exhibits also the strong divergent character tending to infinity It has been therefore argued from the idea of the correspondence principle that this infinite self-energy is due to our assumption of a point electron. But it has been already emphasised by Bhabha and Corben (1941) that the infinite self-energy in the theory of Lorentz lies in his assumption that a point electron can be considered as the limit of a finite electron in which the charge is distributed, the work which is required in compressing a finite charge distributed over a finite volume into an infinitely small volume against the electromagnetic forces being infinite Dirac (1938) and Pryce (1938) and more recently Bhabha and Harish-Chandra (1946) have shown that it is quite possible to construct a scheme of point electron in the classical theory from which the infinite self-energy can be eliminated in a relativistic invariant way But unfortunately the scheme in some cases leads to solutions which are in conflict with the physical ideas (Eliezer, 1943) It has however not been possible as yet to develop an analogous scheme of Dirac for removing in a relativistic invariant way the infinites which appear in the equation characterising the electron in quantum mechanics in spite of the fact that it is treated as a point Dirac's (1942) new method of field quantisation by introducing the λ-limiting process and the negative energy photons to ensure the convergence has also its validity within a limited scope The λ-limiting process fails altogether in the hole theory which alone satisfactorily accounts for the creation and annihilation of electron-position pair (Pauli, 1943) It has further the drawback that it does not lead to the radiation damping which is so significant, particularly in the field theory of meson The discussion of these problems from Dirac's quantum electrodynamics will be taken up in detail in a subsequent paper It is to be noted that the strong divergent character of self energy for an electron in quantum mechanics in the original one electron theory of Dirac, in which it is assumed that all the negative energy states are empty, is obviously due to the emission of very high energy photons in the intermediate states and lies more in our assumption that the electron in the initial state can only emit but not absorb the photon But the self-energy in the hole theory is given by the difference of the self-energy of the electron in the positive energy when the negative energy states are occupied and that of the vacuum electrons filling up all the negative energy states The introduction of vacuum's contribution is just equivalent to allowing the electron also to absorb the photon in the initial state and it is this symmetrisation in the behaviour of the electron that reduces the self-energy to diverge only logarithmically In the present paper we shall give a straight-forward calculation of self-energies for a moving electron from quantum electrodynamics The calculations of self-energy were also undertaken by Weisskopf (1934) by following closely the method of classical electrodynamics, his results, however, remained incomplete due to some oversights in the calculations

1 *Interaction energy of the electron with the electromagnetic field*

The Lagrangian of the Dirac electron in the electromagnetic field is given as usual by

$$L = i\hbar c\left(\psi^{+}\gamma^{\mu}\frac{\partial\psi}{\partial x_{\mu}}+k\psi^{+}\psi\right)+e\psi^{+}\gamma^{\mu}\phi_{\mu}\psi-\frac{1}{16\pi}F_{\mu\nu}F_{\mu\nu} \tag{1}$$

with

$$\psi^{+} = i\psi^{*}\beta,\ \gamma^{k} = -i\beta\alpha^{k},\ \gamma^{4} = \beta,\ k = \frac{mc}{\hbar} \tag{2}$$

and

$$F_{\mu\nu} = \frac{\partial\phi_{\nu}}{\partial x_{\mu}}-\frac{\partial\phi_{\mu}}{\partial x_{\nu}} \tag{3}$$

where ψ is the wave function of the electron, α, β are well known Dirac's matrices and ϕ_{μ} is the 4-vector potential of the field with

$$\phi_{k} = A_{k},\ \phi_{4} = iA_{0},\ \mu,\nu = 1, 2, 3, 4, \text{ and } k = 1, 2, 3 \tag{4}$$

Further

$$F_{4k} = iE_{k},\quad F_{23},\ F_{31},\ F_{12} = H \tag{5}$$

and

$$\frac{1}{16\pi}F_{\mu\nu}F_{\mu\nu} = \frac{1}{8\pi}(H^{2}-E^{2}) \tag{6}$$

E and ***H*** being the electric and magnetic fields It is to be noted that the electromagnetic field here includes besides the field which is produced by the electron itself due to its motion and spin, also the field due to the Zero-point fluctuations of the radiation field

The Hamiltonian function can be readily obtained from the Lagrangian given by (1)

$$H = \int\left(\frac{\partial L}{\partial\left(\frac{\partial\psi}{\partial x_{4}}\right)}\cdot\frac{\partial\psi}{\partial x_{4}}+\frac{\partial L}{\partial\left(\frac{\partial\phi_{k}}{\partial x_{4}}\right)}\cdot\frac{\partial\phi_{k}}{\partial x_{4}}-L\right)d\tau \tag{7}$$

$$= \int \left\{ -i\hbar c\left(\psi^+\gamma^k \frac{\partial \psi}{\partial x_k} + k\psi^+\psi\right) - e\psi^+\gamma^\mu \phi_\mu \psi - \frac{1}{4\pi} F_{4k} \frac{\partial \phi_4}{\partial x_k} - \frac{1}{4\pi} F_{4k} F_{4k} + \frac{1}{16\pi} F_{\mu\nu} F_{\mu\nu} \right\} d\tau \tag{8}$$

or expressing in terms of electric and magnetic field strengths we obtain

$$H = \int \left[\psi^* \left\{ c\alpha\left(p - \frac{e}{c}A\right) + \beta mc^2 \right\} \psi + e\psi^* A_0 \psi - \frac{1}{4\pi} A_0 \operatorname{div} E + \frac{1}{8\pi}(E^2 + H^2) \right] d\tau \tag{9}$$

$$= H_e + H_F + H_D + H_S \tag{10}$$

where H_e and H_F are the usual Hamiltonian for the electron and the electromagnetic field, and H_D and H_S are the interaction energies of the electron with static and dynamic part of the field, the expectation values of which are the self energies

$$H_D = -e \int \psi^* (\alpha A) \psi \, d\tau \tag{11}$$

$$H_S = -e \int \{ \psi^* (\alpha A_l) \psi - \psi^* A_0 \psi \} \, d\tau \tag{12}$$

where A and A_l are the transverse and longitudinal part of the field

2 *Electrodynamic self-energy*

We now proceed to calculate the self-energy given by the interaction (11), where

$$A = \sum_k \sqrt{\frac{2\pi c\hbar^2}{k}} \, e\left(C_k e^{i(kr)/\hbar} + C_k^* e^{-i(kr)/\hbar}\right) \tag{13}$$

and

$$\psi = \sum_p a(p) \, u(p) \, e^{i(pr)/\hbar} \tag{14}$$

Here C_k and C_k^* are absorption and emission operators decreasing and increasing respectively the number of photons by one $a(p)$, $a^*(p)$ are similar operators for the electrons Substituting these values in (11) we obtain for the interaction energy of the electron with the field

$$H_D = \sum_p \sum_{p'} \sum_k -e\sqrt{\frac{2\pi c\hbar^2}{k}} \{ C_k \delta(p - p' + k) + C_k^* \delta(p - p' - k) \} N(p') \Delta^*(p') V(p') V(p) \Delta(p) N(p) \big(u^*(p') \alpha u(p)\big) \quad . \tag{15}$$

where $N(p)$ and $N(p')$ give the number of electrons in the states p and p' $\Delta(p)$, $\Delta(p')$ are operators which operating on $N(p)$, etc , change them to $1 - N(p)$ etc , and $V(p)$ and $V(p')$ are the Jordon-Wigner's Vorzeichenfunctions given by $+1$ or -1 according as the number of occupied states arranged in some definite manner before the state referred to is even or odd Now the expectation value of H_D is given in the first approximation by its diagonal matrix elements which is obviously zero Thus the self-energy is given by the second approximation of the interaction energy

$$W_D = \sum_m \frac{H_{Am} H_{mA}}{E_A - E_m} \quad . \quad . \quad . \quad . \; . \tag{16}$$

where the summation is to be taken over all the intermediate states

(*a*) Hole Theory

The self-energy is given by the difference of the contributions due to the following transitions of the electron in the positive energy state and the vacuum electrons

I The electron $u(\boldsymbol{p}_0)$ emits a photon $\boldsymbol{k}$ in going over to the state $u(\boldsymbol{p}_0-\boldsymbol{k})$, it then absorbs the photon $\boldsymbol{k}$ and comes back to the original state $u(\boldsymbol{p}_0)$

$$E_A-E_I=E(\boldsymbol{p}_0)-ck-E(\boldsymbol{p}_0-\boldsymbol{k}) \quad . \qquad . \tag{17}$$

II The vacuum electron $\underline{u}(\boldsymbol{p}_0+\boldsymbol{k})$ emits a photon $\boldsymbol{k}$ in going over to the positive energy state $u(\boldsymbol{p}_0)$, it then absorbs the photon $\boldsymbol{k}$ and comes back to the original negative energy state

$$E_A-E_{II}=-\big(E(\boldsymbol{p}_0)+ck+E(\boldsymbol{p}_0+\boldsymbol{k})\big) \tag{18}$$

The self energy is thus given by

$$\begin{aligned} W_D &= W_{Vac+1}-W_{Vac} \\ &= \sum_{\boldsymbol{k}}\frac{2\pi e^2\hbar^2 c}{k}\left\{\frac{\big(u(\boldsymbol{p}_0)^*\alpha u(\boldsymbol{p}_0-\boldsymbol{k})\big)\big(u(\boldsymbol{p}_0-\boldsymbol{k})^*\alpha u(\boldsymbol{p}_0)\big)}{E(\boldsymbol{p}_0)-ck-E(\boldsymbol{p}_0-\boldsymbol{k})}\right. \\ &\qquad\left.+\frac{\big(u(\boldsymbol{p}_0)^*\alpha \underline{u}(\boldsymbol{p}_0+\boldsymbol{k})\big)\big(\underline{u}(\boldsymbol{p}_0+\boldsymbol{k})^*\alpha u(\boldsymbol{p}_0)\big)}{E(\boldsymbol{p}_0)+ck+E(\boldsymbol{p}_0+\boldsymbol{k})}\right\} \end{aligned} \tag{19}$$

The summation is over the spin directions of the electrons as well as over all sorts of photons in the intermediate states Now carrying out the summation over the spins in the intermediate states and averaging in the initial states we obtain for the first term in the bracket

$$\frac{1}{8E(\boldsymbol{p}_0-\boldsymbol{k})E(\boldsymbol{p}_0)}Sp\,\alpha\big(E(\boldsymbol{p}_0-\boldsymbol{k})+H(\boldsymbol{p}_0-\boldsymbol{k})\big)\alpha\big(E(\boldsymbol{p}_0)+H(\boldsymbol{p}_0)\big)$$

$$=\frac{1}{8E(\boldsymbol{p}_0-\boldsymbol{k})E(\boldsymbol{p}_0)}Sp\,\alpha\big(c(\alpha,\ \boldsymbol{p}_0-\boldsymbol{k})+\beta mc^2+E(\boldsymbol{p}_0-\boldsymbol{k})\big)\alpha\big(c(\alpha\boldsymbol{p}_0)+\beta mc^2+E(\boldsymbol{p}_0)\big)$$

$$=\frac{1}{8E(\boldsymbol{p}_0-\boldsymbol{k})E(\boldsymbol{p}_0)}\big(-4c^2p_0{}^2+8c^2(\boldsymbol{e}\boldsymbol{p}_0)(\boldsymbol{e}\boldsymbol{p}_0)+4c^2(\boldsymbol{k}\boldsymbol{p}_0)-4m^2c^4+4E(\boldsymbol{p}_0)E(\boldsymbol{p}_0-\boldsymbol{k})\big)$$

Summing over the polarisation of the photons in the intermediate states with the help of the relation

$$\sum_{\boldsymbol{e}}(\boldsymbol{e}\boldsymbol{p})(\boldsymbol{e}\boldsymbol{p})=(\boldsymbol{p}\boldsymbol{p})-\frac{(\boldsymbol{p}\boldsymbol{k})(\boldsymbol{p}\boldsymbol{k})}{k^2} \quad . \tag{20}$$

the first term becomes

$$\frac{1}{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0-\boldsymbol{k})}\left\{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0-\boldsymbol{k})+c^2(\boldsymbol{k}\boldsymbol{p}_0)-\frac{c^2(\boldsymbol{p}_0\boldsymbol{k})(\boldsymbol{p}_0\boldsymbol{k})}{k^2}-m^2c^4\right\} \tag{21}$$

Similarly for the second term in the bracket we obtain

$$\frac{1}{8E(\boldsymbol{p}_0)E(\boldsymbol{p}_0+\boldsymbol{k})}Sp\,\alpha\big(-c(\alpha,\ \boldsymbol{p}_0+\boldsymbol{k})-\beta mc^2+E(\boldsymbol{p}_0+\boldsymbol{k})\big)\alpha\big(c(\alpha\boldsymbol{p}_0)+\beta mc^2+E(\boldsymbol{p}_0)\big)$$

$$=\frac{1}{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0+\boldsymbol{k})}\left\{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0+\boldsymbol{k})+c^2(\boldsymbol{p}_0\boldsymbol{k})+c^2\frac{(\boldsymbol{p}_0\boldsymbol{k})(\boldsymbol{p}_0\boldsymbol{k})}{k^2}+m^2c^4\right\} \quad . \quad .. \tag{22}$$

We now replace the summation over all the photons by integration and multiply thereby the expression (2) by $\frac{k^2dkd\Omega}{8\pi^3\hbar^3}$ which denotes the number of photons lying within the solid angle $d\Omega$ and within the range k and $k+dk$ The expression for the self-energy thus becomes

$$W_D = \frac{e^2c}{4\pi^2\hbar}\int (I_1+I_2)k\,dk \tag{23}$$

where

$$I_1 = \int d\Omega\left[\frac{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0-\boldsymbol{k})+c^2(\boldsymbol{p}_0\boldsymbol{k})-c^2\dfrac{(\boldsymbol{p}_0\boldsymbol{k})(\boldsymbol{p}_0\boldsymbol{k})}{k^2}-m^2c^4}{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0-\boldsymbol{k})\big(E(\boldsymbol{p}_0)-ck-E(\boldsymbol{p}_0-\boldsymbol{k})\big)}\right] \tag{24}$$

$$I_2 = \int d\Omega\left[\frac{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0+\boldsymbol{k})+c^2(\boldsymbol{p}_0\boldsymbol{k})+c^2\dfrac{(\boldsymbol{p}_0\boldsymbol{k})(\boldsymbol{p}_0\boldsymbol{k})}{k^2}+m^2c^4}{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0+\boldsymbol{k})\big(E(\boldsymbol{p}_0)+ck+E(\boldsymbol{p}_0+\boldsymbol{k})\big)}\right] \tag{25}$$

On integrating over the direction of $\boldsymbol{k}$ we have

$$I_1 = \frac{2\pi}{c}\left[-\frac{(E^3+E^2kc+Ek^2c^2+k^3c^3)(E_+-E_-)}{4Ep_0k^3c^3}-\frac{(E-kc)}{Ek}+\frac{(E-kc)(E_+^3-E_-^3)}{12Ep_0k^3c^3}\right.$$
$$\left.+\frac{m^2c^4}{Ep_0kc}\log\frac{E-kc-E_+}{E-kc-E_-}\right] \tag{26}$$

$$I_2 = \frac{2\pi}{c}\left[\frac{(E^3-E^2kc+Ek^2c^2-k^3c^3)(E_+-E_-)}{4Ep_0k^3c^3}+\frac{E+kc}{Ek}-\frac{(E+kc)(E_+^3-E_-^3)}{12Ep_0k^3c^3}\right.$$
$$\left.+\frac{m^2c^4}{Ep_0kc}\log\frac{E+kc+E_+}{E+kc+E_-}\right] \tag{27}$$

where

$$E = c\sqrt{p_0^2+m^2c^2},\quad E_+ = \sqrt{E^2+k^2c^2+2c^2kp_0},\quad E_- = \sqrt{E^2+k^2c^2-2c^2kp_0} \tag{28}$$

Whence we obtain finally

$$W_D = \frac{e^2}{2\pi\hbar}\int k\,dk\left\{\frac{-(E^2+k^2c^2)(E_+-E\ \ )}{2Ep_0k^2c^2}+\frac{2c}{E}-\frac{(E_+^3-E_-^3)}{6Ep_0k^2c^2}\right.$$
$$\left.+\frac{m^2c^4}{Ep_0kc}\log\frac{E^2-(kc+E_+)^2}{E^2-(kc+E_-)^2}\right\} \tag{29}$$

The expression is sufficiently complicated for interpretation We can, however, evaluate it when the kinetic energy of the electron is small compared with its rest energy We have then

$$W_D = \frac{e^2}{2\pi\hbar cE}\mathop{\mathrm{Lt}}_{k\to\infty}\left[c^2k^2-ck\sqrt{E^2+k^2c^2}+m^2c^4\log\frac{kc+\sqrt{E^2+k^2c^2}}{mc^2}\right.$$
$$\left.-p_0^2c^2\left\{\frac{4}{3}\log\frac{kc+\sqrt{E^2+k^2c^2}}{mc^2}+\frac{kc}{\sqrt{E^2+k^2c^2}}-\frac{1}{3}\frac{k^3c^3}{(E^2+k^2c^2)^{3/2}}\right\}\right] \tag{30}$$

$$=\frac{e^2}{2\pi\hbar c}mc^2\left(1-\frac{11}{6}\frac{p_0^2}{m^2c^2}\right)\mathop{\mathrm{Lt}}_{k\to\infty}\log\frac{k}{mc}+\text{finite terms} \tag{31}$$

This approximate result has been given also by Weisskopf The self-energy diverges only logarithmically.

(*b*) Original Theory

It may be of some interest to deduce the expression for self-energy in the original theory of Dirac where it is assumed that all the negative energy states of the electron are empty We have the following processes to consider —

I The same as before

II The electron $u(\boldsymbol{p}_0)$ emits a photon $\boldsymbol{k}$ in going over to the negative energy state $\underline{u}(\boldsymbol{p}_0-\boldsymbol{k})$, it then absorbs the photon $\boldsymbol{k}$ and comes back to the original state $u(\boldsymbol{p}_0)$

$$E_{I}-E_{II}=E(\boldsymbol{p}_0)-ck+E(\boldsymbol{p}_0-\boldsymbol{k}) \tag{32}$$

We have thus

$$W_D=\sum_{\boldsymbol{k}}\frac{2\pi e^2\hbar^2c}{k}\left\{\frac{\left(u(\boldsymbol{p}_0)^*\alpha u(\boldsymbol{p}_0-\boldsymbol{k})\right)\left(u(\boldsymbol{p}_0-\boldsymbol{k})^*\alpha u(\boldsymbol{p}_0)\right)}{E(\boldsymbol{p}_0)-ck-E(\boldsymbol{p}_0-\boldsymbol{k})}\right.$$
$$\left.+\frac{\left(u(\boldsymbol{p}_0)^*\alpha u(\boldsymbol{p}_0-\boldsymbol{k})\right)\left(\underline{u}(\boldsymbol{p}_0-\boldsymbol{k})\alpha u(\boldsymbol{p}_0)\right)}{(E\boldsymbol{p}_0)-ck+E(\boldsymbol{p}_0-\boldsymbol{k})}\right\} \tag{33}$$

Now carrying out the summation over the spins and polarisation in the intermediate states and averaging over the spins of the electron in the initial states as before we obtain —

$$W_D=\frac{e^2c}{4\pi\hbar}\int(I_1+I_2)k\,dk \tag{34}$$

where

$$I_2=\int d\Omega\left[\frac{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0-\boldsymbol{k})-c^2(\boldsymbol{p}_0\boldsymbol{k})+\dfrac{c^2(\boldsymbol{p}_0\boldsymbol{k})(\boldsymbol{p}_0\boldsymbol{k})}{k^2}+m^2c^4}{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0-\boldsymbol{k})\left(E(\boldsymbol{p}_0)-ck+E(\boldsymbol{p}_0-\boldsymbol{k})\right)}\right] \tag{35}$$

which on integration over the direction of $\boldsymbol{k}$ reduces to

$$I_2=\frac{2\pi}{c}\left[\frac{(E^3+E^2kc+Ek^2c^2+k^3c^3)(E_+-E_-)}{4Ep_0k^3c^3}-\frac{(E-kc)}{Ek}-\frac{(E-kc)(E_+^3-E_-^3)}{12Ep_0k^3c^3}\right.$$
$$\left.+\frac{m^2c^4}{Ep_0kc}\log\frac{E-kc+E_+}{E-kc+E_-}\right] \tag{36}$$

and we obtain

$$W_D=\frac{e^2}{\pi\hbar cE}\left[c^2\int_0^\infty k\,dk+\left(\frac{m^2c^4}{2p_0}\log\frac{E+p_0c}{E-p_0c}-Ec\right)\int_0^\infty dk\right] \tag{37}$$

This shows that the self energy diverges The expression (37) was first obtained by Waller

3 *Electrostatic self energy*

In classical electrodynamics the expression for electrostatic self-energy as given by (12) is evaluated with the help of the well-known Maxwell-Lorentz equations

$$\text{rot}\,\boldsymbol{H}-\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t}=\frac{4\pi}{c}\boldsymbol{i},\quad \text{div}\,\boldsymbol{E}=4\pi\rho$$

and,

$$\boldsymbol{E}=-\frac{1}{c}\frac{\partial\boldsymbol{A}}{\partial t}-\text{grad}\,A_0,\quad \boldsymbol{H}=\text{rot}\,\boldsymbol{A} \quad .. \tag{38}$$

together with

$$\text{div}\,\boldsymbol{E}_{\text{trans}}=0,\ \text{rot}\,\boldsymbol{E}_l=0,\ \boldsymbol{H}_l=0 \quad .. \quad .. \quad .. \tag{39}$$

Now for Dirac electron

$$\rho = e\,(\psi^*\psi),\; i = e\,(\psi^*\alpha\psi) \tag{40}$$

Therefore (12) gives

$$H_s = \int \left\{ \rho A_0 - \frac{1}{c}(iA_l) \right\} dr$$

$$= \frac{1}{8\pi} \int \left[E_l^2 + \frac{1}{c}\frac{d}{dt}(A_l E) \right] dr \tag{41}$$

The electrostatic self-energy which is defined as the expectation value of H_s is therefore given by

$$W_s = \overline{H}_s = \frac{1}{8\pi} \int E_l^2 \, dr = \frac{1}{2} \int \frac{\rho(r)\rho(r')}{|r-r'|} \, dr \, dr' \tag{42}$$

For a point electron this becomes infinite We cannot take over this result directly into quantum mechanics It is usually left unquantised and subtracted from the Hamiltonian representing the interaction It may be of some interest, however, to attempt to evaluate the expression (12) exactly in an analogous way as in the transverse part of the self-energy We accordingly express the quantities A_l and A_0 as

$$A_l = \sum_k \sqrt{\frac{2\pi c\hbar^2}{k}} \, n\left(c_k \, e^{i(kr)/\hbar} + c_k^* \, e^{-i(kr)/\hbar}\right) \tag{43}$$

$$A_0 = \sum_k \sqrt{\frac{2\pi c\hbar^2}{k}} \left(c_k' \, e^{i(kr)/\hbar} + c_k'^* \, e^{-i(kr)/\hbar}\right) \tag{44}$$

where $n = \frac{k}{k}$ is the unit vector in the direction of the wave vector k c_k, $c_k'^*$ are operators as before denoting respectively the decrease and the increase in the number of longitudinal photons The interaction energy of the electron with the longitudinal part of the field is obtained from (12) as

$$H_s = \sum_p \sum_{p'} \sum_k -e \sqrt{\frac{2\pi c\hbar^2}{k}} \left\{ c_k' \, \delta(p-p'+k) + c_k'^* \, \delta(p-p'-k) \right\}$$

$$N(p')\Delta^*(p')V(p')V(p)\Delta(p)N(p) \times \left(u(p')^* \alpha_k u(p) - u(p')^* u(p)\right) \tag{45}$$

The self-energy is then obtained in an analogous way as before from the expression

$$W_s = \sum_m \frac{H^{sl}_{Am} H^{sl}_{mA}}{E_A - E_m} - \sum_m \frac{H^{so}_{Am} H^{so}_{mA}}{E_A - E_m} \tag{46}$$

the two parts being the contributions of the longitudinal and the scalar fields respectively

(*a*) Hole Theory

Considering the self-energy as the difference between the self-energy when an electron is present in the positive energy state and the self-energy of vacuum we obtain

$$W_s = \sum_k \frac{2\pi e^2\hbar^2 c}{k} \left\{ \frac{\big(u(p_0)^* \alpha_k u(p_0-k)\big)\big(u(p_0-k)^* \alpha_k u(p_0)\big) - \big(u(p_0)^* u(p_0-k)\big)\big(u(p_0-k)^* u(p_0)\big)}{E(p_0) - ck - E(p_0-k)} \right.$$

$$\left. + \frac{\big(u(p_0)^* \alpha_k \underline{u}(p_0+k)\big)\big(\underline{u}(p_0+k)^* \alpha_k u(p_0)\big) - \big(u(p_0)^* \underline{u}(p_0+k)\big)\big(\underline{u}(p_0+k) u(p_0)\big)}{E(p_0) + ck + E(p_0+k)} \right\} \tag{47}$$

where
$$\alpha_k = \frac{(\alpha \boldsymbol{k})}{k} \tag{48}$$

Making use of Dirac's wave equation we can write

$$u(\boldsymbol{p}_0)^*(\alpha \boldsymbol{k})u(\boldsymbol{p}_0-\boldsymbol{k}) = \frac{1}{c}\left[E(\boldsymbol{p}_0)-E(\boldsymbol{p}_0-\boldsymbol{k})\right]u(\boldsymbol{p}_0)^*u(\boldsymbol{p}_0-\boldsymbol{k})$$

$$u(\boldsymbol{p}_0)^*(\alpha \boldsymbol{k})\underline{u}(\boldsymbol{p}_0+\boldsymbol{k}) = -\frac{1}{c}\left[E(\boldsymbol{p}_0+\boldsymbol{k})+E(\boldsymbol{p}_0)\right]u(\boldsymbol{p}_0)^*\underline{u}(\boldsymbol{p}_0+\boldsymbol{k}), \text{ etc} \tag{49}$$

The expression (47) thus reduces to

$$W_s = \sum_{\boldsymbol{k}} \frac{2\pi e^2\hbar^2}{ck^3}\Big\{\left[E(\boldsymbol{p}_0)+ck-E(\boldsymbol{p}_0-\boldsymbol{k})\right]\big(u(\boldsymbol{p}_0)^*u(\boldsymbol{p}_0-\boldsymbol{k})\big)\big(u(\boldsymbol{p}_0-\boldsymbol{k})^*u(\boldsymbol{p}_0)\big)$$
$$+\left[E(\boldsymbol{p}_0)-ck+E(\boldsymbol{p}_0+\boldsymbol{k})\right]\big(u(\boldsymbol{p}_0)^*\underline{u}(\boldsymbol{p}_0+\boldsymbol{k})\big)\big(\underline{u}(\boldsymbol{p}_0+\boldsymbol{k})^*u(\boldsymbol{p}_0)\big)\Big\} \tag{50}$$

Carrying out the summation and averages as before

$$W_s = \frac{\pi e^2\hbar^2}{4ck^3E(\boldsymbol{p}_0)}Sp\left\{\frac{\big(E(\boldsymbol{p}_0-\boldsymbol{k})+H(\boldsymbol{p}_0-\boldsymbol{k})\big)\big(E(\boldsymbol{p}_0)+H(\boldsymbol{p}_0)\big)\big(E(\boldsymbol{p}_0)+ck-E(\boldsymbol{p}_0-\boldsymbol{k})\big)}{E(\boldsymbol{p}_0-\boldsymbol{k})}\right.$$
$$\left.+\frac{\big(E(\boldsymbol{p}_0+\boldsymbol{k})-H(\boldsymbol{p}_0+\boldsymbol{k})\big)\big(E(\boldsymbol{p}_0)+H(\boldsymbol{p}_0)\big)\big(E(\boldsymbol{p}_0)-ck+E(\boldsymbol{p}_0+\boldsymbol{k})\big)}{E(\boldsymbol{p}_0+\boldsymbol{k})}\right\}$$

Now on evaluating the spur and summing over the intermediate states of the photon we obtain finally

$$W_s = \frac{e^2}{8\pi^2\hbar c}\int (I_3+I_4)\,\frac{dk}{k} \tag{51}$$

where

$$I_3 = \int d\Omega\left[\frac{\big(E(\boldsymbol{p}_0)+ck\big)\big(E(\boldsymbol{p}_0)^2-c^2(\boldsymbol{p}_0\boldsymbol{k})\big)}{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0-\boldsymbol{k})}-\frac{\big(E(\boldsymbol{p}_0)-ck\big)\big(E(\boldsymbol{p}_0)^2+c^2(\boldsymbol{p}_0\boldsymbol{k})\big)}{E(\boldsymbol{p}_0)E(\boldsymbol{p}_0+\boldsymbol{k})}\right] \tag{52}$$

$$I_4 = \int d\Omega\, E(\boldsymbol{p}_0)\left[E(\boldsymbol{p}_0+\boldsymbol{k})-E(\boldsymbol{p}_0-\boldsymbol{k})\right] \tag{53}$$

The integration over the direction of $\boldsymbol{k}$ gives

$$W_s = \frac{e^2}{4\pi\hbar c}\int_0^{\kappa}\left[\frac{(E^2-k^2c^2)(E_+-E_-)}{Ep_0c}+\frac{(E_+^3-E_-^3)}{3Ep_0c}\right]\frac{dk}{k} \tag{54}$$

For small velocity of the electron it reduces to

$$W_s = \frac{e^2}{\pi\hbar c}mc^2\left(1+\frac{p_0^{\,2}}{6m^2c^2}\right)\operatorname*{Lt}_{k\to\infty}\log\frac{k}{mc}+\text{finite terms} \tag{55}$$

(*b*) Original Theory

In the case of one electron theory we have on this model

$$W_s = \sum_{\boldsymbol{k}} \frac{2\pi e^2\hbar^2 c}{k}\left\{\frac{\big(u(\boldsymbol{p}_0)^*\alpha_k u(\boldsymbol{p}_0-\boldsymbol{k})\big)\big(u(\boldsymbol{p}_0-\boldsymbol{k})^*\alpha_k u(\boldsymbol{p}_0)\big)-\big(u(\boldsymbol{p}_0)^*u(\boldsymbol{p}_0-\boldsymbol{k})\big)\big(u(\boldsymbol{p}_0-\boldsymbol{k})^*u(\boldsymbol{p}_0)\big)}{E(\boldsymbol{p}_0)-ck-E(\boldsymbol{p}_0-\boldsymbol{k})}\right.$$
$$\left.+\frac{\big(u(\boldsymbol{p}_0)^*\alpha_k\underline{u}(\boldsymbol{p}_0-\boldsymbol{k})\big)\big(\underline{u}(\boldsymbol{p}_0-\boldsymbol{k})^*\alpha_k u(\boldsymbol{p}_0)\big)-\big(u(\boldsymbol{p}_0)^*\underline{u}(\boldsymbol{p}_0-\boldsymbol{k})\big)\big(\underline{u}(\boldsymbol{p}_0-\boldsymbol{k})^*u(\boldsymbol{p})\big)}{E(\boldsymbol{p}_0)-ck+E(\boldsymbol{p}_0-\boldsymbol{k})}\right\} \tag{56}$$

$$= \frac{\pi e^2 \hbar^2}{ck^3} Sp \left\{ \frac{\big(E(p_0)+ck\big)\big(E(p_0)+H(p_0)\big)-H(p_0-k)\big(E(p_0)+H(p_0)\big)}{2E(p_0)} \right\} \tag{57}$$

Summing over the photons we finally obtain the standard result

$$W_s = \frac{e^2}{\pi\hbar} \int_0^\infty dk, \qquad \ldots \qquad \ldots \tag{58}$$

which is highly divergent

One of us (S N Gupta) is grateful to Sir Maurice Gwyer, the Vice-Chancellor of the Delhi University, for kindly awarding him one of the exhibition scholarships which enabled him to carry out the present investigation

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ON THE CLASS-NUMBER OF THE CORPUS $P(\sqrt{-k})$

By S Chowla, *Govt College, Lahore*

(Communicated by Sir S S Bhatnagar, F R S)

(*Received October 28, 1946*)

§1 In a paper carrying the title of the present paper, Littlewood (1928) proved If the extended Riemann hypothesis (e R h) is true, there exist infinitely many k such that

$$L(1) = \sum_{1}^{\infty} \frac{x(n)}{n} > \{1+0(1)\}\, e^{C} \log\log k$$

where $x(n)$ is a real primitive character (mod k)

This result was proved by Walfisz (1942) *without assuming the e R h* His proof is based on the so-called 'class-number relations' discovered by Kronecker In this paper I use the method developed by me in my paper 'An improvement of a theorem of Linnik and Walfisz' to give another proof of the result *without assuming the e R h*

§2 Throughout we use the notation of my paper 3 In the definitions we only change the definition of b so that

$$\left(\frac{b}{p_r}\right) = +1 \text{ for } 1 \leqq r \leqq (g-1),$$

$$\left(\frac{b}{p_g}\right) = -1,$$

$$b \equiv 1 \pmod 8,$$

$$1 < b < 8a,$$

as before,

$$(1) \qquad T(x) = \sum_{x < n \leqq 2x} \; \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{8an+b}{m}\right),$$

also $S(x)$ is the same sum but with m going up to $x^{\frac{1}{2}}$ in the inner sum The difference between $T(x)$ and $S(x)$ is of the order $x^{\frac{1}{2}}$, as proved in my paper 3

The sum $S(x)$ is split up as before

$$S(x) = S_1(x) + S_2(x) + S_3(x)$$

We find that

$$S_1(x) \sim \tfrac{1}{2} e^{C} x \, (\log\log x),$$

$$S_2(x) = O(x^{\frac{1}{2}}),$$

$$S_3(x) = O\left(\frac{x(\log\log x)^2}{\log x}\right),$$

the sums $S_2(x)$ and $S_3(x)$ are estimated in exactly the same way as in my paper 3, $S_1(x)$ only being different Thus we have

$$\text{(A)} \qquad T(x) \sim \tfrac{1}{2}e^{C}x \log\log x.$$

We now write

$$\text{(2)} \qquad T(x) = T_1(x) + T_2(x)$$

where $T_1(x)$ is defined by (1) with the difference that in the outer sum n is restricted to take values such that $(8an+b)$ is quadratfrei , $T_2(x)$ is defined by the right-hand side of (1) but with n running through values in which $(8an+b)$ is divisible by a square greater than 1 We proceed to estimate $T_2(x)$ Now the numbers $(8an+b)$ cannot be divisible by p_r^2 unless $r > g$ The number of numbers $(8an+b)$ when $x < n \leqq 2x$ such that $8an+b$ is divisible by p_r^2 $(r > g)$ is clearly of the order

$$\sum_{r>g} \left(\frac{x}{p_r^2}\right) = O\left(\sum_{n>g} \frac{x}{n^2 \log^2 n}\right)$$

$$\text{(3)} \qquad = O\left(\frac{x}{g \log^2 g}\right) = O\left\{\frac{x(\log\log x)^2}{\log x(\log\log x)^2}\right\} = O\left(\frac{x}{\log x}\right)$$

Again, as observed by Davenport, we have

$$\sum_{n=u}^{v} x(n) = O(\sqrt{k} \log k)$$

where $x(n)$ is any non-principal character (mod k). It follows, since $a < x^{\frac{1}{10}}$ $(x > x_0)$ proved in my paper 3, that

$$\text{(4)} \qquad \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{8an+b}{m}\right) = O(\log x)$$

for every n with $x < n \leqq 2x$ It now follows from (4) and (3) that

$$\text{(5)} \qquad T_2(x) = O\left(\frac{x}{\log x} \log x\right) = O(x)$$

From (A), (2) and (5) we finally get

$$\text{(6)} \qquad T_1(x) \sim \tfrac{1}{2}e^{C}x \log\log x,$$

i.e

$$\text{(7)} \qquad \sum_{\substack{x < n \leqq 2x \\ (8an+b) \text{ quadratfrei}}} \; \sum_{m=1}^{\infty} \frac{1}{m} \cdot \left(\frac{8an+b}{m}\right) \sim \tfrac{1}{2}e^{C}x \log\log x$$

Since 'almost all' $(8an+b)$ are quadratfrei when $x < n \leqq 2x$, it follows from (7) that there exists a positive integer n with $x < n \leqq 2x$ and such that $(8an+b)$ is quadratfrei and

$$\text{(8)} \qquad \sum_{m=1}^{\infty} \frac{1}{m} \left(\frac{8an+b}{m}\right) > \tfrac{1}{2}e^{C} \{1+0(1)\} \log\log(8an+b)$$

since $\log\log x \sim \log\log(8an+b)$ From the Reciprocity Law for Jacobi's symbol

$$\left(\frac{8an+b}{m}\right) = \left(\frac{m}{8an+b}\right) \text{ when } m \equiv 1 \pmod 2,$$

$$\left(\frac{8an+b}{m}\right) = O \text{ when } m \equiv O (\text{mod } 2),$$

by definition Hence (8) becomes

$$(9) \qquad \sum_{\text{mod } d} \frac{1}{m}\left(\frac{m}{8an+b}\right) > \frac{1}{2}e^{C}\{1+0(1)\} \log\log(8an+b),$$

now

$$\left(\frac{2}{8an+b}\right) = +1 \text{ since } b \equiv 1 \pmod 8$$

Hence

$$\sum_{m=1}^{\infty} \frac{1}{m}\left(\frac{m}{8an+b}\right) = \left(1+\frac{1}{2}+\frac{1}{2^2}+ \quad \right) \sum_{\text{mod } d} \left(\frac{m}{8an+b}\right)\frac{1}{m}$$

$$(10) \qquad = 2 \sum_{\text{mod } d} \frac{1}{m}\left(\frac{m}{8an+b}\right)$$

From (9) and (10) we get

$$(11) \qquad \sum_{m=1}^{\infty} \frac{1}{m}\left(\frac{m}{8an+b}\right) > e^{C}\{1+0(1)\} \log\log(8an+b)$$

for suitable n with $x < n \leqq 2x$, and for all $x > x_0$ since $(8an+b)$ is quadratfrei in (8), (9), (10), (11) we finally get

Theorem 1 For all $x > x_0$ there exists a quadratfrei number $k = 8an+b$ where $x < n \leqq 2x$ such that

$$(12) \qquad \sum_{m=1}^{\infty} \frac{1}{m}\left(\frac{m}{k}\right) > \{1+0(1)\}\, e^{C} \log\log k$$

Since $\left(\dfrac{m}{k}\right)$ is a real primitive character (mod k) in (12), on account of k being quadratfrei, we can write Theorem 1 as

Theorem 2 For all $x > x_0$ there exists a number k between x and x^2 and such that

$$\sum_{1}^{\infty} \frac{x(n)}{n} > \{1+0(1)\}\, e^{C} \log\log k$$

where $x(n)$ is a real primitive character (mod k) Thus Littlewood's Theorem (and more) has been proved without assuming the *e.R h.*

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[Note In a paper entitled On the k analogue of a result in the theory of the Riemann Zeta function' *Mathematische Zeitschrift* (1934) Band **38**, 483–487 I have proved that

$$\sum_{1}^{\infty} \frac{x(n)}{n} = \Omega_R (\log \log k)$$

where $x(n)$ is a real primitive character (mod k)]

Note added during proof correction (*May 7, 1947*)

My paper 'An improvement of a theorem of Linnik and Walfisz has been accepted for publication by the London Math Society There is *very little* difference between the arguments of the present papers and those of the paper to be published by the London Math Society

PROCEEDINGS
OF THE
NATIONAL INSTITUTE OF SCIENCES OF INDIA

JULY—SEPTEMBER, 1947

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11 MAY 1949

A TABLE OF VALUES OF $\tau(n)$.

By HANSRAJ GUPTA, *Government College, Hoshiarpur*

(Communicated by Prof D S Kothari, F N I)

(*Received February 23; read March 7, 1947*)

Ramanujan's function $\tau(n)$ is defined by the relation

$$\sum_{1}^{\infty} \tau(n)x^{n} \equiv x\prod_{1}^{\infty}(1-x^{r})^{24}, \quad |x| < 1$$

Lehmer (1943), in order to verify a conjecture of Ramanujan, computed a table of values of $\tau(n)$ for values of n up to 300

More recently the following interesting congruence properties of this function have been obtained by Hardy, Watson, Mordell, Wilton, Ramanathan, Lahiri, Chowla, Bambah and me —

(1) $\tau(n) \equiv \sigma_{11}(n)$ (mod 2^{8}) if n is odd

(2) $\tau(n) \equiv K\sigma_{7}(n)$ (mod 3^{4})

where $K = n^{2}$ if $n \not\equiv 2$ (mod 3)

and $K = n^{2}+9$ if $n \equiv 2$ (mod 3)

(3) $\tau(n) = 5n^{2}\sigma_{7}(n) - 4n\sigma_{9}(n)$ (mod 5^{3}) if $n \not\equiv 0$ (mod 5)

(4) $\tau(n) \equiv n\sigma_{9}(n)$ (mod 7)

(5) $\tau(n) \equiv 0$ (mod 23) if $\left(\frac{n}{23}\right) = -1$

(6) $\tau(n) \equiv \sigma_{11}(n)$ (mod 691)

where $\sigma_{k}(n) = \sum_{d/n} d^{k}$

The table is here extended to $n = 400$

n	$\tau(n)$	n	$\tau(n)$
1	1	11	5 34612
2	−24	12	−3 70944
3	252	13	−5 77738
4	−1472	14	4 01856
5	4830	15	12 17160
6	−6048	16	9 87136
7	−16744	17	−69 05934
8	84480	18	27 27432
9	−1 13643	19	106 61420
10	−1 15920	20	−71 09760

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n	$\tau(n)$	n	$\tau(n)$
21	−42 19488	61	69564 78662
22	−128 30688	62	12682 36032
23	186 43272	63	19028 38392
24	212 88960	64	26992 96768
25	−254 99225	65	−27904 74540
26	138 65712	66	−32333 33376
27	−732 79080	67	−1 54818 26884
28	246 47168	68	1 01655 34848
29	1284 06630	69	46981 04544
30	−292 11840	70	19409 64480
31	−528 43168	71	97914 85272
32	−1967 06304	72	−96005 60640
33	1347 22224	73	14637 91322
34	1657 42416	74	43731 19536
35	−808 73520	75	−64258 04700
36	1672 82496	76	−1 56936 10240
37	−1822 13314	77	−89515 43328
38	−2558 74080	78	34941 59424
39	−1455 89976	79	3 81168 45680
40	4080 38400	80	47678 66880
41	3081 20442	81	16651 88361
42	1012 67712	82	−73948 90608
43	−171 25708	83	−2 93350 99668
44	−7869 48864	84	62110 86336
45	−5488 95690	85	−3 33556 61220
46	−4474 38528	86	4110 16992
47	26873 48496	87	3 23584 70760
48	2487 58272	88	4 51640 21760
49	−16969 65207	89	−2 49929 17110
50	6119 81400	90	1 31734 96560
51	−17402 95368	91	96736 45072
52	8504 30336	92	−2 74428 96384
53	−15960 55698	93	−1 33164 78336
54	17586 97920	94	−6 44963 63904
55	25821 75960	95	5 14946 58600
56	−14145 33120	96	−4 95699 88608
57	26866 77840	97	7 50135 68546
58	−30817 59120	98	4 07271 64968
59	−51892 03740	99	−6 07549 11516
60	−17916 59520	100	3 75348 59200

n	$\tau(n)$	n	$\tau(n)$
101	8 17429 59102	141	67 72118 20992
102	4 17670 88832	142	−23 49956 46528
103	−22 57551 28648	143	−30 88656 67656
104	−4 88073 06240	144	−11 21810 96448
105	−2 03801 27040	145	62 02040 22900
106	3 83053 36752	146	−3 51309 91728
107	9 02412 58356	147	−42 76352 32164
108	10 78668 05760	148	26 82179 98208
109	7 34826 76310	149	−111 54336 20850
110	−6 19722 23040	150	15 42193 12800
111	−4 59177 55128	151	−82 44472 97848
112	−1 65286 05184	152	90 06767 61600
113	−8 51468 62638	153	78 48110 57562
114	−6 44802 68160	154	21 48370 39872
115	9 00470 03760	155	−25 52325 01440
116	−18 90145 59360	156	21 43084 44672
117	6 56558 79534	157	131 51167 54406
118	12 45408 89760	158	−91 48042 96320
119	11 56329 58896	159	−40 22060 35896
120	10 28256 76800	160	−95 00914 48320
121	4983 19933	161	−31 21629 46368
122	−16 69554 87888	162	−3 99645 20664
123	7 76463 51384	163	−35 78327 59588
124	7 77851 43296	164	−45 35532 90624
125	−35 90011 00500	165	65 07083 41920
126	−4 56681 21408	166	70 40423 92032
127	−26 27172 01024	167	275 48338 92216
128	33 80713 88160	168	−35 64623 46240
129	−43156 78416	169	−145 83791 97393
130	6 69713 88960	170	80 05358 69280
131	63 15287 59932	171	−121 15957 53060
132	−19 83111 13728	172	2 52090 42176
133	−17 85148 16480	173	−95 03874 49578
134	37 15638 45216	174	−77 66032 98240
135	−35 39379 56400	175	42 69590 23400
136	−58 34133 04320	176	52 77347 51232
137	−29 71987 46214	177	−130 76793 42480
138	−11 27545 09056	178	59 98300 10640
139	59 67935 77940	179	168 13842 24780
140	11 90458 21440	180	80 79744 55680

n	$\tau(n)$	n	$\tau(n)$
181	−99 67744 96018	221	398 98204 97292
182	−23 21674 81728	222	110 20261 23072
183	175 30326 22824	223	733 48630 21472
184	157 49836 18560	224	329 36503 54176
185	−88 00903 06620	225	289 78084 26675
186	31 95954 80064	226	204 35247 03312
187	−369 19951 87608	227	−135 98395 65924
188	−395 57769 86112	228	−395 47897 80480
189	122 69849 15520	229	−1182 44112 23170
190	−123 58718 06400	230	−216 11280 90240
191	276 24033 50592	231	−225 57889 18656
192	68 02227 85536	232	1084 77921 02400
193	544 23876 85442	233	−1756 33534 48518
194	−180 03256 45104	234	−157 57411 08816
195	−70 31995 84080	235	1297 98932 35680
196	249 79327 84704	236	763 85079 05280
197	−287 60915 04354	237	960 54451 11360
198	145 81178 76384	238	−277 51910 13504
199	72 83914 02200	239	−713 95774 62960
200	−215 41745 28000	240	120 15024 53760
201	−390 14203 74768	241	−23 13069 09358
202	−196 18310 18448	242	−1 19596 78392
203	−215 00406 12720	243	1340 07966 51732
204	256 17147 81696	244	−1023 99365 90464
205	148 82217 34860	245	−819 63419 49810
206	541 81230 87552	246	−186 35124 33216
207	−211 86773 59896	247	−615 95074 67960
208	−57 03059 78368	248	−446 41908 32640
209	569 97230 69040	249	−739 24451 16336
210	48 91230 48960	250	861 60264 12000
211	−679 31684 39188	251	1298 30535 45252
212	234 93939 87456	252	−280 09781 13024
213	246 74542 88544	253	996 69169 30464
214	−216 57902 00544	254	630 52128 24576
215	−8 27171 69640	255	−840 56266 27440
216	−619 06166 78400	256	−1364 18730 96704
217	88 48060 04992	257	2396 11925 65506
218	−176 35842 31440	258	10 35762 81984
219	36 88754 13144	259	305 09797 29616
220	−380 09630 13120	260	410 75785 22880

n	$\tau(n)$			n	$\tau(n)$		
261	−1459	25146	53090	301	28	67528	54752
262	−1515	66902	38368	302	1978	67351	48352
263	−2427	37284	64488	303	2059	92256	93704
264	1138	13334	83520	304	1052	42714	93120
265	−770	89490	21340	305	3359	97919	37460
266	428	43555	95520	306	−1883	54653	81488
267	−629	82151	11720	307	1531	10928	28556
268	2278	92491	73248	308	1317	66717	78816
269	2583	77065	43670	309	−5689	02924	19296
270	849	45109	53600	310	612	55800	34560
271	−376	79323	60528	311	4987	51605	75912
272	−681	70960	65024	312	−1229	94411	72480
273	243	77585	58144	313	−9948	08327	56438
274	713	27699	09136	314	−3156	28021	05744
275	−1363	21916	75700	315	919	07094	33360
276	−691	56098	88768	316	−5610	79968	40960
277	−1641	89320	05874	317	8336	92483	59366
278	−1432	30458	70560	318	965	29448	61504
279	600	52561	41024	319	6864	77252	77560
280	−683	21949	69600	320	1303	76033	89440
281	2103	57229	07082	321	2274	07971	05712
282	−1625	30837	03808	322	749	19107	12832
283	1671	31763	26532	323	−7362	70628	66280
284	−1441	30663	20384	324	−245	11572	67392
285	1297	66539	67200	325	1473	18712	53050
286	741	27760	23744	326	858	79862	30112
287	−515	91686	80848	327	1851	76344	30120
288	2235	42945	05472	328	2603	00149	40160
289	1342	00281	04723	329	−4499	69632	17024
290	−1488	48965	49600	330	−1561	70002	06080
291	1890	34192	73592	331	−6358	40219	25868
292	−215	47008	25984	332	4318	12667	11296
293	−2392	68589	87458	333	2070	72676	42902
294	1026	32455	71936	334	−6611	60134	13184
295	−2506	38540	64200	335	−7477	72238	49720
296	−1539	33807	66720	336	−416	52085	06368
297	−3917	58755	16960	337	12100	14283	35986
298	2677	04069	00400	338	3500	11007	37432
299	−1077	09266	78736	339	−2145	70093	84776
300	945	87845	18400	340	4909	95333	15840

n	$\tau(n)$			n	$\tau(n)$		
341	−2825	05917	30816	371	2672	43566	07312
342	2907	82980	73440	372	1960	18561	10592
343	6152	23444	10800	373	−5516	17340	23378
344	−144	67798	11840	374	8860	78845	02592
345	2269	18449	47520	375	−9046	82773	26000
346	2280	92987	89872	376	22702	72009	42080
347	−15566	15610	78204	377	−7418	53896	02940
348	−4763	16689	58720	378	−2944	76379	72480
349	−2564	30221	94650	379	14646	31163	22980
350	−1024	70165	61600	380	−7580	01374	59200
351	4233	61091	21040	381	−6620	47346	58048
352	−10516	15505	94048	382	−6629	76804	14208
353	2490	98152	45602	383	23144	95717	33632
354	3138	43042	19520	384	8519	39898	16320
355	4729	28738	63760	385	−4323	59542	74240
356	3678	95739	85920	386	−13061	73044	50608
357	2913	95056	41792	387	194	62168	34244
358	−4035	32213	94720	388	−11041	99728	99712
359	15758	41508	53560	389	−14987	15716	11810
360	−4637	07078	91200	390	1687	67900	17920
361	−282	43824	81819	391	−12874	92059	76048
362	2392	25879	04432	392	−14335	96206	87360
363	12	55766	23116	393	15914	52475	02864
364	−1423	96055	45984	394	6902	61961	04496
365	707	01120	85260	395	18410	43646	34400
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369	−3501	57313	90206	399	−4498	57337	52960
370	2112	21673	58880	400	−2517	12029	69600

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LIFE HISTORY OF *TYPHONIUM TRILOBATUM* SCHOTT.

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(*Received September 25, 1946 , read April 4, 1947*)

The genus *Typhonium* contains about 25 species, distributed over the tropical regions of the world It occurs in Burma, Malaya, Siam, Ceylon, China, Malayasia and North Australia It has been reported from various parts of India, such as Bengal, Bihar, Orissa, Chota-Nagpur, Madras, the Eastern and Western Ghats and Assam

Typhonium trilobatum is one of the common aroids of Bengal It is a perennial herb and is easily recognised by its three-lobed hastate leaves, which appear to arise directly from the ground The plants grow in waste places and profusely flower from about the later part of May to the end of October Rarely they form a pure community More often they grow along with such plants as *Cynodon dactylon*, *Digitaria sanguinales*, *Paspulasm scrobiculatum*, *Ruellia tuberosa*, *Tridax procumbens*, *Boerhaavia repens* and *Colocasia antiquorum*

T trilobatum thrives in moist and partially shaded localities There is, however, considerable variation in size of the leaves according to conditions of growth At the end of the growing season the above ground portion of the plant dies down, and no trace of it is observed above the soil With the advent of the next monsoon, leaves appear again from the underground stem

Two varieties of *Typhonium trilobatum* have been reported to occur Var *genuinum* Engler, has lamina hastately trisect and the apex of the spadix red, whereas the Var *Schotti* (Prain) Engler, has lamina hastately trilobed and the apex of the spadix white The former variety occurs in lower Bengal

Previous Work

The family Araceae has received much attention from botanists Towards the beginning of this century Campbell (1900), Duggar (1900), Gow (1913) and others worked on the embryology of the family Later, Engler (1920) monographed the family, while Solereder (1928) gave a comprehensive account of its anatomical features Jussen (1928) described in detail the haploid generation of some members, while Schnarf (1931) reviewed the relevant literature on the embryology of plants Ertl (1932) studied the nature of the venation Since 1928 a few more important publications on the morphology and cytology of plants belonging to this family have appeared, of which mention may be made of Boodle and Hill's (1929) work on *Typhonodorum Lindleyanum*, Dudley's (1937) on *Calla palustris*, Buell's (1938) investigations on the life history of *Acorus calamus* and Goldberg's (1941) on *Peltandra virginica*

In India, not much work has been done on this family Blatter (1932) has revised the species occurring in the Bombay Presidency Barnes (1934) has described the morphology and the mode of pollination in the genus *Arisaema* occurring in the Nilgiri Hills He (1938 and 1940) has also recorded his observations on right and left handed asymmetry in South Indian Aroids, and has described a new species of *Arisaema* (*A psittacus*). Asana and Sutaria (1935) have recorded the number and the morphology of the somatic and meiotic chromosomes of *Arisaema murrayi*. This was followed by a contribution by the present writer (Banerji, 1937) on the sterility of *Colocasia antiquorum*. Later, Asana and Sutaria (1939) gave an account of the morphology and the number of chromosomes of some Indian aroids McCann (1943) has recently described the light windows in the genus *Cryptocoryne*

MATERIAL AND METHODS

The material used in this investigation was mostly obtained from plants growing under natural conditions A few plants however were grown in the experimental garden for the purpose of closer observation

For anatomical work pieces of petiole leaves tuber spathe spadix and roots were fixed in Formalin acetic alcohol Microtome sections were made of these parts. Free hand sections were also cut and examined The sections were stained either with Safranin and Fast green or Gentian Violet and Bismarck Brown combinations

For cytological studies root tips were collected from corms grown in saw dust and fixed in various modifications of Lewitsky s fluid The morphology of the chromosomes was best seen in material fixed in 1% Chromic acid one part and 10% Formaldehyde 2 parts

For the study of meiosis inflorescences (the staminate portion) in all stages of development were cut into discs 5 to 8 mm thick These were treated with Carnoy s fluid and then fixed in Nawashin s or Belling s modified Nawashin s fluids Other fixatives were also tried but these did not give good results

For embryological work Allen s modified Bouin s fluid and Licent s fluid were chiefly used In this case also the material was cut into small discs to facilitate penetration

Fixation was carried out at different periods of the day It was found that meiotic stages in the sporocytes were best obtained at about 8 a m whereas mitosis in the root tip cells was most common at about 11 a m The fixation was always done in the field and an exhaust pump was used whenever it was found necessary The materials were dehydrated and cleared in the usual way and finally embedded in paraffin Sections were cut 8 to 20μ thick depending on the stage required for study

Heidenhain s Iron alum Haematoxylin and Newton s Gentian Violet Iodine were the stains used for cytological studies For the determination of chromosome nucleolus relationship as also the number of nucleoli in telophase Bhaduri and Sommen s (1939) Feulgen Light Green stain was used Slides showing embryo logical stages were stained with Heidenhain s Haematoxylin Orange G as a counter stain was used for certain preparations

I MORPHOLOGY

The stem of *Typhonium trilobatum* is a subterranean corm of many internodes This is evident from the presence of withered leaf bases and scars on the surface of the corm The corm originates from an axillary or terminal bud of the previous season s corm During the growing season a large number of axillary buds develop on the mother corm these have their bluntly conical apices projecting upwards Sometimes these buds give rise to foliage leaves but generally they do not attain full development in the season They remain in a dormant condition while attached to the mother corm In the next season these buds develop into separate plants, while the mother corm shrivels and disintegrates This accounts for the occurrence of a large number of plants in close aggregation

In form the corm is somewhat sub globose or cylindrical and white in colour. At the lower end is attached the dark brown shrivelled residue of the previous season's corm In size it varies from 1 to 5 cm in diameter The growing point is situated at the top covered by the leaf bases Vegetative buds do not develop in the axils of the leaves of the growing season

Adventitious roots arise in two or more layers below the terminal portion of the corm These roots radiate more or less horizontally and are closely aggregated (Fig 1) They pierce through the leaf-bases in many instances The roots are white in colour except the tips, which are somewhat yellowish The rest of the corm is distinctly free from roots The older roots are slightly fusiform at their base and show corrugated foldings of their surface, which extend from their point of insertion to a distance of about 30 mm or more in certain cases, suggesting thereby their contractile nature Various stages of contraction are noted in different roots and it thus appears that there is no dimorphism in this respect and all the roots originating directly from the corm contract with age These roots branch at some distance The branch roots are slender and non-contractile Where a number of daughter corms develop together, each of them develops its own root system, which agrees in all essential features with that of the mother corm

The lamina is glabrous, characteristically hastate in form and somewhat trisect (Fig 3) The depth of lobation is a variable feature and depends on the size of the leaf The median lobe is slightly larger than the laterals The margin is entire and the apices of the lobes acute The petiole varies in length from 24 to 60 cm It is somewhat circular in outline except at the proximal portion where it becomes markedly grooved due to the development of the basal leaf sheath At the distal end also it shows the presence of a slender furrow on the adaxial side close to the lamina It is green in colour with a few disjoined red streaks on its surface The leaf base encircles the stem at its point of insertion (Fig 2) It encases completely the next leaf and the adjoining inflorescence during the early stages of their development by overlapping of one of the margins Ligular structures noted in some aroids are absent

As is characteristic of aroids, the leaf is reticulately veined The trilobed lamina is characterised by three principal veins All the three primary veins extend up to the apex of the lobes The primary vein (midrib) of the central lobe appears to be a direct continuation of the petiole, whereas the other two, branch at angles varying from 75° to 90° and proceed for about a centimetre or more without throwing off any lateral vein on the outer sides (Fig 3) Thus the lamina is absent at these regions The lateral primary veins then throw off secondary branches as shown in figure 3 The lateral lobes of the leaf being asymmetrical, the secondary and tertiary veins are greater in number and branch more profusely on the outer side In all the three lobes, between the main veins, there are simple cross-connections, which mostly form sharp angles The fields formed in this way are again intergraded into smaller fields by means of irregular running vascular bundles and also by cross-connections It is in these smaller fields that nerve endings are frequently seen and they appear to be branched

Apart from this reticulate type of venation there are two sub-marginal veins, which run parallel in the leaf, coming very close together only at the apices of the lobes The inner sub-marginal vein is placed slightly inwards and delimits the reticulated areas formed by the secondary and tertiary veins, as a result it is somewhat wavy in contour The other sub-marginal vein is placed very close to the edge and follows the outline of the lamina These two veins are connected by a series of almost equidistant parallel veins, which branch slightly at the distal ends

The leaves are arranged on the top of the corm in a pentastichous manner. The sixth leaf occurs directly above the first

The vernation is somewhat peculiar The leaf buds remain completely encased by the cataphylls inside the sheathing base of the petiole of the subtending leaf. Owing to the growth in length of the petiole they emerge through the cataphylls from the bases of the subtending leaves as pointed structures (Fig 5) Close examination reveals that the inner margins of the two lateral lobes of the leaf roll respectively towards the right and left side of the petiole. This process extends as far as their midribs The left median lobe as well as its corresponding lateral then roll sinistrorsely, enclosing the previously rolled portion This process, as noted before in the lateral lobes, extends only up to the midrib. The right median lobe along with its corresponding lateral lobe then rolls over in a dextrose manner and covers the previously rolled portion of the median lobe as a flap In all these processes the inner surface of the leaf always remains inside, but the vernation cannot be said to be convolute or involute

Figure 6 illustrates diagrammatically the nature of ptyxis as observed in serial sections *A* and *B* represent the basal lobes which will be seen to roll in opposite directions *C* represents the left half of the median lobe which rolls in a reverse direction to that of *B* Whereas *D* rolls over *C* and partly encases it, thus being the right half of the median lobe.

The unfolding of the leaf follows the elongation of the petiole and in this process the median lobes first uncoil in reverse directions followed by the laterals (Fig. 4)

The inflorescence is a spadix closely enclosed by the spathe It arises as an axillary structure and during its early developmental stages lies completely encased inside the leaf base It emerges only after the leaf has attained its full dimension In size it is variable, generally it is about 15 cm long The peduncle is comparatively short, it is greenish white in colour with conspicuous red streaks on its upper end It is somewhat oval in outline and shows the presence of a longitudinal groove on the inner surface At the distal end, this groove demarcates the position of the lobes of the spathe

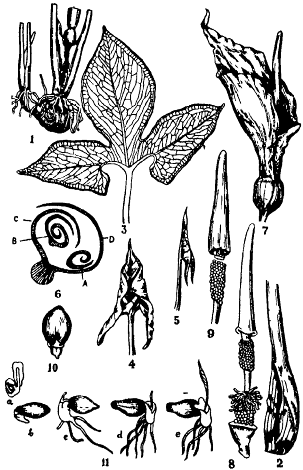
In the bud stage the spathe is convolute and encases the spadix completely The direction of the twist of the spathe as observed previously by Barnes (1938) is both left- and right-handed The latter, however, seems to be more frequent The spathe is about 20 cm long, but its size, as of all other organs, is variable It is broadest at the central region and tapers at the apex When fully expanded it stands out as a conical structure of which one side is greatly extended Below the conical base a barrel-shaped chamber is formed, the top of which is constricted The spathe is green with red streaks on the dorsal surface and scarlet on the ventral surface up to the neck of the constriction This surface is velvety to touch Inside the constricted area the ventral surface is green except for the presence of occasional red streaks

The unfolding of the spathe takes place by the unwinding of the flap above the constricted region, the process being completed in the course of 3 to 4 hours When fully opened, part of the spathe stands out at one side as a scarlet standard behind the crimson coloured appendage of the spadix Below the constricted region the overlap is retained and the barrel-shaped chamber persists

It is interesting to note that only the spathe shows parallel venation This is very clearly seen on the dorsal surface, the veins converging towards the tip

The region of the spadix above the peduncle can be separated into two regions commonly referred to as the fertile and the sterile regions The latter occupies about [illegible] the length of the axis and is situated at the top It is a conical, somewhat swollen, crimson coloured structure, with a broadly pointed end This is commonly known as the appendage Below the appendage the rachis becomes very much attenuated for a very short distance (a few millimetres) and then comes the region of male flowers followed by a short white barren area (about 1 to 2 cm) Immediately below this region lies the neuter flowers, these being represented by long white filiform processes which give a brush-like appearance. The female flowers occur immediately below this and extend up to the base of the spadix The neuter and the female flowers lie inside the barrel-shaped chamber of the spathe, while the male flowers and the appendage lie above the constricted region. Figure 8 shows diagrammatically the nature of the spadix and the spatial distribution of the flowers It would be noted that the area occupied by the male flowers is greater than those occupied by either the neuter or the female flowers.

The male flowers are arranged in an acyclic manner on the axis The staminate inflorescence appears to be red with white dots on the surface This is due to the



FIGS 1-11 *Typhonium trilobatum* Fig 1 The corm, note the axillary bud and the nature and origin of the contractile roots (½ Nat size) Fig 2 The amplexicaul nature of the leaf base (½ Nat size) Fig 3 The leaf, illustrating its form and nature of vention (½ Nat size) Figs 4 and 5 The nature of opening of the leaves (½ Nat size) Fig 6 Diagrammatic representation of a transverse section of a bud showing the nature of ptyxis Fig 7 The spathe with appendage; note the barrel shaped chamber below (½ Nat size) Fig 8 The spadix showing the appendage, male, neuter and the female flowers (½ Nat size) Fig 9. The appendage and the region of male flowers: note some stray anthers at the base of the appendage (½ Nat size) Fig. 10 The mature fruit (×2) Fig. 11*a*. A section of a germinating seed. *b-e* —Stages of germination (×2).

difference in colour of the anther lobes and the connective, the former being pink and the latter white The flowers are without any perianth and consists of a single stamen, with a broad connective and bilocular anthers, the filament being extremely reduced The connective is slightly depressed at the top and also compressed laterally The pollen grains are spherical, rose coloured and show granulations on the surface.

The neuter flowers, as already stated, are represented by white filiform processes, which are about 40 mm long and have rounded ends.

The female flowers are also devoid of any perianth and consist of a pitcher shaped ovary with sessile stigmas The ovary is unilocular and contains a single basal orthotropous ovule

A study of the mode of pollination shows that the flowers are proterogynous and the spathe unfolds itself towards the evening The inner scarlet surface of the spathe, as also the foetid odour given out by the spadix attract insects The barrel-shaped chamber enclosing the female and neuter flowers is not tightly closed at this stage, so that small insects can find easy access Insects alighting on the spathe, slip down to the neck of the chamber on account of the velvety surface of the latter and get inside the chamber Late at night, the constricted neck closes tightly so that the insects (mostly small beetles) find egress impossible and are trapped inside The anthers dehisce late in the evening next day and one notices masses of pollen grains lying at the neck of the constriction Later on, when the constriction opens, the insects come out, their bodies covered with pollen grains These insects when they visit other flowers get entrapped and thus cross-pollination is effected Laboratory experiments on pollen germination show that the pollen grains remain viable for a period of 48 to 72 hours after shedding and thus chances of cross-pollination appear to be great

The spathe shows signs of degeneration on the second day The degeneration of the papillate cells is particularly evident, as a result of which the colour fades and within a week everything above the constricted region degenerates, so that only female flowers remain (the neuter flowers are mostly eaten away by the insects) The mouth of the constriction remains closed The fruit takes from 20 to 25 days to mature When the fruits become mature the spathe unrolls backwards exposing the fruits This facilitates their dispersal

The fruit is an ovoid one-seeded berry, about 10 mm long The distal end is greenish white, while the proximal end is white and glossy Seeds are 4–6 mm long, about 3 mm wide, greyish-black, ovate, broad at the base and slightly constricted at the middle (Fig 10) The funiculus is about 1 mm long Endosperm is present

The seeds germinate in moist saw dust and in ordinary tap water in the course of a week Laboratory experiments as also observations made in the field seem to indicate that the seeds have no period of rest

The first sign of germination is the protrusion of a part of the cotyledon through the micropyle (Fig 11*b*) Sections of germinating seeds at this stage show that the base of the cotyledon protrudes out carrying along with it the plumule and the radicle Part of the cotyledon which remains inside the seed serves as an absorbing organ (Fig 11*a*) The radicle next elongates and gives rise to the primary root, while the plumule gives out the first leaf which appears in a convoluted form through the cotyledonary slit The hypocotyledonary region above the primary root next increases in size and becomes globular This is subsequently transformed into the corm At this stage the primary root generally perishes and is replaced by adventitious roots which arise above the radicle. Figure 11, *b*, *c*, *d* and *e* illustrate various stages of germination.

The first leaf of the seedling is small and cordate in shape The later leaves show an increase in the size of the lateral lobes and gradually become somewhat auriculate in form. From this by gradual stages the typically trilobed structure is reached.

The mode of propagation of the plants appears to be both sexual and vegetative. The latter method has been discussed before. The seed germinates in the soil very soon after dispersal and sends out one or more leaves, which, however, disappear very soon on account of the end of the growing season, the corm which perennates underground sends up the aerial organs next season.

II ANATOMY

Corm —The corm is composed of a compact mass of parenchymatous cells which do not show the presence of intercellular spaces These cells are rich in starch contents Vascular bundles are of the closed collateral type, but do not show the typical scattered arrangement and appear to be disposed more or less in the form of a ring The typical scattered arrangement, however, is met with at the points of insertion of the petiole and the peduncle The corm grows in thickness by the multiplication and enlargement of the ground parenchymatous cells

Periderm formation takes place at a very early stage when the corm has a diameter of about 8 mm This process continues even in mature corms The periderm does not form a continuous cylinder but occurs in isolated patches The phellogen is hypodermal in origin (Fig 18) It cuts off a large number of cork cells on the outside and a few layers of phelloderm cells on the inside

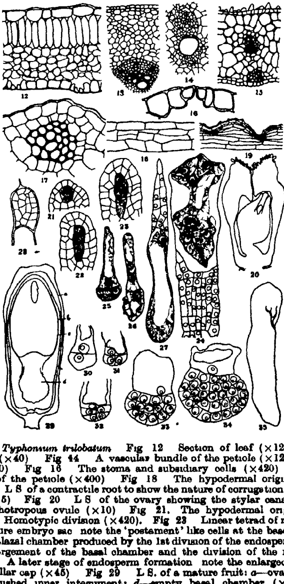
The cells composing the phellem are uniform in shape and radially elongated They are empty, non-living and without pits The cells of the phelloderm show the presence of cytoplasm and nucleus and are loosely arranged The phellogen occurs as a single layer of cells and shows the usual features

Petiole —A cross-section of the petiole shows the presence of slightly elevated ridges and furrows Externally it is bounded by a single layer of somewhat rectangular epidermal cells, the outer walls of which are not thickly cutinised Stomata are few in number and these are found mostly in the furrows and lie at the same level as the epidermal cells Ovoid bands of mechanical tissues are situated as hypodermal bands below the ridges These are composed of a large number of collenchymatous cells (Fig 17) The number of such bands occurring in a single petiole is variable, generally about 20 are present, but this depends mainly on the diameter of the petiole Infrequent union of such bands is noted at the distal end of the petiole The collenchymatous cells are of the prosenchymatous type and superficially resemble sclerenchymatous fibres The cells show unequal thickenings and have pointed ends which are unthickened The length of these cells is very variable They measure from 180 to 700μ, the mean length being about 400μ Transverse sections show that they are of the angular type A single nucleus is present in each cell

Chlorenchymatous cells, two to three layers in thickness, occur below the epidermis in between the bands of collenchymatous cells Interspersed among these are present a variable number of cells containing red anthocyanin pigment These are generally separate from one another The ground tissue is composed of isodiametric parenchymatous cells with intercellular spaces Vascular bundles are closed, collateral and show a scattered arrangement The outermost vascular bundles occur in the form of a ring below the collenchymatous bands with the xylem facing inwards They are, however, separated from these bands by two to three layers of parenchymatous cells The xylem consists of a large annular and two or three smaller spiral and annular vessels Reticulated or pitted elements are entirely absent A few xylem parenchyma cells are also present The phloem is composed of sieve tubes and companion cells, phloem parenchyma being totally absent (Fig. 14)

Leaves —The leaves are dorsiventral. The upper epidermal cells are somewhat rectangular in shape with their outer walls thickly cutinised The cuticle can be separated from the walls when treated with sulphuric acid Stomata occur on both surfaces. The average number of stomata per sq mm was found to be 2 in the upper and 8 in the lower epidermis Subsidiary cells are present along with the guard cells and as usual a respiratory cavity occurs below each stoma (Fig 16) The mesophyll consists of a single layer of palisade cells and 4 to 6 layers of spongy parenchyma (Fig 12) The vascular bundles show the usual structure Associated with each vascular bundle is a hypodermal band of collenchymatous cells, which occur on the dorsal side.

Spathe.—Anatomy of the spathe shows that the inner scarlet covered velvety surface is closely covered by papillose protrusions. These cells have thin walls and contain red anthocyanin pigment in the vacuoles A few stomata (1 per sq. mm. area) occur in between these cells Immediately below this occur rectangular cells in a single layer These cells closely resemble the dorsal epidermal cells, but are somewhat larger Next to this and extending up to the lower epidermal cells, are found chlorenchymatous cells. There is slight development of intercellular spaces.



Figs 12-35 *Typhonium trilobatum* Fig 12 Section of leaf (×125) Fig 13. T S. of part of a root (×40) Fig 14 A vascular bundle of the petiole (×125) Fig 15 T.S. of a spathe (×100) Fig 16 The stoma and subsidiary cells (×420) Fig 17 Collenchymatous band of the petiole (×400) Fig 18 The hypodermal origin of the phellogen (×125) Fig 19 L S of a contractile root to show the nature of corrugation and the disposition of the tissues (×15) Fig 20 L S of the ovary showing the stylar canal and the acentric nature of the orthotropous ovule (×10) Fig 21. The hypodermal origin of the M.M.C. (×420) Fig 22 Homotypic division (×420). Fig 23 Linear tetrad of megaspores (×125) Fig. 24. The mature embryo sac note the 'postament' like cells at the base (×420) Fig. 25 Micropylar and chalazal chamber produced by the 1st division of the endosperm nucleus (×300) Fig 26 The enlargement of the basal chamber and the division of the micropylar nucleus (×300). Fig 27. A later stage of endosperm formation note the enlarged basal cell (×150). Fig 28 The nucellar cap (×45) Fig 29 L S. of a mature fruit: *a*—ovarian wall, *b*—outer integument; *c*—crushed inner integument; *d*—empty basal chamber (×5) Figs. 30-34. Stages in the development of the embryo (×150). Fig. 35. A fully developed embryo (×30).

The outer walls of the lower epidermal cells are unthickened. The average stomatal frequency is 8 per sq mm

Vascular bundles are situated mostly in the central region. Associated with each bundle is present a strengthening band composed of collenchymatous cells These bands occur at the abaxial side of the spathe below the epidermis They are separated from the vascular bundles by a few layers of parenchymatous cells (Fig 15)

A cross-section of the appendage shows it to be circular or ovoid in outline It is bounded by a single layer of epidermal cells, which show slight variation in size Stomata are present, but are few in number The ground tissue is composed of starch-filled parenchymatous cells which are separated by intercellular spaces Wide schizogenous cavities are sometimes present at this region Vascular bundles occur below the starch-filled cells and are of the usual closed collateral type These bundles send out traces diagonally which end at the hypodermal region Elongated crystal sacs containing raphides are disposed radially in one or two series in the peripheral region amongst the starch containing cells They are absent in the central region

Root—The growing point of the root shows the presence of a root cap The root cap and the histogens of the root are derived from a common primordial meristem The cells composing the root cap are larger in size and contain starch grains These are, however, located mainly in the central region (Columella of Němec) of the root cap and the surrounding cells are free from it The three histogenic layers become differentiated a little below the primordial meristem The plerome appears to be wider than the periblem

A t s of the mature root shows the presence of exodermis, which is two-layered at certain regions This is followed by the cortex composed of many layers of parenchymatous cells with slight development of intercellular spaces The endodermal cells have their radial walls suberised Below this occurs a single layer of parenchymatous cells of the pericycle The vascular bundles show the characteristic radial arrangement The number of xylem strands varies from six to eight, the largest number of roots possessing six A single large annular vessel and a few spiral vessels are the elements of the xylem, while phloem is composed of sieve tubes and companion cells, no conjunctive tissue or pith is present.

Figure 13 represents t s of a part of a root which is octarch It will be noted that the central region is occupied by a large vessel Developmental studies indicate that this is a protoxylem element, which is the first to differentiate into an annular vessel Metaxylem elements as in the petiole are absent

A study of the constricted region of the roots shows that the depth of constriction extends radially to about half the cortex and is not nodal in character The folding is of the nature of ridges and furrows, which may be narrow or broad Longitudinal sections show that the outer cortical tissue is alone concerned in the process, the epidermal cells above remain intact at most places. At the point of constriction the cells get extremely compressed laterally and give a lamellated appearance Those lying inside the ridges as well as the cells occurring below the region of constriction appear to have increased in length (Fig 19), the latter being longer than the former

Calcium Oxalate Crystals—Needle shaped crystals of calcium oxalate are present in different parts of the plant body. They are, however, absent in the mature roots and in the axis of the spadix below the appendicular region The size of the crystal sacs occurring in the different parts of the plant is variable The largest are found in the appendage of the spadix, where they range from 65 2 to 234·72μ, the average being 169 52μ. The disposition of the crystals inside a sac is variable, and the occurrence of more than one bundle of crystals inside a sac is not infrequent. The average length of the individual crystal is about 50μ

The first indication of the origin of the crystal sacs is noted by the slight increase in size of the cell, followed by a corresponding increase in size of its nucleus and nucleolus. The cytoplasm is very dense at this stage. At the next stage small vacuoles become apparent in the cytoplasm and the nucleus is pushed to the periphery At this stage the appearance of granular matter is first noted inside the vacuoles Along with the progressive increase in size of the cells, its nucleus and nucleolus also increase in size, and the raphides become apparent The nucleus generally lies at one side of the cell but in a few instances it has been observed to occupy a central position surrounded by groups of crystals When the sac attains its full dimension the nucleus disorganises, and no trace of it or the cytoplasm remains

It is interesting to note in this connection that with the gradual increase in size of the nucleus, the chromosomes become differentiated and generally two or three of them are seen to be attached to the nucleolus The prophasic condition of the nucleus continues till the crystals are organised, when it disappears Thus it appears that the nucleus has a distinct rôle in the formation of the crystals

III EMBRYOLOGY

The early indication of the development of the pistillate flowers is noted by the protrusion of the papillate processes on the axis The growth of the primordia soon becomes arrested and from its base meristematic tissue appears in the form of a ring This grows upwards and later curves inwards to meet at the centre Thus the ovary is organised as a closed chamber The centre of the primordial tissue later gives rise to the placenta from which a single orthotropous ovule develops The growth of the ovule, as compared to that of the ovary, is comparatively rapid In most cases on account of the absence of space inside the ovarian chamber the tip of the ovule lies eccentrically at one side due to the curvature of the funicle as illustrated in figure 20 Later, however, when the ovule becomes fully developed the funiculus straightens itself, still, in most cases the micropyle is eccentrically placed The stigma is sessile and somewhat concave on the outer surface, from which unicellular hairs are produced A very slender stylar canal is present, this is bounded inside the ovary by auricular processes of the ovarian wall

The primordium from which the ovule arises grows upwards from the basal placenta as a hemispherical mass of tissue The inner integument soon becomes differentiated and by its rapid growth encloses the nucellar tissue at an early stage of its development The primordium of the outer integument appears soon after the inner, but its growth is comparatively slow In the mature ovule the nucellus is completely encased by the inner integument the tip of which becomes somewhat swollen and forms a narrow micropyle The outer integument stands at a slightly lower level and the portion next to the funiculus is broader than the apex The inner integument is composed of three layers of cells except at the tip where it is swollen and consists of four to six layers of cells The cells lining the micropyle have dense cytoplasm and are radially elongated The inner layer of the inner integument subsequently forms the tapetal layer of the megagametophyte and is composed of rectangular cells with their longer axis perpendicular to the longitudinal axis of the ovule

Three vascular traces enter each ovule These unite in the funicle and split in the chalazal region into two branches, which enter the outer integument and extend nearly up to the micropyle

A single hypodermal cell at the apex of the ovule becomes differentiated as the archesporial cell and directly functions as the megaspore mother cell (Fig 21) It enlarges during the meiotic divisions and the cytoplasm becomes finely vacuolate. During diakinesis 9 pairs of chromosomes are seen inside the nucleus At this stage the epidermal cell overlying the megaspore mother cell divides periclinally so that it is pushed inside the nucellus Two cells are produced following the first division of the M M C Both of which divide simultaneously and the spindles lie parallel to the longitudinal axis of the ovule (Fig 22) The result of these divisions is the production of a linear tetrad of megaspores (Fig 23)

The micropylar megaspore is the first to degenerate followed by the next two in succession The chalazal megaspore by its activity gives rise to the megagametophyte At the binucleate stage a large vacuole forms between the two nuclei which lie at each end Further divisions of these nuclei lead to the production of a four-, and later, of an eight-nucleate megagametophyte From the binucleate stage onwards degeneration of the surrounding cells of the nucellus is noted and the process is almost complete at the eight-nucleate stage, when the embryo sac is directly in contact with the inner integument The cells derived from the epidermal cell above, however, have divided several times and produced a nucellar cap composed of many cells (Fig 28)

In the eight-nucleate megagametophyte the synergids are somewhat pear-shaped and have the usual basal vacuole The egg is somewhat inconspicuous and generally lies covered up by the synergids (Fig 24) The polar nuclei migrate to the centre of the megagametophyte and fuse Due to the differential growth of the embryo-sac following the fusion of these nuclei it comes to lie adjacent to the antipodals The antipodals are three in number and appear as distinct cells They are about twice the size of the synergids and are triangular in form, with the pointed ends projecting downwards Fig 24 illustrates a mature embryo sac and its component parts

The pollen tube enters the embryo-sac by the way of the micropyle and its remnants can be seen in many preparations showing this stage Stages of fertilisation were, however, not observed After syngamy the synergids and the antipodals degenerate and the egg remains suspended from the micropylar end of the sac It appears to be somewhat elongated at this stage The embryo sac becomes considerably elongated after fertilisation and the primary endosperm nucleus lies close to the base of the sac At this stage the cells of the nucellus lying in contact with the chalazal end of the embryo-sac show a typical 'postament' like appearance (Fig 24) But it should be noted that with the gradual enlargement of the embryo-sac, which occurs during the post-fertilisation stages resorption of the nucellus at the chalazal end takes place and all trace of this strand like tissue is lost.

As in most plants the endosperm nucleus divides before the oospore It migrates slightly towards the centre of the sac before division Karyokinesis is followed by wall formation and the embryo-sac becomes divided into a large micropylar and a small basal chamber (Fig 25) The latter, however, increases very much in size in later stages The micropylar endosperm cell very soon divides transversely (Fig 26) This is followed by divisions in similar planes of the daughter cells At this stage the cells of the lower tiers divide longitudinally while those of the upper might divide several times transversely Later divisions take place in various planes and a typical cellular endosperm is formed capping the large basal cell (Fig 27) Along with the growth of the ovule the endosperm cells multiply rapidly and extend inwards in the form of an arc which, however, later becomes convex in form (Fig 29) The lateral edges of the endosperm tissue which are one cell in thickness extend inwards and line the basal chamber for a considerable distance In the later stages of development the endosperm increases mainly due to the activity of its outermost layers of cells, which line the basal chamber The inner cells become conspicuously vacuolated and elongated With the rapid development of the endosperm tissue the nucellar cap disintegrates and no trace of it is found in later stages The cells of the endosperm show abundant starch grains

The chalazal cell never divides but reaches an enormous size, occupying nearly one-third the space of the embryo-sac cavity, its nucleus also shows corresponding increase in size and is surrounded by dense cytoplasm. The form of the nucleolus appears to be somewhat irregular at this stage. In later stages of the development of the seed it completely disintegrates With the development of the endosperm in the micropylar chamber, the basal chamber gradually becomes reduced in size and in the mature seed it occupies about one-third the space of the embryo-sac The nucellar cells lying immediately below the basal chamber show signs of disintegration as the chamber increases in size This is very conspicuous in the later stages Thus it appears that the basal cell is haustorial in function and brings about a disintegration of the surrounding cells by the secretion of an enzyme In the mature seed the micropylar portion containing the endosperm and embryo are prominent, while the basal chamber with the haustorial cell along with the surrounding coat of the parent sporophyte remains as a withered protuberance at the chalazal end (Fig 10)

The embryo develops more slowly than the endosperm The oospore divides after a few cells have been formed in the micropylar chamber of the endosperm and the first division is periclinal (Fig 30) The micropylar cell is larger and grows slightly in size at later stages but does not undergo any further division and remains as a one-celled suspensor The smaller distal cell gives rise to the embryo proper Its first division is longitudinal, i e , perpendicular to the previous plane of division (Fig 31). This is followed by transverse divisions in both the daughter cells (Fig 32) The next stage observed shows the embryo to be composed of two tiers of cells The immediately succeeding stages have not been observed and a globular embryo is next noted in which the dermatogen appears to have differentiated (Fig 33) Cell division now takes place in various planes and the embryo becomes somewhat ovoid in form Very soon a notch appears at one side of the embryo which separates the terminal cotyledon from the lateral plumule primordium In later stages the terminal cotyledon enlarges rapidly with the result that the plumule is pushed close to the micropylar end of the embryo The radicular portion of the embryo does not appear to be well differentiated even at this stage In the mature embryo, the growing point appears to be a hemispherical projection covered up by a few rudimentary cauline leaves (Fig 35)

A transverse section of the embryo in the region of the plumule shows the hemispherical growing point partly encased by the leaf primordia

Figure 29 represents a longitudinal section of a mature fruit where the comparative size and position of the embryo, endosperm and basal chamber are clearly seen Externally the fruit is covered by the pericarp Below this occurs the outer integument, while the inner integument is seen as a disorganised layer closely adpressed to the endosperm At the lower end, the large empty basal chamber remains surrounded by the partly disorganised cells of the nucellus.

IV CYTOLOGY

Mitosis —The diploid number of chromosomes as determined from root tip cells in *Typhonium trilobatum* is 18. The complement is made up of 6 long, 6 medium and 6 short chromosomes Of these 4 chromosomes possess trabants and two show secondary constrictions The centromere constrictions appear to be differently located Thus of the 6 long chromosomes 4 have sub-terminal constrictions and 2 median, the 6 short chromosomes median, and of the 6 medium chromosomes, 2 have subterminal and 4 median attachment, while the 4 SAT-chromosomes have median constrictions. The Sat-chromosomes show slight difference in length of the filament but the nature of the trabant appears to be the same (Fig 36)

When the onset of prophase the chromosomes appear as small globular bodies which lie mostly adpressed to the nuclear membrane From these the organisation of closely coiled threads inside the nucleus soon becomes apparent. The nucleus as also the nucleolus attain their maximum size at this stage The latter lies somewhat eccentrically and is seen to be connected to a number of chromosomes Careful examination of Feulgen-Light-Green preparations shows that the secondary constricted and Sat-chromosomes are alone attached to the nucleolus Figure 37 shows an attachment by three such chromosomes The number of nucleoli occurring at this stage is variable Generally a single large nucleolus is present, but two nucleoli are frequently seen, while three or more are less frequent Where there are more than one nucleoli, an appreciable size difference is noted

With the disappearance of the nuclear membrane the chromosomes become regularly aligned at the equatorial region of the spindle The nucleolus as a rule disappears at this stage, but in a few instances it has been noted to lie either in the central region or move bodily to one of the poles in advance of the chromosomes In the former case it generally divides into two unequal halves which move to the opposite poles, and are later cast out into the cytoplasm

Polar view of metaphase shows that the chromosomes are double The nature of the twisting of the chromatids could not, however, be made out on account of their small size

The anaphasic movement of the chromosomes appears to be regular and no laggards have been noted The chromosomes on reaching the poles lie free from one another and the homologue of each chromosome can be made out The longer chromosomes do not appear to be situated at the periphery of spindle

Examination of telophase nuclei shows that the chromosomes have undergone a certain amount of expansion and appear as elongated threads In some of the chromosomes the split for the next division is apparent The number of nucleoli in each nucleus appears to be variable at this stage The maximum number of independent nucleoli observed was six (Fig 38) Fusion of these nucleoli commonly takes place and one large or two equal sized nucleoli are generally seen at later stages

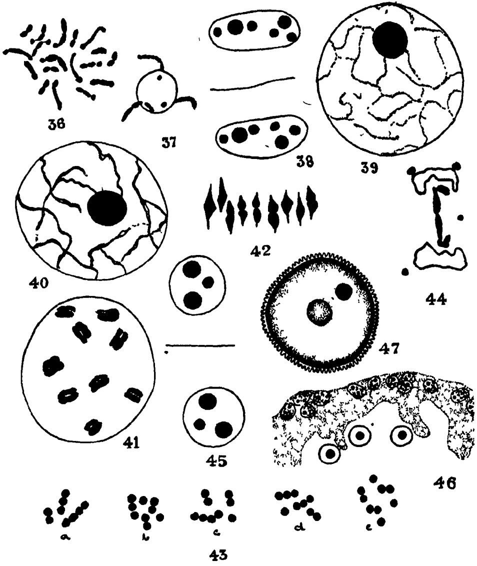
During early telophase the protoplast assumes a phragmoplastic appearance and a hyaline cell plate appears in the central region The phragmoplast soon increases in size and extends laterally Along with this the spindle fibres in the central region disappears, but those at the periphery become conspicuous As the phragmoplast touches the lateral walls of the cell, these striations also disappear and the cell plate extends both ways Thus the phragmoplast brings about a division of the cell by its growth and lateral expansion It is interesting to note that such a mode of cytokinesis has been observed previously by Sharp (1911) in *Physostegia virginiana* and recently by Rao (1942) in *Santalum album*

Meiosis—The primordia of the anthers develop later than those of the ovule, while those producing the filiform processes, the so-called neuter flowers develop last

The initiation of anther primordia is noted by the increase in length and anticlinal division of the hypodermal cells in the staminate region of the spadix The cells in this region become demarcated in groups of actively dividing cells Very soon the surface of the spadix becomes wavy in outline and gradually the anthers protrude out These appear to be quadrilocular in transverse sections

The development of the sporogenous cells could not be definitely traced, but the archesporial cells are hypodermal in origin When the sporogenous cells are first noted they are seen to be separated from the epidermis by six layers of cells Of these the outer four layers are somewhat elongated and form the parietal layers, while the inner two represent the tapetal layers and are somewhat polygonal in shape

The sporogenous cells could be made out by their bigger size, greater chromaticity and dense cytoplasm The nucleus of the mother cell is spherical in shape and has a well-defined nuclear membrane It contains a deeply stained nucleolus and faintly stained chromatic threads, which are spread uniformly throughout the nuclear cavity Prophase of the meiotic division is noted by the greater chromaticity of the threads which appear to be somewhat twisted. The separate threads could not be tracted at this stage. Occasionally, however, free ends are seen. Pairing of the leptotene threads is noted by the appearance of threads which are thicker at certain regions and thinner at others When pairing is complete the threads become thicker and shorter and contract to one side of the nuclear cavity and finally condense into a tight synizetic knot Generally one nucleolus is present at this stage, which may



FIGS. 36–47. *Typhonium trilobatum* Fig 36. The somatic complement of 18 chromosomes (×3500) Fig 37. Attachment of Sat and Sec constricted chromosomes to the nucleolus. (×3500). Fig 38 Telophase nuclei (somatic)—with six nucleoli in each nucleus (×3500). Fig. 39. Prophase of meiotic nucleus (×3500) Fig 40. Pachynema (×3500) Fig 41 Diakinesis (×3500) Fig 42. Configuration of bivalents at metaphase (×3500) Fig 43a–e. Various types of secondary association (×2440) Fig 44. Anaphase I. Chromosome bridge and fragment (×3500) Fig 45. Telophase, I division: note three nucleoli in each nucleus (×3500). Fig. 46. Part of periplasmodium showing the peripheral position of nuclei (×450). Fig 47. Binucleate pollen grain (×3500).

or may not be enclosed in the meshes of the contracted knot Loops are frequently thrown out from the synizetic knot and these are distinctly double in nature Tight synizetic knots as seen in this material are now regarded as artefacts produced by fixing agents accentuating the real contraction of the chromosomes Nevertheless, synizesis represents a very delicate condition of the nucleus On recovery from synizesis the threads become distinctly thicker and lie freely distributed inside the nuclear cavity (Fig 40) The double nature of the chromosomes becomes evident at places A single nucleolus is present, to which three pairs of chromosomes are seen to be attached

Progressive condensation of the chromosomes takes place till the diplotene stage is reached, which is noted by the separation of the paired chromosomes The quadripartite nature of the chromosomes could not, however, be made out, but the chromosomes seem to have an irregular outline At this stage chiasmata are seen to connect the chromatids The chiasmata undergo terminal movement on the condensation of the chromosomes to form the bivalents

At diakinesis terminalisation of the chiasmata is complete and the nine bivalent chromosomes which appear as rod-shaped bodies lie approximately equidistant from one another According to Lawrence (1931) this is due to a repulsion phase which is initiated earlier and continues up to diakinesis In most preparations showing this stage three bivalents are regularly seen to be attached to the nucleolus which is spherical in shape The bivalents appear to be almost similar In no case was multivalent formation noted It is at this stage that the pollen mother cells show signs of rounding off

Towards the close of diakinesis, the nucleolus disappears by progressive diminution in size and the bivalents move towards the centre of the nucleus forming groups or associations This is the commencement of the secondary pairing of the chromosomes which becomes marked in metaphase This association of chromosomes is also observed in metaphase II

An analysis of different types of secondary association as observed in metaphase I is presented in Table I below

TABLE I

Types of Secondary Association

No of cases	No of bivalents in association				Maximum association	Basic number derived from max association
	1	2	3	4		
6	3		2		2(3)+3	5
3	3	3			3(2)+3	6
13	2	2	1		1(3)+2(2)+2	5
7		1	1	1	1(4)+1(3)+1(2)	3
3	1	1	2		2(3)+1(2)+1	4
17	4	1	1		1(3)+1(2)+4	6
2	9				9	
5	2		1	1	1(4)+1(3)+2	4
3	1	4			4(2)+1	5
3	1	2		1	1(4)+2(2)+1	4
1	3	1		1	1(4)+1(2)+3	5
5	1	1	2		2(3)+1(2)+1	4
4	5	2			2(2)+5	7
2	6		1		1(3)+6	7

74 Total,

The maximum number of association between bivalents was found to be 1(4)+1(3)+1(2) It shows one group of four, one group of three and one group of two making three separate associations

Side view of metaphase shows that the bivalent chromosomes group themselves regularly on the equatorial region of the spindle They appear to be equally spaced and the two members of a pair can be easily made out in most cases (Fig 42) Secondary association is maintained to some extent at anaphase Different views have been expressed for the anaphasic separation of the chromosomes Kuwada (1929) believes this is due to polar attraction, while Darlington (1932) is of opinion that it is a 'polar' repulsion, which is essential for metaphasic equilibrium Catcheside (1934) suggests that the attachment constrictions are the regions of localised forces, which lead to a mutual repulsion of the chromosomes of a bivalent Alam (1936) thinks that the anaphasic separation is the result of more than one force 'Repulsion between attachment constrictions and the attraction of the poles '

Separation at anaphase I is normal in most cases In a very few anthers (less than 5%), however, the pollen mother cells show the presence of inversion bridges Each of these bridges produce a dicentric chromosome and an acentric fragment (Fig 44) The acentric fragments are very small indicating that not only the inverted segments are small but also the chiasmata are extremely terminal The fate of these inversion bridges could not be clearly followed, but their absence during second division shows that they are not included in the gametes The few lagging bivalents noticed during first division are due to mechanical difficulty consequent upon chiasma formation in inverted segments All such bivalents later develop into true inversion bridges releasing the acentric fragment incidentally

The chromosomes on reaching the poles organise a telophase nucleus They become somewhat elongated and the split for the 2nd division is apparent in most of the chromosomes Careful examination shows the presence of three nucleoli in each nucleus, of which two are almost equal in size and one smaller (Fig 45) The interkinetic stage seems to be of some duration The protoplast assumes a phragmoplastic appearance and a cell plate appears in the centre, which divides the cell into two equal halves

The two separated daughter nuclei as a rule divide simultaneously Anaphasic separation is normal, the two groups move simultaneously and no lagging univalents are seen On reaching the poles, grand daughter nuclei are organised, each of which shows the presence of two to three nucleoli and slender chromosomes As in telophase of division I, a cell plate appears in the central region at the end of telophase and pollen tetrads are produced Various forms of arrangement of the tetrads are seen This depends mainly on the arrangement of the spindles during II division Tetrahedral and isobilateral modes of arrangement being very common, such diverse modes of arrangement appear to be a common feature of monocotyledonous plants The pollen grains are at first uninucleate The nucleus soon divides and gives rise to a generative and a vegetative nuclei This is the condition in which the pollen grains are shed (Fig 47) Mature pollen grains have an average diameter of 45μ, when examined in lactic acid

Periplasmodium —At the time of differentiation of the sporogenous cells, the tapetum is two-layered The cells are uninucleate and contain dense cytoplasm At an earlier stage some of the tapetal cells have been observed to divide by periclinal walls Such divisions are generally completed before prophasic changes become apparent in the microspore mother cells Binucleate tapetal cells have not been observed

During synizesis the innermost tapetal cells are first noted to protrude into the anther cavity The cytoplasm alone moving inwards The nuclei are spherical and the walls delimiting the cells are distinct at this stage During later stages of meiosis (I division onwards) the cytoplasm progresses further inwards and even reaches across the anther loculus at places, dividing the microsporangium into compartments At this stage the walls delimiting the individual tapetal cells disappear and the nuclei are seen to be situated at the periphery of the anther cavity embedded in a homogeneous cytoplasmic mass At the pollen tetrad stage the tapetal nuclei are seen to leave their peripheral position and migrate into the microsporangium At this stage the plasmodium fills up the anther cavity completely but is not in contact with the pollen tetrads The nuclei of the plasmodium which are irregularly distributed aggregate at places and retain their original form This close aggregation of nuclei very often leads to their fusion resulting in the production of nuclei of variable shape and size When the microspores have been organised the plasmodium comes in contact with the pollen grains but the nuclei remain separate and present a conglomerated appearance Fusion of two or more nuclei is commonly seen at this stage In no case was amitotic division of the nuclei observed With the development of the exine of the pollen grains the plasmodium becomes highly vacuolated and ultimately during the binucleate condition of the pollen grains it completely disappears At this stage the parietal layers of the anther wall become crushed and obliterated, while the endothecial layer becomes radially elongated, except at the tip of the connective, where it is represented by smaller unthickened cells Rupture of these cells brings about the dehiscence of the anthers, which is thus 'porous' in nature

DISCUSSION

Morphology.—The occurrence of contractile roots seems to be a characteristic feature of *Typhonium trilobatum* as all aroids do not possess it Anatomical studies indicate that it is mainly the outer tissue of the cortex, which shows the corrugation while the stele and the inner cortex remain unaffected The limited observations that have been made on the nature of the contractile roots of different plants show that the zone of contraction is restricted to the outer cortex Rimbach (1897) was the first to explain the cause of wrinkling He states 'that the shortening is due to a change of form of the inner cortical cells, which increasing in a radial and tangential direction suffers a great decrease in length ' Woodhead (1904) supports this view, but Arber (1925) working on *Hypoxis* failed to find any evidence in this direction and pointed out the baffling nature of the problem Evidence obtained in the course of the present study supports partially Rimbach & Woodheads' observations, as there is an increase in volume of the outer cortical cells which, however, also show an increase in length Nevertheless, it is difficult to understand how an increase in radial and tangential directions of the outer cortical cells alone could bring about the contraction unless we assume the presence of cells at regular intervals, which retain their original dimensions Such a mode of contraction would present a different appearance to what has been seen in the present material, which indicates that apart from an increase in volume of the cells of the outer cortex, an internal pull is exerted by certain cells at intervals, due to which the cells at these regions present a lamellated appearance This gives rise to the so-called furrows at the constricted regions The origin of this pull might be due to physical or chemical changes in the cell-wall

The nature of ptyxis is also peculiar, but does not appear to be a characteristic feature as it has been noted in *Colocasia antiquorum* and *Alocasia sps.* It appears that aroids with trilobed leaves show this peculiar mode of folding of the leaves

The appendage of the spadix has received much attention from various investigators. Arber (1925) on anatomical evidence suggested that it was composed of the fused bases of the male flowers and represented a region of the inflorescence in which sterilisation is marked. This view has been confirmed by Engler (1881–84) who has shown by a comparative study 'that the club is not a naked axis but it consists of an incompletely developed part of the inflorescence'.

In the course of the present investigation, a spadix was obtained in which a few isolated male flowers (stamens) occurred a little below the appendage and in continuation of the male inflorescence These were comparatively large, being four to six times the size of the normal stamens and of the same colour as the appendage The microsporangia were also larger and contained abundant pollen grains which, however, were of the same size as those found in normal anthers At a slightly higher level and still closer to the appendage occurred other stamens, which were sterile Figure 9 shows the region of the spadix where the sterile flowers are located, while the empty space below indicates the position of the larger fertile stamens This gradual transition indicates that the appendage is derived by the union of such sterile flowers Further evidence in this direction is obtained on anatomical grounds, which shows that vascular traces are given out regularly from the central strands of the appendage The number of such separate traces are quite large and extend to the periphery It is interesting to recall that each stamen is supplied by a single vascular trace, which character persists even in the appendage

Anatomy —The anatomy of the petiole shows difference in the various genera due primarily to the mode of distribution of the mechanical tissue which may be collenchymatous or sclerenchymatous Solereder (1928) states that the mechanical tissue in the petiole may be either disposed in the form of a complete ring below the epidermis, or it may be narrower or broader at different regions and protrude into the ground tissue, or it may occur in isolated patches separated by chlorenchymatous cells The mechanical tissue of *T trilobatum* is composed entirely of collenchymatous cells whose structure has been already described It occurs in isolated patches in the hypodermal region of the petiole, as has also been noted in *Philodendron* It thus forms a sub-epidermal girder system, which is the most suitable type of arrangement for cylindrical inflexible organs This mode of arrangement of the mechanical tissue is also found in *Colocasia antiquorum* where, however, the mechanical strands vary in size and are regularly arranged at the periphery being associated with mestome bundles

An interesting observation made in the course of this study was the absence of metaxylem elements in the vascular bundles of the petiole and root In the bundles of the petiole, generally a single large vessel is present, or there may be one or more smaller ones associated with it In the roots where the exarch arrangement prevails the xylem elements (apart from the xylem parenchyma) consist of an inner bigger vessel and two or three smaller outer ones The size of the larger vessel and its position would seem to indicate that it is a metaxylem element, but developmental studies show that it is the first to differentiate and it shows the annular thickening Further, examination of macerated material from roots and petioles confirmed these observations

The concept of protoxylem and metaxylem has undergone considerable change since Russow (1872) and Van Tieghem (1887) first introduced these terms The original meaning of the words was modified later when the sculpturing of the wall was taken into account Esau (1943) states 'eventually the tendency to ascribe to protoxylem and metaxylem a definite wall morphology became prevalent and it influenced the formulation of concepts of primary xylem by writers of modern reference works' The International Association of wood anatomists recognise the metaxylem as the pitted tracheal elements (the scalariform elements also included). Frey-Wyssling (1940), however, favours the abandonment of delimitation of these two tissues on the basis of wall sculpture and suggests the 'reintroduction of the ontogenetic aspect into the classification' He finds difference in the structure of metaxylem elements in different groups of plants which in certain instances have been noted with spiral secondary thickening Esau (1943) also states that the thickening of the metaxylem elements may vary from spiral to pitted

Popham (1941) has suggested the abandonding of the terms 'protoxylem' and 'metaxylem', because 'in the differentiation of xylem cells, location, time of enlargement, time of secondary wall lignification, time of differentiation and the pattern of the secondary wall do not always bear a specific or constant relationship to the kind of origin, whether primary or secondary'.

Embryology.—A review of the literature shows that the archesporial cell in the family Araceae may be one or many. The former condition, however, appears to be more common and the archesporial cell is hypodermal in most members of the family. In some plants such as *Anthurium crystallinum*, *A. violaceum*, *Symplocarpus foetidus*, it cuts off a parietal cell and then functions as the megaspore mother cell. In *Arum maculatum*, *Homalonema alba*, *Acorus calamus* and others it directly functions as the megaspore mother cell. This condition has been observed in *T. trilobatum*, which comes under the tribe Arineae to which *Arum maculatum* also belongs. The epidermal cell which overlies the megaspore mother cell forms a nucellar cap by repeated divisions. Such nucellar caps have also been noted in *Peltandra virginica*, *Arum maculatum*, *Calla palustris* and in other plants. In *Acorus calamus*, however, the nucellar cap is composed of a single layer of cells formed by the division and radial elongation of the epidermal cells.

More than one type of embryo sac development has been recorded in this family. Schnarf (1931) records the occurrence of 'Lilium type' of development in *Dieffenbachia seguine* and *Anthurium violaceum*. The 'Scilla-type' of development has been found in *Homalonema argenta* and *Nephthytis Grayenreuthii*, whereas, in the majority of plants investigated 'Normal-type' of development prevails. In the tribe Arineae, to which *T. trilobatum* belongs the development of the female gametophyte has so far been recorded in *Arum maculatum* and *Arisaema triphyllum* both of which show normal type of development. Maheshwari (1937) has noted that *Acorus calamus*, *Richardia africana* and *Zantedeschia aethiopica*, which have been recorded as belonging to the 'Adoxa-type' (Lilium type) by early workers, have on re-investigation been found to belong to the 'Normal-type'.

The chalazal macrospore does not always produce the embryosac. Schnarf (1931) states that in *Anthurium crystallinum* and *Spathiphyllum Patinii*, the micropylar megaspore produces the embryo-sac. But it should be noted that in most of the plants, as in *T. trilobatum* it is the chalazal megaspore that functions.

A remarkable feature one comes across in the literature on the embryology of the Araceae is the development of the basal apparatus. The nucleus of the basal chamber may remain undivided or it may divide without the formation of walls or give rise to a number of cells as a result of division. The nature of the basal apparatus in *T. trilobatum* has already been described and it needs only be pointed out in this connection that it agrees closely with Jacobson Palav's (1920) findings in *Arum maculatum*. The 'chalazal cell' (basal apparatus in *T. trilobatum*) increases in size with the development of the seed and brings about a degeneration of the surrounding cells of the nucellus on account of its haustorial nature. But no haustorial processes radiate from this chamber as observed by Boodle and Hill (1929) in *Typhonodorum Lindleyanum*. It remains throughout as a hollow spherical chamber. Consequent on the increase in size of the basal chamber the postament like strand of tissue observed in the early stages disappears completely. In *Acorus calamus*, Buel (1938) notices the postament even in the later stages and it seems to be concerned with the nutrition of the embryo. From the nature of the postament observed in *Acorus calamus* and other plants one is inclined to believe that the strand like tissue observed at the base of the embryo sac in *T. trilobatum* could not be strictly referred to as such.

Cytology.—A vast amount of literature has accumulated on the origin of the nucleolus and its relation to the chromosomes. Wager (1904) working on *Phaseolus* observed that the nucleolus was suspended in a nuclear net-work by numerous strands. It was Latter (1926) who first discovered the nucleolus to be connected to a loop of the spireme in the pollen mother cells of *Lathyrus*. Her observations have later been confirmed in other plants and it is now known that the 'nucleolar bodies' of Latter with their attached chromosomes represent particular chromosomes responsible for organisation of the nucleolus at telophase Working on *Galtonia*, Navashin (1912) observed the nucleolus to be attached to a pair of satellites Sorokin (1924) also reported such chromosome-nucleolus relationship in *Ranunculus acris* Heitz (1931) first showed that the nucleolus was produced either on the satellite stalk or on the secondary constriction of the chromosome Resende (1937) working on *Aloe* confirmed Heitz's findings McClintock (1934) found that in *Zea mays*, the nucleolus is organised around a deeply stained body on the chromosome at the base of the satellite stalk, which she called the nucleolar organising body, responsible for the organisation of nucleolus Similar relationship has been observed in recent years by many workers Gates (1937) has recently reviewed the relevant literature on the subject

A critical study of chromosome-nucleolus relationship has not been made in the present investigation, but the evidences obtained during the course of study show that the four satellited and two secondary constricted chromosomes play an important rôle in the organisation of the nucleolus, as during mitosis these chromosomes have alone been seen to be attached to the nucleolus The organisation of six nucleoli in the telophase nucleus leads one to infer that these nucleoli have been organised independently by these satellited and secondary constricted chromosomes During meiosis also, the number of bivalents seen attached to the nucleolus at diakinesis was three, and the number of nucleoli organised at telophase of I division in each nucleus was again three It is now generally agreed that the Sat-chromosomes and the secondary constricted chromosomes each organise a nucleolus and the number of independent nucleoli found in a nucleus corresponds with the number of such chromosomes Thus Bhaduri (1940) found the constant presence of four nucleoli to correspond to the four secondary constricted chromosomes in species of *Oenothera*

Chromatid bridges in the anaphase of I division has been observed in very few anthers (less than 5 per cent) Such bridges disjoin with difficulty, fragments are seen which indicate that they are formed by the breakdown of the bridges The presence of such dicentric chromatid bridges with acentric fragment may be expected on the basis of crossing over within an inversion Richardson (1936) and others have discussed the processes which lead to the formation of bridges and it is not proposed to discuss it here Upcott (1937) has correlated bridge formation with sterility in *Tulipa* According to her, more than 10 per cent inversion bridges lead to considerable sterility The very low percentage of bridges and the absence of sterile pollen indicate that such aberration in meiosis is not of any significance in *T trilobatum*

Kuwada (1910) was the first to notice the association of bivalents at metaphase I in *Oryza sativa* Ishikawa (1911) also noticed such association of chromosomes at metaphase II of *Dahlia variabilis* Since then the phenomenon has been observed by several investigators, but its real significance was not clearly understood until Lawrence (1931) pointed out that it is an expression of ancestral homology between the associated bivalents Secondary association according to Lawrence is an indication of allopolyploidy The association is best seen at prometaphase and it remains so until interkinesis when a repulsion force develops and keeps the chromosomes away from one another It is apparent again at metaphase II Secondary association of chromosomes has been observed in a large number of plants and it has led to the determination of the primary basic number in certain genera Lawrence (1929) by a study of secondary association has shown that though the lowest haploid number of chromosomes of the Dahlias is 16, it must have been evolved from a species with 8 haploid chromosomes which has now become extinct Nandi (1936) also found evidence to show that $n = 12$ number in *Oryza sativa* has been derived from the basic number five Similarly in *Solanum tuberosum* ($2n = 24$), Muntzing (1938) found the basic number to be six

From Table I it will be seen that excepting two cases out of seventy-four, the bivalent chromosomes during 1st metaphase of the P M C as a rule showed different degrees of secondary association Only in two cases, nine free bivalents were observed It will be further noticed that the maximum secondary association for the present material is 1(4)+1(3)+1(2) and the basic number accordingly should be 3 Assuming the theory of secondary association to be correct, the present observation leads to the conclusion that *T trilobatum* is a secondary polyploid and the haploid number, $n = 9$, is derived from the original basic number 3 Such a low number has, however, not been reported in any other species of *Typhonium* or in any other related genera In *Arum* and *Theriophonum* the haploid number has been found to be $n = 8$ A thorough survey of the chromosome numbers in other related genera might show lower number than 8 The meiotic behaviour of the present species indicates the presence of a perfectly balanced chromosome set The pairing between the homologous chromosomes is complete and the disjunction of the chromosomes is quite regular and normal *T trilobatum* should, therefore, be considered as a balanced secondary polyploid Although evidence of secondary polyploidy could not be gathered from the chromosome numbers of related genera, the study of the SAT-chromosomes and the nucleoli in the present material confirms the above view According to the present conception regarding the phylogenetic significance of the number of nucleoli present in the gametic cells of plants, a true diploid should have only a pair of identical and homologous nucleoli in the body cells corresponding to a pair of homologous and homomorphic SAT-chromosomes present in the somatic complement Increase in the number of nucleoli is brought about by polyploidy, duplication of some chromosomes or through non homologous segmental interchange The presence of three distinct pairs of nucleoli of two different sizes corresponding to two pairs of SAT-chromosomes and another pair of chromosomes with a secondary constriction in each nucleus, shows that *T trilobatum* is a secondarily balanced polyploid and not a true diploid It is also not a structural hybrid except to a certain amount of inversion heterozygosity present in the P M C Further, the complete absence of multivalent formations, absence of any chromosome present in triplicate, and two different sizes of nucleoli rule out the possibility of its being an auto-polyploid species

SUMMARY

The paper gives an account of the morphology, anatomy, embryology and cytology of *Typhonium trilobatum*—a common aroid of Bengal

1 The stem is a sub globose corm of many internodes, axillary buds occur on the surface, these develop into separate plants next season, when the mother corm shrivels and disintegrates

2 Adventitious roots occur in two or more whorls on the crown of the corm The roots are contractile and are spread out almost horizontally in the soil

3 The leaves occur at the top of the corm and enclose completely the growing point which is a dome shaped structure The lamina is characteristically hastate in form and somewhat trisect The petiole is long and leaf base encircles the stem at its point of insertion During developmental stages the younger leaf and inflorescence are completely encased inside the petiole of the subtending leaf

4 The mode of distribution of the veins of the leaf has been described. Free nerve endings are seen, which appear to be branched

5. The phyllotaxis is pentastichous and the nature of ptyxis, which has been described in detail, is of a special type

6 The spathe is constricted in the lower region and forms a barrel shaped chamber inside which the neuter and the female flowers are lodged The spadix has an appendage The male flower is reduced to a stamen, the neuter flower to a filiform process and the female flower to a pistil.

7 The flowers are entomophylous The mode of pollination has been described in detail.

8. The fruit is an ovoid one-seeded berry. Seeds are ovate, greyish black and slightly constricted at the middle from which projects the partially shrivelled basal region

9. The germination of the seed has been studied. Its mode being the same as observed in other plants of the tribe. The plant is propagated both sexually and vegetatively

10 The corm consists of a mass of starch filled parenchymatous cells with the vascular bundles disposed more or less in the form of a ring The corm grows by the multiplication and enlargement of the ground parenchymatous cells

11 Periderm formation is noted at an early stage of the development of the corm The periderm does not form a continuous cylinder but occurs in isolated patches The phellogen is hypodermal in origin

12 Internally the petiole shows the presence of hypodermal bands of collenchymatous cells placed at regular intervals Chlorenchymatous cells occur in between these bands Ground tissue is composed of isodiametrical parenchymatous cells with intercellular spaces

13 The vascular bundles of the petiole are closed and collateral They show a scattered arrangement Xylem consists mainly of annular and spiral vessels reticulated or pitted vessels are absent

14 The leaves show the typical dorsiventral structure Collenchymatous bands are present at the ribs Stomata occur on both suraces

15 The inner surface of the spathe is covered by papillose protrusions The rest of the tissue (excepting the dorsal epidermis) is parenchymatous Vascular bundles are accompanied by collenchymatous bands which occur hypodermally on the abaxial side of the spathe

16 The root shows the normal anatomical features The central region is occupied by one or more large vessels, which differentiate first and show annular thickening In contractile roots the outer cortical region is alone affected At the point of constriction the cells get compressed laterally and present a lamellated appearance

17 The distribution of crystals of calcium oxalate in the different parts of the plant body has been recorded The rôle of nucleus in the development of the crystals has been studied

18 The development of the male and female flowers as also of the ovules has been studied The ovules are orthotropous and bitegmic A nucellar cap is present

19 A single hypodermal archesporial cell differentiates as the megaspore mother cell This produces a linear tetrad of megaspores The chalazal megaspore functions and produces an eight nucleate embryo sac The antipodals are larger than the synergids and are triangular in shape

20 The endosperm nucleus on division produces two chambers The nucleus of the upper chamber produces the entire endosperm tissue, while the lower remains undivided and goes down to the lower part of the embryo-sac and functions as a haustorium

21 The earlier stages in the development of the embryo have been studied The embryo shows the usual monocotyledonous features and has a one-celled suspensor

22 The diploid number of chromosomes is 18 The complement is made up of 6 long, 6 medium and 6 short chromosomes There are two secondary constricted and four Sat-chromosomes

23 The telophase nucleus of the somatic cells shows six nucleoli The phragmoplast appears to play an important rôle in the formation and growth of the cell plate in mitosis

24 During meiosis secondary association of chromosomes has been noted The basic number based on maximum association has been found to be three

25 Chromatid bridges and fragments have been found during the anaphase of division I Chromosome-nucleolus attachment has been observed at different stages of meiosis and mitosis

26 Pollen formation is of the successive type and the pollen grains are binucleate at the time of shedding The pollen grains have a granulated exine

27 The formation and development of the periplasmodium has been followed and the behaviour of the nuclei of the plasmodium recorded Some of the nuclei have been observed to fuse

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ON A PROBLEM OF ANALYTIC NUMBER THEORY

By S Chowla

(Communicated by Sir S S Bhatnagar, F R S)

(*Read January 17, 1947*)

In his *Vorlesungen über Zahlentheorie* Landau raises the problem of an 'elementary' proof of the theorem

Let p denote a prime $\equiv 3 \pmod 4$ *Then there are more quadratic residues than non-residues between* 0 *and* $\frac{p}{2}$ This note contains a reasonably elementary proof of this result We have

Lemma 1

If $0 < x < 1$ *we have*

$$\tfrac{1}{2} - x = \sum_{1}^{\infty} \frac{\sin 2n\pi x}{n\pi}$$

Lemma 2

$$\sum_{n=1}^{p-1} \left(\frac{n}{p}\right) \sin \frac{2mn\pi}{p} = \left(\frac{m}{p}\right)\sqrt{p}$$

where $\left(\frac{n}{p}\right)$ *is Legendre's symbol*

This follows from the Gaussian sum

$$\sum_{1}^{\frac{p-1}{2}} \sin \frac{2n^2\pi}{p} = \tfrac{1}{2}\sqrt{p}$$

of which an elegant and simple proof was given by Estermann in *Journ Lond Math Soc* (1945)

From Lemmas 1 and 2 we immediately get

Lemma 3

$$\sum_{1}^{\infty} \left(\frac{n}{p}\right) \frac{1}{n} = \frac{\pi \Sigma (b-a)}{p\sqrt{p}}, \tag{1}$$

where a *runs through all* $n(1 < n < p)$ *with* $\left(\frac{n}{p}\right) = +1$,

b *runs through all* $n(1 < n < p)$ *with* $\left(\frac{n}{p}\right) = -1$

Lemma 4

$$\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots = \frac{\pi}{4} \text{ or } -\frac{\pi}{4}$$

according as $0 < x < \pi$ *or* $\pi < x < 2\pi$.

From Lemmas 1, 2, 4,

LEMMA 5

$$\sqrt{p}\sum_{n\ \mathrm{odd}}\left(\frac{n}{p}\right)\frac{1}{n}=\frac{\pi}{2}\sum_{0}^{\frac{p}{2}}\left(\frac{n}{p}\right)$$

whence

(2) $$\sqrt{p}\sum\left(\frac{n}{p}\right)\frac{1}{n}=\frac{\pi}{2}\left\{1-\left(\frac{2}{p}\right)\frac{1}{2}\right\}^{-1}\sum_{0}^{\frac{p}{2}}\left(\frac{n}{p}\right)$$

From (1) and (2),

LEMMA 6

(3) $$\sum(b-a)=\frac{p}{\left\{2-\left(\frac{2}{p}\right)\right\}}\sum_{0}^{\frac{p}{2}}\left(\frac{n}{p}\right)$$

From

$$\sum_{1}^{\infty}\left(\frac{n}{p}\right)\frac{1}{n^s}=\prod_{p}\left\{1-\left(\frac{n}{p}\right)\frac{1}{n^s}\right\}^{-1}>0\qquad(s>1)$$

it follows from considerations of continuity that

(4) $$\sum_{1}^{\infty}\left(\frac{n}{p}\right)\frac{1}{n}\geqslant 0$$

From (1) and (4),

(5) $$\Sigma(b-a)\geqslant 0$$

Now

(6) $$\sum(b+a)=\frac{p(p-1)}{2}\equiv 1\ (\mathrm{mod}\ 2)$$

From (6),

(7) $$\Sigma(b-a)\equiv 1\ (\mathrm{mod}\ 2)$$

From (5) and (7),

(8) $$\Sigma(b-a)>0$$

From (3) and (8),

(9) $$\sum_{0}^{\frac{p}{2}}\left(\frac{n}{p}\right)>0$$

q e d

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NOTE ON A CERTAIN ARITHMETICAL SUM

By S. CHOWLA

(Communicated by Sir S. S. Bhatnagar, O.B.E., F.R.S.)

(*Received October 14, read November 23, 1946.*)

I have recently investigated the sum

$$S_r(n) = \sum \sigma(u_1)\sigma(u_2) \cdots \sigma(u_r)$$

where $\sigma(n)$ denotes the sum of the divisors of n and the summation is for all positive integral $u_1, u_2, \ldots, u_r$ such that $u_1+u_2+\cdots+u_r = n$.

In the case when n is equal to a prime p, I find that $S_r(p)$ is a polynomial in p of degree $2r-1$, whenever r is less than 6. Thus

$$S_1(p) = \sigma(p) = p+1 \quad \text{(trivial)},$$

$$(1) \quad S_2(p) = \frac{(p+1)(p-1)(5p-6)}{12}$$

$$(2) \quad S_3(p) = \frac{(p+1)(p-1)^2(p-2)(7p-9)}{192}.$$

(Here p denotes a prime.) Whether $S_r(p)$ is a polynomial in p when r exceeds 5, I am not at present able to determine.

My result (2) is used in a paper by R. P. Bambah and me to be communicated to the *Quarterly Journal of Mathematics* (Oxford) to prove that

$$\tau(p) \equiv 1+p^{11} \pmod{256}$$

where p is an odd prime, and $\tau(n)$, Ramanujan's function, is given by

$$\sum^{\infty} \tau(n)x^n = x\{(1-x)(1-x^2)(1-x^3) \cdots\}^{24} \quad (|x|<1).$$

WIDTH OF NUCLEAR LEVELS

By P L KAPUR, *University of Delhi, Delhi*

(Communicated by Prof D S Kothari, Ph D , F N I)

(*Read November 23, 1946*)

INTRODUCTION

Bohr (1936) has emphasised that the problem of nuclear dynamics is essentially a many-body problem and that for a proper understanding of nuclear transmutations we must regard the process as happening in two distinct stages On account of the close packing and intimate coupling of the nuclear particles the incident particle on colliding with the nucleus immediately shares its energy with the other constituent particles of the bombarded nucleus and gets amalgamated with them, the whole system forming what is called an intermediate compound nucleus Each of the constituent particles in the compound nucleus will have some energy but not, in general, sufficient to enable it to escape from the rest It is only when the energy gets *by chance* concentrated on any of them that it is enabled to escape from the rest Or it might happen that before the energy gets concentrated on any one of the particles constituting the compound nucleus, the system gets down to a stable state either by the emission of radiation (radiative capture) or by breaking into two lighter nuclei (fission) Thus the result of subsequent breaking-up of the intermediate compound nucleus will depend upon a competition between the various disintegration—(including scattering), radiation and fission-processes which are, of course, consistent with the conservation laws

Just as in atomic theory the probability of an atom in an excited state emitting radiation depends upon the width of the level, the width of the intermediate compound nucleus gives us the probability of the emission of particles of any kind—neutrons, protons,. alpha-particles, photons, etc On this view the result of the competition between the various processes of disintegration, etc , of the compound nucleus will depend upon the relative partial widths of the level for the various processes Thus it is that a knowledge of the position and the widths of the levels of the intermediate compound nucleus becomes very important for calculating the cross-section for any nuclear reaction

The width of a nuclear level will depend (among other things) upon the energy of the level, its angular momentum and energy and angular momenta of the products of disintegration To find this dependence we must introduce a special model of the nucleus and then solve its wave equation If we could do this for the case of the intermediate compound nucleus, then at distances very great as compared with the nuclear radius some wave function should correspond to the splitting up of the compound nucleus, i e represent the products of disintegration The calculation for the width of the level for any particular process of disintegration will then be nothing more than a mere evaluation of the matrix element corresponding to a transition from one state of the system to another But unfortunately at the moment we not only do not know the Hamiltonian for a nucleus (all that we know is that the specifically nuclear force is a short range force of the exchange type, we do not know its dependence upon distance) but, even if we knew it, we know no method of solving a many-body problem when the coupling between the particles is very intimate

Sometimes, however, it is required to know the dependence of the width primarily upon the energy and angular momentum of the expelled particle and it is the object of this paper to calculate a limiting value for the width of a nuclear level for very slow particles

CALCULATION OF THE WIDTH

It has already been pointed out that inside a nucleus a particle, be it a neutron or a proton, loses on account of the very close coupling its identity by sharing its energy with the other constituent particles It is just for this reason that we cannot regard the two-body approximation as any good approximation at all Nevertheless when the particle is outside, i e beyond the range of nuclear forces, which it would be if it is at a distance greater than r_0, the nuclear radius, the two-body approximation becomes a very good one and we can write at once for its equation (radial part only)

$$\text{(1)} \qquad \left[\frac{d^2}{dr^2}+k^2-V-\frac{l(l+1)}{r^2}\right]\phi^l(r)=0$$

Where all energies are expressed in units of $2M/\hbar^2$, M being the effective mass of the outgoing particle The other symbols have their usual meaning This equation does not hold for the region $r<r_0$, so that the usual boundary condition $\phi^l(0)=0$ will have to be replaced by a suitable condition at $r=r_0$ If we knew the solution for the region $r<r_0$ this will be fairly straightforward, for all we will have to do will be to join smoothly at $r=r_0$ the solution for the region $r>r_0$ with that for the region $r<r_0$. Since even the equation for the interior of the nucleus is not known, we cannot follow this straight course, and so we will suppose that at the boundary $r=r_0$ the condition to be satisfied by the solution of (1) is

$$\text{(2)} \qquad \left[\frac{r}{\phi^l}\frac{d\phi^l}{dr}\right]_{r=r_0}=A$$

The value of A will depend among other things on the energy of the particle Our object is to find an expression for the width of the nuclear level in terms of this boundary condition or its derivative with respect to energy The method we adopt to find the level-width is to calculate the cross-section at exact resonance and then vary the energy of the incident particle till the cross-section is reduced to one-half its value at resonance The interval through which the energy of the particle has to be varied to reduce the cross-section to one-half its value at exact resonance gives us the half-width for that particular process We will simplify matters further by considering the case of slow neutrons—incidently the most interesting case Now the solution of (1) will be a linear combination of the regular and the irregular solutions, viz.

$$\text{(3)} \qquad \phi^l(r)=\left(\frac{\pi r}{2k}\right)^{1/2}\left[aJ_{l+1/2}(kr)+bJ_{-l-1/2}(kr)\right]$$

Where the coefficients a and b depend upon the energy of the escaping particle If we consider the case of the protons, equation (1) will involve the coulomb potential as well, and consequently the solution will be in terms of the confluent hypergeometric series

If $kr_0 \ll 1$, then in the vicinity of $r=r_0$ (3) can be written as

$$\text{(4)} \qquad \phi^l(r)=\left\{a(kr)^{l+1}+b(kr)^{-l}\right\}\Big/k$$

from which we get

$$\left[\frac{r}{\phi^l}\frac{d\phi^l}{dr}\right]_{r=r_0} = A = \frac{(l+1)a(kr_0)^{l+1}-lb(kr_0)^{-l}}{a(kr_0)^{l+1}+b(kr_0)^{-l}}$$

or

$$b/a = (kr_0)^{l+1}(l+1-A)/(l+A) \tag{5}$$

If at very large distances we write the solution of (1) as

$$\phi^l(r) \sim \frac{1}{k}\sin\left(kr-\frac{l\pi}{2}+\delta_l\right) \tag{6}$$

we know that δ_l is connected with the coefficients a and b by the relation

$$\tan\delta_l = b/a \tag{7}$$

and that the contribution to the cross-section by particles having an orbital angular momentum l is

$$\sigma^l = \frac{4\pi}{k^2}(2l+1)\sin^2\delta_l$$

This with the help of (7) becomes

$$\sigma^l = 4\pi(2l+1)b^2/k^2(a^2+b^2) \tag{8}$$

Evidently σ^l will be a maximum, i e we will get the case of resonance if $a = 0$, i e if

$$A_{\text{res}} = A_0 = -l \tag{9}$$

Starting with the energy of the particle corresponding to resonance if we change the energy (of the particle) the coefficient of the regular solution in (3) namely a will begin to be different from zero till for a certain value of the energy of the particle a equals b When this happens, the value of the cross-section as given by (8) reduces to one-half its value at exact resonance In other words

$$b = a \tag{10}$$

is the condition for obtaining the half-width

So long as the width of nuclear energy levels is small compared to the spacing between them we may write to a first approximation for the value of the boundary condition A in the immediate neighbourhood of resonance

$$A = A_0+\Delta E\frac{dA}{dE} = -l+\Delta E\frac{dA}{dE} \tag{11}$$

Substituting this in (5) we obtain

$$b/a = (kr_0)^{2l+1}\,\frac{(2l+1)-\Delta E\frac{dA}{dE}}{\Delta E dA/dE} \tag{12}$$

The condition (10) now enables us to find the value of ΔE through which the relative energy of the particle must vary for the cross-section to become one-half its value at exact resonance i e the half-width

We thus obtain

$$\Gamma^l \doteq \Delta E = \frac{(2l+1)(kr_0)^{2l+1}}{\{1+(kr_0)^{2l+1}\}dA/dE} \tag{13}$$

CALCULATION OF dA/dE, RESIDUAL NUCLEUS LEFT IN THE GROUND STATE.

To find the value of dA/dE in the immediate neighbourhood of resonance let us consider the wave equation (2) for a slightly different value of the energy k', say, of the escaping particle and denote the wave function for this case by $\phi'^{l}(r)$ Multiplying the equation for ϕ^{l} by ϕ'^{l} and the equation for ϕ'^{l} by ϕ^{l} and subtracting we obtain

$$\frac{d}{dr}\left[\phi^{l}\frac{d\phi'^{l}}{dr}-\phi'^{l}\frac{d\phi^{l}}{dr}\right]=-(k'^{2}-k^{2})\phi^{l}\phi'^{l}$$

Integrating and then multiplying by $r/\phi^{l}\phi'^{l}$ we obtain

$$\text{(14)}\qquad \frac{r}{E-E'}\left[\frac{1}{\phi'^{l}}\frac{d\phi'^{l}}{dr}-\frac{1}{\phi^{l}}\frac{d\phi^{l}}{dr}\right]=-\frac{2M}{\hbar^{2}}\frac{r}{\phi^{l}\phi'^{l}}\int\phi^{l}\phi'^{l}\,dr$$

Proceeding to the limit when $k'\rightarrow k$ we obtain at $r=r_0$

$$\text{(15)}\qquad \frac{d}{dE}\left(\frac{r}{\phi^{l}}\frac{d\phi^{l}}{dr}\right)_{r=r_0}=-\frac{2M}{\hbar^{2}}r_0^{2}\{\textstyle\int(\phi^{l})^{2}dr/r_0(\phi^{l})^{2}\}$$

Taking the factor $\frac{\int(\phi^{l})^{2}dr}{r_0(\phi^{l})^{2}}$ in (15), which in general will be less than unity, to be unity as an approximation we are left with

$$\text{(16)}\qquad \Gamma^{l}=\frac{(2l+1)(kr_0)^{2l+1}}{\{1+(kr_0)^{2l+1}\}}\;\frac{\hbar^{2}}{2Mr_0^{2}}$$

CALCULATION OF dA/dE, RESIDUAL NUCLEUS LEFT IN A NUMBER OF EXCITED STATES

In deducing (15) we have not taken into account the possibility of the residual nucleus being left in a number of excited states Let us now take this possibility into account and see how the value of dA/dE is affected If x (x_1, x_2, x_3) stands for the co-ordinates of the escaping particle and y for all the parameters that may be necessary to describe the rest of the nucleus, the Schrodinger equation for the system is

$$\text{(17)}\qquad \left[\frac{\hbar^{2}}{2M}\Delta_x^{2}-H_y+E-V(r,y)\right]\Psi(x,y)=0,$$

where H_y is the Hamiltonian for the rest of the nucleus and $r=|x|$ Let us write

$$\text{(18)}\qquad \Psi(x,y)=\sum_{i\;l}\frac{1}{r}\phi_i^{l}(r)Y_l(\theta,\phi)\psi_i(y),$$

where ϕ_i's are the solutions of the radial part of the wave equation for the escaping particle in the two-body approximation, viz

$$\text{(19)}\qquad \left[\frac{d^{2}}{dr^{2}}+E_p-V(r,y)\right]\phi_i^{l}=0$$

The centrifugal potential term is absorbed in $V(r,y)$ and the ψ_i's are the solutions of the wave equation for the residual nucleus, viz

$$\text{(20)}\qquad (H_y-E_i)\psi_i=0.$$

If we now multiply (17) by the angle function and integrate over the entire angle space for the escaping particle we get

$$(21)\quad \left\{\frac{\hbar^2}{2M}\frac{d^2}{dr^2}+(E-E_j)-V(r,y)\right\}w_j(r,y) + \sum_{i\neq j}\left\{\frac{\hbar^2}{2M}\frac{d^2}{dr^2}+(E-E_i)-V(r,y)\right\}w_i(r,y)=0$$

where

$$(22)\qquad w_i(r,y)=\phi_i^l(r)\psi_i(y)$$

and the centrifugal potential term has been absorbed in $V(r, y)$

For a slightly different value of the energy of the escaping particle, the nucleus being left in the same energy state, we would get

$$(23)\qquad \left\{\frac{\hbar^2}{2M}\frac{d^2}{dr^2}+(E'-E_j)-V(r,y)\right\}w_j'(r,y) + \sum_{i\neq j}\left\{\frac{\hbar^2}{2M}\frac{d^2}{dr^2}+(E'-E_i)-V(r,y)\right\}w_i'(r,y)=0$$

Multiplying (21) by w'_j and (23) by w_j and subtracting we get after making use of (22) and (19)

$$(24)\qquad \sum_i\frac{w_j}{w_i}\left\{\frac{\hbar^2}{2M}\frac{d}{dr}\left(w_i'\frac{dw_i}{dr}-w_i\frac{dw_i'}{dr}\right)+(E-E')w_iw_i'\right\}=0$$

Writing $E-E'=E_p-E'_p=\varDelta E_p$ where E_p stands for the particle energy, we get on integration with respect to r

$$(25)\quad w_jw_j'\left(\frac{1}{w_j}\frac{dw_j}{dr}-\frac{1}{w_j'}\frac{dw_j'}{dr}\right)+\frac{2M}{\hbar^2}\sum_i\varDelta E_p\int w_jw_i'\,dr + \sum_{i\neq j}\left\{w_jw_i'\left(\frac{1}{w_i}\frac{dw_i}{dr}-\frac{1}{w_i'}\frac{dw_i}{dr}\right)-\int w_iw_i'\left(\frac{1}{w_i}\frac{dw_i}{dr}-\frac{1}{w_i'}\frac{dw_i'}{dr}\right)\frac{d}{dr}(w_j/w_i)dr\right\}=0$$

Now proceeding to the limit when $\varDelta E_p\to 0$, i e the primed and the unprimed states become identical, we obtain after a little simplification

$$(26)\quad -\phi_j^{l2}\frac{d}{dE_p}\left(\frac{1}{\phi_j^l}\frac{d\phi_j^l}{dr}\right)=\frac{2M}{\hbar^2}\int|\phi_j^l|^2dr+\sum_{i\neq j}\frac{\psi_i(y)}{\psi_j(y)}\left\{\frac{2M}{\hbar^2}\int\phi_i^l\phi_j^l\,dr + \phi_j^l\phi_i^l\frac{d}{dE_p}\left(\frac{1}{\phi_i^l}\frac{d\phi_i^l}{dr}\right)-\int|\phi_i^l|^2\frac{d}{dE_p}\left(\frac{1}{\phi_i^l}\frac{d\phi_i^l}{dr}\right)\frac{d}{dr}(\phi_j^l/\phi_i^l)dr\right\}$$

Where there is only one state possible for the residual nucleus the sum $\sum\limits_{i\neq j}$ does not give any contribution and we are left with an expression which is the same as (15).

SUMMARY

An expression for the neutron width of nuclear levels is obtained in terms of the kinetic energy and orbital angular momentum of the neutron The method adopted is to calculate the cross section at exact resonance and then vary the energy of the particle till the cross section is reduced to one half its value at exact resonance This interval through which the energy of the particle has to be varied gives us the half width of the level

REFERENCE

Bohr N (1936) *Nature* **137** 344–348

ON INTEGER ROOTS OF THE UNIT MATRIX.†

By R P BAMBAH *and* S CHOWLA

(Communicated by Sir S S Bhatnagar, F R S)

(*Read January 17, 1947*)

§1 The study of Vaidyanathaswamy's paper (1928) has led us to conjecture that

If p denotes a prime, all the integer matrices X_{p-1} of order $(p-1)$, except E_{p-1} itself, such that

$$[X_{p-1}]^p = E_{p-1}$$

where E_{p-1} is the unit matrix of order $(p-1)$, can be expresed as

$$\Delta^{-1} M_{p-1} \Delta$$

[i e transform of M_{p-1} by Δ] where Δ is an integer matrix of order $(p-1)$ and determinant ± 1, and

$$M_{p-1} = \begin{bmatrix} -1 & -1 & & -1 & -1 \\ 1 & 0 & & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ & & & & \\ 0 & 0 & & 1 & 0 \end{bmatrix}$$

(I) In this paper we prove this conjecture for $p = 3$

§2 In this section we prove that the necessary and sufficient conditions for the integer matrix

$$X_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \neq E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

to be a cube root of E_2 are

(II) $$a+d = -1 \quad \text{and} \quad ad-bc = 1$$

Consider the transformation

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}(x, y) = ax+by,\ cx+dy$$

It can be easily verified that

$$A^3 = (a^3+2abc+bcd)x+b(a^2+ad+d^2+bc)y,$$
$$c(a^2+ad+d^2+bc)x+(d^3+2bcd+bca)y$$

Therefore the necessary and sufficient conditions for X_2 to be a cube root of E_2 are (except when $X_2 = E_2$)

$$a^3+2abc+bcd = 1 \qquad \text{(i)}$$
$$a^2+ad+d^2+bc = 0 \qquad \text{(ii)}$$
$$d^3+2bcd+bca = 1 \qquad \text{(iii)}$$

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† All letters in this paper denote integers.

It can be easily seen that

$$(iii) = (i) + (d-a)(ii)$$

Therefore we obtain the necessary and sufficient conditions as

$$a^3 + (2a+d)bc = 1 \qquad (i)$$

$$a^2 + ad + d^2 + (bc) = 0 \qquad (ii)$$

Eliminating bc from these equations we have

$$(a+d)^3 = -1 \quad \text{or} \quad (a+d) = -1$$

as a necessary condition

Also from (ii) we have

$$\begin{aligned} bc &= -(a^2+ad+d^2) = ad-(a+d)^2 \\ &= ad-1 \end{aligned}$$

Hence $ad-bc = 1$, is another necessary condition

That these two conditions are sufficient can be easily verified with the help of (i) and (ii)

§3 On account of (II) the proof of (I) reduces to showing that

$$X_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

where $a+d = -1$ and $ad-bc = 1$, can be expressed as

$$\Delta^{-1} M_2 \Delta$$

where

$$\Delta = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad AD-BC = \pm 1$$

and

$$M_2 = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$$

This we shall prove in this section

We shall consider only the case when b is negative The case when b is positive can be similarly treated by taking Δ of determinant -1 That b cannot be zero can be easily seen

We can easily prove the following lemmas

(1) If $\Delta = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ and $AD-BC = 1$,

$$\Delta^{-1} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

(2) $\Delta^{-1} M_2 \Delta = \begin{bmatrix} -CD-AD-AB & -D^2-BD-B^2 \\ C^2+AC+A^2 & CD+BC+AB \end{bmatrix}$

Now

$$ad-bc = 1,$$

therefore

$$\begin{aligned} -bc &= 1-ad = 1-a(-a-1) \\ &= a^2+a+1. \end{aligned}$$

It is well known that all factors of a^2+a+1 are of the form k^2+l^2+kl

We choose A, B C and D such that

$$-b = D^2+BD+B^2 \tag{iv}$$

$$c = C^2+AC+A^2 \tag{v}$$

and

$$AD-BC = 1 \tag{vi}$$

That A, B, C and D can be chosen to satisfy (vi) will be proved in §4

Now we shall show that

$$a = -CD-AD-AB \tag{vii}$$

and

$$d = CD+BC+AB \tag{viii}$$

As

$$a^2+a+1 = -bc = (B^2+D^2+BD)(A^2+C^2+AC)$$

we have

$$a^2+a+[1-(B^2+D^2+BD)(A^2+C^2+AC)] = 0$$

The roots of this equation in a are

$$-CD-AD-AB \quad \text{and} \quad CD+BC+AB,$$

for

$$\begin{aligned}(-CD-AD-AB)+(CD+BC+AB) &= -(AD-BC)\\ &= -1\end{aligned}$$

and

$$\begin{aligned}&\{(-CD-AD-AB)(CD+BC+AB)\}\\ &-[1-(B^2+D^2+BD)(A^2+C^2+AC)]\\ &= -1+(AD-BC)^2 = 0\end{aligned}$$

We, therefore, have

$$a = (-CD-AD-AB) \quad \text{or} \quad (CD+BC+AB)$$

In case a has the second value, replace A C B and D by C A $-D$ and $-B$ respectively Obviously (iv) (v) and (vi) are unaffected while a has the value $-CD-AD-AB$

So that in all cases we have

$$a = -CD-AD-AB$$

and hence

$$\begin{aligned}d = -1-a &= -1+CD+AD+AB\\ &= CD+BC+AB\end{aligned}$$

From (iv), (v), (vi), (vii) and (viii) it follows that we can choose A B C and D to satisfy

$$X_2 = \Delta^{-1}M_2\Delta$$

where

$$\Delta = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \text{and} \quad AD-BC = 1$$

§4

THEOREM

(1) If $$n^2+n+1 = m_1m_2$$

we can choose A, B, C and D such that

(2) $$m_1 = A^2+C^2+AC,$$

$$m_2 = B^2+D^2+BD, \tag{3}$$

and

$$AD-BC = \pm 1 \tag{4}$$

It is well known that all factors of n^2+n+1 are of the form k^2+l^2+kl Therefore we have only to prove that A, B, C and D can be chosen to satisfy (4)

Lemma 1 —If m_1 is prime, the theorem is true

Let

$$m_1 = A^2+C^2+AC$$

As

$$n^2+n+1 = m_1 m_2,$$

$$m_2 = \frac{n^2+n+1}{m_1} = \frac{(n^2+n+1)(A^2+C^2+AC)}{m_1^2}$$

$$= \left(\frac{An-C}{m_1}\right)^2 + \left[\frac{C(n+1)+A}{m_1}\right]^2 + \left(\frac{An-C}{m_1}\right)\left[\frac{C(n+1)+A}{m_1}\right]$$

Now

$$\begin{aligned}(An-C)(Cn-A) &= ACn^2-n(A^2+C^2)+AC\\ &\equiv \{ACn^2-n(A^2+C^2-A^2-C^2-AC)+AC\} \pmod{m_1}\\ &\equiv AC(n^2+n+1) \pmod{m_1}\\ &\equiv 0 \pmod{m_1}\end{aligned}$$

m_1 being a prime, one at least of $(An-C)$ and $(Cn-A)$ is a multiple of m_1

In case it is only the latter, replace C by A and A by C so that in all cases we have

$$m_1 = A^2+C^2+AC$$

and

$$An-C \equiv 0 \pmod{m_1} \tag{5}$$

Now

$$\begin{aligned}0 &\equiv n(An-C) \pmod{m_1}\\ &\equiv An^2-Cn \pmod{m_1}\\ &\equiv -Cn+A(n^2-n^2-n-1) \pmod{m_1}\\ &\equiv -Cn-A-An \pmod{m_1}\\ &\equiv -Cn-A-C \pmod{m_1}\end{aligned}$$

[because of (5)]

Therefore

$$m_2 = B^2+D^2+BD$$

where B and D are integers given by

$$B = \frac{An-C}{m_1} \quad \text{and} \quad D = \frac{C(n+1)+A}{m_1}$$

Now

$$AD-BC = \frac{ACn+AC+A^2-ACn+C^2}{m_1}$$

$$= 1.$$

Therefore the lemma is true.

LEMMA 2.—If

$$n^2+n+1 = m_1 m_2$$

and

(i) $m_1 = mp_2$, p_2 being a prime number

(ii) $m = a^2+c^2+ac$

(iii) $p_2 m_2 = b^2+d^2+bd$

(iv) $ad-bc = 1$,

then we can choose A, B, C and D such that

$$m_1 = A^2+C^2+AC$$
$$m_2 = B^2+D^2+BD$$

and

$$AD-BC = \pm 1$$

p_2, being a factor of n^2+n+1, is equal to e^2+f^2+ef where e and f are suitable integers.

As

$$p_2 m_2 = b^2+d^2+bd$$

$$m_2 = \frac{b^2+d^2+bd}{e^2+f^2+ef} = \frac{(b^2+d^2+bd)(e^2+f^2+ef)}{p_2^2}$$

$$= \left(\frac{be-df}{p_2}\right)^2 + \left(\frac{bf+de+df}{p_2}\right)^2 + \left(\frac{be-df}{p_2}\right)\left(\frac{bf+de+df}{p_2}\right)$$

Now

$$(be-df)(de-bf) = bde^2+bdf^2-efb^2-efd^2$$
$$\equiv [bd(e^2+f^2-e^2-f^2-ef)-ef(b^2+d^2)] \pmod{p_2}$$
$$\equiv -ef(b^2+d^2+bd) \pmod{p_2}$$
$$\equiv 0 \qquad \pmod{p_2}$$

p_2 being a prime, one at least of $(be-df)$ and $(de-bf)$ is a multiple of p_2. In case it is not the former, replace e and f by $-f$ and $-e$ respectively so that in all cases we have

$$p_2 = e^2+f^2+ef$$

and

$$be-df \equiv 0 \pmod{p_2}$$

Now

$$0 \equiv be-df \pmod{p_2}$$
$$\equiv b^2e-bdf \pmod{p_2}$$
$$\equiv e(b^2-b^2-d^2-bd)-bdf \pmod{p_2}$$
$$\equiv -d(ed+be+bf) \pmod{p_2}$$
$$\equiv -d(ed+df+bf) \pmod{p_2}$$

Therefore $ed+df+bf$ is a multiple of p_2 for, if not, d must be a multiple of p_2 and hence on account of (iii) lemma 2, b and therefore $ed+df+bf$ is a multiple of p_2.

Now we have

$$m_2 = B^2+D^2+BD$$

where B and D are integers given by

$$B = \frac{be-df}{p_2}, \quad D = \frac{bf+df+de}{p_2}$$

Also

$$\begin{aligned} m_1 = mp_2 &= (a^2+c^2+ac)(e^2+f^2+ef) \\ &= (ae-cf)^2+(af+ce+cf)^2+(ae-cf)(af+ce+cf) \\ &= A^2+C^2+AC \end{aligned}$$

where

$$A = ae-cf \quad \text{and} \quad C = af+ce+cf$$

$$\begin{aligned} AD-BC &= \frac{(ae-cf)(bf+df+de)-(be-cf)(af+cf+ce)}{p_2} \\ &= ad-bc = 1, \end{aligned}$$

the lemma is true

The main theorem of this section (with $AD-BC = 1$) can now be proved by combining the two lemmas and using the method of induction

To prove that the theorem is true with $AD-BC = -1$ also we have only to replace A and C by $-A$ and $-C$ respectively

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ON A TREATMENT OF IMPERFECT GAS AFTER FERMI'S MODEL

By M DUTTA

(Communicated by Prof N R Sen, D Sc , Ph D , F N I)

(*Received December 12, 1946 , read April 4, 1947*)

ABSTRACT

In this paper, a statistical theory of imperfect gases has been developed by application of a principle analogous to Pauli's Exclusion Principle and method of Fermi The equation of state as given by Saha and Bose has been deduced Further, Van der Waal's equation up to first approximation has been obtained by accounting for the boundary effect and cohesion by equivalent increase in total volume

1 INTRODUCTION

The method of counting the total number of thermodynamic complexions of a set of particles in phase-space, and then, of introduction of entropy, after Planck, by application of Boltzmann's principle is the simplest and finest of all the methods, used in the statistical theories of thermodynamics This method is not only successful in explaining the behaviour of the ideal gas completely but is also equally successful in quantum statistics Few attempts have, however, to our knowledge, been so far made to extend this method to the theory of an imperfect gas, or to that of a liquid The present work represents an endeavour to apply this method to the theory of an imperfect gas We have here obtained for an imperfect gas a new equation of state from which the equation of state of an ideal gas as well as that of Van der Waal's gas follows by suitable approximation We start with the accepted view that the physical behaviour of an imperfect gas is attributable to two factors, namely, the finite size of the molecules, and the forces of molecular cohesion We propose, accordingly, to develop our theory in two stages, in the first of which account is taken of the finite size of the molecules, and in the second, of the cohesive forces

In the present method, the only dynamical property of particle, which plays an important rôle in distribution, is its energy Now in the case of an imperfect gas, the total energy is separable into kinetic energy (depending on momenta only) and potential energy (depending on configurational co-ordinates only) (Fowler 1936.) As the distribution of the particles is random with respect to kinetic energy and potential energy separately (we mean thereby that the positional and the momenta co-ordinates of the particles are unconnected), the positional and the momentum space can be considered separately with much convenience The product of the thermodynamic probabilities corresponding to the two spaces will evidently give the total thermodynamic probability

2 EFFECT OF FINITE DIMENSION OF MOLECULES

We consider an assembly of N molecules, in an enclosure of volume V, each of which is supposed to possess a rigid volume b of exclusion There is no association or dissociation in the assembly

Now, the total volume V is divided into space-cells of volume b, the number of such cells is then V/b and is evidently a large number, b being small Then, the effect of volume of exclusion can be stated as, a cell of the physical space cannot contain more than one particle. The cell may be vacant or occupied by only one particle.

Then, the total number of ways in which N molecules may be distributed amongst the (V/b) cells is

$$\frac{(V/b)!}{N!\,(V/b-N)!}$$

Turning now to the consideration of the distribution of N molecules in momenta space, we remark that the volume of a cell in phase-space being h^3, the volume of a cell in momenta space is to be taken as h^3/b, since the volume of a cell in the physical space is, by hypothesis, b As the energy of a molecule is given by

$$\epsilon = \frac{1}{2m}\left(p_1^2+p_2^2+p_3^2\right),$$

so the number of cells h^3/b in the microcanonical layer

$$(\epsilon_r,\ \epsilon_r+d\epsilon) = \frac{b}{h^3}\ 2\pi\,(2m)^{\frac{3}{2}}\,\epsilon_r{}^{\frac{1}{2}}d\epsilon$$

Now, if a_r denotes number of molecules in the layer $(\epsilon_r,\ \epsilon_r+d\epsilon)$ then the number of ways, in which molecules may be distributed, is

$$\frac{N!}{a_1!\,a_2!\ \ \ a_r!}$$

where $\sum a_r = N,\quad \sum a_r\epsilon_r = E$

Then the thermodynamic probability is

$$W = \frac{(V/b)!}{(V/b-N)!\ N!}\ \frac{N!}{a_1!\,a_2!\ \ \ a_r!}$$

To get the entropy, this expression is to be maximised, subject to above-mentioned restrictions

This gives

$$a_r = e^{-\lambda-\mu\epsilon_r},$$

where λ, μ are, as usual, undetermined constants

and $$S = k\,[V/b \log V/b-(V/b-N)\log(V/b-N)+N\lambda+\mu E].$$

Now,

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V = k\mu, \quad \therefore \mu = \frac{1}{kT}$$

and

$$N = \sum a_r = \sum_r e^{-\lambda-\epsilon_r/kT} = e^{-\lambda}\iiint_{-\infty}^{\infty} e^{-\left(p_1^2+p_2^2+p_3^2\right)/2mkT}\ \frac{dp_1dp_2\,dp_3}{h^3/b}$$

or

$$\lambda = \log\left\{\frac{b}{Nh^3}(2\pi mkT)^{\frac{3}{2}}\right\},$$

and

$$E = \sum_r a_r \epsilon_r = e^{-\lambda} \int\!\!\int\!\!\int_{-\infty}^{\infty} \frac{1}{2m} \left(p_1^2+p_2^2+p_3^2\right) e^{-\left(p_1^2+p_2^2+p_3^2\right)/2mkT} \frac{dp_1 dp_2 dp_3}{h^3/b}$$

$$= \frac{3}{2} NkT$$

Then, finally,

$$S = k\left[\frac{V}{b}\log\frac{V}{b} - \left(\frac{V}{b}-N\right)\log\left(\frac{V}{b}-N\right) + \frac{3}{2}N + \frac{3}{2}N\log T - N\log\lambda + N\log\left\{\frac{b}{h^3}(2\pi mk)^{\frac{3}{2}}\right\}\right],$$

and

$$\Psi = S - \frac{E}{T}$$

$$= k\left[\frac{V}{b}\log\frac{V}{b} - \left(\frac{V}{b}-N\right)\log\left(\frac{V}{b}-N\right) + \frac{3}{2}N\log T - N\log N + N\log\left\{\frac{b}{h^3}(2\pi mk)^{\frac{3}{2}}\right\}\right]$$

Then as usual (Saha and Bose Equation),

$$p = T\left(\frac{\partial\Psi}{\partial V}\right)_T = -\frac{kT}{b}\log\left(1-\frac{Nb}{V}\right)$$

In the limiting case, $\frac{Nb}{V} \to 0$, the equations give expressions for ideal gas as

$$S = kN\left[\frac{5}{2}+\frac{3}{2}\log T + \log\left\{\frac{V}{N}\frac{(2\pi mk)^{\frac{3}{2}}}{h^3}\right\}\right],\ p = \frac{kNT}{V}$$

Up to 1st approximation, these give,

$$S = kN\left[\frac{5}{2}+\frac{3}{2}\log T + \log\frac{V-\frac{1}{2}Nb}{N} + \log\left\{\frac{(2\pi mk)^{\frac{3}{2}}}{h^3}\right\}\right],$$

and

$$p = \frac{kNT}{V}\left(1+\frac{1}{2}\frac{Nb}{V}\right)$$

These are the expressions for entropy and pressure of a Van der Waals' gas, when cohesive force is ignored, and only, the correction for the finite size of the molecules is made

3. Correction for Cohesive Forces

The cohesive forces are assumed here to be of Van der Waals' type, i e , they are short-ranged isotropic forces of attraction The effect of the cohesive forces may, therefore, be described as amounting to the production of a molecular density in the interior of the enclosure slightly greater than that in the thin surface layer (parallel to the walls of enclosure) Were the cohesive forces removed, and replaced by a uniform field such that the potential energy of every particle is the same as that in the surface layer, then the volume requirement of the N molecules under the same pressure and at the same temperature will be slightly greater than V Writing a for the small additional volume, we may say that an actual volume V with the forces of cohesion becomes equivalent, under the same conditions of temperature and pressure, to a volume $(V+a)$ without forces of cohesion Full account of the forces of cohesion may, on this view be taken, if the actual volume V is replaced by $(V+a)$ in the above thermodynamic expressions So, the thermodynamic expressions can be written as

$$S = k\left[\frac{V+a}{b}\log\frac{V+a}{b} - \left(\frac{V+a}{b} - N\right)\log\left(\frac{V+a}{b} - N\right) + N\log\left\{\frac{b}{Nh^3}(2\pi mkT)^{\frac{3}{2}}\right\} + \frac{E}{kT}\right],$$

$$\Psi = S - \frac{k}{T} = k\left[\frac{V+a}{b}\log\frac{V+a}{b} - \left(\frac{V+a}{b} - N\right)\log\left(\frac{V+a}{b} - N\right) + \frac{3}{2}N\log T - N\log N + N\log\left\{\frac{b}{h^3}(\pi mk)^{\frac{3}{2}}\right\}\right],$$

and

$$p = T\left(\frac{\partial\Psi}{\partial V}\right)_T = \frac{kT}{b}\log\left(1 - \frac{Nb}{V+a}\right)\left\{1 + \left(\frac{\partial a}{\partial V}\right)_T\right\}$$

4 APPROXIMATE EVALUATION OF a AND p

Let n_i, n_s be number densities, and, w_i, w_s be potential energies of particles in the interior of the enclosure and in the surface layer respectively, where $n_s < n_i$ Let V_s be volume of the surface layer

Then,

$$(V+a)n_s = (V - V_s)n_i + V_s n_s$$

Therefore

$$a = \frac{(V - V_s)(n_i - n_s)}{n_s}$$

Assuming density law for gas, we have

$$n_i \propto e^{-\frac{w_i}{kT}}, \quad n_s \propto e^{-\frac{w_s}{kT}}$$

and so

$$w_i < w_s$$

Therefore

$$a = (V - V_s)\left(e^{\frac{w}{kT}} - 1\right),$$

where

$$w = w_s - w_i > 0$$

Therefore

$$a = (V - V_s)\frac{w}{kT}, \quad \text{(approximately)}$$

Now, in the assembly of Van der Waals' gas,

$$V_s \simeq At,$$

where A is the area, and t the thickness of layer less than the radius of the sphere of influence of Van der Waals' force which is generally very small. Therefore

$$V_s \ll V$$

Up to 1st approximation,

$$a = \frac{Vw}{kT}$$

Now, it is easy to see that

$$w = \frac{N}{V} c,$$

where c only depends upon mass of molecule and nature of the force. Up to this approximation,

$$\left(\frac{\partial a}{\partial V}\right)_T = 0$$

Then

$$p = -\frac{kT}{b} \log \left(1 - \frac{Nb}{V+a}\right)$$

$$= \frac{NkT}{V}\left[1 + \frac{\frac{1}{2}Nb}{V} - \frac{a}{V}\right], \text{ correct up to 1st approximation}$$

or

$$p + \frac{\alpha}{V^2} = \frac{NkT}{V-\beta},$$

where

$$\alpha = NkTa = N^2 c,$$

where c and hence α is independent of volume and temperature. This is the Van der Waals' equation.

The following point may be noted in connection with the above process. The difference of energies of the assembly of N molecules enclosed in a volume V under cohesive force and of the assembly of the same particles in a volume $(V+a)$ under no cohesive force and under uniform field is

$$-Nw$$

On the other hand, the change of energy due to change of volume under constant pressure is

$$\int_V^{V+a} p\,dV = pa = \frac{NkT}{V} \frac{Vw}{kT} = Nw$$

Thus, in the change as pictured here the total energy remains unchanged. The original and the altered assemblies have both the same N and E values.

5 CONCLUSION

This present paper is different from the previous discussions on this topic mainly on two points, (i) division of the physical space of particles into cells of volume equal to the volume of exclusion, (ii) consideration of the effect of cohesion as a correction in volume only

So far as the first idea is concerned, it is only a new way of looking at the effect of rigid volume of exclusion This idea makes the treatment of volume of exclusion so easy that it is likely it will prove very suitable in simplifying the theories of imperfect Gases and Liquids

So far as the second idea is concerned, it may be added that, the increase of volume due to boundary effect can also be easily and clearly visualised according to modern picture of boundary effect According to Millikan (1923) in the light of various experiments done by them, during the collisions with the boundary, the particles are adsorbed by the surface, and, then, re-emitted with random velocities This in a way may be looked upon as increasing the effective volume of the assembly of the particles pictured in this paper As the number of molecules adsorbed is proportional to number density, so, the constant a is proportional to N But, as the idea is not yet fully developed in this line, a correct expression of the boundary effect cannot be obtained at this stage

The writer takes this opportunity to express his gratitude and thanks to Dr S C Kar, Prof N R Sen and Prof S N Bose for helpful discussions and keen interest

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PROCEEDINGS
OF THE
NATIONAL INSTITUTE OF SCIENCES OF INDIA

OCTOBER—DECEMBER, 1947

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PUBLISHED
BY THE
NATIONAL INSTITUTE OF SCIENCES OF INDIA
UNIVERSITY BUILDINGS, DELHI

Price Rs 6-14-0

P 51

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11 MAY 1948

MALLOPHAGA (ISCHNOCERA) INFESTING BIRDS IN THE PUNJAB (INDIA)

By M Atiqur Rahman Ansari, *M Sc , Ph D (Punjab), Research Student, Laboratory of the Imperial Entomologist, Imperial Agricultural Research Institute, New Delhi.*

(Communicated by Dr Hem Singh Pruthi, O B E , M Sc , Ph D & Sc D (Cantab.), F.R A S B , F.N I)

(*Received June 10th, 1943, read April 4th, 1947*)

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Introduction.

The group Mallophaga has been much neglected by workers in India There are in India about twenty-five hundred species and subspecies of birds Of these, up to the present, only about one hundred bird hosts have been examined and a few species of Mallophaga recorded from them

The first Mallophagan to be described from India was *Esthiopterum* (= *Pediculus*) *tantali* (*Fab*), from the Painted Stork (*Ibis l leucocephalus*, Pennant), (1798, *Ent Syst. Suppl* , p 571). Thereafter, *Lipeurus himalayensis* Rudow, from the Western Horned Pheasant (*Tragopan melanocephalus*, Gray), (1870, *Zeit f g Nat* , 36, p. 123) and *Menopon acutovulvatum* Piaget from the Indian Large Pied Hornbill (*Hydrocissa malabarica malabarica*, Gmel), (1881, *Tijd. v Ent* , 24, p 5, pl 1, fig 4) were described, probably from the material collected from the Indian birds

There is no further mention of the Indian Mallophaga for more than thirty years, when Kellogg and Paine (1914) published 'Mallophaga from birds (mostly of Corvidae and Phasianidae) of India and neighbouring countries', which contained the description of about nine new species and forty records of old species found within the Indian limits A year later Kellogg and Nakayama (1915) published 'Additional Mallophaga from the Indian Museum (Calcutta)', from 32 bird hosts

Waterston (1928) published a very valuable paper on the Mallophaga of the Sand-grouse and described six species of *Syrrhaptoecus* collected from skins of the Indian Sand-grouse (Pteroclididae) in the British Museum Recently, Qadri (1935) started work on this group and has so far described eleven species Clay and Meinertzhagen (1935–1943) have added 27 species to the Indian fauna, and Sen (1942) has added 2 more species to the list

During an investigation of the food-habits of birds undertaken at the Punjab Agricultural College and Research Institute, Lyallpur, a collection of the Mallophaga was made at the suggestion of Prof. M Afzal Husain. The lice were collected from about one hundred species of birds belonging to 83 genera, 38 families and

Linlithgow Library.

10 different orders. Of these, the Mallophaga from only 22 species of birds had been previously recorded from India. Those of the remaining 78 hosts are recorded here for the first time Collection was made from the freshly killed birds and bird skins by Mr H R Bhalla and myself The birds were identified by the Bombay Natural History Society and the Indian Museum, Calcutta. The help received is gratefully acknowledged I acknowledge with gratitude the share of Mr. Bhalla in the extensive collection, so laboriously made and excellently preserved

Specimens soaked in 5% caustic potash for about 24 hours and ultimately mounted in canada balsam were used for all the measurements recorded here The specimens were not artificially pressed but were mounted under light, No 6A ¾″-circular microscope cover-glasses Apart from the errors inherent in the method owing to distortion or fixation, measurement made of some species mounted in glycerine showed that an accuracy of well within ±4% was usually attained All linear measurements were taken along the medium line, while the breadth recorded is the maximum for each body part Measurements were made under a microscope, by means of an eyepiece micrometer The drawings were made by the author with the aid of camera lucida The types have been deposited in the collection of the Entomological Laboratory of the Punjab Agricultural College and Research Institute, Lyallpur

ACKNOWLEDGMENTS

The present work would not have been possible but for the guidance and valuable help of Prof M Afzal Husain (formerly Entomologist to the Punjab Government and Vice-Chancellor, Punjab University) I am greatly indebted to him for his unfailing interest in this work and for the advice which he gave me, throughout these studies My thanks are also due to Dr Khan A Rahman (Entomologist, Punjab Agricultural College and Research Institute, Lyallpur) for placing the entire collection of Mallophaga at the Institute at my disposal and for his constant help in several other ways

My respectful gratitude is due to Dr Hem Singh Pruthi, O B E, Imperial Entomologist, for facilities for work in his laboratory and for his keen interest in the progress of this work My sincere thanks are also due to Dr M L Roonwal and Mr M S Mani for going through the manuscript, giving me the benefit of their valuable criticism and for help in its preparation for the press

The grant-in-aid to carry out this work at the Imperial Agricultural Research Institute, New Delhi, sanctioned by the Punjab University, is gratefully acknowledged.

SYSTEMATIC ACCOUNT

ISCHNOCERA

1896 *Ischnocera*, Kellogg, *Proc Calif Acad Sci*, VI(2), p 63

Harrison (1916) divided the bird-infesting Ischnocera into six sub-families, but Ewing (1929) discarded this classification on the ground that a logical sub-grouping was doubtful In the present paper, however, I propose, for the sake of convenience, to treat the Ischnocera under five main groups, viz., Esthiopteridae, Philopteridae, Degeeriellidae, Lipeuridae and Goniodidae The table given below will be found useful in recognising the genera dealt with in this paper.

I KEY TO GENERA

1 Forehead usually rectangular or almost trapezoidal in shape; clypeo lateral margin slightly concave, or straight, never convex; trabeculae often present and usually pronounced; temporal lobes rounded 2

Forehead not rectangular or trapezoidal in shape, clypeo-lateral margin convex, trabeculae almost absent or only as very small protuberances 23

2 Ptero-thorax rectangular, sides almost sub-parallel, posterior margin almost straight: ESTHIOPTERIDAE 3
Ptero-thorax (fused meso and meta-thorax) broader than long, sides strongly divergent and with posterior margin angulate or outwardly rounded 8
3 Clypeal signature (pigmented blotch) bearing numerous crescentic papillae on the upper side *Ardeicola* Clay
Clypeal signature without such character 4
4 Clypeal signature large, bearing a longitudinal slit, gular plate large, IX abdominal segment in female bifid, partly flanked on each side by pointed prolongation of segment VIII *Fulicoffula* Clay & Mein
Without such characters 5
5 Forehead with 4–6 circular incrassations on the lateral margins *Falcolipeurus* Bedford
Forehead without circular incrassations on the lateral margins 6
6 Clypeal suture deeply emarginate, the cavity so formed margined with hyaline produced preantennal region and furnished with a strong spine and 3 4 fine hairs *Anaticola* Clay
Clypeal suture without such characters 7
7 Forehead narrow, sides sub-parallel, trabeculae almost absent, I antennal segment longest, eyes flatly rounded *Columbicola* Ewing
Forehead wider, sides strongly diverging, preantennal area broad, trabeculae conspicuous, small lobes, II antennal segment longest, eyes protruding *Turturicola* Clay & Mein
8 Stouter species with large head and comparatively short, broad abdomen PHILOPTERIDAE 9
Slender species DEGEERIELLIDAE 19
9 Trabeculae short, ventral, female abdomen with three irregularly shaped dark chitinised plates in the centre of segment VII, dorsal and ventral abdominal hairs lanceolate *Aegypoecus* Clay & Mein
Trabeculae large, movable; abdomen without such characters 10
10 Forehead with hyaline flaps 11
Forehead without hyaline flaps 17
11 Preantennal region very narrow, clypeal front deeply notched or forcipated and flanked with hyaline flap 12
Preantennal region truncate, short, clypeal region with hyaline margin 14
12 Abdomen bearing many short stout spines on ventral aspect of segment I and II and sometimes on III and IV also *Echinophilopterus* Ewing
Abdomen not bearing spines on the abdominal sternum 13
13 Tergal plates I–VIII entire, pleurites with straight narrow re-entrant heads, I abdominal segment rectangular *Picophilopterus* gen nov
Tergal plate VIII entire; pleurites with curved re entrant heads *Alcedoffula* Clay & Mein
14 Hyaline anterior portion of head originates each side of the level of clypeal suture 15
Clypeal region expanded with hyaline free margin, evenly rounded throughout 16
15 Clypeal margin broadly emarginate in front, and notch flanked with hyaline flaps which touch each other at tips *Incidifrons* Ewing
Clypeal margin concave, flanked with hyaline flap *Falcoecus* Clay & Mein
16 Clypeal suture passing inwards and forming median suture, anterior hyaline margin projecting beyond the contour of the preantennal margin *Alcedoecus* Clay & Mein
Clypeal region expanded with hyaline margin throughout, hyaline flap not projecting beyond the contour of the preantennal margin, two small peg like dorsal spines, one on each side of the posterior apex of the signatural plate *Anatoecus* Cummings
17. Male genitalia with parameres curved and protruding beyond the mesosome, the latter consisting of flattened plate with central penis *Penenirmus* Clay & Mein
Male genitalia with very small parameres which do not protrude beyond the mesosome, the mesosome a flattened plate with central penis, which is usually projecting *Philopterus* Nitzsch.
18 Head circumfasciate, parabolic to subogival 19
Head quadrangulate or interrupto-fasciate 26
19. Antennae showing sexual dimorphism *Psittaconirmus* Harrison
Antennae similar in the two sexes 20
20. Forehead with clypeal signature 21
Clypeal signature absent 23
21. Clypeal suture present; hyaline margin arising anterior to clypeal suture, median vertical preantennal suture present *Quadraceps* Clay & Mein.
Hyaline margin arising at clypeal suture, without median vertical suture 22
22. With transverse preantennal suture; hyaline margin narrow *Lunaceps* Clay & Mein
With vertical preantennal suture, hyaline margin broad *Carduiceps* Clay & Mein.
23. Tergal plates interrupted in the middle 24
Tergal plates not interrupted in the middle 25

24 Forehead completely rounded, curved transverse suture across preantennal region present *Cuculicola* Clay & Mein
Forehead parabolic or subogival, transverse suture absent *Syrrhaptoecus* Waterston
25 Forehead parabolic, pleural plates with straight wedge-shaped intermittent heads, highly pigmented, reaching as far as 2/3rds of the preceding segment, tails not highly pigmented *Upupicola* Clay & Mein
Forehead completely or flatly rounded, pleural plates bent towards the median line, with blunt intermittent heads uniformly pigmented throughout *Kelerinirmus* Eichler
26 Head quadrangular 27
Head interrupto fasciate *Painjunirmus* gen nov
27 Male genitalia highly pigmented with broad endomeral plate, with strongly built pointed and incurved parameres *Bruelia* Keler (= *Degeeriella* Nitzsch)
Male genitalia feebly sclerotic with poorly developed endomeral plate and almost straight parameres *Trashorella* gen nov
28 Head longer than broad with projecting forehead and rounded temples LIPEURIDAE 29
Head usually as broad as or even broader than long forehead equal to at least one third of the post antennal region of head GONIODIDAE 30
29 Male genitalia characteristic with broad basal plate parameres short inwardly curved endomeral plate longer than parameres *Cuclotogaster* Carriker (= *Gallipeurus* Clay)
Basal plate long narrow parameres long, usually narrowly pointed endomeral plate short not extending beyond the middle of parameres *Lipeurus* Nitzsch
30 Third segment of male and sometimes first also with an appendage *Goniodes* Nitzsch
Third segment of male never appendiculate 31
31 Two large recurved frontal processes on each side of head present male genital armature nirmoid type highly built *Paragoniocotes* Cummings
Without frontal processes, male genital armature simple, rod like and narrow *Goniocotes* Burmeister

II ESTHIOPTERIDAE

1 Ardeicola gaibagla,[1] sp nov

Female (Text fig 1*a*) elongate, creamy white with yellowish brown pleural plates

Head long and narrow, triangular forehead trapezoidal, rounded in front, clypeal suture distinct, entire dorsally and confined marginally on the ventrum, clypeal signature bearing about fifty seven crescentic papillae or ridges (Text fig 1*b*), chaetotaxy scarce, disposed as in figure Trabeculae short, conical and immovable Temples rounded with narrow, indistinct, yellowish marginal band and a short seta disposed as shown in figure Occipital margin sinuous with a small seta, gular plate weak Antennae 5 jointed, filiform, bearing cluster of fine setae at the tip Eyes protruding, rounded, ocular blotch distinct, small Pharyngeal glands and sclerite well developed

Prothorax small, lateral margins straight with a small seta in posterior lateral angle Meso and meta thorax fused into a pterothorax, slightly wider than pro thorax and bearing 4 pustulated hairs on the posterior margin Sternal plates indistinct, intercoxal plates not well developed Legs as shown in figure, concolorous with the body

Abdomen elongated, broadest in the V segment, gradually tapering towards the posterior end Segments I VII well marked, segments VIII IX fused, last segment bilobed posteriorly Pleural plates I–VII distinct, narrow Tergal plates yellowish, indistinct Chaetotaxy scarce as shown in figure Sternal plates indistinct

Male similar to female but smaller Antennae 5 segmented, I joint enlarged and III joint has a small lateral protuberance Male genitalia characteristic basal plate long, evenly and feebly chitinised, almost double the size of parameres and mesosome, slightly narrowing towards the anterior, parameres long, well developed but not well chitinised, furnished with a sensory seta, endomeral plate of simple structure, penis short, rod shaped reaching as far as the middle of parameres

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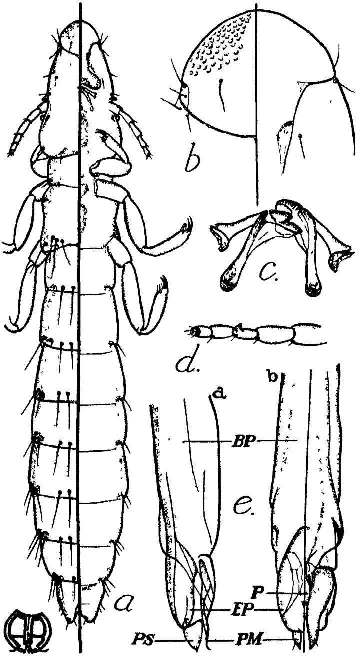
[1] Gaibagla' in vernacular means the Indian Cattle Egret



TEXT-FIG 1 *Ardeicola gaibagla*, sp. nov

a Dorsal and ventral aspects of female, *b* Clypeal region, showing crescentic papillae, *c*. Mandibles, *d*. Antenna of male, *e*. Male genital armature (a) side view, (b) dorso-ventral aspect.

Measurements (mm.).

	Female (Holotype)	Male (3) [1]
Body	2 84×0 48	2 51 2 62×0 40
Head	0 65×0 34	0 61 0 62×0 32 0 34
Prothorax	0 18×0 28	0 15 0 17×0 24 0 28
Pterothorax	0 28×0 34	0 30×0 26 0 30
Abdomen	1 73×0 48	1 45-1 53×0 40

Holotype (female) and *Allotype* (male) both from Lyallpur, 29-iii-1930, from the Indian Cattle Egret (*Bubulcus ibis coromandus*, Bodd), mounted together on slide No MI 139 *Paratypes* 2 males mounted together on slide No MI 139 P (same data as above)

This louse closely resembles *Ardeicola ardea* (Linn) from the Heron (*Ardea c, cinerea* Linn) and *Ardeicola episcopi* (Qadri) from the Indian White-necked Stork (*Dissoura e episcopa* Bodd) It differs from the allied forms in size, chaetotaxy and tergal plates

2. **Fulicoffula luridum** (Nitzsch)

1818 *Lipeurus luridus*, Nitzsch, *Germ Mag*, III, p 292

This species has been recorded on the Coot (*Fulica a atra* Linn), from England (Denny, 1842), and Germany (Nitzsch, 1818, Mjoberg, 1910), on *Fulica americana*, from the United States of America (Osborn, 1896), on *Gallinula chloropus*, from England (Denny, 1842).

One male and one female were obtained from the Coot (*Fulica a atra* Linn), shot in Lyallpur, 16-ii-1928

Measurements (mm)

	Female	Male
Body	2 56×0 43	1 87×0 32
Head	0 57×0 31	0 57×0 31
Thorax	0 47×0 32	0 44×0 27
Abdomen	1 52×0 43	0 86×0 32

Denny (1842) gave the length of female as 2 54 mm, while Piaget (1880) and Taschenberg (1882) gave it as 3 04 mm and 3 2 mm respectively

3. **Falcolipeurus quadripustulatus** (Nitzsch)

1818 *Lipeurus quadripustulatus*, Nitzsch, *Germ Mag*, III, p 293

This species has been recorded from different vultures from Europe, Africa, Asia and the U S A. Most of the hosts recorded from outside India are also present within India, viz, the Cinereous Vulture (*Aegypius monachus*, Linn), the European Griffon (*Gyps fulvus fulvus*, Hab), the Bearded Vulture (*Gypaëtos barbatus*, Linn) the Golden Eagle (*Aquila chrysaëtos daphanea*, Hodgs), the White-tailed Sea-Eagle (*Haliaëtus albicillus*, Linn.), and the Upland Buzzard (*Buteo hemilasius* Temm.).

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[1] Figures in parenthesis indicate the number of individuals measured in each case.

My specimens are from the Cinereous Vulture (*Aegypius monachus*, Linn), 26-iii 1928, and the Himalayan Griffon (*Gyps himalayensis* Hume), 9 i-1930, both shot in Lyallpur

Measurements (mm)

	Female (3)	Male (2)
Body	3 450–4 069 × 0 563–0 774	3 886–3 985 × 0 60–0 676
Head	0 732–0 873 × 0 493–0 563	0 817 × 0 493–0 563
Thorax	0 563–0 591 × 0 563–0 714	0 534–0 591 × 0 590–0 614
Abdomen	2 155–2 605 × 0 563–0 774	2 535–2 577 × 0 600–0 676

Piaget (1880) and Mjoberg (1910) gave the measurements of female as 3 25 mm × 0 7 mm and 4 3375 mm × 0 75 mm, while of male as 3 0 mm × 0 6 mm and 3 537 mm × 0 612 mm respectively

4 **Anaticola crassicorne** (Scopoli)

1763 *Pediculus crassicorne* Scopoli *Ent carn* p 383

This long known species is very widely distributed on various species of ducks On account of slight variations, parasites from different hosts have been given varietal status, resulting in a long synonymy The specimens referred to this species were obtained from the Ruddy Sheldrake (*Casarca f ferruginea*, Vroeg) shot in Lyallpur, 21 ii 1933, and Kulu, 21 x 1939 the Common Teal (*Nettion c crecca*, Linn) shot in Lyallpur, 20 ii 1933, and Kulu 21 x 1939 and the Dun Bird (*Nyroca f ferina* Linn) shot in Lyallpur 14 xi 1932 One female was also obtained from the Himalayan Whistling Thrush (*Myophonus coeruleus temmincki* Vigors) shot in Kulu, 15 ix 1928 This is undoubtedly straggler

Measurements (mm)

	Female (3)	Male (1)
Body	2 82–3 29 × 0 35–0 57	2 56 × 0 44
Head	0 56–0 61 × 0 34–0 43	0 55 × 0 42
Thorax	0 42–0 59 × 0 35–0 44	0 55 × 0 38
Abdomen	1 84–2 09 × 0 35–0 57	1 46 × 0 44

Piaget (1880) and Kellogg (1896) gave the measurements of female as 2 85 mm × 0 5 mm and 3 3 mm × 0 62 mm respectively, while Piaget's (1880) male was 2 5 mm × 0 39 mm

5 **Columbicola columbae** (Linn).

1758 *Pediculus columbae*, Linnaeus, *Syst Nat* p 164

This is one of the commonest species all over the world There is a certain amount of variation coincident with geographical areas and hosts, but they are almost entirely those of size, which make it difficult to define the various sub-species

Numerous specimens were obtained from the Bengal Green Pigeon (*Crocopus p. phoenicopterus*, Lath) shot in Ambala, the Indian Blue Rock Pigeon (*Columba *livia intermedia* Strick) shot in Lyallpur, 27-viii-1928, the Indian Spotted Dove (*Streptopelia chinensis suratensis*, Gmel), 8-iii-1928, the Little Indian Brown Dove (*Streptopelia senegalensis cambayensis*, Gmel), 21-viii-1929, the Indian Ring Dove (*Streptopelia d decaocta*, Frival), 27-iii-1928, and the Red Turtle Dove (*Oenopopelia t tranquebarica*, Herm), 6-iv-1932, all shot in Lyallpur

Measurements (mm)

	Female (3)	Male (5)
Body	2 239–2 453 × 0 29–0 40	1 786–2 012 × 0 200–0 266
Head	0 553–0 560 × 0 20–0 33	0 480–0 546 × 0 116–0 213
Thorax	0 373–0 400 × 0 20–0 29	0 306–0 400 × 0 186–0 213
Abdomen	1 333–1 493 × 0 29–0 40	1 000–1 066 × 0 200–0 266

Piaget (1880) and Taschenberg (1882) gave the size of female as 2 1 mm × 0 37 mm and 2 44 mm × 0 39 mm , while of male as 1 8–1 9 mm × 0 3 mm and 2 28 mm. × 0 36 mm respectively Kellogg's (1896) female was 2 5 mm × 0 37 mm

6 **Turturicola salimalii** Clay & Mein

1937 *Turturicola salimalii*, Clay & Meinertzhagen, *Entomologist*, LXX, p 278, f 1

Clay and Meinertzhagen (1937) found it on *Streptopelia d decaocta*, Frival and *Oenopopelia t tranquebarica*, Herm , both from Rajputana The present specimens were obtained from the Indian Blue Rock Pigeon (*Columba livia intermedia* Strick), 27-viii-1928, the Indian Ring Dove (*Streptopelia d decaocta*, Frival), 27-iii-1928, the Indian Spotted Dove (*Streptopelia chinensis suratensis*, Gmel), 8-iii-1928, and the Little Indian Brown Dove (*Streptopelia senegalensis cambayensis*, Gmell), 21-viii-1928.

I also found it on the Bengal Jungle Babbler (*Turdoides t terricolor*[1] Hodgs), 16-iii-1932, the Burmese White browed Fantail Fly-catcher (*Leucocirca aureola burmanica* Hume), 21 ii-1928, the Common Indian Myna (*Acridotheres t tristis*, Linn), 16-xii-1931, the Common Indian House Sparrow (*Passer domesticus indicus* Jard & Selby) and the Rose-ringed Paroquet (*Psittacula krameri manillensis*, Bechst), 7-iii-1931 All the hosts were shot in Lyallpur It is difficult to explain the presence of this louse on such widely separated hosts Some are possibly stragglers and might have reached the hosts while they were feeding in close association, breeding in closer proximity, huddling together on perches, or they might have used deserted nests of pigeons and doves

Measurements (mm)

	Female (7)	Male (5)
Body ..	2 180–2 370 × 0 306–0 426	1 786–1 893 × 0 226–0 306
Head	0 520–0 546 × 0 299–0 333	0 466–0 493 × 0 280–0 293
Thorax ..	0 333–0 450 × 0 240–0 305	0 360–0 400 × 0 213–0 253
Abdomen ..	1 266–1·440 × 0 306–0 426	0 946–1 000 × 0 226–0·306

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[1] It has been pointed out that the Punjab form appears to be *Turdoides terricolor sindianus* Ticehurst.

III PHILOPTERIDAE

7 Aegypoecus brevicollis (Nitzsch).

1838 *Docophorus brevicollis*, Nitzsch, in Burmeister, *Handbuch der Ent*, II, p 424

This species was first described from the Cinereous Vulture (*Aegypius monachus*, Linn) and has since been recorded on the type host from many parts of the world The specimens referred to here were obtained from the type host shot in Lyallpur, 26-iii-1928

Measurements (*mm*)

	Female (1)	Male (1)
Body	2 198 × 1 266	1 959 × 1 130
Head	0 666 × 0 933	0 666 × 0 880
Thorax	0 466 × 0 733	0 360 × 0 666
Abdomen	1 066 × 1 266	0 933 × 1 130

Piaget (1880) gave its length as 1/2'''

8 Aegypoecus griffoneae, sp nov

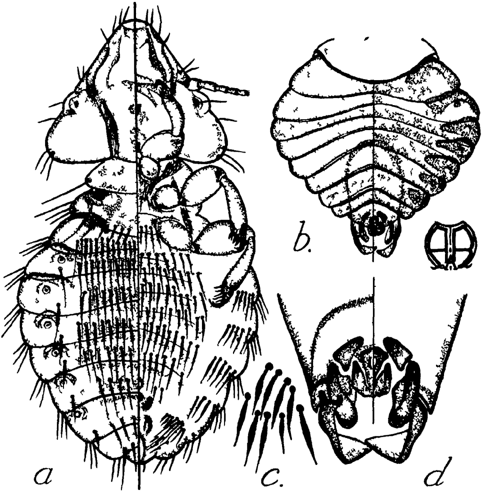
Female (Text-fig 2*a*) golden yellow with brown markings on head, thorax and abdomen, abdomen almost circular

Head slightly wider than long, triangular, clypeal front uncoloured, very slightly dilated, narrow, straight to slightly concave, two short hairs at the anterior angle, two hairs near the distinct clypeal suture, two dorsal hairs just before the clear uncoloured clypeal region, clypeal band deep yellow, clypeal signature indistinct, reaching as far as the mandibles, internal band conspicuous, antennal band dark brown, distinct, running along the margin, posteriorly straight, fusing with occipital band, a black blotch at the base of trabeculae, trabeculae ventral, short and not projecting beyond the contour of head, antennae short and slender, temporal margins projecting outwardly, posterior angle rounded with two short hairs, temporal band narrow, slightly conspicuous at the base of eye, eyes protruding, each with a short sub-basal hair, occipital margin almost straight furnished with two central hairs, occipital band indistinct, occipital signature shield-shaped and yellowish brown

Prothorax narrow, with acutely rounded lateral angles, each bearing a short hair, posterior margin slightly convex, bare, marginal band indistinct, brown inter-coxal plates showing through Meso- and meta-thorax completely fused into a pterothorax, slightly projecting laterally, acutely rounded lateral angles, each bearing three hairs, posterior margin markedly convex with three median and three submarginal lanceolate hairs on each half, marginal bands indistinct, with a black, distinct, anterior blotch at the inter thoracic suture, intercoxal and pericoxal plates highly chitinised and showing through Legs with claws unequal in size and bearing a number of pedunculate spines at the distal end of tibia, dark rings at the distal end of femoral and tibial segments distinct

Abdomen almost circular or broadly elliptical, with dark brown transverse marginal intersegmental lines, tergal plates not distinctly marked, posterior margin of segments I–VIII concave in the middle and convex submarginally, segment IX uncoloured, narrow and emarginate, segments I–VII with numerous dorsal median rows of lanceolate hairs; segments II–VII with 1–3 lateral hairs Ventrum with similar lanceolate hairs on I–V segments, and segments III–VI with lateral 5-6 lanceolate hairs. Anal orifice distinct, segment VII with three brown plates, one central, trapezoidal, two lateral, one on each side, bean-shaped, genital plate distinct with few marginal hairs.

Male (Text-fig 2*b*) similar to female, the tergal plates very well marked, confined to the submarginal region Last segment rounded and thickened Genital armature (Text fig 2*d*) well developed, general characters as for the genus but the parameres are strongly developed, twisted inwards from the middle and shear shaped



TEXT FIG 2 *Aegypoecus gryffoneae*, sp nov

a Dorsal and ventral aspects of female, *b* Dorsal and ventral aspects of abdomen of male, *c* Lanceolate hairs clothing body, *d* Male genital armature

Holotype (female) on slide No MI 043 H, *Allotype* (male) on slide No MI 043A, obtained from Himalayan Griffon Vulture (*Gyps himalayensis* Hume), both shot in Lyallpur, 9 i 1930 *Paratypes* 3 females and one male on slide No MI 043 P (same data as above)

This species is similar to *Aegypoecus* (= *Helluo*) *neophron* (Clay and Mein), but differs in the narrow, uncoloured fronto-clypeaus and indistinct, weakly pigmented

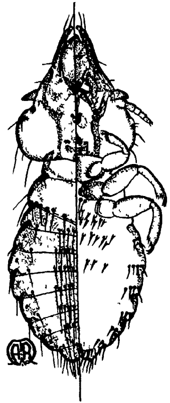
Measurements (mm)

	Female (Holotype)	Female (3)	Male (2)
Body	2 083 × 1·000	2·027–2 199 × 1 000–1 093	1 732–1 946 × 0 760–1 113
Head	0 791 × 1·010	0 718–0 853 × 0 986–1 010	0 731–0 760 × 0 906–0 933
Prothorax	0 120 × 0 506	0 120–0 169 × 0 506–0 591	0 133–0 169 × 0 480–0 521
Pterothorax	0 226 × 0 666	0 221–0 253 × 0 666–0 704	0 173–0 211 × 0 634–0 693
Abdomen	0 951 × 1 000	0 887–1 000 × 1 000–1 093	0 774–0 826 × 0 760–1 113

clypeal signature, prothorax with slightly angulate lateral margin and so is the pterothorax, male genitalia with very well-developed parameres which are twisted inwards in the middle and are flat and shear shaped

9. Echinophilopterus tota,[1] sp nov

Female (Text-fig 3) golden yellow with brown markings on head, thorax and abdomen; abdomen oval with angularly emarginate terminal segment



TEXT-FIG 3 *Echinophilopterus tota*, sp nov
Dorsal and ventral aspects of female

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[1] 'Tota' in vernacular means the Indian Rose ringed Paroquet.

Head resembling Piaget's (1880) interesting group *forficulati*, triangular with narrow anteriorly tapering clypeus, clypeal front deeply notched and flanked with anteriorly produced lateral flaps which meet in the middle, anterior angle with two ventrally situated hairs showing through the dorsum, clypeal band prominent, clypeal suture distinct, extending far beyond clypeal signature, deeply concave; clypeal signature distinct, reaching as far as the mandibles, internal band conspicuous, antennal band dark brown, distinct, turned inwards and reaching as far as the sub-median region, trabeculae large, long, projecting and blunt at the tips; antennae short, temporal margins projecting outward, rounded, with two ocular setae, and two long hairs and a seta at the posterior angle, temporal bands not conspicuous, pale yellow with slightly distinct ocular blotch, eyes not protruding, occipital margin a little concave marginally and convex medially, bare, occipital band distinct reaching as far as the middle of the head and fused with the antennal band Occipital signature is triangular on a rectangular base

Prothorax large, rectangular with acute latero-posterior angles, a small seta in the posterior angle, posterior margin straight, bare, marginal band distinct, continuous in the head, intercoxal plates highly developed, reaching as far as the middle line, sternum well developed, oblong, bare Meso- and metathorax completely fused into pterothorax, pterothorax slightly projecting laterally, latero-posterior angle blunt bearing two long hairs, posterior margin angulate on abdomen with 5 hairs distributed as shown in figure, lateral bands distinct, sternum weak with two hairs, pericoxal plates well developed Legs normal with slightly developed marginal bands

Abdomen ovate, with dark yellow latero-transverse plates and dark brown marginal bands on segments I–VII, segment VIII with entire blotch, segments I–VII with 4 5 hairs on each side of the median line, confined in the middle, one hair on sublateral margin at the base of the lateral plates, and 1-2 hairs in the lateral angles, segment VIII deeply emarginate posteriorly and with about 6 hairs on each lobe Ventrum with short setae on segments I, II and III as shown in figure, hairs on posterior margin, genital plate well developed with a series of short hairs

Measurements (mm)

Female	(Holotype)	(Paratype)
Body	1 892 × 0 720	1 665 × 0 680
Head	0 760 × 0 613	0 653 × 0 546
Prothorax	0 133 × 0 374	0 106 × 0 333
Pterothorax	0 266 × 0·533	0 240 × 0 480
Abdomen	0 733 × 0 720	0 666 × 0 680

Holotype and *Paratype* 2 females from Lyallpur *ex* the Indian Rose-ringed Paroquet (*Psittacula krameri manillensis*, Bechst) mounted together on slide No MI 034, shot in Lyallpur, 17-iv-1931

10 Echinophilopterus, sp.

Several nymphs were collected from the Indian Large Paroquet (*Psittacula eupatria nepalensis* Hodgs) shot in Lyallpur, 21-ii-1931

11 Alcedoffula alcedinis (Denny).

1842 *Docophorus alcedinis*, Denny, *Anop. Brit* , p 111, pl 6, f 1.

This species was first described from the European King-fisher (*Alcedo atthis ispida*) Only one immature specimen was obtained by me from the Egyptian White-breasted King-fisher (*Halcyon s. smyrnensis*, Linn.) shot in Lyallpur, 17-ii-1928.

Picophilopterus, gen. nov.

This genus is erected for the reception of a new species collected from the Himalayan Scaly-bellied Green Woodpecker (*Picus s squamatus* Vigors) shot in Kulu, 15-ix-1928 and 6-x-1939

Description of the genus —Elongate nirmoid Philopteridae with very moderately sclerotised body. Head large, clypeus narrow, separated from preantennal region by a distinct suture, clypeal front forcipated because of the lateral clypeal bands extending beyond the signature, area between flanked with hyaline flap which is entire, clypeal signature not distinctly chitinised, concave anteriorly, bluntly pointed posteriorly, reaching as far as the anterior margin of labrum Trabeculae large, not extending beyond the I antennal segment Antennae filiform, showing no sexual dimorphism Occipital signature small, triangular, pharyngeal sclerite and glands well developed Prothorax small, far reaching in the occipital margin, pterothorax with projected sides, small, broadly convex posteriorly, sternum and precoxal plates present Legs short Abdomen ovate, with rectangular I segment, pleural plates II-VI straight, narrow, re entrant heads straight, tergal plates well marked, entire on segments II-VIII Female with deeply emarginate IX segment, with slightly coloured blotch, male with rounded IX segment, with distinct marginal band

Genital plate in female conspicuous, notched and fringed with hairs Male genital armature with narrow basal plate Parameres are short, pointed, curved, endomeral plate broad with narrow chitinised margins, in the centre of the endomeral plate lies a compound structure with curved triangular ears near the tip and a hollow tube beyond

This genus can be separated from *Alcedoffula* by the distinctly different form of genital armature and from *Echinophilopterus* by the absence of strong spines on ventral aspect of abdomen

This genus is apparently confined to the Woodpeckers (Picidae) and should contain the species belonging to Piaget's (1880) group *angustifrontes* and *Philopterus evagans* (Kellogg), *P jungens* (Kellogg) and *P californiensis* (Kellogg)

Genotype —*Picophilopterus tuktola* sp nov (*vide infra*) ex the Himalayan Scaly-bellied Green Woodpecker (*Picus s squamatus* Vigors)

12 Picophilopterus tuktola,[1] sp nov.

Female (Text-fig 4*a*) elongate, nirmoid body, narrow clypeal front, pale with dark marginal markings

Head a little longer than broad, triangular, with narrow, anteriorly tapering front, clypeus incised in front because of the lateral clypeal bands which extend beyond the signatural plate, anterior angle with two long hairs, one hair in front of suture, one dorsal, submarginal hair between the suture and the front, one such hair on the tail of internal band, two hairs just in front of trabeculae, antennal bands distinct, broken at the clypeal suture, trabeculae large, acute, hardly reaching the I antennal segment, antennae normal, similar in the two sexes, eyes prominent, flatly rounded, with central seta, ocular blotch blackish, ocular band distinct, temples broadly rounded with narrow marginal band, two long hairs and a prickle, occipital margin sinuous, bare, occipital band distinct but not highly pigmented, occipital signature short, triangular, pharyngeal sclerite and glands distinct

Prothorax short, projecting considerably beneath head, posterior angle rounded with one hair, posterior margin flatly convex, true pterothorax, short, projecting laterally, latero-posterior angle with one short and two long hairs, posterior margin obtusely angulate on the abdomen with two sub-median hairs on each side, marginal bands distinct sternum distinct intercoxal plates well developed Legs normal, concolorous with the body

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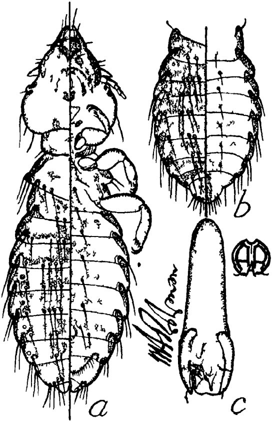
[1] 'Tuktola' in vernacular means the Himalayan Scaly-bellied Woodpecker.



TEXT FIG 4 *Picophilopterus tuktola* sp nov

a Dorsal and ventral aspects of female *b* Dorsal and ventral aspects of thorax and abdomen of male *c* Male genital armature

Abdomen elongate oval with dark marginal bands on segments I–VIII, transverse bands entire on segments I VIII but on I II emarginate medially on anterior margin segments III–VIII with long hairs on slightly projecting posterior angles segments II–VII with one submarginal hair segments I VII with median row of hairs segment VIII with 3 hairs on the posterior margin segment IX deeply emarginate almost colourless and bearing no hairs Sternal plates confined in the middle chaetotaxy scarce genital plate on segment VIII as shown in figure

Male (Text fig 4*b*) similar to female last segment projecting rounded, with long hairs on posterior margin and a distinct band Genital armature (Text-fig 4*c*) distinct showing through segments VI–IX

Holotype (female) and *Allotype* (male) mounted together on slide No MI 080 From Kulu *ex* the Himalayan Scaly bellied Woodpecker (*Picus s squamatus* Vigors), 15-ix 1928 *Paratypes* Numerous males and females from the type host shot in Kulu, 15-ix 1928 and 6 x 1939

Measurements (mm)

	Female (Holotype)	Female (3)	Male (2)
Body	2 266 × 0 740	2 173–2 266 × 0 560–0 746	1 719–1 839 × 0 600–0 610
Head	0 666 × 0 533	0 613–0 666 × 0 533–0 560	0 573–0 586 × 0 480–0 490
Prothorax	0 160 × 0 320	0 146–0 160 × 0 280–0 320	0 133 × 0 280
Pterothorax	0 240 × 0 573	0 240–0 254 × 0 533–0 573	0 200 × 0 406–0 466
Abdomen	1 200 × 0 740	1 133–1 200 × 0 560–0 746	0 800–0 933 × 0 600–0 610

13 **Incidifrons pertusus** (Nitzsch)

1818 *Docophorus pertusus*, Nitzsch, *Germ Mag*, III, p 290

This species was described from the Coot (*Fulica atra* Linn) by Schrank (1803) and since then has been recorded from the type host from many parts of the world Kellogg (1896) recorded it from the American Coot (*Fulica americana*) and the Ruddy Duck (*Erismatura rubida*) which are constant associates in nature

My specimens were recorded from the Coot (*Fulica a atra* Linn) shot in Lyallpur, 16-ii-1928

Measurements (mm)

	Female (1)	Male (1)
Body	1 666 × 0 586	1 126 × 0 493
Head	0 560 × 0 466	0 380 × 0 400
Thorax	0 306 × 0 400	0 266 × 0 306
Abdomen	0 800 × 0 586	0 480 × 0 493

Piaget (1880) gave its length to be 1/2''', i e 1 27 mm and 2/3''', i e 1 693 mm for male and female respectively, while Kellogg's (1896) female specimens were slightly bigger and measured 2 0 mm × 0 92 mm

14 **Falcoecus ?milvi** (Mjoberg)

1910 *Docophorus milvi*, Mjöberg, *Ark Zool*, VI, p 109, pl 3, f 1, tf 63

I provisionally refer to this species a number of specimens obtained from the Common Pariah Kite (*Milvus migrans govinda* Sykes) shot in Lyallpur, 5-iv-1933 The type-host of this species is *Milvus aegyptius*

Measurements (mm.)

	Female (2)	Male (1)
Body	2 093–2 213 × 0 893–0 933	1 799 × 0·800
Head	0 800 × 0 800–0 84	0 733 × 0 733
Thorax	0 453–0 480 × 0 60	0 333 × 0 546
Abdomen	0 840–0 933 × 0 893–0 933	0 733 × 0 800

Mjöberg's (1910) female and male specimens measured 2 3125 mm × 1 0875 mm. and 1 925 mm × 0 9625 mm. respectively.

15 **Alcedoecus capistratus** (Neumann).

1912 *Philopterus capistratus*, Neumann, *Arch Parasit*, XV, p. 375, f 20.

Numerous specimens from the Egyptian White-breasted King-fisher (*Halcyon s smyrnensis*, Linn) shot in Kulu, 9-x-1939, and Lyallpur, 21-i-1929 This species was described from specimens taken off *Halcyon leucoçephala* Bedford (1919) recorded it from the Brown-hooded Kingfisher (*Halcyon albiventris*) from Transvaal and Natal

Measurements (mm)

	Female (2)	Male (3)
Body	1 679–1 746 × 0 453–0 600	1 439–1 492 × 0 533–0 560
Head	0 546–0 560 × 0 506–0 533	0 453–0 493 × 0 480–0 493
Thorax	0 306–0 320 × 0 453–0 466	0 280–0 306 × 0 400–0 426
Abdomen	0 813–0 880 × 0 453–0 600	0 666–0 706 × 0 533–0 560

16 **Anatoecus dentatus** (Scopoli)

1763 *Pediculus dentatus*, Scopoli, *Ent Carn*, p 383

This is the commonest parasite of ducks, recorded under different names from various parts of the world A large number of its recorded hosts occur within Indian limits, viz, the White-fronted Goose (*Anser a albifrons*, Scop), the Mallard (*Anas platyrhyncha*, Linn), the Wigeon (*Mareca penelop*, Linn), the Common Teal (*Nettion c crecca*, Linn), the Shoveller (*Spatula clypeata*, Linn), the Red Crested Poohard (*Netta rufina*, Pallas), the Pochard (*Nyroca f ferina*, Linn), the Scaup (*Nyroca m marila*, Linn), the Tufted Pochard (*Nyroca f fuligula*, Linn), the Smew (*Mergellus albellus*, Linn) and the Goosander (*Mergus merganser merganser* Linn)

My specimens were obtained from the Brahminy Duck (*Casarca ferruginea*, Vroeg), 5-iv-1933, and the Dun Bird (*Nyroca f ferina*, Linn), 14-xi-1932, both shot in Lyallpur

Measurements (mm)

	Female (2)	Male (3)
Body	1 479 × 0 533	1 118 × 0 466
Head	0 506 × 0 426	0 426 × 0 400
Thorax	0 280 × 0 360	0 226 × 0 333
Abdomen	0 693 × 0 533	0 466 × 0 466

Kellogg (1896) gave the measurements of female as 1 4 mm × 0 52 mm

17 **Penenirmus subflavescens** (Geoffroy)

1762 *Pediculus subflavescens*, Geoffroy, *Hist Ab Ins*, II, p 599

This species has been recorded from a number of passerine birds Giebel (1874) listed 29 passerine birds representing 15 genera, Picaglia (1885) gave an exhaustive list of synonymy and listed 43 species of European birds from which this species was collected up to that time Kellogg *et al.* (1896–1899) recorded it from about 45 American Passeriformes Harrison (1916) has referred to 21 synonyms. Provisionally I refer numerous specimens to this group of closely allied species from the following birds shot in Lyallpur. Variation among these specimens was marked and certainly of specific value, but the uncertain condition of this species kept me from attempting further diagnosis. The Bengal Jungle Babbler (*Turdoides terricolor terricolor* Hodgs), the Common Babbler (*Argya c caudata*, Dumont), the Punjab Red-vented Bulbul (*Molpastes cafer intermedius*, Jerdon), the Himalayan White-cheeked Bulbul (*Molpastes l leucogenys*, Gray), the Western Red Spotted Blue-throat (*Cyanosylvia s suecica*, Linn), the Brown-backed Indian Robin (*Saxicoloides fulicata cambaiensis*, Lath), the Indian Great Grey Shrike (*Lanius excubitor lahtora*, Sykes), the Rufous-backed Shrike (*Lanius schach erythronotus*, Vigors), the Brown Willow Warbler (*Phylloscopus collybitus tristis* Blyth), the Red headed Bunting (*Emberiza brunniceps* Brandt), the Indian White Wagtail (*Motacilla alba dukhunensis*, Sykes) and the Indian Crested Lark (*Galerida cristata chendoola* Frankl).

Measurements (*mm*)

	Female (8)	Male (3)
Body	1 289–1 462 × 0 53–0 613	1 126–1 329 × 0 428 0 546
Head	0 453–0 560 × 0 426–0 530	0 463 × 0 400
Thorax	0 263–0 266 × 0 360 0 493	0 200–0 266 × 0 333–0 386
Abdomen	0 530–0 733 × 0 530–0 613	0 463–0 600 × 0 426–0 516

18 **Penenirmus ornatus** (Nitzsch)

1866. *Docophorus ornatus*, Nitzsch, in Giebel, *Zeit*, *f ges*, *Nat*, XXVII, p 116

This species was described by Nitzsch (in Giebel 1866) from specimens taken off the European Golden Oriole (*Oriolus o oriolus*, Linn). The specimens referred to here were immature and obtained from the Indian Golden Oriole (*Oriolus o kundoo* Sykes), shot in Lyallpur, 5-vii-1928.

19 **Penenirmus raji**,[1] sp nov

Female (Text-fig 5*a*) well built, yellowish with brownish body markings, with conspicuous tergal plates and ventral transversal blotches.

Head slightly longer than broad, forehead narrow, concave, clypeal suture distinct, anterior angle with two hairs, 3 hairs beyond it on the preantennal region, antennal band and internal bands well formed, narrow, trabeculae large, acute, reaching as far as two-third of the II antennal segment, antennae normal, similar in the two sexes, eyes prominent with a basal hair, ocular bands distinct, temples rounded with one long hair and a few setae as shown in figure, occipital signature small, pharyngeal glands and sclerite well developed.

Prothorax well developed, projecting considerably beneath the head, posterior angle with a hair, marginal band well pigmented, intercoxal plates showing through. True pterothorax, projecting laterally, latero-posterior angle with two hairs, posterior margin characteristically angulate on the abdomen with a series of long hairs, marginal bands distinct, sternum distinct, intercoxal plates well developed. Legs normal, concolorous with body.

Abdomen ovate, with slightly projecting segments, marginal bands I–VII distinct, with turned inwards heads, tergal plates I–VII submarginal, tergal plate VIII entire, segment IX emarginate, colourless. Chaetotaxy scarce. Sternal

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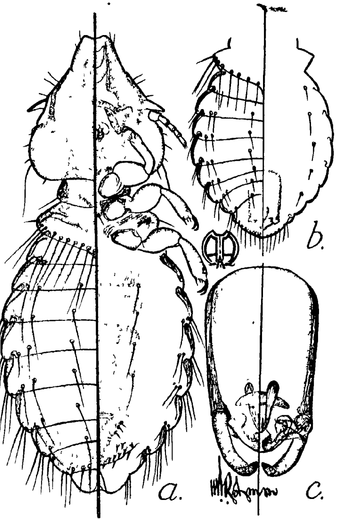
[1] 'Raji' in vernacular means the Indian Yellow-throated Sparrow

plates distinctly marked, genital plate on segment VIII with pustulated setae on the margins.



TEXT-FIG 5 *Penenirmus raji*, sp nov

a Dorsal and ventral aspects of female, *b* Dorsal and ventral aspects of thorax and abdomen of male; *c* Male genital armature

Male (Text-fig 5*b*) similar to female, last segment rounded behind with narrow posterior border and numerous hairs Genitalia (Text-fig 5*c*) showing through segments VI–IX

Holotype (female) mounted on slide No. MI 025H, from Lyallpur, 13-v-1933, *ex* the Yellow-throated Sparrow (*Gymnoris x xanthocollis*, Burt) *Allotype* (male) on slide No MI 025A from Lyallpur, 5-v-1931, *ex* the Indian House Sparrow (*Passer domesticus indicus* Jard. and Selby.). *Paratypes* (2 females) mounted together

Measurements (mm).

	Female (Holotype)	Female (8)	Male (1)
Body	1 357 × 0 581	1 325–1 346 × 0 506–0 600	1 052 × 0 413
Head	0 413 × 0 411	0 413–0 440 × 0 413	0 400 × 0 360
Prothorax	0 072 × 0 221	0 066–0 080 × 0 213–0 240	0 066 × 0 200
Pterothorax	0 157 × 0 335	0 160 × 0 280–0 346	0 080 × 0 28
Abdomen	0 715 × 0 581	0 666–0 746 × 0 506–0 606	0 506 × 0 413

on slide No MI 025P from Lyallpur, 5 v-1931, *ex* Yellow-throated Sparrow (*Gymnoris x xanthocollis*, Burt), and numerous males and females from Lyallpur, 13-v-1931, *ex* the Indian House Sparrow (*Passer domesticus indicus* Jard and Selby)

20. Philopterus crassipes (Nitzsch)

1838 *Docophorus crassipes*, Nitzsch, in Burmeister, *Handbuch der Ent* , II, p 425, f 7

The type-host of this species is the Nutcracker (*Nucifraga caryocatactes*) Kellogg and Paine (1914) recorded it from the Kashmir Magpie (*Pica pica bactriana* Bonap) from the Punjab The present specimens were obtained from the Himalayan Nutcracker (*Nucifraga caryocatactes hemispila* Vigors) shot in Lyallpur, 12-ii-1928, and Kulu, 14-x-1939

Measurements (mm)

	Female (2)	Male (1)
Body	2 160–2 199 × 1 000	1 825 × 0 890
Head	0 560–0 666 × 0 680–0 693	0 626 × 0 666
Thorax	0 400 × 0 586–0 603	0 400 × 0 533
Abdomen	1 183–1 200 × 1 000	0 800 × 0 890

Piaget (1880) gave the measurements of female and male as 2 0–2 1 mm × 0 85 mm. and 1 7 mm × 0 75 mm respectively

21 Philopterus rotundatus (Piaget)

1880. *Docophorus rotundatus*, Piaget, *Les Pediculines*, p 47, pl 3, f 5

The type-host is the Carrion Crow (*Corvus corone*). Kellogg and Paine (1914) recorded it from the House Crow (*Corvus s splendens*) from Nepal The present specimens were obtained from the Common Indian House Crow (*Corvus s splendens* Vieill) shot in Lyallpur, 11-ii-1930

Measurements (mm.).

	Female (10)	Male (5)
Body	1 933–2 246 × 0 826–0 828	1 586–1 780 × 0 613–0 773
Head	0 640–0 660 × 0 532	0 586–0 653 × 0 559–0 573
Thorax	0 427–0 453 × 0 532–0 559	0 307–0 354 × 0 427–0 493
Abdomen	0 866–1 130 × 0 826–0 828	0 693–0 773 × 0 613–0 773

Piaget (1880) gave the measurements of female and male as 1 7–1 8 mm ×0 73 mm and 1 6 mm ×0 7 mm respectively

22 **Philopterus garruli** (Boisd & Lacord.)

1835 *Docophorus garruli*, Boisduval & Lacordaire, *Faun Ent., Paris*, p 120

This species is found quite commonly upon the Jay (*Garrulus glandarius*) Kellogg and Paine (1914) recorded it from the Yellow-billed Magpie (*Urocissa f flavirostris*, Blyth), the Indian Treepie (*Dendrocitta rufa rufa*, Lath), the Himalayan Treepie (*Dendrocitta sinensis himalayensis* Blyth), the Black-throated Jay (*Garrulus lanceolatus* Vigors) and the Large Spotted Nutcracker (*Nucifraga multipunctata* Gould), all from India

The present specimens are from the Indian Red-billed Blue Magpie (*Urocissa erythrorhyncha occipitalis*, Blyth) and the Bengal Treepie (*Dendrocitta r rufa*, Lath) both shot in Lyallpur, 16-viii-1928 and 25-viii-1928 respectively

Measurements (mm)

	Female (1)
Body	2 106 ×0 80
Head	0 600 ×0·573
Thorax	0 466 ×0·546
Abdomen	1 04 ×0 80

Piaget (1880) gave the measurements of male and female as 1 5 mm ×0 67 mm. and 1 9 mm ×0 8 mm respectively

23 **Philopterus corvi** (Linn)

1758 *Pediculus corvi*, Linnaeus, *Sys Nat* , II, p 612.

This species was recorded by Denny (1842) from the Common Rook (*Corvus frugilegus*) and the Hooded Crow (*Corvus cornix*) It has also been recorded by Kellogg and Paine (1914) from crows in India

The present specimens were obtained from the Punjab Raven (*Corvus corax laurencei*, Hume), the Eastern Rook (*Corvus frugilegus tschusii*, Hartert), the Common Indian House Crow (*Corvus splendens splendens* Vieill), all shot in Lyallpur 21-vi-1938, 21-ii-1929 and 11-ii-1930 respectively

Measurements (mm)

	Female (8)	Male (1)
Body	1 746–2 220 ×0 820–0 880	1 913 ×0 76
Head	0 610–0 680 ×0 660–0 773	0 660 ×0 66
Thorax	0 333–0 414 ×0 560–0 613	0 307 ×0 56
Abdomen	0 800–1 200 ×0 828–0 880	0 946 ×0 76

Piaget (1880) gave the measurements of female and male as 2 2 mm. ×0 94 mm. and 1·8–1 9 mm. ×0 94 mm. respectively.

24 Philopterus sturni (Schrank)

1776. *Pediculus sturni*, Schrank, *Beit zur Nat.*, p 118, f 11–14

This species was described from specimens obtained from the Starling (*Sturnus vulgaris* Linn) The specimens referred to were obtained from the Rose-coloured Starling (*Pastor roseus*, Linn), the Himalayan Starling (*Sturnus vulgaris humii* Brooks), the Black-Headed Mayna (*Temenuchus pagodarum*, Gmel), the Common Mayna (*Acridotheres t tristis*, Linn), and the Bank Mayna (*Acridotheres ginginianus*, Lath), all shot in Lyallpur.

Measurements (mm)

	Female (5)	Male (2)
Body	1 306–1 613 × 0 613–0 733	1 189 × 0 653
Head	0 530–0 600 × 0 533–0 560	0 506 × 0 533
Thorax	0 240–0 280 × 0 440–0 493	0 240 × 0 400
Abdomen	0 613–0 813 × 0 613–0 733	0 440 × 0 463

Piaget (1880) gave the measurements of the female and male as 1 5 mm × 0 72 mm and 1 2–1 3 mm × 0 54 mm respectively

IV DEGEERIELLIDAE

25 Psittaconirmus chandabani,[1] sp nov

Female (Text-fig 6*a*) yellowish white with distinct, brownish yellow pleural plates and indistinct tergal markings

Head longer than broad, clypeal front rounded, small hairs distributed as shown in figure, clypeal band narrow, antennal band broad, highly pigmented, clypeus with two small papillae as shown in figure, clypeal signature absent, internal bands travelling half way towards the clypeal signature, trabeculae distinct, small, triangular, antennae filiform, temporal lobes rounded, as broad as the base of preantennal region, eyes distinct, ocular fleck black with a seta, temporal margin narrowly banded Occipital margin concave, occipital signature distinct, large, pharyngeal sclerite and glands present

Prothorax rectangular, large, a long hair in the posterior margin and a small seta towards the anterior, posterior margin straight or slightly convex, bare, sternal plate narrow, precoxal margin pigmented showing through on the dorsum Meso- and metathorax with a distinct marginal suture, otherwise fused, pterothorax trapezoidal, projecting laterally a little, posterior angle outwardly rounded with two small hairs, posterior margin angulate on the abdomen with a distinct median denticle with four long hairs on the margin, marginal bands distinct, brown, intercoxal plates showing through on the dorsum Sternal plates distinct, broad, with a small seta Legs long, lipeuroid

Abdomen elongate, marginal bands brown, conspicuous on segments I–VII, chaetotaxy very scarce, confined submarginally only, median areas almost bare, ventrum with a well built genital plate arising from the base of segment VII with four thick setae and numerous delicate hairs, terminal segment notched

Male (Text-fig 6*b*) similar to female, but shorter, antennae appendiculate (Text-fig 6*c*), abdomen widening posteriorly to segment VI, segment VII a little narrower than VI, and segments VIII and IX narrowing more rapidly, segment VIII with 18–20 dorsal hairs arranged in a semicircle, segment IX broadly rounded

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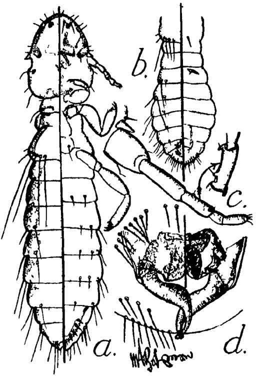
[1] 'Chandabani' in vernacular means the Indian Large Paroquet.

posteriorly with a median notch and profusely setacious, ventrum with a delicate spur on segment III Genitalia (Text-fig. 6*d*) characteristic with interlocking and well built parameres



TEXT FIG 6 *Psittaconirmus chandabani*, sp nov

a Dorsal and ventral aspects of female, *b* Dorsal and ventral aspects of thorax and abdomen of male, *c* Male antenna, *d* Male genital armature

Measurements (mm.)

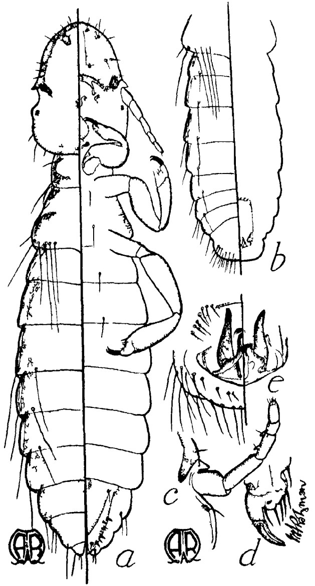
	Female (Holotype)	Female (3)	Male (2)
Body	1 918 × 0 466	1 705–1 825 × 0 386–0 426	1 412–1 425 × 0 346–0 373
Head	0 493 × 0 366	0 466–0 493 × 0 360–0 373	0 400–0 426 × 0 306–0 333
Prothorax	0 133 × 0 226	0 133 × 0 213–0 226	0 093–0 106 × 0 178–0 200
Pterothorax	0 226 × 0 350	0 200–0 216 × 0 333–0 360	0 226 × 0 293–0 306
Abdomen	1 066 × 0 466	0 893–1 026 × 0 386–0 426	0 680 × 0 346–0 373

Holotype (female) and *Allotype* (male) from Lyallpur, 18-iii-1931, *ex* the Indian Large Paroquet (*Psittacula eupatria nepalensis* Hodgs) mounted on slide No MI. 035, *Paratypes*: numerous males and females (same data as above) preserved in alcohol.

This species closely resembles *Lipeurus circumfasciatus* Piaget from *Platycercus melanurus*, but differs in numerous details given above.

26 Psittaconirmus lybartota,[1] sp nov

Female (Text fig 7*a*) yellowish white with distinct brownish yellow pleural bands and indistinct tergal markings very similar to *Psittaconirmus chandabani* sp nov (*vide supra*) but the following characters separate it



TEXT FIG 7 *Psittaconirmus lybartota* sp nov

a Dorsal and ventral aspects of female b Dorsal and ventral aspects of thorax and abdomen of male; c Male antenna d Hind tarsus e Male genital armature

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[1] Lybartota in vernacular means the Indian Rose ringed Paroquet

(1) Male (Text-fig 7b) with almost subparallel sides, (2) segment VIII with 6-7 hairs on each side of the circle, median area definitely bare, (3) male genitalia (Text-fig 7e), as seen in extending condition, is different, with short parameres which are not interlocking, (4) ventrum without any spur on segment III

Measurement (*mm*)

	Female (Holotype)	Female (3)	Male (1)
Body	1 645 × 0 333	1 600–1 652 × 0 306–0 36	1 293 × 0 280
Head	0 440 × 0 293	0 413–0 440 × 0 280–0 32	0 400 × 0 280
Prothorax	0 093 × 0 213	0 080–0 120 × 0 173–0 20	0 093 × 0 173
Pterothorax	0 226 × 0 306	0 200–0 226 × 0 293–0 32	0 200 × 0 293
Abdomen	0 886 × 0 333	0 880–0 893 × 0 306–0 36	0 600 × 0 280

Holotype (female) and *Allotype* (male) from Lyallpur, 6-x-1931, mounted together on slide No MI 036 *ex* the Indian Rose-ringed Paroquet (*Psittacula krameri manillensis*, Bechst) *Paratypes* numerous males and females preserved in alcohol (same data as above)

27. Quadraceps kekra,[1] sp nov

Female (Text-fig 8a) yellowish white with dark black marginal markings on head, thorax and abdomen, and chestnut-brown occipital signature and abdominal blotches

Head elongate, conical, clypeus truncate, front concave, feebly emarginate, a short hair at each anterior angle, two lateral ones, one in the middle and one at the clypeal suture, lateral band well defined, black, clypeal signature wanting, inner band absent, antennal band represented by two conspicuous black blotches, one just near the clypeal suture and another at the base of trabeculae, trabeculae short, conical and distinct, antennae short and slender, second segment longest, terminal segment pigmented, temporal margins weakly convex, with one long and two short hairs at the rounded posterior angle, temporal band black, continuous with the ocular blotch and bar Eyes weakly developed, each with a spine, hyaline cornea distorted Occipital margin concave, bare, occipital signature chestnut brown

Prothorax quadrangular with short hair on the posterior angle, lateral margin black, the black area running anteriorly in the head as far as the occipital signature, and posteriorly along the posterior margin as far as the middle line Meso- and metathorax completely fused into pterothorax Pterothorax pentagonal, posterior angles projecting, anterior lateral margin chestnut brown, intercoxal, curved sternal blotches, showing through, posterior angles with 3-4 short hairs, two long marginal hairs on the posterior Legs as in *Degeeriella felix* (Gb)

Abdomen nearly parallel sided for most of the length, segment I narrow, sides slightly concave, posterior margin angulate on segment II with a black spot at the large pleurite re-entrant head; sternum with a pointed half-moon shaped brownish blotch Segments II–VII with marginal bands thickened on the anterior ends and with black spots at the large pleurite re-entrant heads, dorsum with small median blotches and ventrum with transverse blotches, segment VIII with a blotch and a bar at the lateral margin, and a median blotch just near the posterior margin, segment IX bilobed posteriorly with a black blotch on each lobe. Chaetotaxy scarce Anal orifice in female very conspicuous

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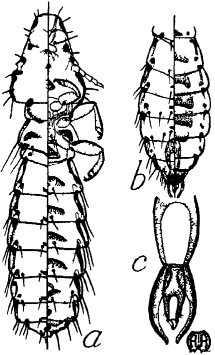
[1] 'Kekra' in vernacular means the Egyptian Gull billed Tern

Male (Text fig 8*b*) agrees with the female except in size and abdominal sternal blotches which are wider last abdominal segment broadly rounded with chestnut marginal band and with about 10 12 long marginal hairs and 4–4 submarginal hairs Genitalia (Text fig 8c) well developed short parameres sword shaped, turned inward with a small seta at the tip endomeral plate well developed with well chitinised lateral margin having two small setae at the postal end penis well developed and enclosed in a sheath



TEXT FIG 8 *Quadraceps kekra* sp nov
a Dorsal and ventral aspects of female *b* Dorsal and ventral aspects of abdomen of male, *c* Male genital armature

Measurements (*mm*)

	Female (Holotype)	Male
Body	1 816 × 0 394	1 440 × 0 366
Head	0 507 × 0 360	0 466 × 0 333
Prothorax	0 080 × 0 218	0 080 × 0 186
Pterothorax	0 173 × 0 280	0 133 × 0 226
Abdomen	1 056 × 0 394	0 761 × 0366

Holotype (female) and *Allotype* (male) from Lyallpur, 11 viii 1931, *ex* the Egyptian Gull billed Tern (*Gelochelidon n nilotica*, Gmel) mounted on slide Nos MI 102H and MI 102A respectively *Paratype* one female mounted on slide (same data as above)

These examples closely resemble *Degeeriella praestans* (Kellogg) and *Degeeriella felix* (Gb), but are distinctly of a smaller size, different type of marginal bands on the abdomen, transverse abdominal blotches and also a slight variation in chaetotaxy distinguish it from the two species

28. Quadraceps cursorius (Mjöberg).

1910 *Nirmus cursorius*, Mjöberg, *Ark Zool* , VI, p 141, pl 1, f 4

Mjöberg (1910) obtained it from the Cream-coloured Courser (*Cursorius gallicus* Gmelin = *cursor cursor*, Lath) in Europe The specimens referred to here were obtained from the type-host (*Cursorius c cursor*, Lath), shot in Lyallpur, 13-xii-1930 I transfer it to the genus *Quadraceps* because of the characters given by Clay and Meinertzhagen (1939)

Measurements (mm)

	Female (3)
Body	2 013–2 226 × 0 401–0 533
Head	0 560–0 640 × 0 401–0 426
Thorax	0 293–0 333 × 0 400–0 401
Abdomen	1 160–1 293 × 0 401–0 533

Mjoberg (1910) gave the measurement of female as 2 3 mm × 0 6125 mm

29. Quadraceps hiaticulae (Müller)

1780. *Pediculus hiaticulae*, Müller, in Fabricius, *Faun Groen* , p 220

This species is recorded from various birds belonging to the family Charadriidae The following of its bird-hosts are also common within Indian limits The Ringed Plover (*Charadrius hiaticola* Linn), the Little Plover (*Charadrius dubius* Scop), the Avocet (*Recurvirostra avocetta avocetta* Linn) and the Lapwing (*Vanellus vanellus* Linn) My specimens were obtained from the Indian Red-wattled Lapwing (*Lobivanellus i indicus*, Bodd), shot in Lyallpur, 15-iii-1929

Measurements (mm)

	Female (1)	Male (1)
Body	1 772 × 0 360	1 319 × 0 346
Head	0 466 × 0 333	0 440 × 0 320
Thorax	0 306 × 0 280	0 293 × 0 240
Abdomen	1 000 × 0 360	0 586 × 0 346

Piaget (1880) gave the measurements of female and male as 1 5 mm × 0 4 mm. and 1 3–1 4 mm. respectively

30. Quadraceps signata (Piaget).

1880 *Nirmus signatus*, Piaget, *Les Pediculines*, p 186, pl 15, f 8

This species was first described from the Avocet (*Recurvirostra a avocetta* Linn.) Waterston (1914) recorded it on the same host in South Africa The present specimens were taken off the Black-winged Stilt (*Himantopus h. himantopus*, Linn.)

Measurements (mm)

	Female (1)	Male (1)
Body	2 211 × 0 451	1 857 × 0 451
Head	0 478 × 0 309	0 478 × 0 338
Thorax	0 338 × 0 309	0 295 × 0 295
Abdomen	1 395 × 0 451	1 084 × 0 451

Piaget (1880) gave the measurements of female and male as 2 0–2 2 mm × 0 52 mm. and 1 6–1 7 mm × 0 46 mm respectively

31. Quadraceps holophaea (Nitzsch).

1838 *Nirmus holophaeus*, Nitzsch, in Burmeister, *Handbuch der Ent*, II, p 427

It has been recorded from most parts of the world on the type-host, the Ruff and Reeve (*Philomachus pugnax*, Linn) My specimens were taken off the type-host, shot in Lyallpur, 13-viii-1929

Measurements (mm)

	Female (1)	Male (1)
Body	1 562 × 0 295	1 243 × 0 352
Head	0 394 × 0 225	0 380 × 0 267
Thorax	0 239 × 0 211	0 218 × 0 239
Abdomen	0 929 × 0 295	0 645 × 0 352

Piaget (1880) gave the measurements of female and male as 1 7–1 8 mm × 0 34 mm and 1 6–1 7 mm × 0 33 mm respectively

32. Quadraceps furva (Nitzsch).

1838 *Nirmus furvus*, Nitzsch, in Burmeister, *Handbuch der Ent*, II, p 427

Recorded by Piaget (1880) from the Greenshank (*Glottis nebularia*, Gum), the Spotted or Dusky Redshank (*Tringa erythropus*, Pallas), the Common Sand-piper (*Tringa hypoleucus* Linn), the Green Plover or Lapwing (*Vanellus vanellus* Linn) and the Chinese Little Ringed Plover (*Charadrius d dubius* Scop) It has also been recorded by European authors on the Turnstone (*Arenaria i interpres*, Linn), the Kentish Plover (*Leucopolius a alexandrinus*, Linn), the Large Sand Plover (*Cirrepidesmus leschenaultii*, Lesson), the Black-winged Stilt (*Himantopus h himantopus*, Linn.), the Bar-tailed Godwit (*Limosa lapponica lapponica* Linn), the Green Sand-piper (*Tringa ochrophus* Linn), the Curlew-Stint or Pigmy Sand-piper (*Erolia testacea*, Pallas), and the Dunlin (*Erolia a alpina*, Linn) All these hosts occur within Indian limits

My specimens were obtained from the Green Sand-piper (*Tringa ochrophus* Linn), 24-iv-1933, and the Black-winged Stilt (*H h himantopus*, Linn) 25-iv-1933, both shot in Lyallpur

Measurements (mm.)

	Female (1)	Male (1)
Body	1 689 × 0 295	1 210 × 0 253
Head	0 422 × 0 239	0 239 × 0 197
Thorax	0 239 × 0 218	0 197 × 0 182
Abdomen	1 028 × 0 295	0 774 × 0 253

Piaget (1880) gave the measurements of female and male as 1 5–1 6 mm × 0 37 mm and 1 2–1 3 mm × 0 29 mm respectively

33. **Lunaceps actophila** (Kell & Chap)

1899 *Nirmus actophilus*, Kellogg & Chapman, *Proc Calif Acad Sci* (2) VI, p 78, pl 6, f 4

This species was described from the Sanderling (*Calidris arenaria*) shot in California Bedford (1920) recorded it from the Curlew Sand-piper (*Erolia testacea*, Pallas) and the Little Stint (*Erolia minuta minuta*, Leist) from South Africa Both these birds are also distributed within Indian limits My specimens were obtained from the Little Stint (*Erolia m minuta*, Leist), shot in Lyallpur, 25-iv-1933

Measurements (mm)

	Female (1)	Male (1)
Body	1 464 × 0 239	1 295 × 0 211
Head	0 338 × 0 225	0 352 × 0 182
Thorax	0 253 × 0 169	0 225 × 0 169
Abdomen	0 873 × 0 239	0 718 × 0 211

Kellogg and Chapman (1899) gave the measurements of female as 1 59 mm × 0 4 mm

34. **Carduiceps cingulatus zonarius** (Nitzsch).

1838 *Nirmus cingulatus zonaria*, Nitzsch, in Burmeister, *Handbuch der Ent* , II, p 438

This species was described by Denny (1842) from specimens taken off the Black-tailed Godwit (*Limosa l limosa*, Linn) Piaget (1880) recorded it from the Sanderling (*Calidris arenaria*) and the Little Stint (*Erolia m minuta*, Leist) The Godwit and the Little Stint are migrants to North-West India The specimens referred to below were obtained from the Little Stint (*Erolia m minuta*, Leist), shot in Lyallpur, 5-iv-1933

Measurements (mm)

	Female (1)
Body	1 280 × 0 338
Head	0 309 × 0 253
Thorax	0 197 × 0 182
Abdomen	0 774 × 0 338

Piaget (1880) gave the measurements of female and male as 1 3 mm ×0 3 mm and 1·1–1 2 mm, respectively while *Nirmus cingulatus* Nitzsch is given as 1 5 mm. ×0 42 mm and 1 3–1 4 mm ×0 38 mm respectively

35. Cuculicola latirostris (Burmeister)

1838 *Nirmus latirostris*, Burmeister, *Handbuch der Ent*, II, p 429

It was described from specimens taken off the European Cuckoo (*Cuculus canorus* Linn), which migrates in winter to North-West India My specimens were obtained from the Common Hawk Cuckoo (*Hierococcyx varius* Vahl), 9-iv-1928, and the Indian Pied Crested Cuckoo (*Clamator j jacobinus*, Bodd), 22-viii-1929, both shot in Lyallpur

Measurements (mm)

	Female (4)	Male (4)
Body	1 620–1 758 × 0 360–0 440	1 599–1 613 × 0 440–0 466
Head	0 477–0 506 × 0 333–0 346	0 480–0 493 × 0 346
Thorax	0 200–0 240 × 0 305–0 320	0 186–0 200 × 0 333
Abdomen	0 946–1 066 × 0 360–0 440	0 920–0 933 × 0 460–0 480

Piaget (1880) gave the measurements of female and male as 1 5–1 6 mm × 0 43 mm and 1 4 mm ×0 39 mm respectively

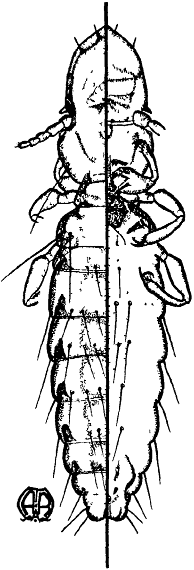
36. Syrrhaptoecus emahusaini, sp nov

Female (Text-fig 9) elongate, about five times as long as wide, smoky brown with fuscous black bands on the margin of head, thorax and abdomen and fuscous transverse abdominal blotches, chaetotaxy scarce

Head stout, about two times as long as wide, forehead broader at posterior aspect, with an angulation on meson, lateral margins straight bearing two short hairs, antennal bands fuscous black, continuous with yellowish brown clypeal bands, trabeculae movable, short, triangular, slightly longer than broad, antennae simple, eyes hyaline, prominent, cornea rounded, ocular seta wanting, ocular band well formed, temples convergent, lateral margins almost straight or flatly rounded with a long median hair, marginal band narrow, not very distinct but near the eyes, posterior margin flatly concave, occipital band dark yellow to brown

Thorax about half as long as head Prothorax narrow, short, trapezoidal, anterior portion deeply inserted under the occipital margin, lateral margins projecting, straight, bare, lateral bands distinct, posterior margin straight Mesothorax narrow, collar-like, with distinct lateral bands, completely fused posteriorly with metathorax Pterothorax well developed, broad, parallel sided, lateral bands broad, latero-posterior angles rectangular with two postulated long hairs, posterior margin straight, bare Legs concolorous with the thorax, marginal markings on femora and tibia narrow Sternal plates well developed, as shown in figure

Abdomen elongated, broadest in the III segment, gradually tapering towards the last segment, segment I shortest, segment II longest and segments III–V long, nearly equal in length, segments VI-VII short, segments VIII and IX fused, latero-posterior angles of II–VIII segments with one hair, posterior margins straight, terminal segment with median notch, segments I–VI bearing transverse row of 2-3 long hairs, lateral bands well developed, each pleurite articulating with pleurite in front of it by means of an inner, capitate condyle, transverse bands confined to the middle Sternal plates I–V with 1-2 short hairs, sternite VI with distinct genital blotch and longitudinal bands.



TEXT FIG 9 *Syrrhaptoecus emahusaini* sp

Dorsal and ventral aspects of female.

Holotype (female) on slide No. MI 129 from Lyallpur, *ex* the Indian Common Sandgrouse (*Pterocles exustus erlangeri* Neum), shot on 28-viii-1931. ***Paratypes:*** 3 females (same data as above).

Measurements (*mm*)

	Female (Holotype)	Female (3)
Head index (breadth: length)	0 594	0 650–0 658
Body	1 642×0·381	1 598–1 680×0 373 0 426
Head	0 516×0 301	0 506–0 533×0 333–0 346
Prothorax	0 079×0 198 }	0 201–0 226×0 293–0 333
Pterothorax	0 135×0 254 }	
Abdomen	0 912×0 381	0 866–0 946×0 373–0 426

This species closely resembles *Syrrhaptoecus digonus* Waterston, but sufficient difference, however, exists in the size of the body, general chaetotaxy and other important details

37. Syrrhaptoecus falcatus Waterston.

1928 *Syrrhaptoecus falcatus*, Waterston, *Proc Zool Soc*, *London*, p 345, t f 2*a*–10*b*

Waterston recorded it from *Pterocles senegalensis*, Licht var *orientalis* [1]

Two females and one male were obtained from the Indian Common Sandgrouse (*Pterocles exustus erlangeri* Neum), shot in Lyallpur, 28-viii-1931 They differ from the type in some minor details, viz , in being smaller and in having evenly parabolic forehead in the male and slightly angulate in the female

Measurements (*mm*)

	Female (2)	Male (1)
Head index	0 77–0 85	0 73
Body	2 065–2 105×0 466	1 453×0 333
Head	0 466–0 493×0 360–0 373	0 400×0 293
Thorax	0 333–0 346×0 333	0 240×0 240
Abdomen	1 266×0 466	0 813×0 333

Waterston (1928) gave the measurements of female and male as 2 14–2 37 mm ×0 51–0 58 mm and 1 43–1 69 mm ×0 4–0 45 mm respectively, while head index as 0 76–0 77 and 0 76–0 80 respectively

38. Upupicola melanophrys (Nitzsch).

1866 *Nirmus melanophrys*, Nitzsch, in Giebel, *Zeit f ges Nat*, XXVIII, p 369

It was originally described from the specimens taken from the European Hoopoe (*Upupa epops epops* Linn) The present specimens were obtained from the Indian Hoopoe (*Upupa epops orientalis* Stuart Baker), shot in Lyallpur, 19-ii-1928 It has also been recorded from the African Hoopoe (*Upupa epops africana*), by Waterston (1914) and Bedford (1919)

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[1] *Pterocles senegalensis* Licht (name preoccupied) = *P exustus* Temm (F B I, V, 271) There appears to be some discrepancy in Waterston's record (1928), as there seems to be no such Sandgrouse as *P senegalensis orientalis* Hass from within Indian region By *P s orientalis* Hass., the author probably means *P. exustus orientalis* Hartert (Stuart Baker, 1928, *Faun Brit. Ind.*, V, p. 271)

Measurements (mm)

	Female (4)	Male (2)
Body	1 811–1 866 × 0 506–0 530	1 719–1 836 × 0 466–0 488
Head	0 546–0 573 × 0 426–0 440	0 493–0 530 × 0 400
Thorax	0 293–0 333 × 0 360–0 400	0 293–0 333 × 0 360–0 400
Abdomen	0 930–1 000 × 0 506–0 530	0 933–0 973 × 0 466–0 480

Piaget (1880) gave the measurements of female and male as 1 7 mm × 0 48 mm and 1 6 mm × 0 46 mm respectively

39. **Kelerinirmus fusca** (Nitzsch).

1842 *Nirmus fuscus*, Nitzsch, in Denny, *Anop Brit*, p 118, pl 9, f 8

This is a long known species, recorded under different names, from all over the world, from numerous diurnal birds of prey (Accipitres) Most of the hosts are known to occur within Indian limits, viz, the Lesser Kestril (*Cerchneis naumanni*, Fleisch), the Booted Eagle (*Hieraetus pennatus*, Gmel), the Black Kite (*Milvus migrans migrans*, Bodd), the Black-winged Kite (*Elanus coeruleus*, Desf), the Marsh-Harrier (*Circus a aeruginosus*, Linn), the Goshawk (*Astur g gentilis*, Linn).

The present specimens were obtained from the Indian Red-headed Merlin (*Falco c chiquera*, Dauden), 4 i-1929 and 16-iii-1928, the White-eyed Buzzard Eagle (*Butastur teesa*, Frankl), 26-iii-1928, 4-v-1928 and 14-vi-1929, and the Common Pariah Kite (*Milvus migrans govinda* Sykes), 4-i-1929, 15-iii-1930 and 15-iv-1930, all from Lyallpur

Measurements (mm)

	Female (3)	Male (2)
Body	1 892–2 100 × 0 460–0 573	1 705–1 953 × 0 400–0 490
Head	0 546–0 600 × 0 410–0 453	0 493–0 530 × 0 360–0 400
Thorax	0 266–0 333 × 0 400–0 480	0 266–0 333 × 0 360–0 426
Abdomen	0 950–1 200 × 0 460–0 573	0 946–1 000 × 0 401–0 493

Piaget (1880) gave the measurements of female and male as 1 7–1 8 mm × 0 48 mm and 1 5–1 6 mm × 0 43 mm respectively, while Kellogg's (1896) female was 2 4 mm × 0 62 mm

40. **Kelerinirmus rufa** (Nitzsch).

1838 *Nirmus rufus*, Nitzsch, in Burmeister, *Handbuch der Ent*, II, p 430

This species has been recorded from various Accipitres, many of which also occur within Indian limits, viz, the Eastern Peregrine Falcon (*Falco peregrinus calidus*, Lath), the Hobby (*F s subbuteo*, Linn), the European Kestrel (*Cerchneis t. tinnunculus*, Linn), the Indian Crested Hawk Eagle (*Spizaetus* (= *Limnaetops*) *c cirrhatus*, Gmel), the Montagu's Harrier (*Circus pygargus*, Linn), the Pale Harrier (*Circus macrourus*, Gmel), the Hen Harrier (*Circus cyaneus cyaneus*, Linn), the Marsh Harrier (*Circus a aeruginosus*, Linn.), the Desert Buzzard (*Buteo vulpinus*, Gloger), the Sparrow Hawk (*Accipiter nisus*, Linn), etc.

The specimens referred to below were obtained from the Lagger Falcon (*Falco jugger* Gray), 29-iii-1928, 5-i-1929 and 11-iv-1929, and the Himalayan Kestrel (*Cerchneis tinnunculus interstinctus*, McClell), 19-iv-1936, both shot in Lyallpur

Measurements (*mm*)

	Female (2)	Male (4)
Body	1 876–2 079 × 0 440–0 560	1 599–1 810 × 0 440–0 460
Head	0 530–0 573 × 0 400–0 440	0 453–0 506 × 0 306–0 370
Thorax	0 266–0 306 × 0 373–0 440	0 266–0 280 × 0 320
Abdomen	1 080–1 200 × 0 440–0 560	0 880–1 026 × 0 440–0 460

Piaget (1880) gave the measurements of female and male as 1 9 mm × 0 53 mm and 1 6 mm × 0 46 mm respectively

Painjunirmus, gen nov.

This genus is distinguished from the other Degeeriellidae by the shape of the head and abdomen, narrow marginal bands, absence of tergal plates, and male genitalia Head conical, clypeal margins bordered with yellowish brown to black bands, clypeal signature entirely absent, internal band absent, trabeculae small, narrow, antennae filiform in both sexes, temporal margins rounded, slightly extended beyond the lateral clypeal margin, occipital band absent, occipital signature triangular but not sclerotic Cephalic band conspicuous Prothorax quadrangular, pterothorax with strongly diverging sides and rounded posterior margin Abdomen elongated, with sub-parallel sides, not tapering posteriorly until after segments VII-VIII, last segment rounded posteriorly in male and bilobed in female, pleurites distinct, with re-entrant heads, tergal plates not distinct, chaetotaxy very scarce, female genital plate conspicuous on segment VII, with fine row of posterior hairs Male genitalia characteristic as shown in figure

The genus is erected to include species of the type usually referred to *interrupto fasciati* Piaget. Specimens of *Painjunirmus* have been examined from various Passerine genera

Genotype —*Painjunirmus pengya* sp nov (*vide infra*) *ex* the Bengal Jungle Babbler (*Turdoides terricolor terricolor*, Hodgs) [1]

41 **Painjunirmus pengya**,[2] sp nov

Female (Text-fig 10*a*) yellowish white with distinct, narrow, deep yellow, lateral bands and marginal markings, median abdominal blotches well pigmented, body otherwise poorly sclerotised

Head conical, slightly longer than broad, clypeal front narrow, parabolic, angularly concave, one short hair at the anterior angle, one such on the submarginal region, four hairs along the latero-frontal margin, clypeal band deep yellow-brown, fused with the antennal band which turns angularly inwards at antennal fossae, clypeal signature indistinct, internal bands not well marked trabeculae small, extending in length to the middle of the first antennal segment, antennae small, segment II longest, I smaller than II but robust, III-V smallest and subequal;

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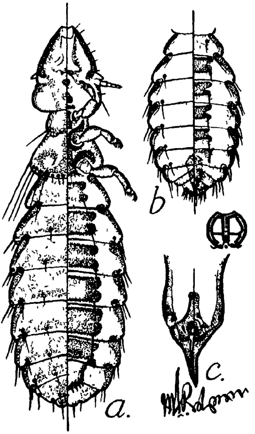
[1] It has been pointed out that the Punjab form appears to be *Turdoides terricolor sindianus* Ticehurst I retain the name *Turoides t terricolor* on the authority of Bombay Natural History Society who identified this specimen (*Jour Bombay Nat Hist Soc.*, 1937, XXXIX, p 2)

[2] 'Pengya-maina' in vernacular means the Bengal Jungle Babbler.

temporal margin about half the frontal region in length, well rounded, with a long hair and a seta towards the posterior angle, temporal band narrow but distinct, occipital margin slightly convex, bare, occipital signature triangular, not well marked Eyes protruding, each with a sub-basal seta, ocular blotch not distinct, ocular band narrow, conspicuous



Text fig 10 *Painjunirmus pengya*, sp nov

a. Dorsal and ventral aspects of female, *b* Dorsal and ventral aspects of male; *c* Male genital armature

Prothorax narrow, quadrangular with slightly rounded lateral margin; lateral band yellowish brown, distinct, a single hair in posterior lateral angle, posterior margin convex marginally and concave in the middle, bare, inter-coxal plates well developed, sternal plate elongate, hexagonal, bare Meso- and metathoraces completely fused into pterothorax, trapezoidal, projecting laterally, posterior angles rounded with five long hairs, posterior margin convex with 3 submarginal hairs, marginal markings narrow, sternal plate bottle-shaped with two hairs in the centre, intercoxal plates deeply pigmented, sclerotic Legs short, concolorous with the body, marginal markings slightly darker, annular markings at the distal end of femoral and tibial segments

Abdomen elongate, slender, sides sub-parallel, tapering posteriorly to segments VIII-IX, segment I without and others with a single weak hair in posterior angles, and 1 2 weak submarginal hairs posterior margin with one submedian hair segment VIII with one latero marginal hair in the middle and one in the posterior angle Posterior margin with 2 such hairs segment IX broadly rounded with angular emargination one fine seta set in each posterior angle segments I VII with distinct narrow lateral marginal band Sternum with pale broad rectangular transverse blotches on segments II–VIII segment IX uncoloured genital plate distinct on segment VIII with 3 4 peg like spines on each half

Male (Text fig 10*b*) similar to female but shorter abdominal segment VIII shorter and concave segment IX protruding rounded with fine posterior hairs genital armature (Text fig 10*c*) well developed short extending as far as segment VI parameres well developed narrow and run more or less parallel to each other median endomeral plate well developed with concave posterior margin

Measurements (*mm*)

	Female (Holotype)	Female (7)	Male (5)
Body	1 848×0 572	1 720 1 883×0 533–0 610	1 431 1 617×0 427–0 520
Head	0 451×0 36	0 420–0 466×0 347 0 373	0 400–0 444×0 293–0 373
Prothorax	0 121×0 231	0 120–0 126×0 231 0 242	0 120–0 126×0 220–0 40
Pterothorax	0 176×0 301	0 171–0 184×0 280–0 347	0 146–0 181×0 293–0 307
Abdomen	1 100×0 572	1 000 1 107×0 533–0 610	0 780–0 866×0 427–0 520

Holotype (female) and *Allotype* (male) from Lyallpur 16 iii 1933 *ex* the Bengal Jungle Babbler (*Turdoides t terricolor* Hodgs)[1] mounted together on slide No MI 057 *Paratypes* numerous females and males *ex* type host and the Common Babbler (*Argya c caudata* Dumont) shot in Lyallpur

This small species of Piaget s (1880) group *interrupto fasciati* is allied to *Degeeriella vulgatus* (Kellogg) (*New Mallophaga* II 1896 p 496) and other similar forms

42 **Painjunirmus vulgata** (Kellogg)

1896 *Nirmus vulgatus* Kellogg *Proc Calif Aca Sci* (2) VI p 496 pl 67 f 5

Kellogg (1896) and Kellogg and Chapman (1899) recorded it from a number of Passeriformes in U S A Waterston (1914) recorded it from the Cape Sparrow (*Passer melanurus*) and the Red headed Finch (*Amadina erythrocephala*) from South Africa The present specimens referred to were obtained from the White throated Munia (*Uroloncha malabarica* Linn) shot in Lyallpur 11 v 1928 and the Brown backed Indian Robin (*Saxicoloides fulicata cambaiensis* Lath) shot in Lyallpur 5 ix 1939

Measurements (*mm*)

	Female (2)
Body	1 532–1 629×0 413–0 427
Head	0 444–0 466×0 266–0 280
Thorax	0 239–0 266×0 293–0 333
Abdomen	0 800–0 946×0 413–0 427

Kellogg (1896) gave its measurement as 1 62 mm ×0 41 mm

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[1] See foot note 1 on page 33

43. **Painjunirmus iliaci** (Denny)

1842. *Nirmus iliaci*, Denny, *Anop Brit*, p 130, pl 9, f 4

Denny (1842) described this species from the Rose-coloured Starling (*Pastor roseus*, Linn) from England, and also from the Red Wing (*Turdus iliacus*) Numerous specimens of the Mallophaga referred to this species were obtained from the Rosy Pastor (*Pastor roseus*, Linn), 1932–1936, the Blackheaded Mayna (*Temenuchus pagodarum*, Gmel , 11-v-1928, the Common Mayna (*Acridotheres tristis tristis*, Linn), 9-iv-1931, and the Bank Mayna (*Acridotheres ginginianus*, Lath) 29-iii-1930, all shot in Lyallpur

Measurements (*mm*)

Female	Rosy Pastor	Black headed Mayna	Common Mayna
Body	1 613–1 640 × 0 386–0 400	1 613 × 0 360	2 346 × 0 466
Head	0 414 × 0 280	0 414 × 0 266	0 427 × 0 320
Thorax	0 239–0 266 × 0 266–0 288	0 250 × 0 266	0 253 × 0 320
Abdomen	0 960 × 0 386–0 400	0 946 × 0 360	1 066 × 0 466

Male	Rosy Pastor	Bank Mayna	Common Mayna
Body	1 545 × 0 400	1 479 × 0 347	1 439 × 0 400
Head	0 400 × 0 280	0 400 × 0 266	0 373 × 0 280
Thorax	0 239 × 0 293	0 213 × 0 266	0 253 × 0 266
Abdomen	0 906 × 0 400	0 866 × 0 347	0 813 × 0 400

Denny (1842) gave its length as ¾‴, i e 1 905 mm

44. **Painjunirmus cyclothorax** (Nitzsch).

1838 *Nirmus cyclothorax*, Nitzsch, in Burmeister, *Handbuch der Ent.*, II, p 429

This species was first described from the Tree Sparrow (*Passer montanus*, Linn) from Europe and the House Sparrow (*Passer domesticus* Linn) and the Brambling (*Fringilla montifringilla*, Linn) All the three birds also occur within Indian limits My specimens were obtained from the Indian Yellow-throated Sparrow (*Gymnoris x xanthocollis*, Burt), 13-vi-1933, the Indian House sparrow (*Passer domesticus indicus* Jard & Selby), 11-iv-1931, and the Indian Pipit (*Anthus richardi rufulus* Vieill), 20-iii-1928, all shot in Lyallpur

Measurements (*mm*)

	Female (3)	Male (1)
Body	1 500–1 653 × 0 307–0 400	1 354 × 0 330
Head	0 360–0 413 × 0 253–0 280	0 354 × 0 247
Thorax	0 247–0 257 × 0 253–0 307	0 200 × 0 253
Abdomen	0 893–1 000 × 0 307–0 400	0 800 × 0 333

Denny (1842) gave the length as ¾''', i e 1 905 mm , while Piaget (1880) gave it as 4/7''', i e 1 45 mm

45. Bruelia (= Degeeriella) varia (Nitzsch).

1838 *Nirmus varius*, Nitzsch, in Burmeister, *Handbuch der Ent* , II, p 430

It is a familiar parasite of crows and has been reported from all over the world Kellogg and Paine (1914) recorded it from the Raven (*Corvus corax* Linn) from Yarkand and Gilgit, the Eastern Rook (*C frugilegus* Linn) from Gilgit and Herat, the Jackdaw (*C monedula* Linn) from Yarkand and Gilgit, and the Magpie (*Pica rustica* Blanf) from Gilgit and Ladak

My specimens were obtained from the Punjab Raven (*Corvus corax laurencei* Hume), the Eastern Rook (*Corvus frugilegus tschusii* Hartert), 21 ii-1929 and the Common Indian House Crow (*Corvus s splendens* Vieill), 1928 1929, all from Lyallpur

Measurements (*mm*)

	Female (3)	Male (3)
Body	1 692–1 933 × 0 560–0 666	1 453–1 652 × 0 506–0 573
Head	0 453–0 560 × 0 440–0 493	0 440–0 480 × 0 401–0 460
Thorax	0 306–0 360 × 0 466–0 533	0 280–0 333 × 0 440–0 460
Abdomen	0 920–1 013 × 0 560–0 666	0 693–0 893 × 0 506–0 573

Piaget (1880) gave the measurements of female and male as 1 5 mm × 0 54 mm and 1 3 mm × 0 52 mm respectively

46 Bruelia (= Degeeriella) munda (Nitzsch).

1866 *Nirmus mundus*, Nitzsch, in Giebel, *Ziet f ges Nat* , XXVIII, p 366

This species was described from the European Golden Oriole (*Oriolus o oriolus* Linn) which is a migrant into North-Western India in winter Several immature specimens of this species were obtained from the Indian Golden Oriole (*Oriolus o kundoo* Sykes), shot in Lyallpur, 5-vii-1928, and the Indian Bush Chat (*Saxicola torquata indica* Blyth), shot in Lyallpur, 5-viii-1939 Piaget's specimens were ½''', i e 1 27 mm long

47 Bruelia (= Degeeriella) marginalis (Nitzsch)

1938 *Nirmus marginalis*, Nitzsch, in Burmeister, *Handbuch der Ent* , II, p 131, f 37

It was described *ex* the Field Flare (*Arceuthornis pilaris* Linn) from Europe Kellogg and Paine (1914) recorded it from *Dendrocitta r rufa*, Lath , *Dendrocitta sinensis himalayensis* Blyth , and *Urocissa malanocephala occipitalis*, Blyth , all from India.

The present specimens were obtained from the Indian Red-billed Blue Magpie (*Urocissa erythrorhyncha occipitalis*, Blyth), shot in Lyallpur, 16-ix-1928, the Yellow-billed Magpie (*Urocissa f flavirostris*, Blyth), shot in Kulu, 5-iv-1939, and the Simla Streaked Laughing Thrush (*Trochalopteron lineatum griseescentior*, Hart), shot in Kulu, 14-iv-1934 and 6-x-1939.

Measurements (*mm*)

	Female (1)	Male (1)
Body	1 548 × 0 436	1 126 × 0 422
Head	0 422 × 0 309	0 380 × 0 338
Thorax	0 239 × 0 380	0 211 × 0 309
Abdomen	0 887 × 0 436	0 535 × 0 422

Piaget (1880) gave the measurements of female and male as 1 3–1 35 mm × 0 48 mm and 1 0–1 1 mm × 0 44 mm respectively

48 **Bruelia (= Degeeriella) myiophoneae** (Clay)

1935 *Degeeriella myiophoneae*, Clay, *Proc Zool Soc*, London, p 911

This species was described from the specimens taken off the Himalayan Whistling Thrush (*Myophonus coeruleus temminckii* Vigors), shot in Kashmir I obtained only one mutilated specimen from the type-host in Kulu, 10-vi-1939

49 **Bruelia (= Degeeriella)**, sp

Several immature specimens were collected from the Pied Chat (*Oenanthe picata* Blyth), shot in Lyallpur, 9-xii-1930

Traihoriella, gen. nov

This genus is proposed to accommodate the species described below from *Megalaima virens marshallorum* Swinh The species may be distinguished from other *Degeeriella* by the shape of its head, pterothorax, pleural plates and male genitalia The description of the type species is given below

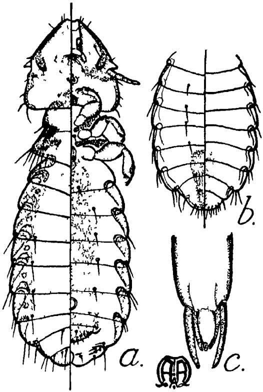
Genotype —*Traihoriella punjabensis* sp nov, *vide infra, ex* the Himalayan Great Barbet (*Megalaima virens marshallorum* Swinh)

50 **Traihoriella punjabensis**, sp nov

Female (Text-fig 11*a*) yellowish white with pale yellow to golden yellow body markings

Head more or less equilatero-triangular in shape, clypeal front shallow, concave, uncoloured, two inconspicuous hairs on the anterior angle, one a short distance behind, clypeal band deep yellow, slightly brown anteriorly, clypeal suture entirely absent, clypeal signature indistinct or absent, antennal band darker, short, not running beyond antennal fossa, trabeculae short, but well developed, antenna short, concolorous with the body, temporal lobes flatly rounded, not swollen, posterior angles with two long hairs, a short hair between the eye and the posterior angle and one such hair in the base of the eye, temporal bands narrow, not very conspicuous, eyes small, not conspicuous, occipital margin almost straight, bare, occipital band absent, occipital signature shield-shaped, yellowish, internal band well formed

Prothorax rectangular with distinct pale yellow lateral border, re-entrant head showing through the occipital margin, sternum narrow, intercoxal plate well-formed, highly pigmented Meso- and metathorax fused to form pterothorax, pterothorax strongly projecting laterally, lateral borders distinct with squat re-entrant heads, latero-posterior angle rounded with two long hairs; posterior margin angulated with 5 long hairs along the projecting latero-posterior margin, intercoxal blotch well pigmented, sternal plates narrow Legs short with distinct outer bands.



TEXT FIG 11 *Trathoriella punjabensis*, sp nov
a Dorsal and ventral aspects of female, *b* Dorsal and ventral aspects of abdomen of male, *c* Male genital armature

Abdomen narrow anteriorly, wider posteriorly to segment VI, tergal plates not distinct, pleural plates absent, marginal bands distinct with re entrant heads conspicuously showing through in the preceding segment, chaetotaxy very scarce Venter with distinctly coloured median area, genital plate distinct on the VII segment Last segment bilobed

Male (Text-fig 11*b*) mutilated, similar to female, shorter, the abdomen oval, last abdominal segment entire, rounded with small marginal hairs Genitalia as shown in text-fig 11*c*

Measurements (*mm*)

	Female (Holotype)	Female (2)
Body	1 839 × 0 613	1 760–1 786 × 0 526
Head	0 480 × 0 466	0 466 × 0 453–0 466
Prothorax	0 133 × 0 293	0·093–0 120 × 0 253–0 266
Pterothorax	0 186 × 0 453	0 200–0 201 × 0 400–0 426
Abdomen	1 046 × 0·613	1 000 × 0·526

Holotype (female) mounted on slide No. MI 082 *Allotype* (male) preserved in alcohol *Paratypes* 2 females on slide and 3 females in spirit All from Kulu, 16-ix-1928, *ex* the Himalayan Great Barbet (*Megalaima virens marshallorum* Swinh)

V LIPEURIDAE

51 Cuclotogaster (= Gallipeurus) heterographus (Nitzsch)

1866 *Lipeurus heterographus*, Nitzsch, in Giebel, *Zeit f ges* , *Nat* , XXVIII, p 381

This is a long known species of lice infesting domestic fowl (*Gallus g domesticus* Linn) all over the world My specimens were obtained from the Common Domestic Fowl (*Gallus g domesticus* Linn), the Northern Chukor (*Alectoris graeca pallescens*, Hume), shot in Hoshiarpur, 14-vii-1928, and Kulu, 9-x-1939, and the Common Pariah Kite (*Milvus migrans govinda* Sykes) shot in Lyallpur, 21-v-1932 On the latter bird the parasite is probably a straggler and appears to have been transferred from a Gallinaceous bird, probably chicken, on which it preyed prior to having been shot

Measurements (mm)

	Female (15)	Male (5)
Body	1 958–2 146 × 0 666–0 693	1 892–2 093 × 0 506–0 560
Head	0 600–0 626 × 0 400–0 518	0 520–0 600 × 0 493
Thorax	0 266–0 333 × 0 400–0 493	0 306–0 400 × 0 440–0 490
Abdomen	1 066–1 200 × 0 666–0 693	1 066–1·093 × 0 506–0 560

Piaget (1880) gave the measurements of female and male as 1 85 mm × 0 68 mm and 1 7–1 8 mm × 0 5 mm respectively

52 Lipeurus pavo Clay.

1938 *Lipeurus pavo*, Clay, *Proc Zool Soc London* (B), CVIII, p 125

Clay's (1938) specimens were taken off the Common Pea Fowl (*Pavo cristatus* Linn) from Nepal and Yorkshire My specimens were obtained from the type-host (*Pavo cristatus* Linn), shot in Hoshiarpur and Atari (near Lahore)

Measurements (mm)

	Female (3)	Male (3)
Body	2 733–2 745 × 0 422–0 451	2 381–2 409 × 0 338–0 366
Head	0 634–0 704 × 0 352–0·408	0 563–0 606 × 0 296–0 309
Thorax	0 408–0 422 × 0 324–0 381	0 324–0 381 × 0 281–0 324
Abdomen	1 619–1 691 × 0 422–0 451	1 437–1 479 × 0 338–0 366

53 Lipeurus caponis (Linn).

1758 *Pediculus caponis*, Linnaeus, *Syst Nat* , p 614.

This cosmopolitan species has been recognised under various varietal names according to the host and geographical area. These varieties are entirely based upon size, intensity and presence or absence of body markings and slight variations in chaetotaxy I prefer to retain the specific name only It has been recorded from the domestic fowl and other Gallinaceous birds from all over the world Kellogg and Paine (1914) recorded it from India on *Gennaeus melanotus*, *G swinhoe*, *Argusianus argus*, *Phasianus torquata* and *Pavo nigripennis* The specimens in my collection were obtained from the Domestic Fowl (*Gallus g domesticus* Linn)

54 **Lipeurus tropicalis** Peters

1931 *Lipeurus tropicalis*, Peters, *Ent News*, XLII, p 195, f 1, 2

Peters (1931) described it from domestic fowls in Bahama Islands, Caicos Islands, Venezuela and Liberia He also recorded it from five species of wild guinea fowls in Africa Bedford (1932) recorded a male from domestic fowl in Onderstepoort (South Africa) My specimens were taken off the domestic fowl (*Black Minorca*) at Lyallpur, 9-11-1936, and were markedly longer than Peters' specimens, especially the males.

Measurements (*mm*)

	Female (1)	Male (1)
Body	3 35 × 0 85	3 44 × 0 61
Head	0 80 × 0 53	0 82 × 0 53
Thorax	0 55 × 0 56	0 61 × 0 56
Abdomen	2 00 × 0 85	2·01 × 0 61

Peters (1931) gave the measurements of female and male as 3 264 mm × 0 837 mm and 3 196 mm × 0 637 mm respectively

55 **Lipeurus cinereus** Nitzsch

1874 *Lipeurus cinereus*, Nitzsch, in Giebel, *Ins Epiz* , p 221

This species was first obtained from *Perdix coturnix* My specimens were obtained from the Common Grey Quail (*Coturnix c coturnix*, Linn), shot in Lyallpur, 28-1-1928, and differ from Piaget's figure in some important particulars, viz , the shape of the head, and the last abdominal segment of the male The shape of the head in this case resembles *Lipeurus unicolor* Piaget (*Pediculines*, p 354, pl 28, f. 6), while the last abdominal segment in male is entire and furnished with four long hairs In other particulars the present examples agree well The species may be recognised by its pointed head with dark-brown bands, body with dark-black lateral bands on thorax and abdomen, and by its yellowish ground colour and yellowish brown transverse blotches

Measurements (*mm*)

	Female (4)	Male (3)
Body	1 692–1 906 × 0 333–0 506	1 478–1 665 × 0 293–0 333
Head	0 506–0 560 × 0 320–0 346	0 466–0 506 × 0 266–0·293
Thorax	0 226–0 280 × 0 240–0 320	0 226–0 266 × 0 226–0 266
Abdomen	0 933–1 066 × 0 333–0 506	0 786–0 946 × 0 293–0 333

Piaget (1880) gave the measurements of female and male as 1 8 mm ×0 52 mm and 1 5 mm. ×0 31 mm respectively

56 Lipeurus caponis var delta Piaget

1880 *Lipeurus caponis* var *delta*, Piaget, *Les Pediculines*, p 366, pl 29, f 5

Piaget (1880) described it from *Francolinus capensis* One male was obtained by me from the Indian Black Partridge (*Francolinus f asiae* Bonap) shot in Lyallpur, 12-xi-1933 It differs from *delta* Piaget in minor details, such as the presence of a long hair behind the eyes, two long hairs on the temporal margin of the head, abdomen with two longitudinal submedian rows of hairs, each segment being furnished with four median hairs and one marginal hair It stands midway between *L caponis formosanus* Sugimoto (*Rept Dept Agric*, *Formosa*, XLIII, 1926, p 53, pl 6, f 4, 5) and *L introductus* Kellogg (*Proc Calif Acad Sci*, (2) VI, (1896), p 500, pl 68, f 1, 5).

Measurements (mm)

	Male
Body	1 546 ×0 400
Head	0 466 ×0 306
Thorax	0 280 ×0 333
Abdomen	0 800 ×0 400

Piaget (1880) gave the measurements of female as 1 75 mm ×0 39 mm

VI GONIODIDAE

57 Goniodes minor Piaget

1880 *Goniodes minor*, Piaget, *Les Pediculines*, p 256, pl 21, f 3

This parasite of doves and pigeons has been recorded from most parts of the world The following recorded hosts of it are also found within Indian limits *Columba livia livia* Gmel , the Burmese Spotted Dove (*Streptopelia chinensis tigrina*, Temm) and Ring Dove (*Streptopelia d decaocta*, Frivalszky) My specimens were obtained from the Indian Spotted Dove (*Streptopelia chinensis suratensis*, Gmel) and the Indian Ring Dove (*Streptopelia d decaocta*, Frival), both shot in Lyallpur, 23-iii-1929 and 14-v-1928 respectively

Measurements (mm)

	Female (3)	Male (1)
Body	1 746–1 853 ×0 566–0 733	1 392 ×0 506
Head	0 533 ×0 600–0 626	0 426 ×0 493
Thorax	0 280–0 320 ×0 426–0 466	0 240 ×0 360
Abdomen	0 933–1 000 ×0 566–0 733	0 726 ×0 506

Piaget (1880) gave the measurements of female and male as 1 7 mm ×0·73 mm. and 1·45 mm ×0 65 mm respectively.

58 **Goniodes pavonis** (Linn)

1758 *Pediculus pavonis* Linnaeus *Syst Nat* p 613

This is one of the best known *Goniodes* and has been recorded from the Common Pea Fowl (*Pavo cristatus*) from all over the world Numerous specimens were obtained by me from the Common Pea Fowl shot in Hoshiarpur 14 v 1928 Amritsar 9 x 1935 and Attari (near Lahore) 9 vi 1935

Measurements (*mm*)

	Female (7)	Male (4)
Body	3 506–3 799 × 1 600 2 100	3 066 3 265 × 1 86
Head	0 840–0 933 × 1 200 1 240	0 80) 0 866 × 1 04–1 06
Thorax	0 866–0 933 × 0 800 1 026	0 86)–0 933 × 1 066
Abdomen	1 800 1 933 × 1 600–2 100	1 333 1 533 × 1 86

Piaget (1880) gave the measurements of female and male as 3 3 mm × 1 8 mm and 3 05 mm × 1 74 mm respectively

59 **Goniodes dissimilis** Nitzsch

1818 *Goniodes dissimilis* Nitzsch *Germ Mag* III p 294

This species has been recorded from all parts of the world on *Gallus domesticus G furcatus G bankiva* It is evidently rare in the Punjab as only four females were taken by me off the Domestic Fowl (*Black Minorca*) at Gurdaspur 3 vii 1934

Measurements (*mm*)

	Female (1)
Body	2 69 × 1 51
Head	0 90 × 1 13
Thorax	0 61 × 0 78
Abdomen	1 18 × 1 51

Piaget (1880) and Sugimoto (1929) gave the measurement of female as 2 6 mm × 1 36 mm and 2 8 mm × 0 85 mm respectively

60 **Goniodes breviantennatus** Piaget

1885 *Goniodes breviantennatus* Piaget *Les Pediculines Suppl* p 50 pl 5 f 8

The type host of this species is the Chukor (*Alectoris graeca chukor* Grey) The specimens in my collection were obtained from the type host shot in Kulu and Hoshiarpur 12 vi 1938 and 14 vii 1928 respectively They agree well with Piaget s figure in general shape and scriptural markings on the abdomen but differ in three particulars, viz , the shape of the pterothorax thoracic hairs and size The ptero thorax in these examples is obtusely angulate on the abdomen with a series of long hairs on posterior margin, the dorsal surface of the abdomen is medially beset with hairs Piaget s figure does not show these details

Measurements (mm)

	Female (3)	Male (1)
Body	2 366–2 493 × 1 098–1 126	1 661 × 0 915
Head	0 718–0 803 × 0 943–1 000	0 605 × 0 803
Thorax	0 338–0 437 × 0·704–0 705	0·253 × 0 577
Abdomen	1 141–1 309 × 1·098–1 126	0 803 × 0 915

Piaget (1885) gave the measurements of female and male as 3 5 mm × 1 64 mm and 2 5 mm × 1 25 mm. respectively

61 Goniodes asterocephalus (Nitzsch)

1874 *Goniocotes asterocephalus*, Nitzsch, in Giebel, *Ins Epiz*, p 182, pl 13, f 3 4

This species was first described from specimens obtained from the Common Grey Quail (*Coturnix c coturnix* Linn) Since then it has been recorded on the type-host from most parts of the Old World My specimens were obtained from the Common Indian Grey Quail (*C c coturnix* Linn), shot in Lyallpur, 14-vii-1927

Measurements (mm)

	Female (5)	Male (1)
Body	2 354–2 563 × 0 591–0 845	2 051 × 0 605
Head	0 591–0 634 × 0 563–0 605	0 635 × 0 591
Thorax	0 338–0 381 × 0 437–0 490	0 290 × 0 486
Abdomen	1 425–1 549 × 0 591–0 845	1 126 × 0 605

Piaget (1880) gave the measurements of female as 2 9 mm × 0 8 mm, while Taschenberg (1882) gave it as 3 06 mm × 1 08 mm His male was 2 21 mm × 0 75 mm

62 Paragoniocotes, spp

I obtained one immature specimen from the Indian Rose-ringed Paroquet (*Psittacula krameri manillensis*, Bechst.), 2-ii-1928, and one immature specimen from the Egyptian White-breasted King-fisher (*Halcyon s smyrnensis*, Linn), 17-ii-1928, both shot in Lyallpur

63 Goniocotes bidentatus (Scopoli).

1763. *Pediculus bidentatus*, Scopoli, *Ent carn*, p 385

This familiar species, has been recorded from America, Europe, Africa and Asia on various pigeons and doves Several immature specimens were obtained from the Indian Blue Rock Pigeon (*Columba livia intermedia* Strick) and the Indian Ring Dove (*Streptopelia d. decaocta*, Frival), both shot in Lyallpur, 22-iii-1929 and 10-ii-1936 respectively

Measurements (mm)

	Female (immature)
Body	1 225 × 0 451
Head	0 338 × 0 366
Thorax	0 169 × 0 195
Abdomen	0 718 × 0 415

Piaget (1880), Kellogg (1896) and Sugimoto (1929) gave the measurements of female as 1 4 mm ×0 55 mm , 1 06 mm ×0 48 mm and 1 3–1 5 mm ×0 55 mm respectively

64 Goniocotes rectangulatus Nitzsch

1818 *Goniocotes rectangulatus*, Nitzsch, *Germ Mag* , III, p 294

This species was first recorded from *Pavo cristatus*, *Pavo spiciferus* and *Numida meleagies* My specimens were taken off the Common Pea Fowl (*Pavo cristatus* Linn), shot in Hoshiarpur, 14-v-1928

Measurements (mm)

	Female (3)
Body	1 064–1 168 × 0 465–0 493
Head	0 324–0 394 × 0 408–0 436
Thorax	0 197–0 211 × 0 324–0 366
Abdomen	0 563 × 0 465–0 493

Piaget (1880) gave the measurements of female and male as 1 05 mm ×0 52 mm and 0 8 mm ×0 45 mm respectively

65 Goniocotes hologaster Nitzsch

1818 *Goniocotes hologaster*, Nitzsch, *Germ Mag* , III, p 294

This familiar species has been recorded, from practically all over the world, on the fowls (*Gallus domesticus* Linn and others) Piaget (1880) also recorded it from *Gallus bankiva* Tem , *Ortyx virginianus* and *Euplocamus cuvieri* Bedford (1932) recorded it from the Bush Partridge (*Dendroperdix* (= *Francolinus*) *sephaena*), *Numida papillus transvaalensis* and *Pternistis castaneiventer krebsi*, all from Africa Numerous specimens were taken by me from the type-host (*Gallus g domesticus* Linn)

Measurements (mm)

	Female (6)	Male (4)
Body	**1·228–1 338 × 0 563–0 704**	**0 815–0 886 × 0 422–0 450**
Head	**0 352–0·394 × 0 422–0·464**	**0 266–0·309 × 0 309–0 352**
Thorax	**0 225–0 282 × 0 352–0 380**	**0 141–0 183 × 0 268–0 297**
Abdomen	**0 636–0 704 × 0 563–0 704**	**0 366–0 408 × 0 422–0 450**

Piaget (1880), Mjöberg (1910) and Sugimoto (1929) gave the measurements of female as 1 3 mm ×0 66 mm, 1 3375 mm ×0 625 mm and 1 3–1 6 mm ×0 66–0 88 mm respectively, while male of Piaget (1880) and Sugimoto (1929) was 0 8–0 9 mm ×0 5 mm only

66 Goniocotes (? Goniodes) gigas Taschenberg

1869 *Goniocotes gigas*, Taschenberg, *Zeit f ges Nat*, LII, p 104, pl 1, f 10

It has been recorded from all over the world from the Domestic Fowl (*Gallus domesticus* Linn) Bedford (1932) took it from *Numida coronata*, *N papillus* and *N p transvaalensis* My specimens were collected from the Domestic Fowl (*Gallus g domesticus* Linn)

Measurements (mm)

	Female (8)	Male (5)
Body	2 960–4 015 × 1 370–1 840	2 676–3 280 × 1 366–1 500
Head	0 845–1 090 × 1 000–1 200	0 845–0 910 × 0 873–1 100
Thorax	0 636–0 650 × 0 732–0 970	0 564–0 580 × 0 774–0 910
Abdomen	1 479–2 356 × 1 370–1 840	1 267–1 790 × 1 366–1 500

Piaget (1880) and Taschenberg (1882) gave the measurements of female as 3 3 mm ×1 6 mm and 4 05 mm ×2 07 mm, while of male as 2 9 mm ×1 5 mm and 3 33 mm ×1 95 mm respectively.

67 Goniocotes alatus Piaget

1885 *Goniocotes alatus*, Piaget, *Les Pediculines*, *Suppl*, p 45, pl 5, f 4

Piaget (1885) obtained it from the Chukor (*Perdix* (= *Caccabis*) *chukor*) I collected one female from the Northern Chukor (*Alectoris graeca pallescens*, Hume), shot in Hoshiarpur, 9-x-1939

Measurements (mm)

	Female
Body	1·084 × 0 493
Head	0 324 × 0 422
Thorax	0 243 × 0 352
Abdomen	0 517 × 0 493

Piaget (1885) gave the measurements of female as 1 5 mm ×0 63 mm

68. Goniocotes jirufti,[1] sp nov

Female (Text-fig 12*a*) small, head subpentagonal, prothorax narrow; pterothorax acutely angulate posteriorly, medianly overlapping the first abdominal segment; abdomen similar to *Goniocotes hologaster* with simple markings

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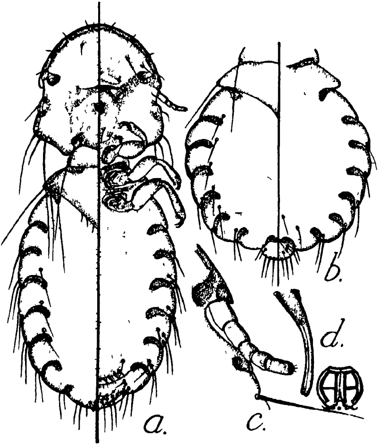
[1] 'Jirufti' in vernacular means the Indian Black Partridge.

Head somewhat hexagonal, usually longer than broad, sometimes slightly wider than long, front broad, convex, with ten short hairs distributed as in *G hologaster*, clypeal band deep yellow, narrow, ending posteriorly into a big antennal blotch, trabeculae small, blunt and immovable, antennae normal, eyes not protruding, flatly rounded, bearing small hair at the base, ocular blotch deep yellow, temporal margin straight with a small hair and two long hairs at the pointed anterior angle, marginal band narrow, not very distinct but near the eye, posterior margin concave in the middle with obtuse angles at each end of concavity, bare, occipital band sinuous, dark yellow to brown



Text fig 12 *Goniocotes jirufti*, sp nov

a Dorsal and ventral aspects of female, *b* Dorsal and ventral aspects of thorax and abdomen of male, *c* Antenna of male, *d* Male genital armature

Prothorax narrow, short, trapezoidal, with anteriorly converging lateral margins, posterior angle slightly produced, acute, bearing a long hair, posterior margin straight, slightly concave in the middle, bare, meso- and metathorax fused, pterothorax with blunt lateral angles, bearing 2 long hairs, posterior margin acutely angulate on the first abdominal segment, bare, except two long hairs near the lateral angles.

Abdomen broadly elliptical, segments subequal, marginal band indistinct, intersegmental bands at the pleural re-entrant heads, deep yellow, narrow, transversal bands absent, sternal blotches absent, chaetotaxy scarce, segment II–VIII with 1–3 long, finely pointed hairs on the slightly projecting angles, segments III–V with one submarginal hair at the curved lateral band, dorsum bare, ventrum with marginal hairs on segments II–VIII, last segment angularly emarginate with short hairs

Male (Text-fig 12*b*) similar to female, abdomen rather rounded, last segment truncate and globular, posterior margin with tuft of hairs Genital armature (Text fig 12*d*) distinct, with narrow feebly chitinised basal plate, parameres short, median endomeral plate slightly broad, concave posteriorly, penis elongated, enclosed in a tube with an aperture at its tip

Measurements (*mm*)

	Female (Holotype)	Female (3)	Male (3)
Body	1 108 × 0 493	0 999–1 106 × 0 460–0 493	0 666–0 679 × 0 320–0 360
Head	0 350 × 0 333	0 346–0 360 × 0 333–0 386	0 253–0 266 × 0 280–0 293
Prothorax	0 055 × 0 200	0 040–0 066 × 0 160–0 200	0 040 × 0 120–0 143
Pterothorax	0 103 × 0 266	0 080–0 120 × 0 240–0 280	0 080–0 093 × 0 213
Abdomen	0 600 × 0 493	0 533–0 566 × 0 460–0 493	0 293–0 306 × 0 320–0 360

Holotype (female) and *Allotype* (male) from Lyallpur, 14-viii-1928, *ex* the Indian Black Partridge (*Francolinus francolinus asiae* Bonap) both mounted together on slide No MI 157 *Paratypes* 3 females and 2 males in spirit (same data as above)

SUMMARY

An account of sixty five species of Ischnoceron Mallophaga, belonging to thirty two genera, is given This includes the description and figures of three new genera, viz , *Picophilopterus*, *Painjunirmus* and *Traihoriella* and twelve new species, viz , *Ardeicola gaibagla* from the Cattle Egret (*Bubulcus ibis coromandus* Bodd), *Aegypoecus gryffoneae* from the Himalayan Griffon Vulture (*Gyps himalayensis* Hume), *Echinophilopterus tota* from the Rose ringed Paroquet (*Psittacula krameri manillensis* Bechst), *Picophilopterus tuktola* from the Himalayan Scaly-bellied Woodpecker (*Picus s squamatus* Vigors), *Penenirmus raji* from the Indian Yellow-throated Sparrow (*Gymnoris x xanthocollis*, Burt), *Psittaconirmus chandabani* from the Large Parrot (*Psittacula eupatria nepalensis* Hodgs), *Psittaconirmus lybartota* from the Indian Rose-ringed Paroquet (*Psittacula krameri manillensis* Bechst), *Quadraceps kekra* from the Gullbilled Tern (*Gelochelidon n nilotica* Gmel), *Syrrhaptoecus emahusaini* from the Indian Sand Grouse (*Pterocles exustus erlangeri* Neum), *Painjunirmus pengya* from the Bengal Jungle Babbler (*Turdoides t terricolor* Hodgs), *Traihoriella punjabensis* from the Himalayan Great Barbet (*Megalaima virens marshallorum* Swinh) and *Goniocotes jirufti* from the Indian Black Partridge (*Francolinus f asiae* Bonap) About fifty species have been recorded for the first time from the Punjab A few of the recorded species differ from the description and figures of previous works in certain morphological details and size and seem well differentiated to warrant their being treated as varieties, but as type specimens were not available, no attempt has been made to alter their existing status

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ON SOME NEW KERNELS AND FUNCTIONS SELF-RECIPROCAL IN THE HANKEL TRANSFORM

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(Communicated by Prof. D. S. Kothari, M.Sc., Ph.D.)

(*Received September 20, read November 23, 1946.*)

§1 Introduction

Adopting the usual notation of Hardy and Titchmarsh a function $f(x)$ is said to be R_ν if it is self-reciprocal in the Hankel Transform of order ν, i.e.,

$$f(x) = x^{\frac{1}{2}} \int_0^\infty y^{\frac{1}{2}} J_\nu(xy)\, f(y)\, dy$$

Starting with an R_ν function a number of theorems have been given from time to time for finding another R_ν function with the help of some kernel or other.

The object of this paper is to discover some new kernels by using Hardy's formula (1935)

$$g(x) = \int_0^\infty P_1(y)\, P_2(xy)\, dy$$

where $P_1(x)$ and $P_2(x)$ are two Fourier kernels.

The main results are stated in the form of three theorems. The importance of these theorems lies in the fact that they are useful in identifying the nature of the resultant of two kernels, as transforming a self-reciprocal function of a known order to another of a different order.

The theorems have been discussed in §2–§4. Only the proof of Theorem (1) is given in detail. The other two theorems can be proved similarly.

Finally, in §5–§7, I have used the kernels of §§2–4 to investigate certain new self-reciprocal functions.

§2 Theorem 1.

The resultant,

$$K(x) = \int_0^\infty P_1(y)\, P_2(xy)\, dy,$$

of two kernels of the form (Titchmarsh, 1937)

$$P_1(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \Gamma\left(\frac{1}{4}+\frac{\nu}{2}+\frac{s}{2}\right)\Gamma\left(\frac{3}{4}+\frac{\mu}{2}-\frac{s}{2}\right)\chi(s)x^{-s}\,ds \tag{2.1}$$

and

$$P_2(x) = \frac{1}{2\pi i} \int_{c'-i\infty}^{c'+i\infty} 2^s\,\Gamma\left(\frac{1}{4}+\frac{\nu}{2}+\frac{s}{2}\right)\Gamma\left(\frac{1}{4}+\frac{\rho}{2}+\frac{s}{2}\right)\omega(s)x^{-s}\,ds \quad . \tag{2.2}$$

where, $0<c<1, \quad 0<c'<1$

and $\chi(s) = \chi(1-s), \quad \omega(s) = \omega(1-s)$

is a kernel transforming R_μ to R_ρ and vice versa

In order to prove this consider the two kernels (2.1) and (2.2)

The resultant is,

$$K(x) = \frac{1}{2\pi i}\int_0^\infty P_1(y)\,dy\int_{c'-i\infty}^{c'+i\infty} 2^s\Gamma\left(\frac{1}{4}+\frac{\nu}{2}+\frac{s}{2}\right)\Gamma\left(\frac{1}{4}+\frac{\rho}{2}+\frac{s}{2}\right)\omega(s)x^{-s}y^{-s}\,ds$$

$$= \frac{1}{2\pi i}\int_{c-i\infty}^{c'+i\infty} 2^s\Gamma\left(\frac{1}{4}+\frac{\nu}{2}+\frac{s}{2}\right)\Gamma\left(\frac{1}{4}+\frac{\rho}{2}+\frac{s}{2}\right)\omega(s)x^{-s}\,ds\int_0^\infty P_1(y)y^{-s}\,dy \qquad (2.3)$$

provided the change in the order of integration is permissible

Now, applying Mellin's Inversion formula to (2.1) and changing s to $(1-s)$ in it, we get

$$\int_0^\infty P_1(y)y^{-s}\,dy = \Gamma\left(\frac{3}{4}+\frac{\nu}{2}-\frac{s}{2}\right)\Gamma\left(\frac{1}{4}+\frac{\mu}{2}+\frac{s}{2}\right)\chi(1-s)$$

Therefore, from (2.3)

$$K(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} 2^s\Gamma\left(\frac{1}{4}+\frac{\nu}{2}+\frac{s}{2}\right)\Gamma\left(\frac{1}{4}+\frac{\rho}{2}+\frac{s}{2}\right)\Gamma\left(\frac{3}{4}+\frac{\nu}{2}-\frac{s}{2}\right)\Gamma\left(\frac{1}{4}+\frac{\mu}{2}+\frac{s}{2}\right)\omega(s)\times \chi(1-s)x^{-s}\,ds$$

$$= \frac{1}{2\pi i}\int_{c'-i\infty}^{c'+i\infty} 2^s\Gamma\left(\frac{1}{4}+\frac{\rho}{2}+\frac{s}{2}\right)\Gamma\left(\frac{1}{4}+\frac{\mu}{2}+\frac{s}{2}\right)\psi(s)x^{-s}\,ds \qquad (2.4)$$

where

$$\psi(s) = \Gamma\left(\frac{1}{4}+\frac{\nu}{2}+\frac{s}{2}\right)\Gamma\left(\frac{3}{4}+\frac{\nu}{2}-\frac{s}{2}\right)\omega(s)\chi(1-s)$$

Since, $\psi(s) = \psi(1-s)$ we have that $K(x)$ is a kernel transforming R_μ to R_ρ and *vice versa*

It only remains to justify the change in the order of integration in (2.3)

Putting $s = c+it$ and applying the formula,

$$\left|\Gamma\left(A+\frac{it}{2}\right)\right| \sim ce^{-\frac{\pi}{4}|t|}|t|^{A-\frac{1}{2}}$$

to the integral (2.3) we see that it is less than

$$\left(\frac{2}{x}\right)^c\int_{-\infty}^{\infty} e^{-\frac{\pi}{2}|t|}|t|^{Rl\left[\frac{\nu+\rho}{2}\right]-\frac{1}{2}+c}|\omega(c+it)|\,dt\int_0^\infty |P_1(y)|y^{-c}\,dy$$

Since, $\omega(c+it) = O(e^{[\pi/2-\alpha+\eta]|t|})$

we have that the t-integral is uniformly convergent Also, the y-integral converges uniformly since $P_1(y)$ belongs to the class of analytic functions $A(\omega, \alpha)$ Also since, (2 4) exists, the change in the order of integration is justified.

Example —To illustrate the above theorem, let us take,

$$\chi(s) = \frac{1}{2\Gamma\left(1+\frac{\mu+\nu}{2}\right)}, \qquad 0 \leq -\frac{1}{2}-\mu < Rl(s) < \frac{1}{2}-\frac{\mu}{2}+\frac{\nu}{2}$$

Then (2 1) gives

$$P_1(x) = \frac{x^{\mu+\frac{1}{2}}}{(1+x^2)^{\frac{\mu+\nu}{2}+1}},$$

a known kernel transforming R_μ to R_ν (Baily, 1931).

Again, putting

$$\omega(s) = \frac{1}{\Gamma\left(\frac{\mu}{2}-\frac{s}{2}+\frac{7}{4}\right)\Gamma\left(\frac{\mu}{2}+\frac{s}{2}+\frac{5}{4}\right)},$$

$$0 < Rl(s)+\mu+\frac{1}{2} < \mu+\frac{5}{2}$$

and $\qquad \nu = \mu+2$

we have from (2 2),

$$P_2(x) = x^{-\frac{1}{2}} J_{\mu+1}(x),$$

a known kernel transforming R_μ to $R_{\mu+2}$ and *vice versa* (Varma, 1939)

The resultant kernel is given by,

$$K(x) = x^{-\frac{1}{2}} \int_0^\infty \frac{y^\mu J_{\mu+1}(xy)}{(1+y^2)^{\frac{\mu+\nu}{2}+1}}\, dy \text{ (Watson, 1944)}$$

$$= \frac{x^{\mu+\frac{1}{2}}\Gamma(\mu+1)\Gamma\left(\frac{\nu-\mu}{2}\right)}{2^{\mu+2}\Gamma\left(\frac{\mu+\nu}{2}+1\right)\Gamma(\mu+2)}\, {}_1F_2\left(\mu+1\,, \frac{\mu-\nu}{2}+1, \mu+2\,, \frac{x^2}{4}\right)$$

$$+ \frac{x^{\nu+\frac{1}{2}}\Gamma\left(\frac{\mu-\nu}{2}\right)}{2^{\nu+2}\Gamma\left(\frac{\mu+\nu}{2}+2\right)}\, {}_1F_2\left(\frac{\mu+\nu}{2}+1\,, \frac{\mu+\nu}{2}+2, \frac{\nu-\mu}{2}+1\,, \frac{x^2}{4}\right)$$

valid for $\qquad \nu \neq \mu\,, Rl(\mu) > -1\,, Rl(\nu) > -1, Rl\left(\nu+2\mu+\frac{9}{2}\right) > 0$

Our theorem at once gives that $K(x)$ is a kernel transforming R_ν to $R_{\mu+2}$ and *vice versa*.

§3. Theorem 2.

The resultant,

$$K(x) = \int_0^\infty P_1(xy)\, P_2(y)\, dy$$

of two kernels of the form,

$$P_1(x)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} 2^s\Gamma\left(\frac{1}{4}+\frac{\mu}{2}+\frac{s}{2}\right)\Gamma\left(\frac{1}{4}+\frac{\nu}{2}+\frac{s}{2}\right)\chi(s)x^{-s}\,ds \tag{3 1}$$

and,

$$P_2(x)=\frac{1}{2\pi i}\int_{c'-i\infty}^{c'+i\infty} 2^s\Gamma\left(\frac{1}{4}+\frac{\mu}{2}+\frac{s}{2}\right)\Gamma\left(\frac{1}{4}+\frac{\lambda}{2}+\frac{s}{2}\right)\omega(s)x^{-s}\,ds \quad . \tag{3 2}$$

where, $$0<c<1, \quad 0<c'<1,$$

$$\omega(s) = \omega(1-s), \quad \chi(s) = \chi(1-s)$$

is a kernel transforming R_ν to R_λ

In order to illustrate the above theorem, let us take in (3 1),

$$\chi(s) = \frac{1}{4\sqrt{2}\Gamma\left(\frac{\mu}{2}+\frac{3}{4}-\frac{s}{2}\right)\left(\frac{\mu}{2}+\frac{1}{4}+\frac{s}{2}\right)\left(\frac{\mu+\nu}{2}+1\right)},$$

$$Rl\left(-\nu-\frac{3}{2}\right) < Rl(s) < Rl(\mu)+\frac{3}{2}$$

and replace μ by $(\mu+2)$, we get

$$P_1(x)=\frac{x^{\mu+\frac{1}{2}}\Gamma(\mu+1)\Gamma\left(\frac{\nu-\mu}{2}\right)}{2^{\mu+2}\Gamma\left(\frac{\mu+\nu}{2}+1\right)\Gamma(\mu+2)}\ {}_1F_2\left(\mu+1,\ \frac{\mu-\nu}{2}+1,\ \mu+2,\ \frac{x^2}{4}\right)$$

$$+\frac{x^{\nu+\frac{1}{2}}\Gamma\left(\frac{\mu-\nu}{2}\right)}{2^{\nu+2}\Gamma\left(\frac{\mu+\nu}{2}+2\right)}\ {}_1F_2\left(\frac{\mu+\nu}{2}+1,\ \frac{\mu+\nu}{2}+2,\ \frac{\nu-\mu}{2}+1,\ \frac{x^2}{4}\right)$$

a kernel transforming R_ν to $R_{\mu+2}$ and *vice versa* Again taking,

$$\omega(s) = 2^{\frac{\mu+\nu}{2}-\frac{3}{2}} \text{ in (3.2), } Rl(s)+\frac{1}{2} > Rl(-\mu)$$

we get

$$P_2(x) = x^{\frac{\mu+\nu}{2}+\frac{1}{2}}K_{\frac{1}{2}[\nu-\mu]}(x),$$

a known kernel transforming R_μ to R_ν and *vice versa.*

The resultant kernel is given by

$$K(x) = \int_0^\infty \left[\frac{(xy)^{\mu+\frac{1}{2}}\Gamma(\mu+1)\Gamma\left(\frac{\nu-\mu}{2}\right)}{2^{\mu+2}\Gamma(\mu+2)\Gamma\left(\frac{\mu+\nu}{2}+1\right)} {}_1F_2\left(\mu+1\,,\ \frac{\mu-\nu}{2}+1,\ \mu+2\,,\ \frac{x^2y^2}{4}\right) + \frac{(xy)^{\nu+\frac{1}{2}}\Gamma\left(\frac{\mu-\nu}{2}\right)}{2^{\nu+2}\Gamma\left(\frac{\mu+\nu}{2}+2\right)} {}_1F_2\left(\frac{\mu+\nu}{2}+1\,,\ \frac{\mu+\nu}{2}+2,\ \frac{\nu-\mu}{2}+1\,,\ \frac{x^2y^2}{4}\right)\right] \times y^{\frac{1}{2}(\mu+\nu+1)}K_{\frac{1}{2}(\nu-\mu)}(y)\,dy$$

Expanding the ${}_1F_2$ function and integrating term by term, by the help of the integral,

$$\int_0^\infty t^{\mu-1}K_\nu(t)\,dt = 2^{\mu-2}\Gamma\left(\frac{\mu-\nu}{2}\right)\Gamma\left(\frac{\mu+\nu}{2}\right), \qquad Rl(\mu) > |\,Rl(\nu)\,|$$

we get the new kernel,

$$K(x) = \frac{\pi \operatorname{cosec} \pi\left(\frac{\nu-\mu}{2}\right)}{2^{2-\frac{\mu+\nu}{2}}\Gamma\left(\frac{\mu+\nu}{2}+1\right)} \left\{ x^{\mu+\frac{1}{2}}\,{}_3f_2{}^{*}\left(\begin{array}{c} \mu+1,\ \mu+1,\ \frac{\mu+\nu}{2}+1, \\ \frac{\mu-\nu}{2}+1,\ \mu+2, \end{array}\ x^2\right) - x^{\nu+\frac{1}{2}}\,{}_3f_2\left(\begin{array}{c} \frac{\mu+\nu}{2}+1,\ \frac{\mu+\nu}{2}+1,\ \nu+1, \\ \frac{\mu+\nu}{2}+2,\ \frac{\nu-\mu}{2}+1, \end{array}\ x^2\right)\right\}$$

valid for,

$$Rl(\mu) > -1,\ Rl(\mu+\nu+2) > 0,\ |\,x\,| < 1,\ \nu \neq \mu$$

Our theorem gives that $K(x)$ is a kernel transforming $R_{\mu+2}$ to R_μ

The term by term integration is valid since

(i) ${}_1F_2$ is an integral function and hence is uniformly convergent in any arbitrary interval (O, A) of x

(ii) $\qquad K_\nu(x) = O(x^{-\frac{1}{2}}e^{-x})$ for $|\,x\,|$ large,

(iii) The integrated series of ${}_3F_2$ functions is uniformly convergent for $|\,x\,| < 1$

§4. Theorem 3.

The resultant,

$$K(x) = \frac{1}{2\pi i}\int_0^\infty P_1(xy)\,P_2(y)\,dy,$$

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* I shall henceforth use the abbreviated notation ${}_pf_q(a_1 \quad a_p\,,\ b_1 \quad b_q\,;\ x)$ to denote

$$\frac{\Gamma(a_1) \quad \Gamma(a_p)}{\Gamma(b_1) \quad \Gamma(b_q)}\,{}_pF_q(a_1 \quad a_p\,,\ b_1 \quad b_q\,,\ x)$$

of two kernels of the form,

$$P_1(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \Gamma\left(\frac{1}{4}+\frac{\nu'}{2}+\frac{s}{2}\right)\Gamma\left(\frac{3}{4}+\frac{\rho}{2}-\frac{s}{2}\right)\omega(s)x^{-s}\,ds \quad . \qquad (4.1)$$

and,

$$P_2(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c'+i\infty} \Gamma\left(\frac{1}{4}+\frac{\nu'}{2}+\frac{s}{2}\right)\Gamma\left(\frac{3}{4}+\frac{\mu'}{2}-\frac{s}{2}\right)\chi(s)x^{-s}\,ds \qquad (4\ 2)$$

where,

$$0<c<1,\ 0<c'<1,$$

$$\omega(s) = \omega(1-s),\ \chi(s) = \chi(1-s)$$

is a kernel transforming $R_{\mu'}$ to R_ρ

To illustrate the above theorem, let us put in (4 1) and (4 2)

$$\omega(s) = \chi(s) = \frac{1}{2\Gamma\left(1+\frac{\mu+\nu}{2}\right)}\ ,\quad 0 \leq -\tfrac{1}{2}-\mu < Rl(s) < \tfrac{1}{2}-\frac{\mu-\nu}{2},$$

and

$$\nu' = \mu,\ \rho = \mu' = \nu,$$

then we have,

$$P_1(x) = P_2(x) = \frac{x^{\mu+\frac{1}{2}}}{(1+x^2)^{\frac{\mu+\nu}{2}+1}}\ ,$$

a known kernel transforming R_μ to R_ν

The resultant kernel is given by

$$K(x) = x^{\mu+\frac{1}{2}}\int_0^\infty \frac{y^{2\mu+1}\,dy}{[(1+y^2)(1+x^2y^2)]^{\frac{\mu+\nu}{2}+1}}$$

$$= \frac{x^{\mu+\frac{1}{2}}}{2\pi i\,\Gamma\left(\frac{\mu+\nu}{2}+1\right)}\int_0^\infty \frac{y^{2\mu+1}dy}{(1+y^2)^{\frac{\mu+\nu}{2}+1}}\int_{-i\infty}^{i\infty}\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(-s)(x^2y^2)^s\,ds$$

Changing the order of integration and integrating, we get

$$\frac{x^{\mu+\frac{1}{2}}}{4\pi i\left\{\Gamma\left(\frac{\mu+\nu}{2}+1\right)\right\}^2}\int_{-i\infty}^{i\infty}\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(\mu+1+s)\Gamma(-s)\Gamma\left(\frac{\nu-\mu}{2}-s\right)x^{2s}\,ds \qquad (4\ 3)$$

$$= \frac{x^{\mu+\frac{1}{2}}\pi \operatorname{cosec} \pi\left(\frac{\nu-\mu}{2}\right)}{2\left\{\Gamma\left(\frac{\mu+\nu}{2}+1\right)\right\}^2}\left\{x^{\nu-\mu}{}_2f_1\left(\nu+1,\ \frac{\mu+\nu}{2}+1;\ \frac{\nu-\mu}{2}+1,\ x^2\right)\right.$$

$$\left. -{}_2f_1\left(\mu+1,\ \frac{\mu+\nu}{2}+1,\ 1-\frac{\nu-\mu}{2},\ x^2\right)\right\}$$

valid for $Rl(\nu) > -1,\ Rl(\nu-\mu+1) > 0,\ \nu \neq \mu$

Our theorem at once gives that $K(x)$ is a kernel transforming R_ν into itself

The change in the order of integration is permitted since the y and the s-integrals converge uniformly for $Rl(\nu-\mu+1)>0$ and $|x|<1$ respectively and also since the integral (4 3) exists

§5 Let us take the $R_{\rho+2m}$ function,

$$2^{\frac{m+1}{2}} x^{m+\rho-\frac12} e^{-\frac14 x^2} \quad W_{\rho+\frac{m+1}{2},\,\frac12 m}(\tfrac12 x^2) \qquad \text{(Varma, 1938)} \quad (5\ 1)$$

ρ being an even integer or zero and $\rho+2m>-1$

Putting $\rho=\nu-2m$, we get the R_ν function

$$2^{\frac{m+1}{2}} x^{\nu-m-\frac12} e^{-\frac14 x^2} \quad W_{\nu-\frac32 m+\frac12,\,\frac12 m}(\tfrac12 x^2)$$

Applying the kernel of §2 to this R_ν function we get the $R_{\mu+2}$ function,

$$F(x)=\frac{\pi \operatorname{cosec} \pi\left(\frac{\nu-\mu}{2}\right)}{\Gamma\left(\frac{\mu+\nu}{2}+1\right)} \int_0^\infty \left\{ \frac{(xy)^{\mu+\frac12}}{2^{\mu+2}}\, {}_1f_2\left(\begin{matrix}\mu+1, & \\ \frac{\mu-\nu}{2}+1,\ \mu+2, & \end{matrix} \frac{x^2y^2}{4}\right)\right.$$

$$\left. -\frac{(xy)^{\nu+\frac12}}{2^{\nu+2}}\, {}_1f_2\left(\begin{matrix}\frac{\mu+\nu}{2}+1, & \\ \frac{\mu+\nu}{2}+2,\ \frac{\nu-\mu}{2}+1, & \end{matrix} \frac{x^2y^2}{4}\right)\right\} \times 2^{\frac{m+1}{2}} y^{\nu-m-\frac12} e^{-\frac14 y^2} W_{\nu-\frac32 m+\frac12,\,\frac12 m}(\tfrac12 x^2)$$

$$=\frac{\pi \operatorname{cosec} \pi\left(\frac{\nu-\mu}{2}\right)}{\Gamma\left(\frac{\mu+\nu}{2}+1\right)} \left\{ \frac{x^{\mu+\frac12}}{2^{\mu-\frac{m}{2}+\frac32}} \sum_{n=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{2n}\Gamma(\mu+1+n)}{n\,!\,\Gamma\left(\frac{\mu-\nu}{2}+1+n\right)\Gamma(\mu+2+n)}\right.$$

$$\times \int_0^\infty y^{\mu+\nu+2n-m} e^{-\frac14 y^2} W_{\nu-\frac32 m+\frac12,\,\frac12 m}(\tfrac12 y^2)\, dy$$

$$-\frac{x^{\nu+\frac12}}{2^{\nu-\frac{m}{2}+\frac32}} \sum_{n=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{2n}\Gamma\left(\frac{\mu+\nu}{2}+1+n\right)}{\Gamma\left(\frac{\mu+\nu}{2}+2+n\right)\Gamma\left(\frac{\nu-\mu}{2}+1+n\right)n\,!}$$

$$\left. \times \int_0^\infty y^{2\nu+2n-m} e^{-\frac14 y^2} W_{\nu-\frac32 m+\frac12,\,\frac12 m}(\tfrac12 y^2)\, dy \right\}$$

Changing the variable and applying Goldstein's result

$$\int_0^\infty x^{l-1} e^{-\frac12 x}\, W_{k,\,m}(x)\, dx = \frac{\Gamma(l+m+\frac12)\Gamma(l-m+\frac12)}{\Gamma(l-k+1)}$$

for $Rl(l\pm m+\frac{1}{2})>0$, we get

$$F(x)=\frac{\pi\operatorname{cosec}\pi\left(\frac{\nu-\mu}{2}\right)}{4\Gamma\left(\frac{\mu+\nu}{2}+1\right)}\left\{\begin{array}{l}\dfrac{x^{\mu+\frac{1}{2}}}{2^{\frac{\mu-\nu}{2}}}\,{}_3f_3\left(\begin{matrix}\mu+1,\ \frac{\mu+\nu}{2}+1,\ \frac{\mu+\nu}{2}-m+1, \\ \frac{\mu-\nu}{2}+1,\ \mu+2,\ \frac{\mu-\nu}{2}+m+1,\end{matrix}\ \frac{x^2}{2}\right) \\ -\dfrac{x^{\nu+\frac{1}{2}}}{1}\,{}_3f_3\left(\begin{matrix}\frac{\mu+\nu}{2}+1,\ \nu+1,\ \nu+1-m, \\ \frac{\mu+\nu}{2}+2,\ \frac{\nu-\mu}{2}+1,\ m+1;\end{matrix}\ \frac{x^2}{2}\right)\end{array}\right\}\quad . \tag{5.2}$$

where $Rl\left(\frac{\mu+\nu}{2}+1\right)>0,\quad Rl\left(\frac{\mu+\nu}{2}-m+1\right)>0,$

$$Rl(\nu+1)>0,\quad Rl(\nu+1-m)>0$$

and $(\nu-2m)$ is an even integer or zero, and $2m\neq$ an integer or zero

The term by term integration is valid, since,

(i) ${}_1F_2(a,\ b,\ c,\ x)$ is an integral function and hence is uniformly convergent in any arbitrary interval $(O,\ A)$ of x

(ii) $W_{k,m}(x)\sim O(x^k e^{-\frac{1}{2}x})$, for $|x|$ large

(iii) The integrated series of ${}_3f_3$ functions is uniformly convergent

Alternatively, applying the same kernel to the $R_{\mu+2}$ function (5.1), we get the R_ν function,

$$F(x)=\frac{\pi\operatorname{cosec}\pi\left(\frac{\nu-\mu}{2}\right)}{\Gamma\left(\frac{\mu+\nu}{2}+1\right)}\int_0^\infty\left\{\frac{(xy)^{\mu+\frac{1}{2}}}{2^{\mu-\frac{m}{2}+\frac{3}{2}}}\,{}_1f_2\left(\begin{matrix}\mu+1, \\ \frac{\mu-\nu}{2}+1,\ \mu+2,\end{matrix}\ \frac{x^2y^2}{4}\right)\right.$$

$$\left.-\frac{(xy)^{\nu+\frac{1}{2}}}{2^{\nu-\frac{m}{2}+\frac{3}{2}}}\,{}_1f_2\left(\begin{matrix}\frac{\mu+\nu}{2}+1, \\ \frac{\mu+\nu}{2}+2,\ \frac{\nu-\mu}{2}+1,\end{matrix}\ \frac{x^2y^2}{4}\right)\right\}$$

$$y^{\mu+\frac{1}{2}-m}e^{-\frac{1}{2}y^2}W_{\mu-\frac{3}{2}m+\frac{5}{2},\frac{1}{2}m}(\tfrac{1}{2}y^2)\,dy$$

Integrating as before, we get,

$$F(x)=\frac{\pi\operatorname{cosec}\pi\left(\frac{\nu-\mu}{2}\right)}{2\Gamma\left(\frac{\mu+\nu}{2}+1\right)}\left\{\begin{array}{l}x^{\mu+\frac{1}{2}}\,{}_2f_2\left(\begin{matrix}\mu+1,\ \mu+2-m, \\ \frac{\mu-\nu}{2}+1,\ m,\end{matrix}\ \frac{x^2}{2}\right) \\ -\dfrac{x^{\nu+\frac{1}{2}}}{2^{\frac{\nu-\mu}{2}}}\,{}_2f_2\left(\begin{matrix}\frac{\mu+\nu}{2}+1,\ \frac{\mu+\nu}{2}+2-m, \\ 1+\frac{\nu-\mu}{2},\ \frac{\nu-\mu}{2}+m,\end{matrix}\ \frac{x^2}{2}\right)\end{array}\right\} \tag{5.3}$$

valid for, $Rl(\mu+2)>0,\quad Rl(\mu+2-m)\quad>0$

$$Rl\left(\frac{\mu+\nu}{2}+2\right)>0,\quad Rl\left(\frac{\mu+\nu}{2}+2-m\right)>0$$

and $(\mu+2-2m)$ is an even integer or zero and $\nu\neq\mu$.

The process of term by term integration is valid as before

§ 6. Applying the kernel of § 3 to the $R_{\mu+2}$ function (Baily, 1930),

$$x^{\mu+\frac{1}{2}}e^{-\frac{1}{2}x^2}{}_1F_1\left(-2n,\ \frac{\mu}{2}+2-n,\ \tfrac{1}{2}x^2\right),$$

where n is a positive integer, we get the R_μ function

$$F(y)=\frac{\pi\operatorname{cosec}\pi\left(\frac{\nu-\mu}{2}\right)}{2^{2-\frac{\mu+\nu}{2}}\Gamma\left(\frac{\mu+\nu}{2}+1\right)}\int_0^\infty\left\{\begin{array}{l} x^{\mu+\frac{1}{2}}{}_3f_2\left(\begin{matrix}\mu+1,\ \mu+1,\frac{\mu+\nu}{2}+1, \\ \frac{\mu-\nu}{2}+1,\ \mu+2,\end{matrix}\ x^2\right) \\ -x^{\nu+\frac{1}{2}}{}_3f_2\left(\begin{matrix}\frac{\mu+\nu}{2}+1,\ \frac{\mu+\nu}{2}+1,\ \nu+1, \\ \frac{\mu+\nu}{2}+2,\ \frac{\nu-\mu}{2}+1,\end{matrix}\ x^2\right)\end{array}\right\}$$

$$\times(xy)^{\mu+\frac{1}{2}}e^{-\frac{1}{2}x^2y^2}{}_1F_1\left(-2n,\ \frac{\mu}{2}+2-n,\ \frac{1}{2}\ x^2y^2\right)dx$$

$$=\frac{\pi\operatorname{cosec}\pi\left(\frac{\nu-\mu}{2}\right)}{2^{2-\frac{\mu+\nu}{2}}\Gamma\left(\frac{\mu+\nu}{2}+1\right)}\left[\frac{y^{\mu+\frac{1}{2}}}{2\pi i}\int_0^\infty x^{2\mu+3}e^{-\frac{1}{2}x^2y^2}{}_1F_1\left(-2n,\ 2+\frac{\mu}{2}-n,\ \tfrac{1}{2}x^2y^2\right)dx\right.$$

$$\times\int_{-i\infty}^{i\infty}\frac{\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\{\Gamma(\mu+1+s)\}^2\Gamma(-s)}{\Gamma(\mu+2+s)\Gamma\left(\frac{\mu-\nu}{2}+1+s\right)}(-x^2)^s\,ds$$

$$-\frac{y^{\mu+\frac{1}{2}}}{2\pi i}\int_0^\infty x^{\mu+\nu+3}e^{-\frac{1}{2}x^2y^2}{}_1F_1\left(-2n,\ 2+\frac{\mu}{2}-n,\ \tfrac{1}{2}x^2y^2\right)dx$$

$$\left.\times\int_{-i\infty}^{i\infty}\frac{\left\{\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\right\}^2\Gamma(\nu+1+s)\Gamma(-s)(-x^2)^s}{\Gamma\left(\frac{\mu+\nu}{2}+2+s\right)\Gamma\left(\frac{\nu-\mu}{2}+1+s\right)}\,ds\right]$$

Changing the order of integration and integrating term by term we get

$$F(y)=\frac{\pi\operatorname{cosec}\pi\left(\frac{\nu-\mu}{2}\right)}{2^{2-\frac{\mu+\nu}{2}}\Gamma\left(\frac{\mu+\nu}{2}+1\right)}y^{\mu+\frac{1}{2}}\left[\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\{\Gamma(\mu+1+s)\}^2\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(-s)(-)^s}{\Gamma(\mu+2+s)\Gamma\left(\frac{\mu-\nu}{2}+1+s\right)}ds\right.$$

$$\times\sum_{m=0}^{\infty}\frac{\Gamma(-2n+m)(y^2/2)^m}{m!\,\Gamma\left(2+\frac{\mu}{2}-n+m\right)}\int_0^\infty x^{2s+2m+2\mu+3}e^{-\frac{1}{2}x^2y^2}\,dx$$

$$-\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\left\{\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\right\}^2\Gamma(\nu+1+s)\Gamma(-s)(-)^s}{\Gamma\left(\frac{\mu+\nu}{2}+2+s\right)\Gamma\left(\frac{\nu-\mu}{2}+1+s\right)}\,ds$$

$$\left.\times\sum_{m=0}^{\infty}\frac{\Gamma(-2n+m)(y^2/2)^m}{m!\,\Gamma\left(2+\frac{\mu}{2}-n+m\right)}\int_0^{\infty}x^{2s+2m+\mu+\nu+3}\,e^{-\frac{1}{2}x^2y^2}\,dx\right]$$

$$=\frac{\pi\operatorname{cosec}\pi\left(\frac{\nu-\mu}{2}\right)}{2^{2-\frac{\mu+\nu}{2}}\,\Gamma\left(\frac{\mu+\nu}{2}+1\right)}$$

$$\times\left\{\frac{2^{\mu+1}y^{-\mu-\frac{1}{2}}}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\{\Gamma(\mu+1+s\}^2\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(-s)(-)^s\left(\frac{2}{y^2}\right)^s}{\Gamma\left(\frac{\mu-\nu}{2}+1+s\right)\Gamma(\mu+2+s)}\,ds\right.$$

$$\times{}_2F_1\left(-2n,\ \mu+2+s\,,\ 2+\frac{\mu}{2}-n\,,\ 1\right)$$

$$-\frac{2^{\frac{\mu+\nu}{2}+1}y^{-\nu-\frac{1}{2}}}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\left\{\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\right\}^2\Gamma(\nu+1+s)\Gamma(-s)(-)^s\left(\frac{2}{y^2}\right)^s}{\Gamma\left(\frac{\mu+\nu}{2}+2+s\right)\Gamma\left(\frac{\nu-\mu}{2}+1+s\right)}\,ds$$

$$\left.\times{}_2F_1\left(-2n,\ \frac{\mu+\nu}{2}+2+s\,,\ 2+\frac{\mu}{2}-n\,,\ 1\right)\right\}\quad\ldots\quad(6\,1)$$

$$=\frac{\pi\operatorname{cosec}\pi\left(\frac{\nu-\mu}{2}\right)\Gamma\left(2+\frac{\mu}{2}-n\right)}{2^{1-\frac{\mu+\nu}{2}}\,\Gamma\left(\frac{\mu+\nu}{2}+1\right)\Gamma\left(2+\frac{\mu}{2}+n\right)}$$

$$\times\left\{2^{\mu}y^{-\mu-\frac{1}{2}}{}_4f_3\left(\begin{matrix}\mu+1,\ \mu+1,\ \frac{\mu+\nu}{2}+1,\ 1+\frac{\mu}{2}+n\,; \\ \frac{\mu-\nu}{2}+1,\ \mu+2,\ 1+\frac{\mu}{2}-n\,;\end{matrix}\ \frac{2}{y^2}\right)\right.$$

$$\left.-2^{\frac{\mu+\nu}{2}}y^{-\nu-\frac{1}{2}}{}_4f_3\left(\begin{matrix}\frac{\mu+\nu}{2}+1,\ \frac{\mu+\nu}{2}+1,\ \nu+1,\ 1+\frac{\nu}{2}+n\,, \\ \frac{\mu+\nu}{2}+2,\ \frac{\nu-\mu}{2}+1,\ 1+\frac{\nu}{2}-n\,,\end{matrix}\ \frac{2}{y^2}\right)\right\}\quad\ldots\quad(6.2)$$

The integral being convergent for

$$Rl(\mu)>-\tfrac{1}{2},\quad Rl(\mu+\nu+4)>0,$$

$$Rl(\nu)>Rl(\mu)\,;\ n>Rl(\nu),\ \text{and}\ |y|>\sqrt{2}.$$

The change in the order of integration is valid, since,

(i) $$\int_0^\infty \left| x^{2\mu+3+2s}\, e^{-\frac12 x^2y^2}\, {}_1F_1\!\left(-2n\,,\ 2+\frac{\mu}{2}-n\,,\ \tfrac12 x^2y^2\right)\right| dx$$

is convergent for $Rl(\mu) > -2$ and $\frac12\, Rl(\mu)+1-n<0$

(ii) $$\int_{-i\infty}^{i\infty} \left| \frac{\{\Gamma(\mu+1+s)\}^2\,\Gamma\!\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(-s)}{\Gamma\!\left(\frac{\mu-\nu}{2}+1+s\right)\Gamma(\mu+2+s)}\,(-x^2)^s\right| ds$$

is uniformly convergent for $|x|<1$

(iii) The repeated integrals exist

Similar reasoning holds for the other integral

The term by term integration is valid since

(i) ${}_1F_1$ is an integral function and hence is uniformly convergent in any arbitrary interval (O, A) of x

(ii) The integrated function reduces to ${}_2F_1\,(a\,,b\,,\ c,\ 1)$

§7 Lastly, applying the kernel of §4, to the known R_ν function

$$x^{\frac16-\frac{\nu}{3}}\, e^{-\frac14 x^2}\, I_{\frac{\nu}{3}-\frac16}\,(\tfrac14 x^2),$$

we get the R_ν function,

$$F(x)=\frac{\pi\,\mathrm{cosec}\,\pi\left(\frac{\nu-\mu}{2}\right)}{2\left\{\Gamma\!\left(\frac{\mu+\nu}{2}+1\right)\right\}^2}\int_0^\infty\left\{\begin{array}{l} y^{\nu+1}{}_2f_1\!\left(\nu+1,\ \frac{\mu+\nu}{2}+1\,,\ \frac{\nu-\mu}{2}+1\,,\ y^2\right)\\ -y^{\mu+1}{}_2f_1\!\left(\mu+1,\ \frac{\mu+\nu}{2}+1\,,\ 1-\frac{\nu-\mu}{2},\ y^2\right)\end{array}\right\}$$

$$\times(xy)^{\frac16-\frac{\nu}{3}}\, e^{\ \frac14 x^2y^2}\, I_{\frac{\nu}{3}\ \frac16}\,(\tfrac14 x^2y^2)\, dy \qquad (7\ 1)$$

Using the formula,

$$\Gamma(\nu+1)\, e^{-z}\, I_\nu\,(z) = (z/2)^\nu\, {}_1F_1\,(\nu+\tfrac12\,,\ 2\nu+1\,,\ -2z)$$

and substituting the integral (4 3) for the ${}_2f_1$ functions in (7 1), we get

$$F(x)=\frac{x^{\frac13-\frac{\nu}{3}}}{4\pi i\left\{\Gamma\!\left(\frac{\mu+\nu}{2}+1\right)\right\}^2}\int_0^\infty y^{\mu-\frac{\nu}{3}+\frac23}\left\{\frac{(x^2y^2/8)^{\frac{\nu}{3}-\frac16}}{\Gamma\!\left(\frac{\nu}{3}+\frac56\right)}\right.$$

$$\left.\times{}_1F_1\!\left(\frac{\nu}{3}+\frac13\,,\ \frac{2\nu}{3}+\frac23\,,\ -\frac12 x^2y^2\right)\right\}dy$$

$$\times\int_{-i\infty}^{i\infty}\Gamma\!\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(\mu+1+s)\Gamma(-s)\Gamma\!\left(\frac{\nu-\mu}{2}-s\right)y^{2s}\, ds$$

$$F(x) = \frac{x^{\frac{\nu}{3}-\frac{1}{6}}}{2^{\nu+\frac{1}{2}}\left\{\Gamma\left(\frac{\mu+\nu}{2}+1\right)\right\}^2\Gamma\left(\frac{\nu}{3}+\frac{5}{6}\right)}\int_0^\infty y^{\mu+\frac{\nu}{3}+\frac{1}{3}}\,{}_1F_1\left(\frac{\nu}{3}+\frac{1}{3},\ \frac{2\nu}{3}+\frac{2}{3},\ -\frac{1}{2}x^2y^2\right)dy$$

$$\times\int_{-i\infty}^{i\infty}\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(\mu+1+s)\Gamma(-s)\Gamma\left(\frac{\nu-\mu}{2}-s\right)y^{2s}\,ds \quad \ldots \quad (7\ 2)$$

Changing the order of integration, we get

$$F(x) = \frac{x^{\frac{\nu}{3}-\frac{1}{6}}}{2^{\nu+\frac{1}{2}}\left\{\Gamma\left(\frac{\mu+\nu}{2}+1\right)\right\}^2\Gamma\left(\frac{\nu}{3}+\frac{5}{6}\right)}$$

$$\times\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(\mu+1+s)\Gamma(-s)\Gamma\left(\frac{\nu-\mu}{2}-s\right)ds$$

$$\times\int y^{\frac{\nu}{3}+\frac{1}{3}+\mu+2s}\,{}_1F_1\left(\frac{\nu}{3}+\frac{1}{3},\ \frac{2\nu}{3}+\frac{2}{3},\ -\frac{1}{2}x^2y^2\right)dy$$

Since,

$${}_1F_1(\alpha\,,\ \beta\,,\ x) = \frac{\Gamma(\beta)e^x}{\Gamma(\alpha)\Gamma(\beta-\alpha)}\int_0^1 t^{\beta-\alpha-1}(1-t)^{\alpha-1}e^{-xt}\,dt$$

$$[Rl(\beta) > Rl(\alpha) > 0]$$

we get, on substitution,

$$F(x) = \frac{x^{\frac{\nu}{3}-\frac{1}{6}}\ \Gamma\left(\frac{2\nu}{3}+\frac{2}{3}\right)}{2^{\nu+\frac{1}{2}}\left\{\Gamma\left(\frac{\mu+\nu}{2}+1\right)\right\}^2\Gamma\left(\frac{\nu}{3}+\frac{5}{6}\right)\left\{\Gamma\left(\frac{\nu}{3}+\frac{1}{3}\right)\right\}^2}$$

$$\times\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(\mu+1+s)\Gamma(-s)\Gamma\left(\frac{\nu-\mu}{2}-s\right)ds$$

$$\times\int_0^\infty y^{\frac{\nu}{3}+\frac{1}{3}+\mu+2s}\,e^{-\frac{1}{2}x^2y^2}\,dy\int_0^1 [t(1-t)]^{\frac{\nu}{3}-\frac{2}{3}}\ e^{\frac{1}{2}x^2y^2t}\ dt.$$

'Again, changing the order in integration of the y and t integrals since both are uniformly and absolutely convergent, and integrating the y-integral, we get,

$$F(x) = \frac{x^{\frac{\nu}{3}-\frac{1}{6}}\ \Gamma\left(\frac{2\nu}{3}+\frac{2}{3}\right)2^{\frac{\mu}{2}-\frac{5\nu}{6}-\frac{5}{6}}}{\left[\Gamma\left(\frac{\mu+\nu}{2}+1\right)\Gamma\left(\frac{\nu}{3}+\frac{1}{3}\right)\right]^2\Gamma\left(\frac{\nu}{3}+\frac{5}{6}\right)}$$

$$\times \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} 2^{s}\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(\mu+1+s)\Gamma(-s)\Gamma\left(\frac{\nu-\mu}{2}-s\right)ds$$

$$\times \int_{0}^{1} \Gamma\left(\frac{\nu}{6}+\frac{\mu}{2}+s+\frac{2}{3}\right) x^{-\left(\frac{\nu}{3}+\mu+2s+\frac{4}{3}\right)}\, t^{\frac{\nu}{3}-\frac{2}{3}}\, (1-t)^{\frac{\nu}{6}-\frac{\mu}{2}-\frac{4}{3}}\ {}^{s}\, dt$$

$$= \frac{x^{-\mu-\frac{1}{2}}\, 2^{\frac{\mu}{2}-\frac{\nu}{6}-\frac{7}{6}}}{\left\{\Gamma\left(\frac{\mu+\nu}{2}+1\right)\right\}^{2}}\ \frac{1}{2\pi i}\int_{i\infty}^{i\infty} \Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(\mu+1+s)\Gamma\left(\frac{\nu}{6}+\frac{\mu}{2}+s+\frac{2}{3}\right)\Gamma(-s)$$

$$\times\Gamma\left(\frac{\nu}{6}-\frac{\mu}{2}-\frac{1}{3}-s\right)\left(\frac{2}{x^{2}}\right)^{s} ds$$

Changing s to $(-s)$ and evaluating the residues at poles of the integrand which lie on the positive half of the real axis, we get

$$F(x) = \frac{2^{\frac{\mu}{2}-\frac{\nu}{6}-\frac{7}{6}}\, x^{-\mu-\frac{1}{2}}}{\left\{\Gamma\left(\frac{\mu+\nu}{2}+1\right)\right\}^{2}}$$

$$\times\left[\left(\frac{x^{2}}{2}\right)^{\frac{\mu+\nu}{2}+1}\sum_{n=0}^{\infty}\frac{\Gamma\left(\frac{\mu-\nu}{2}-n\right)\Gamma\left(\frac{\mu+\nu}{2}+1+n\right)\Gamma\left(-\frac{\nu}{3}-\frac{1}{3}-n\right)\Gamma\left(n+\frac{2\nu}{3}+\frac{2}{3}\right)}{n!}\left(-\frac{x^{2}}{2}\right)^{n}\right.$$

$$+\left(\frac{x^{2}}{2}\right)^{\mu+1}\sum_{n=0}^{\infty}\frac{\Gamma\left(\frac{\nu-\mu}{2}-n\right)\Gamma(\mu+1+n)\Gamma\left(\frac{\nu}{6}-\frac{\mu}{2}-\frac{1}{3}-n\right)\Gamma\left(\frac{\mu}{2}+\frac{\nu}{6}+\frac{2}{3}+n\right)}{n!}\left(-\frac{x^{2}}{2}\right)^{n}$$

$$\left.+\left(\frac{x^{2}}{2}\right)^{\frac{\mu}{2}+\frac{\nu}{6}+\frac{2}{3}}\sum_{n=0}^{\infty}\frac{\Gamma\left(\frac{\nu}{3}+\frac{1}{3}-n\right)\Gamma\left(\frac{\mu}{2}-\frac{\nu}{6}+\frac{1}{3}-n\right)\Gamma\left(\frac{\nu}{6}+\frac{2}{3}+\frac{\mu}{2}+n\right)\Gamma\left(\frac{\nu}{3}+\frac{1}{3}+n\right)}{n!}\left(-\frac{x^{2}}{2}\right)^{n}\right]$$

$$F(x) = \frac{2^{\frac{\mu}{2}-\frac{\nu}{6}-\frac{7}{6}}\, x^{-\mu-\frac{1}{2}}}{\left\{\Gamma\left(\frac{\mu+\nu}{2}+1\right)\right\}^{2}}$$

$$\times\pi^{2}\left[\left(\frac{x^{2}}{2}\right)^{\frac{\mu+\nu}{2}+1}\operatorname{cosec}\pi\left(\frac{\mu-\nu}{2}\right)\operatorname{cosec}\pi\left(-\frac{\nu}{3}-\frac{1}{3}\right){}_{2}F_{2}\left(\begin{matrix}\frac{\mu+\nu}{2}+1,\ \frac{2\nu}{2}+\frac{2}{3}, \\ \frac{\nu}{3}+\frac{4}{3},\ 1-\frac{\mu-\nu}{2},\end{matrix}\ -\frac{x^{2}}{2}\right)\right.$$

$$+\left(\frac{x^{2}}{2}\right)^{\mu+1}\operatorname{cosec}\pi\left(\frac{\nu-\mu}{2}\right)\operatorname{cosec}\pi\left(\frac{\nu}{6}-\frac{\mu}{2}-\frac{1}{3}\right){}_{2}F_{2}\left(\begin{matrix}\mu+1,\ \frac{\mu}{2}+\frac{\nu}{6}+\frac{2}{3}, \\ 1-\frac{\nu-\mu}{2},\ \frac{4}{3}-\frac{\nu}{6}+\frac{\mu}{2};\end{matrix}\ -\frac{x^{2}}{2}\right)$$

$$+\left(\frac{x^2}{2}\right)^{\frac{\nu}{6}+\frac{2}{3}+\frac{\mu}{2}} \operatorname{cosec}\pi\left(\frac{\nu}{3}+\frac{1}{3}\right)\operatorname{cosec}\pi\left(\frac{\mu}{2}-\frac{\nu}{6}+\frac{1}{3}\right){}_2f_2\left(\begin{matrix}\frac{\nu}{3}+\frac{1}{3}, \frac{\nu}{6}+\frac{2}{3}+\frac{\mu}{2}, \\ \frac{2}{3}-\frac{\nu}{3}, \frac{2}{3}-\frac{\mu}{2}+\frac{\nu}{6},\end{matrix} -\frac{x^2}{2}\right)\Bigg]$$

which is the required R_ν function valid for $Rl(\nu)>-1$ and $Rl(\mu)>-1$ and $Rl(\nu+3\mu+4)>0$ and $Rl(\nu-3\mu+1)>0$

The change in the order of integration in (7 2) is permissible, since,

(i) $$\int_{-i\infty}^{i\infty}\left|\Gamma\left(\frac{\mu+\nu}{2}+1+s\right)\Gamma(\mu+1+s)\Gamma(-s)\Gamma\left(\frac{\nu-\mu}{2}-s\right)y^{2s}\right|ds$$

exists for $0 \leq y \leq A$

(ii) $$\int_0^\infty \left|y^{\frac{\nu}{3}+\frac{1}{3}+\mu+2s}{}_1F_1\left(\frac{\nu}{3}+\frac{1}{3}, \frac{2\nu}{3}+\frac{2}{3}, -\frac{1}{2}x^2y^2\right)\right|dy$$

also exists for $Rl(3\mu+\nu+4)>0$ and $Rl(\nu-3\mu+1)>0$

(iii) The integral (7 2) exists for $Rl(\nu)>-1$ and $Rl(3\mu+\nu+4)>0$

ACKNOWLEDGMENT

The author wishes to express his respectful thanks to Dr R S Varma, D Sc, for his generous aid and suggestions leading to the preparation of this paper

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ON THE STRONG SUMMABILITY OF A FOURIER SERIES AND ITS CONJUGATE SERIES

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(Communicated by Dr B N Prasad)

(*Received October 10, read October 18, 1946*)

1 Let the Fourier series corresponding to a function $f(x)$, periodic and integrable (L) with period 2π, be

$$\tag{1 1} \tfrac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

The conjugate series of (1 1) is

$$\tag{1 2} \sum_{n=1}^{\infty} (b_n \cos nx - a_n \sin nx)$$

We denote by $S_n(x)$ and $\bar{S}_n(x)$ the partial sums of the series (1 1) and (1 2) respectively Let

$$\bar{f}_n = \bar{f}_n(x) = \frac{1}{\pi} \int_{\frac{1}{n}}^{\pi} \psi(t) \cot \tfrac{1}{2}t \, dt,$$

where

$$\psi(t) = \psi(t, x) = \tfrac{1}{2}\{f(x+t) - f(x-t)\},$$

and put

$$\tag{1 3} f(x) = \lim_{n \to \infty} \bar{f}_n(x),$$

whenever the latter exists

A series

$$a_0 + a_1 + a_2 +$$

with partial sum A_n, is said to be *strongly summable* with index 2 to s, or summable H_2, if there exists a finite s such that

$$\sum_{\nu=0}^{n} (A_\nu - s)^2 = o(n).$$

The strong summability of a Fourier series and of its conjugate series has been discussed of late by a number of writers like Hardy and Littlewood (1913, 1926, 1935), Fejér (1938), Carleman (1923), Sutton (1925), Szász (1940), Marcinkiewicz (1939), Wang (1944, 1945), Wang (1945) has recently obtained a general condition for the summability H_2 of Fourier series. The object of this paper is to obtain an analogous result for the conjugate Fourier series We prove the following theorem:

Theorem A *If*

$$\int_0^t |\psi(u)|\,du = o\left(\frac{t}{\left\{\log\frac{1}{t}\right\}^{\alpha}}\right), \quad as\ t\to 0 \tag{1 4}$$

for some $\alpha > \frac{1}{2}$, *then the conjugate series* (1 2) *is summable* H_2 *to the sum* $\bar{f}(x)$ *provided that the limit* (1 3) *exists*

My thanks are due to Dr B N Prasad for his kind interest and advice in the preparation of this paper

2 We shall require the following lemma in order to prove the above theorem

Lemma 1 *If*

$$\int_0^t |\psi(u)|\,du = o(t), \quad as\ t\to 0, \tag{2 1}$$

then

$$\sum_{\nu=1}^{n}\{\bar{S}_\nu(x)-\bar{f}_n(x)\}^2 = \frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t^2}\,dt\int_{\frac{1}{n}}^{t}\psi(u)\frac{\sin n(u-t)}{u-t}\,du+o(n)$$

Proof

It is known that

$$\bar{S}_\nu(x) = \frac{1}{\pi}\int_0^{\pi}\psi(t)\cot\tfrac{1}{2}t(1-\cos\nu t)\,dt+o(1),$$

$$\text{or}\quad \bar{S}_\nu(x) = \frac{1}{\pi}\int_0^{\frac{1}{n}}\psi(t)\cot\tfrac{1}{2}t(1-\cos\nu t)\,dt+\frac{1}{\pi}\int_{\frac{1}{n}}^{\pi}\psi(t)\cot\tfrac{1}{2}t(1-\cos\nu t)\,dt+o(1)$$

$$= \frac{2}{\pi}\int_0^{\frac{1}{n}}\frac{\psi(t)}{t}\left(2\sin^2\frac{\nu t}{2}\right)dt+\frac{1}{\pi}\int_{\frac{1}{n}}^{\pi}\psi(t)\cot\tfrac{1}{2}t\,(1-\cos\nu t)\,dt+o(1)$$

Since

$$\left|\frac{2}{\pi}\int_0^{\frac{1}{n}}\frac{\psi(t)}{t}\left(2\sin^2\frac{\nu t}{2}\right)dt\right| \le \frac{2}{\pi}\int_0^{\frac{1}{n}}\nu\,|\psi(t)|\,dt = \nu o\left(\frac{1}{n}\right) = o(1),$$

we have

$$(\bar{S}_\nu(x)-\bar{f}_n) = -\frac{1}{\pi}\int_{\frac{1}{n}}^{\pi}\psi(t)\cot\tfrac{1}{2}t\cos\nu t\,dt+o(1)$$

$$= -\frac{2}{\pi}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\cos\nu t\,dt+o(1),$$

where δ is small, but fixed.

Hence

$$\sum_{\nu=1}^{n} (\bar{S}_\nu(x)-\bar{f}_n)^2 = \frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\,\frac{\psi(u)}{u}\left\{\sum_{1}^{n}\cos\nu t\cos\nu u\right\}dt\,du+o(n)$$

$$= \frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)\,\psi(u)}{tu}\left\{\sum_{1}^{n}(\cos\nu(u+t)+\cos\nu(u-t))\right\}dt\,du+o(n)$$

$$= \frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)\,\psi(u)}{tu}\left[\frac{\sin(n+\frac{1}{2})(u-t)}{2\sin\frac{1}{2}(u-t)}-\frac{1}{2}\right]dt\,du \tag{2 2}$$

$$+\frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)\,\psi(u)}{tu}\left[\frac{\sin(n+\frac{1}{2})(u+t)}{2\sin\frac{1}{2}(u+t)}-\frac{1}{2}\right]dt\,du+o(n)$$

Now

$$\int_{\frac{1}{n}}^{\delta}\frac{|\psi(t)|}{t}\,dt\int_{\frac{1}{n}}^{\delta}\frac{|\psi(u)|}{u}\,du = o\{(\log n)^2\} = o(n)$$

For, putting $\Psi(t)=\int_0^t |\psi(u)|\,du$ and integrating by parts, we have

$$\int_{\frac{1}{n}}^{\delta}\frac{|\psi(u)|}{u}\,du = \left[\frac{\Psi(u)}{u}\right]_{\frac{1}{n}}^{\delta}+\int_{\frac{1}{n}}^{\delta}\frac{\Psi(u)}{u^2}\,du$$

$$= O(1)+\int_{\frac{1}{n}}^{\delta}o\left(\frac{1}{u}\right)du$$

$$= O(1)+o(\log n)$$

$$= o(\log n)$$

Hence we obtain from (2 2)

$$\sum_{\nu=1}^{n}(\bar{S}_\nu(x)-\bar{f}_n)^2 = \frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\,dt\int_{\frac{1}{n}}^{\delta}\frac{\psi(u)}{u}\cdot\frac{\sin n(u-t)}{u-t}\,du$$

$$+\frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\,dt\int_{\frac{1}{n}}^{\delta}\frac{\psi(u)}{u}\cdot\frac{\sin n(u+t)}{u+t}\,du+o(n) \tag{2 3}$$

$$= J_1+J_2+o(n)$$

Now

$$\frac{1}{u(u-t)}=\frac{1}{t}\left[\frac{1}{u-t}-\frac{1}{u}\right]$$

and

$$\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\,dt\int_{t}^{\delta}\frac{\psi(u)}{u}\,\frac{\sin n(u-t)}{u-t}\,du=\int_{\frac{1}{n}}^{\delta}\frac{\psi(u)}{u}\,du\int_{\frac{1}{n}}^{u}\frac{\psi(t)}{t}\,\frac{\sin n(u-t)}{u-t}\,dt$$

$$\therefore J_1=\frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\,dt\int_{\frac{1}{n}}^{\delta}\frac{\psi(u)}{u}\,\frac{\sin n(u-t)}{u-t}\,du$$

$$=\frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\,dt\int_{\frac{1}{n}}^{t}\frac{\psi(u)}{u}\,\frac{\sin n(u-t)}{u-t}\,du+\frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\,dt\int_{t}^{\delta}\frac{\psi(u)}{u}\,\frac{\sin n(u-t)}{u-t}\,du$$

$$=\frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\,dt\int_{\frac{1}{n}}^{t}\frac{\psi(u)}{u}\,\frac{\sin n(u-t)}{u-t}\,du+\frac{2}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(u)}{u}\,du\int_{\frac{1}{n}}^{u}\frac{\psi(t)}{t}\,\frac{\sin n(u-t)}{u-t}\,dt$$

$$=\frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t}\,dt\int_{\frac{1}{n}}^{t}\frac{\psi(u)}{t}\left[\frac{1}{u-t}-\frac{1}{u}\right]\sin n(u-t)\,du$$

$$=\frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t^2}\,dt\int_{\frac{1}{n}}^{t}\psi(u)\,\frac{\sin n(u-t)}{u-t}\,du-\frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t^2}\,dt\int_{\frac{1}{n}}^{t}\psi(u)\,\frac{\sin n(u-t)}{u}\,du.$$

Hence

$$J_1=\frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t^2}\,dt\int_{\frac{1}{n}}^{t}\psi(u)\,\frac{\sin n(u-t)}{u-t}\,du+O\left\{\int_{\frac{1}{n}}^{\delta}\frac{|\psi(t)|}{t^2}\,dt\int_{\frac{1}{n}}^{t}\frac{|\psi(u)|}{u}\,du\right\}. \tag{2.4}$$

Now integrating by parts, we get

$$\int_{\frac{1}{n}}^{\delta}\frac{|\psi(t)|}{t^2}\,dt\int_{\frac{1}{n}}^{t}\frac{|\psi(u)|}{u}\,du=\int_{\frac{1}{n}}^{\delta}\frac{|\psi(t)|}{t^2}\left\{\left[\frac{\Psi(u)}{u}\right]_{\frac{1}{n}}^{t}+\int_{\frac{1}{n}}^{t}\frac{\Psi(u)}{u^2}\,du\right\}dt$$

$$=\int_{\frac{1}{n}}^{\delta}\frac{|\psi(t)|}{t^2}\left\{\frac{\Psi(t)}{t}+o(1)+o(\log nt)\right\}dt$$

$$=O\left\{\int_{\frac{1}{n}}^{\delta}\frac{|\psi(t)|}{t^2}\log nt\,dt\right\}.$$

Again

$$\int_{\frac{1}{n}}^{\delta} \frac{|\psi(t)|}{t^2}\log nt\,dt = \left[\frac{\Psi(t)}{t^2}\log nt\right]_{\frac{1}{n}}^{\delta} - \int_{\frac{1}{n}}^{\delta}\frac{\Psi(t)}{t^3}\,dt + 2\int_{\frac{1}{n}}^{\delta}\frac{\Psi(t)}{t^3}\log nt\,dt$$

$$= o(n) + o\left(n\int_{1}^{n\delta}\frac{dv}{v^2}\right) + o\left(n\int_{1}^{n\delta}\frac{\log v}{v^2}\,dv\right)$$

$$= o(n).$$

Thus from (2.4) we get

$$J_1 = \frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t^2}\,dt\int_{\frac{1}{n}}^{t}\psi(u)\,\frac{\sin n\,(u-t)}{u-t}\,du + o(n) \tag{2.5}$$

Now

$$J_2 = \frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t^2}\,dt\int_{\frac{1}{n}}^{t}\psi(u)\left[\frac{1}{u}-\frac{1}{u+t}\right]\sin n\,(u+t)\,du$$

$$= \frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t^2}\,dt\int_{\frac{1}{n}}^{t}\frac{\psi(u)}{u}\sin n\,(u+t)\,du - \frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t^2}\,dt\int_{\frac{1}{n}}^{t}\frac{\psi(u)}{u+t}\sin n\,(u+t)\,du$$

$$|J_2| \le \frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{|\psi(t)|}{t^2}\,dt\int_{\frac{1}{n}}^{t}\frac{|\psi(u)|}{u}\,du + \frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{|\psi(t)|}{t^2}\,dt\int_{\frac{1}{n}}^{t}\frac{|\psi(u)|}{u}\,du$$

Hence

$$J_2 = O\left\{\int_{\frac{1}{n}}^{\delta}\frac{|\psi(t)|}{t^2}\,dt\int_{\frac{1}{n}}^{t}\frac{|\psi(u)|}{u}\,du\right\} = o(n) \tag{2.6}$$

Thus Lemma 1 follows from (2.5) and (2.6)

3 Proof of Theorem A.

From Lemma 1, we have

$$\sum_{\nu=1}^{n}(\bar{S}_\nu(x)-\bar{f}_n)^2 = \frac{4}{\pi^2}\int_{\frac{1}{n}}^{\delta}\frac{\psi(t)}{t^2}\int_{\frac{1}{n}}^{t}\frac{\psi(u)}{u-t}\sin n(u-t)du + o(n) \tag{3.1}$$

Now

$$\int_{\frac{1}{n}}^{t}\psi(u)\,\frac{\sin n\,(u-t)}{u-t}\,du = O\left\{n\int_{0}^{t}|\psi(u)|\,du\right\}$$

$$= o\left\{\frac{nt}{[\log(1/t)]^{\alpha}}\right\} \text{ (from (1.4)).}$$

Thus

$$\int_{\frac{1}{n}}^{\delta} \frac{\psi(t)}{t^2}\, dt \int_{\frac{1}{n}}^{t} \psi(u)\, \frac{\sin n\,(u-t)}{u-t}\, du = o\left\{ n \int_{\frac{1}{n}}^{\delta} \frac{|\,\psi(t)\,|}{t\{\log 1/t\}^{\alpha}}\, dt \right\}. \tag{3 2}$$

But

$$\int_{\frac{1}{n}}^{\delta} \frac{|\,\psi(t)\,|}{t\left(\log\frac{1}{t}\right)^{\alpha}}\, dt = \left[\frac{\Psi(t)}{t\left(\log\frac{1}{t}\right)^{\alpha}} \right]_{\frac{1}{n}}^{\delta} + \int_{\frac{1}{n}}^{\delta} \frac{\Psi(t)}{t^2\left(\log\frac{1}{t}\right)^{\alpha}}\, dt - \alpha \int_{\frac{1}{n}}^{\delta} \frac{\Psi(t)}{t^2\left(\log\frac{1}{t}\right)^{\alpha+1}}\, dt$$

$$= O(1) + O\left(\int_{\frac{1}{n}}^{\delta} \frac{dt}{t\left(\log\frac{1}{t}\right)^{2\alpha}} \right) + O\left(\int_{\frac{1}{n}}^{\delta} \frac{dt}{t\left(\log\frac{1}{t}\right)^{2\alpha+1}} \right)$$

$$= O(1) + O\left(\int_{\frac{1}{\delta}}^{n} \frac{dv}{v(\log v)^{2\alpha}} \right) + O\left(\int_{\frac{1}{\delta}}^{n} \frac{dv}{v(\log v)^{2\alpha+1}} \right)$$

$$= O(1) \tag{3 3}$$

The theorem follows from (3 1), (3 2) and (3 3)

4 It will be observed that Wang's theorem (*loc cit*) about Fourier series referred to above, may be put in a different form also It is

Theorem B If for some $\beta > \frac{1}{2}$,

$$\int_{0}^{t} |\,\phi(u)\,|\, du = O\left(\frac{t}{\left(\log\frac{1}{t}\right)^{\beta}} \right), \quad as\ t \to 0 \tag{4 1}$$

where $\phi(t) = \phi(x, t) = \frac{1}{2}\{f(x+t)+f(x-t)-2s\}$,

then the Fourier series (1 1) is summable H_2 *to the sum* s

It can easily be seen that (4 1) implies

$$\int_{0}^{t} |\,\phi(u)\,|\, du = o\left(\frac{t}{\left(\log\frac{1}{t}\right)^{\alpha}} \right), \text{ as } t \to 0, \tag{4 2}$$

for $\frac{1}{2} < \alpha < \beta$

Hence the theorem will hold in view of Wang's condition

5. The corresponding form of the theorem for the conjugate series is

Theorem B′. If for some $\beta > \frac{1}{2}$,

$$\int_{0}^{t} |\,\psi(u)\,|\, du = O\left(\frac{t}{\left(\log\frac{1}{t}\right)^{\beta}} \right), \tag{5.1}$$

then the conjugate series (1.2) is summable H_2 *to the sum* $\bar{f}(x)$, *provided that the limit (1 3) exists*

The proof of this theorem is similar to that of Theorem B.

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STUDIES IN THE ASSOCIATION OF PLANT CHARACTERS AND PEST INCIDENCE.

I NATURE OF LEAF SURFACE AND MITE ATTACK IN SUGARCANE

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(*Received February 11, read April 4, 1947*)

I INTRODUCTION

Till recently, attack of mites on sugarcane had not received much attention, evidently due to its minor importance as a plant pest It has been recorded on sugarcane from various parts of the world (Wolcott, 1921, Swezy, 1923, Williams, 1931, Holloway, 1936) including places within India*, viz, Coimbatore (Cherian, 1934), Punjab (Rahman and Sapra, 1940) and Sind (Haroon Khan and Bhatia, 1946) In all these places, it has been found attacking sugarcane, generally during the drier part of the season, viz, April to June and disappearing with the onset of monsoon rains So far it has not been recorded from Bihar or Eastern India

Though not considered as a serious pest, the attack of mites considerably reduces the effective functioning area of the leaf surface by the innumerable webs built on the lower surface The portions affected often become colourless and spotted with reddish specks, evidently due to a discolouration or disintegration of the chloroplasts in those portions The pest usually attacks fully developed and mature leaves and it is not improbable that its attack hastens their drying up and prevents the plant from getting full benefit of these leaves While its deleterious effect is not perceptible it is probable, as stated by Swezy (1923), that there is a check at least in growth, particularly because the attack coincides with a period of stress, when the plant is trying to tide over the hot dry season

II OBSERVATIONS.

The attack of mites was first noticed on a clump of *S spontaneum* collected from Bikna-Thoree in 1943 and grown at the Central Sugarcane Research Station, Pusa The attack was seen about the middle of May in a clump of the above species, transplanted and grown near the net house in the laboratory area In a few days the attack progressed to a neighbouring clump of another form of *S spontaneum* and also to a few pots of sugarcane seedlings kept inside the net house By the end of May, the attack became widespread and progressed to some Coimbatore canes grown in the vicinity The varieties attacked included among others Co 313, Co 331, and Co 513 The attack was so rapid that the leaves of almost all the above-mentioned varieties became spotted by the innumerable webs, which appeared as a number of small whitish patches, on the under-surface of the lamina (Pl III, fig 1) These patches on examination showed all stages of the pest, ranging from eggs and just hatched larvæ to well-developed adults The latter were often freely moving about and evidently became responsible for fresh infestations of other leaves and plants in the neighbourhood. The infestations generally started along the margins of the leaves or from the tips and progressed both inwards and backwards, until the lower surface of the leaves was literally spotted with the innumerable patches of webs (Pl III, fig 1) The webs were confined only to the blade portions of the leaves and were not found on the midribs

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* The mites attacking sugarcane and the wild *Saccharums* in Bihar have been identified as *Paratetranychus indicus* Hirst.

The most interesting feature noticed about the infestation of this mite was the peculiarly complete immunity from its attack of three clumps, one of *S arundinaceum* and two others of two different forms of *S munja*, which were growing side by side with the forms of *S spontaneum*, in which the attack was first noticed and was severe throughout the season While the pest had gone and attacked sugarcane seedlings inside the net house and Coimbatore canes standing farther away, they had surprisingly enough, left unattacked the above clumps which were all standing and growing more or less touching the above forms of *S spontaneum* As is well known the spread of this pest takes place by the migration of the females to fresh leaves, aided by contact of such leaves with the infested ones or by the dispersal of mites by wind It is interesting, therefore, that the above-mentioned clumps of *S arundinaceum* and *S munja* did not show any *infestation throughout* the period of nearly six months—May to November—when the attack was spreading around them to various Coimbatore varieties grown in the vicinity More or less similar immunity was noticed in the Jhilli Nursery also where an attack of this mite was noticed only on the forms of *S spontaneum* grown in the wild *Saccharum* block and not on any forms of *S arundinaceum* or *S munja* growing along with them

In order to understand and to find out any possible plant characters which may be associated with this variation in varietal susceptibility, an examination was made of the leaves of different varieties, including those of Coimbatore canes which were found most susceptible Since the attack generally took place on the lower surface of the blade portion of the lamina, a study was made of the epidermal characters of the under-surface of the leaves, both in sections taken across and in peelings obtained by maceration In all cases, wherever infestation by the mites was noticed, particularly in the forms of *S spontaneum* and Coimbatore canes, the lower surface was characterised by the presence of well-marked stomatal grooves, often lined by characteristic spinous outgrowths on the ridges adjoining the grooves (Pl III, figs 4–7, Pl IV, figs 8, 11-13), while in the forms of *S arundinaceum* and *S munja* which were found to be free from infestation, these grooves and spinous outgrowths were completely absent and the surface was nearly smooth (Pl III, figs 2 and 3, Pl IV, figs. 9 and 10) The difference between the varieties regarding these features is noteworthy, since it appears probable that their susceptibility to mite attack is evidently closely related to them From what is known already, these mites have the habit of selecting generally sheltered places on the under-surface of the leaves, particularly concave areas between two veins (Rahman and Sapra, 1945) or between the midrib and the lamina for spinning their webs. The stomatal grooves, wherever they are present are located between the veins and evidently afford an ideal place for shelter and the spinous outgrowths on either side evidently give proper holds for the webs to be spun This evidently explains the preference shown by the mites towards those varieties, viz , *S spontaneum* and Coimbatore canes which have stomatal grooves and spinous projections on the under-surface of their leaves and the immunity of the forms of *S. arundinaceum* and *S. munja*, which show an entire absence of the above features Even amongst the susceptible varieties, it is interesting to note the preference shown by the mites to the margins and tips of the leaves affected It is not improbable that this attraction may be due to the closer approximation of stomatal grooves towards the margins and tips and the presence of better developed spines and aspirites, which as suggested already give suitable place of shelter and proper hold for spinning webs by the pest.

Lastly, it is noteworthy to record here the presence of this mite on a common meadow grass, *Dicanthium annulatum*, on which it has been noticed almost throughout the season. An examination of the under-surface of the leaves of this grass also shows the presence of tubercled hairs and a slight grooving between veins which as stated already, afford suitable shelter and hold for the spinning of webs as in the canes described above

III Summary.

The paper records the incidence of mites (*Paratetranychus indicus* Hirst) on some sugarcane varieties and wild *Saccharums* at Pusa, Bihar The attack was first noticed in clumps of *S spontaneum* and spread later on to some other varieties of sugarcane in the vicinity Forms of *S munja* and *S arundinaceum* growing in the same area were found to be free from the attack throughout the season An examination of certain plant characters associated with the attack of mites indicated the probability that the presence or absence of stomatal grooves on the under-surface of the leaves might be responsible for this variation in susceptibility While forms of *S. spontaneum* and varieties of sugarcane that were attacked, have stomatal grooves and spinous outgrowths protecting them on the under-surface of their lamina, forms of *S munja* and *S arundinaceum* which were throughout free from attack, have no such stomatal grooves or spinous outgrowths It is suggested that the presence of stomatal grooves between the veins afford suitable shelter for the mites and the spinous outgrowths afford suitable hold for spinning their webs and thus explain the susceptibility of sugarcane varieties and forms of wild *Saccharums* having these features, in contrast to the immunity of those forms showing an absence of such characters

IV Acknowledgments

The work was carried out as part of the Sugarcane Research Scheme in Bihar being financed jointly by the Indian Central Sugarcane Committee and the Bihar Government to whom grateful thanks are due

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Explanation of Plates

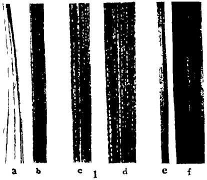
Plate III.

Fig 1. Leaves of different varieties of sugarcane and wild *Saccharums* showing attack of mites; (*a*) and (*b*) *S spontaneum*, (*c*) *Co* 513, (*d*) *Co* 313, (*e*) *S munja* and (*f*) *S arundinaceum* The last two varieties are altogether free from attack

Figs 2–7 Cross section of the lamina of the different varieties of sugarcane and wild *Saccharums* showing the absence of stomatal grooves on the under surface of the leaves of *S munja* (Fig 3) and *S arundinaceum* (Fig 2) and their presence in *S spontaneum* (Fig 4) and Co. varieties, *Co* 513 (Fig 5), *Co* 313 (Fig 6) and *Co* 331 (Fig 7)

Plate IV.

Figs. 8–13 Epidermal peelings of the lower surface of the lamina of the different varieties of sugarcane and wild *Saccharums* showing the presence or absence of stomatal grooves, Fig. 8, *S spontaneum*; Fig 9, *S munja*; Fig 10, *S. arundinaceum*; Fig 11, *Co* 313; Fig 12, *Co* 331 and Fig. 13, *Co* 513.



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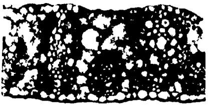

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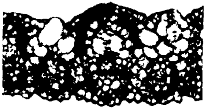

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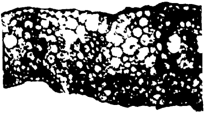

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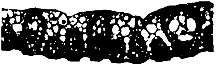

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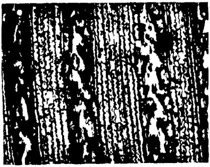

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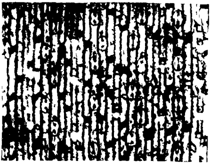

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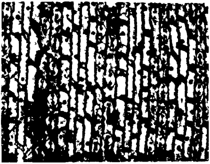

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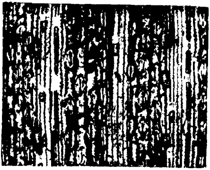

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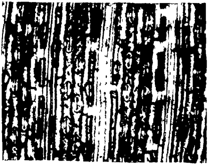

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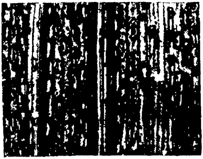

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ON THE STRUCTURE AND DEVELOPMENT OF CTENOID SCALES IN CERTAIN INDIAN FISHES.

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(Communicated by Rai Bahadur Dr S L Hora, D Sc, F.N I)

(*Received June 13, read August 1, 1947*)

ABSTRACT

The paper gives the structure and development of ctenoid scales in certain Indian fishes The formation of spines is not influenced by the nature of the circuli Each spine is an integral part of a ctenoid scale, consisting of several component basal parts, the apices of which may be pointed or displaced The one rowed nature of ctenii by secondary distortion gives a false impression that they are multi rowed The ctenoid scale is an advance over the cycloid from which the former is evolved The spines and circuli are formed from the same substance but independently It is suggested that the ctenii have arisen under the influence of a physical factor, namely, differential movements of the various parts of the body of the fish

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I. INTRODUCTION.

The scales of fishes have been studied by a number of investigators, both from the standpoint of evolution and practical application to the question of determination of age. All the previous researches on scales reveal that there still exists a large scope for investigation into the genesis of ctenoid scales Besides, the relation of the ctenoid scales to the cycloid has so far been imperfectly understood An attempt has been made in this paper to determine the nature and evolution of the ctenoid scales in certain Indian fishes.

In 1945, the Zoology Department of the Calcutta University collected earlier stages of a number of species of fish, such as, *Sciaena coitor*, *Anabas testudineus*, *Mugil spp.*, *Colisa lalius*, *Colisa fasciata*, and other species possessing ctenoid scales in the adult stage. The materials had been preserved in 4% formalin and were found quite good for study of the earlier stages of scale development. The results embodied here are based on an examination of at least five specimens of each stage. A few of the scales were stained with Borax Carmine, and were compared with untreated scales. Average length of the fishes studied is stated in respective cases.

II. ACKNOWLEDGMENTS

We are grateful to Prof H K Mookerjee for placing the materials at our disposal and for his guidance in course of the study We are also thankful to Mr D Mukerji for his comments and criticism Mr S N Banerjee was kind enough to prepare for us the photomicrographs and Mr U Parui made some of the illustrations We offer our sincere thanks to them

III TERMINOLOGY

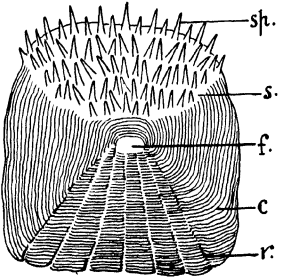
In our account we have followed the terminology used by Taylor (1914–16), and for convenience of reference, we give below a brief definition of the terms used in this article

Circuli are relief lines on the exterior surface of scales, arranged in concentric or nearly concentric rings.

Focus is the area of the scale round which circuli are concentrically arranged The focus usually lies at the centre of the scale

Radii are markings, radiating from the periphery to the focus of a scale They are found usually on the anterior surface, but sometimes on the posterior field too

Ctenii are spinous outgrowths in the posterior field of the scales of certain Actinopterygian fishes and are the characteristic features of the ctenoid scales



TEXT-FIG 1 —Diagrammatic representation of a ctenoid scale of *Sciaena coitor*

sp —Spines (ctenii) undergone displacement giving the multi rowed appearance, *s* —Space between the spines and the circuli, *f* —Focus, *c* —Circuli, *r* —Radii

Anterior field **is the portion of the scale that lies in the scale pocket, and is directed toward the inferior end of the fish.**

Posterior field **is the portion of the scale which is exposed and directed towards the posterior end of the fish. In ctenoid scales, this portion is covered with spines.**

Lateral fields **are the portions of the scale directed dorsally and ventrally in relation to the position of the fish.**

Inferior side **of a scale is that which lies close to the body and is plain.**

Superior side **of a scale is that which is exposed and sculptured.**

IV. STRUCTURE OF SCALES IN *SCIAENA COITOR* (HAMILTON)

Sciaena coitor is an Acanthopterygian fish commonly found in the larger rivers of India and Burma. Though it attains only a foot in length, it is a valuable food fish.

According to the nature of the scales, the surface region of the fish can be distinguished into the following four areas :—

(i) *Typical cycloid scales without radii* (Pl. VI, Fig. 7).—Head region.
(ii) *Cycloid scales with a few radii* (Pl. VI, Fig. 8).—Scales covering the fins and the anal region.
(iii) *Ctenoid scales with radiating spines* (Pl. VI, Fig. 9).—On the lateral line. The scales on the lateral line are peculiar in their structure. The spines of these scales are continuous radiating rays or spines projecting into the posterior field. A space exists in between the circuli and the place of origin of the spines. These radiating rays may often be broken, but generally they are uninterrupted. The epidermis covers the scales on the entire length of the lateral line on either side.
(iv) *Typical ctenoid scales* (Pl. VI, Fig. 11) covering the rest of the body, though the scales on the caudal peduncle are somewhat different from those in other parts of the body.

In a typical body scale the spines are arranged in a single row, each spine being made up of several component basal parts. The apical block is pointed, while the remaining ones may be blunt. On the caudal peduncle, the component parts of the spine are displaced or distorted so as to create a false impression of the arrangement of spines into several rows.

Attention may here be invited to Gunther's (as depicted by Parker and Haswell, 1940) drawing of a ctenoid scale in which spines are shown as many-rowed structures. This is usually not the normal condition as many-rowed structures are secondarily produced during development (*vide infra*), and may give a false notion that the spines are many-rowed structure.

V. DEVELOPMENT OF TYPICAL CTENOID SCALES IN *SCIAENA COITOR*

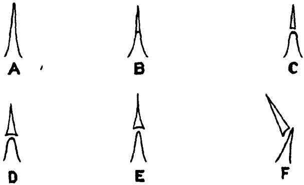
No trace of scale in the fish was found up to a length of 14 mm., but later patches of scales appeared here and there on the body. At this stage, a scale shows a lamellar structure with few circuli (Pl. V, Fig. 1). The circuli appear as deposits on the *superior* surface of the lower layer. This secreted layer has been variously termed as superior layer (Kuntzmann, 1824, Mandl, 1864, Ussow, 1897), hyalodentine (Hofer, 1889) and cell-less ganoin (Parker and Haswell, 1940). The superior side of the scale lacks a shining surface.

16 mm. Stage (Pl. V, fig. 2).—The superior side of the scale develops a shining surface, the number of circuli increases, and radii appear as cracks on the upper surface due to the folding of the lamella. Generally, radii develop in the posterior field and divide the circuli centripetally starting from the periphery and ending toward the focus.

17 mm. Stage (Pl. V, Fig. 3).—At this stage, the circuli become interrupted in the posterior field, while the secreted substance forming the circuli is drawn out in a radial direction instead of being laid out in the form of a complete ring, so as to give rise to the formation of a spine.

20 mm. Stage (Pl. V, Figs. 4, 5, 6).—With the growth of the scale, more and more circuli appear, but in the posterior field the continuity of ring formations is interrupted by the formation of spines. The number of spines formed seems to depend on the space available between the ends of the circuli that were interrupted, but there is no correspondence between the number of spines and the position of the successive circuli It would, therefore, appear that the formation of spines is independent of that of the circuli, though both are derived from the same material

Owing to the continued deposition of the material secreted the spines go on increasing in size It soon begins to project beyond the edge of the scale The projecting part becomes separated by a crack at the base of the spine Owing to the flexibility of the apical part of the spine, the crack assumes an arc-like shape somewhat resembling a *synovial cavity* As growth proceeds, the apical portion of the spine is pushed out more and more, and more cracks appear in the basal part Thus in the adult stage each spine appears as composed of several basal component halves It will also follow that the earlier formed spines will be larger than those developing subsequently



TEXT FIG 2

Diagrammatic representation of the formation of the component basal parts of ctenii

A—an individual spine at a very early stage of development, *B*—later a crack appears in the spine, *C*—the crack becomes prominent and separates the individual spine into an anterior apical and a posterior basal component, *D*—the crack assumes an arc-like shape resembling somewhat synovial cavity, *E*—the apex of the basal component is also being pointed, *F*—a displacement due to some physical factor (*viz* differential movement of the body of the fish) has caused the apex and the corresponding basal component of the spine to move wide apart, thus giving the so called multi rowed nature

Thus, in the posterior field of a ctenoid scale, there are spines of different ages and consequently of different lengths The older the spine, the larger the number of basal components it is likely to have

The fully formed structure of ctenoid scale is attained at the 20 mm stage though additional circuli, radii and spines continue to be formed with the further growth of the fish.

VI. SCALES OF *ANABAS TESTUDINEUS*

(i) *Structure* (Pl VI, Fig 10).—Though Cockerell (1912-13) gave an account of the ctenoid scales in *Anabas testudineus*, he did not report that the scales in the same fish on the head, operculum and nape region remained cycloid throughout life, as has been observed by us He also recorded that the lateral line scales have their spines continuous at the base with the circuli We find that these remain detached, although in other respects we confirm his observations

(ii) *Development* —The development of scales is noticed at the 11 mm stage, and it follows the same course as outlined above for *Sciaena coitor* The spines appear at the 17 mm stage In the earlier stages, the spines have one-rowed components arranged in a single row, but later on, due to displacement, the basal components superficially appear as made of several rows.

VII. SCALES OF *MUGIL SPP*, *COLISA LALIUS* AND *COLISA FASCIATA*

Preliminary observations regarding the development of scales in *Mugil spp*, *Colisa lalius* and *Colisa fasciata* show that the ctenoid pattern is developed more or less in the same way as noted above in the case of *Sciaena coitor*

VIII ORIGIN OF SPINES

We agree with Klaatsch (1890), Duncker (1896) and Creaser (1926) that the ctenoid condition of a scale is an advance on the cycloid nature and we disagree with Boudelot (1873), who regarded cycloid condition as a secondary feature produced through the dropping of spines in certain cases

According to Boudelot, the spines have their origin in the serrae on the edges of the posterior circuli Cockerell and Moore (1910) considered that the spines arise through the modification of the circuli, which in the apical region retain their vertical position In their opinion, a scale with completely transverse apical circuli cannot be and cannot become ctenoid Later, Cockerell (1913) opined that the ctenoid feature appears to be derived from the longitudinal apical circuli which are modified and become segmented In the development of the ctenoid scales, we, however, find that the circuli and the spines develop independently (Pl VI, Fig 6, Pl VI, Fig 11) We have came across several instances of vertical circuli of the discontinuous type without showing the least indication of the formation of the spine For instance, photographs of the scales of the Pacific Salmon, published by Gilbert (1912-13) show vertical circuli but no spines It may also be pointed out that the scales of the species described here are provided with completely concentric circuli in the earlier stages though later they become ctenoid It would thus appear that the assumptions of Cockerell and Moore (1910) are not compatible with the results obtained by us Moreover, in adult *S coitor* the circuli are not at all longitudinal but show a tendency to become concentric (Pl VI, Fig 11) and in case of *A testudineus* (Pl VI, Fig 10) the spines are formed in between the apical ends of two successive, longitudinal circuli In view of this, it is difficult to conceive how Cockerell (1913) postulated the formation of spines from the longitudinal apical circuli

From the observations we have recorded above, we are led to conclude that both the circuli and the spines are independently formed by the deposition of a secretion of the lower layer of the scale Thus the spines constitute an integral part of ctenoid scale as already stated by Kuntzmann (1824), and Ussow (1897). We are not in agreement with Mandl (1840), who considered that the spines are comparable to true teeth, or with Peters (1841) and Salsbey (1868), who regarded them as the extension of osseous corpuscles We treated some scales with dilute hydrochloric acid and found the spines vanishing leaving a sculptured surface on the matrix, each component of which represented an individual spine A faint axial projection has been marked at the centre of each component If the spines are composed of osseous corpuscles, such disappearance of spines could not have been possible, as in that case the calcium compound that impregnates the corpuscles might have been dissolved by the action of the acid but the corpuscles which formed the spines should have remained intact

IX EVOLUTION OF SPINES

When one finds fishes with cycloid and ctenoid scales living together in the same span of water, naturally one wonders what functions could these tiny spines perform. Duncker (1896) has shown that ctenoid scales are developed from cycloid scales only when the posterior edge of the latter is raised out of the enclosing epithelium so that a layer of substance bearing ctenii may be laid over the surface of the scale Creaser (1926) has supported Duncker's view in his studies of the scales of Sunfish. He has stated

> 'In the case of Sunfish it appears that only those scales which extend posteriorly into the epidermis as they become imbricated develop ctenii When the scale is first formed, it is an embedded cycloid plate, but as soon as it goes into the epidermis by the rapid extension of its posterior margin in the oblique direction taken by overlapping scales, ctenii begin to be formed on the surface '

It would thus appear that some physical factor rather than a biological cause is responsible for the development of spines Ryder (1893), Taylor (1914–16) and Creaser (1926) have attributed the production of spines to the movements of the body of the fish We are also inclined to support this view as we did not find even radii and spines on the scales of those parts of the body, such as head, which do not show any flexuous movement Taylor (1914–16) has already expressed the view that the number of radii is directly proportional to the flexion of that part of the fish from which the scale is taken We believe the presence or absence of spines is perhaps also attributable to the same cause, namely, the differential movements of the various parts of the body.

X SUMMARY

1 Ctenoid scale is an advance over the cycloid one and the former is evolved from the latter

2 Once evolved, the ctenoid scale is never transformed again to cycloid one.

3 The spines are actually not many-rowed as they look superficially Really there is primarily one row of spines which, owing to some mechanical means (probably constant flexuous movement of the fish's body), crack and finally break into basal component halves to cope with the pressure exerted In many adult fish it is very difficult to detect the one-rowed nature of the spine unless either the developmental history of the scale or the specimens from different regions of the body are studied

4 The spines never originate from the serrae on the edge of the posterior field, nor through the modifications of the apical ends of the vertical circuli as stated by previous authors These are quite separate structures developed independently of the circuli, of course from the same substance The spines in scales do not commence to grow all simultaneously The particular formative stage occurs in greater abundance in the length of the fish specified

5. We are in entire agreement with Taylor that the flexibility of the body is the main cause of the formation of the radii More flexible is the body, more the number of radii in its scales

6 Spines are always found in the scales of these regions of the fish which show strong flexions of the body Probably the spines are formed by the physical phenomenon, namely, the differential movements of the body of the fish

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XII. EXPLANATION OF PLATES

PLATE V

FIG 1 A scale taken from *S cotitor* of 15 mm stage showing the lamellar structure with few circuli Radii and spines are not yet developed It is non shining (×280)

FIG 2 A scale taken from *S cotitor* of 16 mm stage showing development of radii on the posterior field centripetally towards the focus It has developed a shining surface (×260)

FIG 3 A scale taken from *S cotitor* of 17 mm stage, showing increase in number of circuli and a single spine (×250)

FIG 4 A scale taken from *S cotitor* of 18 mm stage, showing the number of spines increased to 3 (×240)

FIG 5 A scale taken from *S cotitor* of 19 mm stage, showing 4 spines (×230)

FIG 6 A scale taken from *S cotitor* of 18 mm stage, showing the crack of the spines into anterior apical and corresponding component basal halves (×230)

PLATE VI

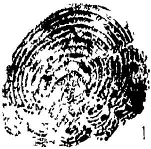
FIG 7 A scale taken from the head region of adult *S cotitor* showing the cycloid nature without radii (×175)

FIG 8 A scale taken from the anal region of adult *S cotitor* showing a cycloid scale with radii (×175)

FIG 9 A scale taken from the lateral line of adult *S cotitor* showing the radiating ctenii on the posterior field (×175)

FIG 10 A scale taken from the caudal peduncle of adult *A testudineus* showing the so called multi rowed appearance of the spines caused secondarily due to displacement of the basal components (×200)

FIG 11 A scale taken from the body of adult *S cotitor* showing the detailed arrangement of apical and the corresponding component basal parts of each spine, between the two respective component parts an arc like space resembling synovial cavity is seen Distinct space exists between the circuli and the spinous region (×240)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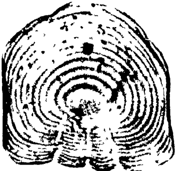
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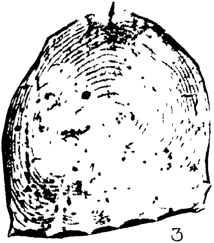
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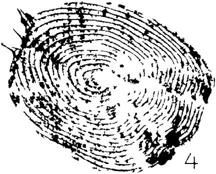
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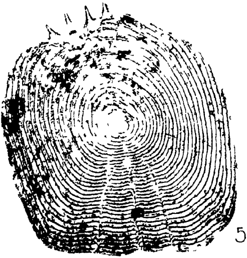
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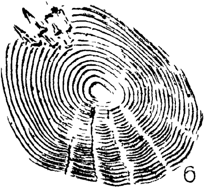
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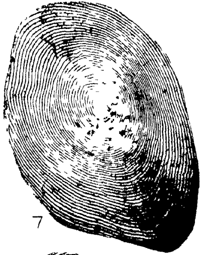

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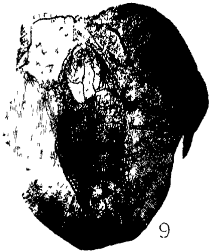

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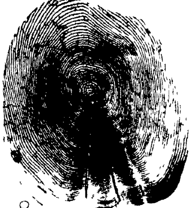

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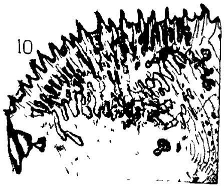

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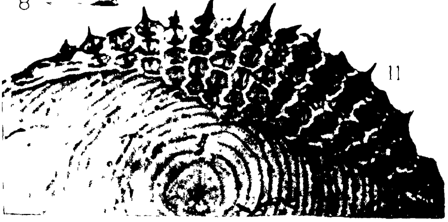

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ON ACETYLATION OF CELLULOSE IN RAW JUTE FIBRE

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(Communicated by Prof M N Saha, F R S)

(*Received July 26, read December 5, 1947*)

Abstract

Results of X ray analysis of acetylated raw jute fibre are compared with those for acetylated delignified jute fibre It is found that the two patterns differ from each other although the length of the *b* axis is the same in the two cases The degree of acetylation at different instants during the period of treatment for acetylation extending up to 48 hours has also been determined by chemical analysis and has been found to be less than that observed in the case of ramie by Hess and Trogus It has been observed that the rate of acetylation reduces to almost zero after two acetyl groups are taken up by each glucose unit and the formation of triacetate begins after twenty hours of treatment The results are explained on the hypothesis that lignin is present in the acetylated product and it is responsible for the difference in the structure from that of pure cellulose acetate It is further shown that the rate of acetylation at the two stages of reaction can be explained by assuming that the reagent enters the micelles by diffusion

Introduction

It is well-known that various compounds of cellulose are used in industry nowadays and cellulose acetate is probably the most important among them It is mainly used in making photographic films, plastics, lacquers and artificial silk The advantages of this ester over cellulose nitrate are its transparency, light-resistance and non-inflammability

The esterification of cellulose is brought about by the esterifying agents through the available hydroxyl groups The number of hydroxyl groups available for acetylation in glucose residue is three, two of which are secondary and one is primary In course of acetylation, it has been observed that with the progress of time, the number of acetyl groups in the cellulose molecule gradually increases giving first the mono-, then di-, and finally the triacetate

The source of cellulose generally used for preparing the cellulose acetate is chiefly cotton in which a fairly high percentage of cellulose free from other impurities is available Hess and Trogus (1932) used ramie as source of cellulose for preparing cellulose acetate They found the acetic acid content in the cellulose acetate prepared by them to be about 60 per cent, a little below the theoretical maximum value which is 62 5 per cent

In Bengal, jute is one of the main sources of cellulose, as it is much cheaper and more abundant than cotton and ramie But the main difficulty in using it as source of cellulose is that jute contains about 11 to 15 per cent of lignin as impurities while the lignin-content of ramie is below 3%

The question whether jute fibre can be used as a source of cellulose for preparing cellulose derivatives was investigated formerly by Chowdhury and Basu (1932) by chemical methods, but they prepared the cellulose derivatives from delignified jute. The object of the present investigation is to find out by X-ray analysis combined with chemical methods to what extent the raw jute fibre containing lignin is acetylated when it is subjected to the same treatment as cotton or ramie for obtaining cellulose acetate

EXPERIMENTAL

In order that jute fibres even after acetylation may retain its fibrous form and properties the process of fibrous acetylation used by Hess and Trogus (1932) was followed. In the present method 5 gms of well-combed, cleaned and air-dried raw jute fibre was placed in flask in which a mixture of 50 gms of glacial acetic acid, 200 gms of acetic anhydride and 60 gms of fused potassium acetate were taken and the whole mixture was refluxed. At the end of second, fourth, seventh, eleventh, twentieth, thirty-second and forty-eighth hour about 0.7 gm of fibre was withdrawn every time, washed in running tap water and then dried in air for about two days. When they were fully dried, about 15 to 20 strands of acetylated fibre were taken from each sample. Each of these samples was subjected to both chemical analysis and X-ray examination. X-ray photographs were taken after making all the strands parallel by pressing them mildly with fingers and holding them taut during exposure. The photographs were taken with a very fine slit of 0.5 mm bore and 5 cm. in length using Cu Kα radiation from a Hadding tube. The photographs are reproduced in Plate VII.

Some quantity of raw jute fibre was next delignified by treating it with chlorine peroxide for a long time. This method was adopted because of the fact that ClO_2 affects the cellulose least. The delignified fibre was then acetylated for 50 hours by the method mentioned above and the final product was analysed chemically and its X ray diffraction pattern was also photographed. This photograph is reproduced in Fig. 9, Plate VII.

As regards the degree of acetylation at certain intervals the results obtained from X-ray photographs were compared with those obtained by chemical analysis. The chemical method adopted for the estimation of acetyl groups was that of Perkin (1904)*. The estimation of acetyl groups in acetylated delignified jute fibre was also made by the same method under identical experimental conditions.

RESULTS AND DISCUSSION.

(a) *Results of chemical analysis of the product*

The results obtained with raw jute fibre are given graphically in Fig 11, in which Curve I is drawn with acetic acid content as ordinate and the reaction period in hours as abscissa. In order to compare these results with those obtained by Hess and Trogus in the case of ramie, the results obtained by them are shown in Curve II, Fig 11. It can be seen that the curve obtained in the present investigation with acetylated raw jute fibre is approximately of the same nature as that obtained by Hess and Trogus in case of ramie fibre. But from the general appearance of the two curves it can be seen that after the treatment for a particular period under identical conditions raw jute fibre is not acetylated to the same extent as ramie fibre. For instance after the treatment for two hours the product obtained with raw jute fibre is found to contain 19% of acetic acid while that obtained with ramie contains 33% of acetic acid. The saturation value, i e the maximum quantity of acetic acid which the fibre takes up after 48 hours of treatment is lower in case of jute than in ramie. The values of acetic acid content observed by Hess and Trogus are also given in Table I.

It has already been mentioned that cellulose acetates are formed stepwisely. In Curve I it can be seen that at the eleventh and twentieth hour the amounts of acetic acid found in the acetylated fibre are 39.3% and 39.8% respectively. This flat portion of the curve indicates that the jute fibre after being acetylated to a certain extent refuses to take up any more acetic acid for a considerable length of time after which it again begins to react until the final saturation value is reached Since this takes place after the product shows 39 3% of acetic acid content and the

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* The author is indebted to Dr. B. K. Bhattacharyya and P. Sen Gupta for their kind help in the chemical analysis.

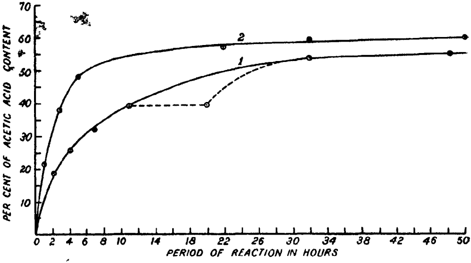

FIG 11

saturation value is 57 6%, these results suggest that two of the hydroxyl groups take up acetyl radical easily while the third hydroxyl group in the glucose residue cannot be so easily acetylated It appears that some time elapses before the third hydroxyl group begins to take up the acetyl group It is evident that the transition phase from diacetate to triacetate begins at this stage

TABLE I

Acetylated raw jute fibre

Time in hours	Per cent Acetyl content	Per cent Acetic acid	Per cent Acetic acid in acetylated ramie (Hess and Trogus)
2	13 43	18 74	32 5
4	18 55	25 8	38 5
7	22 8	31 8	52
11	28 32	39 51	55
20	28 52	39 78	57 5
32	38 83	54 2	59 8
48	39 9	55 6	60 5

It can be seen further from Fig 11 that the curve becomes almost parallel to the time axis after 32 hours This shows that the acetic acid content becomes maximum at this stage and a saturation value is very slowly approached This value observed in the case of raw jute fibre is, however, slightly lower than that for pure cellulose acetate

(b) *X-ray investigation.*

In Plate VII, Fig 1 represents the photograph of untreated raw jute fibre Fig 2 corresponds to the fibre acetylated for two hours. In this latter photograph after careful observation it can be seen that 101 and 10$\bar{1}$ reflections are not so sharp as in Fig 1 and the spot 020, on the second layer line has become elongated It shows that lattice has not yet been disturbed, but the acetylation has begun and it has disorientated the micelles So at this stage the reaction is purely micellar surface reaction This photograph (Fig 2) is mainly of cellulose rather than cellulose acetate. Fig 3 which corresponds to acetylation for 4 hours is nearly the same as Fig 2, but the pattern shows that the acetylation has advanced further However, the pattern still retains some characteristics of that for cellulose Fig 4 represents the diagram of acetylation for seven hours, which is also a mixed diagram Fig 5 corresponds to the diagram of acetylation for eleven hours and Fig 6 represents that for 20 hours These two photographs are practically the same Here the oblique reflections have nearly disappeared and 101 and 10$\bar{1}$ have become so diffuse that they cannot be identified The shape of 002 spot has also remarkably changed From all these it seems that herefrom the acetylation phase, i e the formation of triacetate has just begun This is also evident from the curve drawn with data obtained by chemical analysis Figs 7 and 8 represent the photographs corresponding to the acetylation for 32 hours and 48 hours respectively. It can be seen from these photographs that the original oblique reflections have totally disappeared and a new elongated structure has appeared on the different layer lines The equatorial spots have also changed remarkably in size and shape, but the position of the 002 spots remains the same Fig 8 which corresponds to the acetic acid content of about 55 6% is the diagram for the final acetylated product obtained from raw jute fibre This diagram differs appreciably from that obtained by Hess and Trogus in the case of acetylated cellulose with the same acetic acid content It appears that this difference in the nature of the X-ray diffraction patterns of acetylated ramie and raw jute fibre obtained by treating the fibres under identical conditions for the same time is due to the presence of high percentage of lignin in jute fibre. From the constitutional point of view, both of cellulose and lignin, it is seen that lignin cannot be so easily acetylated as cellulose So the percentage of acetic acid content after complete acetylation becomes different Since lignin acetate is present along with cellulose acetate in the product obtained with raw jute fibre, the crystal structure of the product is different from that of pure cellulose acetate The X-ray diagram of cellulose acetate obtained from delignified jute fibre is reproduced in Fig 9 It can be seen that the structure is the same as that of cellulose acetate obtained from ramie by Hess and Trogus (Fig 10), but it is different from that obtained from raw jute fibre This shows that in the acetylated raw jute fibre lignin is present and it makes the structure different from that of pure cellulose acetate In the case of delignified jute fibre the percentage of acetic acid content has been found to be 60% which is slightly below the theoretical maximum value and is the same as that obtained by Hess and Trogus. After calculating the spacings it has been found that the identity period, i e the length of *b*-axis, remains the same, e g. 10 3Å in the case of cellulose acetate obtained from raw jute fibre So it is evident that lignin plays an important part in the acetylation process but it keeps the *b*-axis unchanged even after complete acetylation

The difference in X-ray patterns obtained from acetylised jute fibre and acetylised ramie is due also to the size of micelles In a previous communication (Sirkar and Saha, 1946) it has been shown that the length of micelles in ramie is about 1000Å and the width is 60 to 70Å, while in the case of jute the micelles are at most 100Å long After acetylation the micelles in jute fibre being shorter, become more disorientated than those in ramie fibre. So the spots in the pattern for acetylated jute fibre are more elongated than in the pattern for cellulose acetate obtained from ramie

(c) *The kinetics of the reaction*

The course of reaction is graphically represented in Curve I, Fig 11 The reaction appears to be a bit complicated The velocity constant K_1 for unimolecular reaction has been calculated from the equation,

$$\frac{dx}{dt} = K_1(a-x) \quad . \quad .. \quad . \quad (1)$$

where a the initial concentration of hydroxyl groups and x is the amount converted The velocity constant K_2 for bimolecular reaction is obtained by using the equation,

$$\frac{dx}{dt} = K_2(a-x)^2 \quad . \quad .. \quad . \quad . \quad (2)$$

An attempt has been made to calculate the values of K_1 and K_2 from the curve in Fig 11 These values are given in Table II

TABLF II

Time of treatment in hours	$K_1 \times 10^3$ (unimolecular)	$K_2 \times 10^4$ (bimolecular)
2	178	34 2
4	133	28 3
7	101	23 7
11	91	24 7
20	50	14 1
32	63	32 3
48	45	26 7

It can be seen easily from the above table that neither K_1 nor K_2 is constant throughout the course of reaction If we consider the first portion of the Curve I (Fig 11) before the discontinuity, K_2 varies slowly but K_1 varies very rapidly Hence the reaction is more of bimolecular type than unimolecular But some other factor must be considered in order to explain the variation of K_2

It may be that at the beginning of the reaction, the esterifying agent first reacts with the surface layer and gradually it enters into the interior of the fibre where again the reaction sets in So it is to be conceived that the reagent enters by the process of diffusion In diffusion phenomena it is well known as shown by Ostwald (*vide* Sakurada) that the course of diffusion of an organic liquid can be expressed in the form,

$$l = Kt^m$$

where l is the diffusion path and t the diffusion time, K and m are constants. As the direct measurement of diffusion path is not practicable, here in this case x, the amount of reagent reacted, has been taken instead of l, as has been done by Sakurada (1932),

$$x = Kt^m \quad . \quad . \quad .. \quad (3)$$

m and K can be found by plotting log x against log t. The values for the two portions of the Curve I, Fig. 11 obtained from Fig 12 are given in Table III

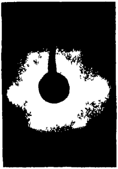

FIG. 1

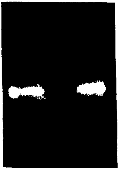

FIG. 2

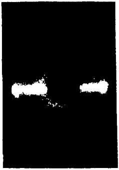

FIG. 3

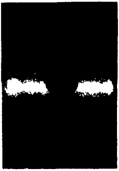

FIG. 4

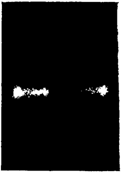

FIG. 5

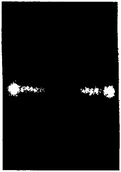

FIG. 6

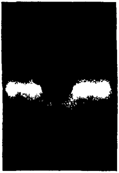

FIG. 7

FIG. 8

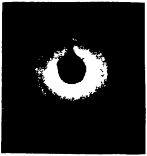

FIG. 9

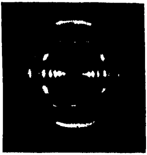

FIG. 10

X-ray Diffraction Patterns

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PUBLISHED
BY THE
NATIONAL INSTITUTE OF SCIENCES OF INDIA
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Price Rs 1-14-0

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23 NOV 1948

STUDIES ON FERRIC-THIOSULPHATE COMPLEX BY COLORIMETRIC METHOD

By Barun Chandra Haldar, *M Sc*, *and* Sukumar Banerjee, *M.Sc.*, *University College of Science, Calcutta*

(Communicated by Dr P B Sarkar, D Sc, F N I.)

(*Received May 27; read August 1, 1947*)

Introduction

It has been observed by all that a deep violet colour is developed instantaneously when ferric chloride solution is added to sodium thiosulphate solution or *vice versa* Copper salts have an appreciable accelerating influence on this reaction so that the deep violet colour disappears much more rapidly in the presence of copper salts, than in copper-free solutions When the reaction is carried out in the presence of thiocyanates which serve as indicators for the ferric salts and also slow up the reactions, very small amounts of copper can be detected, by comparing the time of decolorisation by thiosulphate of a copper-containing and copper-free ferric salt solution This method was actually employed by Hahn and Leimbach (1922) for the detection of small amounts of copper and the limit of identification was found to be 0.02% copper The appearance of the violet colour is usually assumed to be due to an unstable complex formed between ferric chloride and sodium thiosulphate, but the exact nature of it is not known very precisely Holluta and Martini (1924) tried to trace the initial acceleration of the reaction, ferric chloride-sodium thiosulphate solutions, but were not successful They, however, attempted to explain the violet colour formation as due to a complex $Fe(S_2O_3)_2^-$ where the complex ion is negatively charged. Schmid (1930) is of opinion that the complex is $Fe(S_2O_3)^+$ and not $Fe(S_2O_3)_2^-$. He has also calculated the instability constant of the complex $Fe(S_2O_3)^+$ and gives its value as $K = 66.6 \times 10^{-8}$ at 18°C. where $K = \dfrac{(Fe^{+++})(S_2O_3^{=})}{[Fe(S_2O_3)^+]}$ The chief difficulty in determining the nature of the complex and its instability constant was to find out a suitable method to follow the reaction Although Schmid tried to solve the problem by potential measurement at different fixed points in a glass tube through which flowed under constant pressure a mixture of acid ferric-ferrous-chloride mixture and thiosulphate solutions, it is expected that the e m f. noted by him may not be the true e m f of the system ferric chloride-sodium thiosulphate solutions under static condition Moreover, the potential recorded within such a small period of time (within 3 mins) is not expected to be the true reversible potential of the system So we have tried to follow the ferric chloride-sodium thiosulphate reaction by colorimetric method. By the help of a photoelectric colorimeter the decrease in intensity of the violet colour with time can be easily noted. Now this decay in colour noted in transmission scale is found to give a linear relation with time. Considering the fact that the development of the violet colour is very rapid compared to its decay, it can be assumed that the concentration of the complex is greatest at zero-time. So if the straight line obtained by plotting percentage transmission against time be extrapolated to zero-time it will cut the transmission axis at a point where the concentration of the complex is greatest Thus the amount of complex formed by mixing known volumes of the reacting solutions is known in transmission scale. By applying P Job's (1928) continuous variation method, it is now possible to know the nature of the complex and also to determine its instability constant

Theory —The continuous variation method applicable to the imperfect molecular and ionic aggregates is due to P Job According to it, the variation in one physical property of a mixture of two solutions from which the complex is formed, is studied against composition of one of the reacting constituent keeping the total volume of the mixture constant Frequently it so happens that a suitably chosen property becomes a maximum or minimum for a mixture of a particular composition One is then to admit that this particular composition is the index of the formation of a complex whose formula precisely corresponds to this 'maximum composition' This method is called the method of 'continuous variation' Suppose the complex which we want to study is formed according to the equation

$$mA + nB \rightleftharpoons AmBn \qquad (1)$$

A solution of A of molar concentration C and of B of concentration C', where $C' = RC$, are prepared A volume x of the second is mixed with a volume $(1-x)$ of the first, and let us assume that this mixture is produced without any contraction or expansion in volume

Let C_1, C_2 and C_3 be concentrations of A, B and $AmBn$ respectively. For any mixture, the following equations apply —

$$C_1^m \times C_2^n = KC_3 \qquad (2)$$

$$C_1 + mC_3 = C(1-x) \qquad (3)$$

$$C_2 + nC_3 = RCx \qquad (4)$$

The concentration C_3 of the complex depends only upon the composition x of the given mixture It will be sufficient, therefore, in order to obtain the maximum composition to write that

$$dC_3/dx = 0 \qquad (5)$$

By differentiating equations (2), (3) and (4) and combining the resulting differential equations with (2), (3) and (4) we get the general equation

$$\frac{C^{m+n-1} \times R^{n-1}\left[(Rm+n)x-n\right]^{m+n}}{m^{n-1}\, n^{m-1}\, (R-1)^{m+n-1}} = K[n-(m+n)x] \qquad (6)$$

When $R = 1$, that is, for equimolecular solutions the left-hand side is zero. Now K cannot be zero, therefore

$$x(m+n) - n = 0 \text{ or } \frac{m}{n} = \frac{1-x}{x} \qquad (7)$$

Hence from a knowledge of the maximum composition x, the formula of the complex can be determined from the ratio m/n taking the simplest values for m and n.

When the solutions are not equimolecular, the maximum composition depends both upon the concentrations of the two primary solutions and on the instability constant K. The value of this maximum composition determines, with the help of the equation (6), the instability constant K.

The absorption of monochromatic light is a suitable property for this method, because complex ions are often differently coloured from their components. Also the absorption of light is proportional to the concentration of the absorbing species which is one of the necessary conditions of the continuous variation method.

Generally the value P of this property will depend in a more or less simple manner on the concentrations C_1, C_2, C_3 of the three constituents A, B and $AmBn$ in solution giving $P = f(C_1, C_2, C_3)$ When the property in question depends only upon the concentration of the complex, as in our case, the value of the property as a function of the composition of the mixture, passes through a maximum or minimum for the maximum composition itself, i e. $dP/dx = 0$

EXPERIMENTAL DETAILS

Percentage of light transmitted due to the complex-formation was measured with Lumetron photoelectric colorimeter model 400G of Photo-volt Corporation, New York The apparatus is so designed and calibrated that the scale gives percentage transmission directly Reagents used were of extra-pure quality All solutions were made in copper-free redistilled water (tested for copper by Rubeanic acid reagent) Solutions of sodium thiosulphate were standardised against standard dichromate solution iodometrically Ferric chloride solutions were also standardised with standard dichromate solution using diphenylamine sulphonate as an internal indicator Dilute solutions were all prepared by diluting stock solutions of strength $N/10$ kept in Jena bottles and were used immediately after their preparation The absorption due to the complex in the visible region is between 450 $m\mu$ to 550 $m\mu$ * and so the measurements were made at two different wave-lengths 490 $m\mu$ and 530 $m\mu$ Moreover, the absorption due to unreacted ferric chloride is negligible in this region Each measurement was carried out as follows —

The galvanometer needle was set exactly on zero mark of the transmission scale by means of zero adjustment knob The power cord was then connected to a six volt battery On and off switch was then thrown off to the on-position and the instrument was allowed to warm for a few minutes Now the desired filter was placed in the path of the light beam The tube containing redistilled water was placed in the light path and the needle was set exactly on 100 mark of the transmission scale by means of the controlling knobs The test tube containing redistilled water was then replaced by a test tube containing a known volume (10 c c to 18 c c) of one of the reactants Definite volume of the other reactant was added from a graduated pipette while stirring with a dry glass stirrer The total volume of the solution was 20 c c The initial stirring was continued for 5 secs. only and then the stirrer was removed The first reading was taken after 10 secs Then after each 5 or 10 secs the position of the galvanometer needle was noted for 30 to 80 secs Percentage transmission was then plotted against time and the straight line obtained was extrapolated to zero-time In this way different graphs were obtained by mixing different proportions of the reactants and noting the transmission with time The transmissions at zero-time were then plotted against composition and the resulting curve showed a minimum So from the position of the minimum in the curves of different equi-molecular solutions the formula of the complex was fixed The instability constant of the complex was then determined from the minima in the curves of non-equimolecular solutions with the help of the general equation (6)

Each reading was repeated thrice and the results were found to agree within half-a-division of the transmission scale.

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* We are grateful to Mr B Mukherjee, M.Sc , for kindly photographing the absorption spectra of the complex.

TABLE 1

Strength of $FeCl_3$ solution = $M/75$, Strength of sodium thiosulphate = $M/75$; Wavelength used = 490 mμ

(*a*) Vol of $FeCl_3$ solution in c c = 15. Vol of Na-thiosulphate solution in c c = 5.

(*b*) Vol of $FeCl_3$ solution in c c = 14 Vol of Na thiosulphate solution in c c = 6.

Time in secs	% trans	Time in secs	% trans
10	30·5	10	25·0
20	35·0	20	32·5
30	41·0	30	40·5
40	47·0	40	48·5
50	54·0	50	56·5
60	60·0	60	64·0
70	66·5	70	71·5
80	73·0	80	78·5
90	79·5	90	84·0
100	85·0	100	89·0

(*c*) Vol of $FeCl_3$ solution in c c = 13 Vol of Na thiosulphate solution in c c = 7.

(*d*) Vol of $FeCl_3$ solution in c c = 12. Vol of Na thiosulphate solution in c c = 8.

Time in secs	% trans	Time in secs	% trans
10	23·5	10	21·0
20	30·5	20	31·5
30	39·0	30	41·5
40	47·0	40	51·5
50	54·0	50	60·5
60	61·0	60	68·5
70	68·0	70	75·5
80	75·0		
90	80·0		
100	85·0		

(*e*) Vol of $FeCl_3$ solution in c c = 11. Vol of Na thiosulphate solution in c c = 9

(*f*) Vol of $FeCl_3$ solution in c c = 10 Vol of Na-thiosulphate solution in c c. = 10.

Time in secs	% trans	Time in secs.	% trans
10	21·0	10	21·0
20	32·0	15	28·0
30	42·0	20	35·0
40	51·5	25	41·5
50	60·5	30	47·5
60	69·0	35	53·5
		40	58·5
		45	63·0
		50	67·0

(*g*) Vol of $FeCl_3$ solution in c.c = 9
Vol of Na-thiosulphate solution in c c = 11

Time in secs	% trans
10	23·0
15	29·5
20	37·0
25	44·5
30	50·5
35	57·5
40	62·5
45	67·5

(*h*) Vol of $FeCl_3$ solution in c c. = 8.
Vol. of Na-thiosulphate solution in c c = 12

Time in secs	% trans
10	27·0
15	35·0
20	44·0
25	52·0
30	59·0
35	66·0
40	71·5
45	76·5

(*i*) Vol of $FeCl_3$ solution in c c = 7
Vol of Na thiosulphate solution in c.c = 13.

Time in secs	% trans
10	31·0
15	38·0
20	47·5
25	53·0
30	62·0
35	67·0
40	73·0

(*j*) Vol of $FeCl_3$ solution in c c = 6
Vol of Na-thiosulphate solution in c c = 14.

Time in secs.	% trans.
10	31·5
15	39·5
20	48·0
25	56·0
30	63·5
35	69·5
40	74·5

(*k*) Vol. of $FeCl_3$ solution in c.c. = 5.
Vol. of Na-thiosulphate solution in c.c. = 15.

Time in secs	% trans.
10	38·5
15	45·5
20	53·0
25	59·5
30	66·5
35	71·8

TABLE 2.

Strength of $FeCl_3$ soln $M/75$.
Strength of $Na_2S_2O_3$ soln = $M/75$
Wavelength used = 490 $m\mu$.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero-time
5	15	23 0
6	14	18 0
7	13	16 0
8	12	12 0
9	11	10 0
10	10	8 0
11	9	10 0
12	8	13 0
13	7	15 5
14	6	19 0
15	5	23 0

TABLE 3.

Strength of $FeCl_3$ soln $M/75$.
Strength of $Na_2S_2O_3$ soln. $M/75$.
Wavelength used = 530 $m\mu$.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero-time
5	15	20 0
6	14	16 0
7	13	13 5
8	12	12 0
9	11	11 0
10	10	9 0
11	9	12 5
12	8	16 0
13	7	20 0
14	6	24 0
15	5	30 0

TABLE 4

Strength of $Na_2S_2O_3$ soln = $M/100$
Strength of $FeCl_3$ soln = $M/100$.
Wavelength used = 530 $m\mu$.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero-time
5	15	42 0
6	14	31 0
7	13	25 0
8	12	22 5
9	11	21 0
10	10	20 0
11	9	23·0
12	8	27 5
13	7	34·0
14	6	39·0
15	5	46·0

TABLE 5

Strength of $Na_2S_2O_3$ soln = $M/50$.
Strength of $FeCl_3$ soln = $M/50$.
Wavelength used = 530 $m\mu$.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero-time
2	19	31·0
4	16	10·0
5	15	6 5
6	14	4 0
8	12	1 5
10	10	0
11	9	4 5
12	8	12 0
13	7	18·5
14	6	25·0

TABLE 6

Strength of $Na_2S_2O_3$ soln = $M/100$.
Strength of $FeCl_3$ soln = $M/50$.
Wavelength used = 530 $m\mu$.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero-time
5	15	13
6	14	10
7	13	8
8	12	6
9	11	4
10	10	3
11	9	2
12	8	2
13	7	9
14	6	15
15	5	20
16	4	26

TABLE 7

Strength of $Na_2S_2O_3$ soln. = $M/100$.
Strength of $FeCl_3$ soln = $M/50$.
Wavelength used = 490 $m\mu$.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero-time
2	18	44
3	17	28
4	16	19
5	15	14
6	14	9
7	13	6
8	12	5
9	11	4
10	10	3
11	9	2
12	8	1
13	7	4
14	6	8
15	5	12
16	4	17
17	3	32 5
18	2	43

TABLE 8

Strength of $Na_2S_2O_3$ soln. = $M/500$.
Strength of $FeCl_3$ soln = $M/100$.
Wavelength used = 530 $m\mu$.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero-time
2	18	45
3	17	31
4	16	22
5	15	20
6	14	17
7	13	12
8	12	9
9	11	13
10	10	17
11	9	20
12	8	27
13	7	29
14	6	34
15	5	40
	4	

TABLE 9

Strength of $Na_2S_2O_3$ soln = $M/50$.
Strength of $FeCl_3$ soln = $M/100$.
Wavelength used = 490 $m\mu$.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero-time
2	18	42·0
3	17	29·0
4	16	21·0
5	15	18·0
6	14	14·0
7	13	11 0
8	12	8·0
9	11	11 5
10	10	15 0
11	9	19 0
12	8	23 0
13	7	28·0
14	6	32 5
15	5	38 0

TABLE 10.

Strength of $Na_2S_2O_3$ soln = $M/20$.
Strength of $FeCl_3$ soln = $M/200$.
Wavelength used = 530 mμ.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero time
0 5	19 5	55·0
1 0	19 0	33 5
1 5	18 5	23 0
2 0	18 0	19 5
2 5	17 5	14 0
3 0	17 0	12 0
3 5	16 5	10 0
4 0	16 0	7 5
4 5	15 5	5 0
5 0	15 0	9 0
5 5	14 5	13 0
6 0	14 0	17 0
7 0	13 0	27 0

TABLE 11

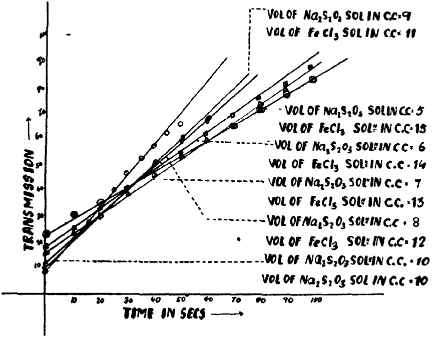
Strength of $Na_2S_2O_3$ soln = $M/250$.
Strength of $FeCl_3$ soln = $M/25$.
Wavelength used = 530 mμ.

Vol of $Na_2S_2O_3$ soln in c c	Vol of $FeCl_3$ soln in c c	%transmission at zero-time
6 0	14 0	35
7 0	13·0	31
8 0	12·0	26
9 0	11 0	19
10 0	10 0	15
11 0	9 0	12
12 0	8 0	11
13 0	7 0	10
14 0	6 0	8
15 0	5 0	7
16 0	4 0	17
17 0	3 0	22
17 5	2 5	26
18 0	2 0	34
18 5	1 5	46
19 0	1 0	61

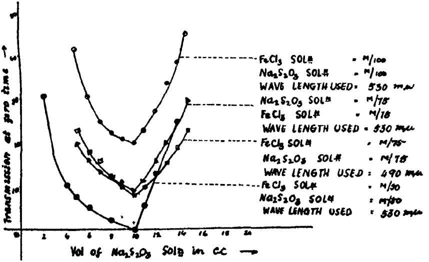
TABLE 12

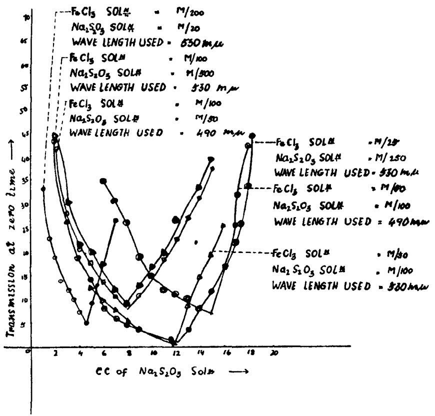
Instability constant of the complex $Fe(S_2O_3)^+$

Exp No	Concentration of Ferric Chloride solution	x	R	Wave-length used.	K (instability constant).	Mean K
1	0·01M	0 4	2	530 mμ	$2\ 0 \times 10^{-3}$	$2{\cdot}22 \times 10^{-3}$
2	0·02M	0 595	0 5	530 mμ	$2\ 43 \times 10^{-3}$	
3	0 04M	0 75	0 1	530 mμ	$2\ 7 \times 10^{-3}$	
4	0 005M	0 225	10·0	530 mμ	$2{\cdot}19 \times 10^{-3}$	
5	0·01M	0 400	2	490 mμ	$2{\cdot}0 \times 10^{-3}$	
6	0·02M	0 6	0 5	490 mμ	$2{\cdot}0 \times 10^{-3}$	



STRENGTH OF $Na_2S_2O_3$ SOL = M/75
STRENGTH OF $FeCl_3$ SOL = M/75
WAVE LENGTH USED = 490 mμ

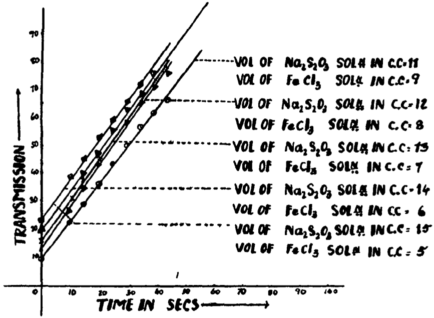




DISCUSSION

Careful examination of the curves which present the studies with solutions of different but equimolecular concentrations, reveal that the maximum composition x is independent of the concentrations of the primary solutions and of wavelength used, since all these curves show minima at the same point, i e ferric chloride-sodium thiosulphate equal to 1 1 The instability constants determined at four different concentrations of ferric chloride and sodium thiosulphate solutions are fairly in agreement with each other considering the fact that the temperature was not kept constant (varied from 24° to 26°C) and in calculating the constant we used concentration terms and not activities It is also independent of wavelength of light absorbed by the complex Although our conclusion as to the nature of the complex is in agreement with that of Schmid, our value of the instability constant differs from that of the latter A slight difference in the two values of the instability constant is expected since Schmid's value of $K = 66\,6 \times 10^{-8}$ is at 18°C whereas our value is between 24° and 26°C Such a large difference is, however, mainly due to the different experimental conditions and method employed by Schmid Moreover, the potentials measured within such a short period of time and under flowing conditions may not be the true reversible e.m f of the system. So it is expected that our value of K is likely to be nearer the true value than that of Schmid.

In spite of the remarkable concordance observed in the above determinations of K, the precision of these experiments must not be over-estimated. Though for



STRENGTH OF THE $Na_2S_2O_3$ SOL - M/75
STRENGTH OF THE $FeCl_3$ SOL - M/75
WAVE LENGTH USED 490 mμ

the estimation of the intensity of the colour much improved and precise method has been employed yet it is not possible to determine the maximum composition x with a very great exactness Let us determine, for the six experiments, the relative error made in the determination of K. We have

$$K = \frac{C[(R+1)x-1]^2}{(R-1)(1-2x)} \qquad . \qquad . \quad (8)$$

$$\frac{1}{K}\,dK/dx = 2\left[\frac{(R+1)}{(R+1)x-1}+\frac{1}{1-2x}\right] \qquad .. \quad (9)$$

It is found that the above values for dK/K are 40 dx, $-38.5\ dx$, $-16.5\ dx$, $11{\cdot}2\ dx$, 40 dx, $-40\ dx$. It is impossible to measure the maximum composition to more than 1·0%, that is to say, that dx is at least equal to 0 01, the most favourable experiment then entails an error of about 11 2% Thus the common idea that the intermediate violet colour is due to the complex $Fe(S_2O_3)_2^-$ which has found place even in standard text-books (*Text Book Of Qualitative Chemical Analysis* by Vogel;

Text Book Of Inorganic Chemistry by Partington) cannot be supported by physico-chemical evidences.

SUMMARY

1 The reaction between ferric chloride and sodium thiosulphate solutions, has been studied by colorimetric method with the help of a photoelectric colorimeter

2 The intermediate deep-violet colour developed by the interaction of ferric chloride and sodium thiosulphate *is due to the complex* $Fe(S_2O_3)^+$

3 The instability constant of the positively charged complex has been determined by Job's continuous variation method and the mean value is found to be $2\ 22 \times 10^{-3}$ at temperatures 24° to 26°C

Our best thanks are due to Prof P B Sarkar, for his keen interest, helpful suggestions, and all laboratory facilities during the progress of the work

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A CYTOLOGICAL INVESTIGATION ON THE GENUS *PHLOEOBA* (ACRIDIDAE)

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(Communicated by Dr P N Bhaduri, M Sc, Ph D, F N I)

(*Received June 27, read August 1, 1947*)

INTRODUCTION

During a course of investigation on the phylogenetic relationship between the various groups of Indian Acridids, we encountered with two different forms of grasshoppers belonging to the genus *Phloeoba* Both the types were captured from the field adjoining the Biological Laboratories of the Calcutta University In this paper we shall call them form 'A' and 'B' Form 'B' is morphologically distinguishable from form 'A' by (1) the presence of two yellowish stripes across each of the compound eyes and (2) a pair of parallel stripes of nearly the same shade running across the two lateral aspects of the thorax and the genae and meeting at the frons (Figs 1 and 1*a*) In all other respects they were exactly similar The specimens were sent to Dr B P Uvarov of the Imperial Bureau of Entomology, London, for identification Dr Uvarov identifies them as two separate species without, however, assigning any specific names for them They were also sent to Dr B R Seshachar of Bangalore, and he is of opinion that both of them belong to the same species and identifies them as *Phloeoba angustidorsis*

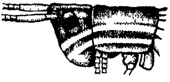


FIG 1 *Phloeoba sp* Form 'B'

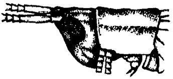


FIG 1*a* *Phloeoba sp* Form 'A'

A thorough cytological investigation of the two forms was thought desirable to find out, if possible, any constant difference between them either in the morphology of the chromosomes or in the details of their chiasma behaviour during meiosis. Detailed cytological studies of form 'A' was done earlier and is being published elsewhere (Ray Chaudhuri and Dutt, 1947)

The present paper reports a detailed study of form 'B' and compares the data so obtained with those of the other form studied previously

MATERIAL AND METHODS

Adult testes were dissected out in Ringer 'A' and fixed in medium Flemming for sectioning. Belling's modification of Nawaschin mixture was found suitable for smear preparations. Sections were cut at 25 to 30 micra in thickness and stained in iodine-crystal-violet and Feulgen stain An overnight premordanting in 1% chromic acid was found necessary to give the best result with crystal-violet staining.

OBSERVATIONS

The chromosome complement —The number of chromosomes is 23 in the male, with 11 pairs of acrocentric autosomes and a similar X chromosome There are two dot shaped chromosomes and they lie in most cases at the centre of the plate (Fig 2) The second division metaphase chromosomes are more suitable for metrical studies because they are generally very well spread Five such selected nuclei were drawn under camera lucida and the lengths of the different chromosomes were measured Table 1 gives the mean lengths of the chromosomes in micra

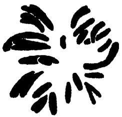


FIG 2 Spermatogonial metaphase of *Phloeoba* sp × 2062

TABLE I

Tabulation of mean lengths of second spermatocyte chromosomes
The figures in italics represent the sex chromosomes

Chromosome type	Mean length in micra	
	Form 'B'	Form 'A'
Long	4 77	4 77
	4 36	4 44
	4 03	4 12
Medium	*3 42*	*3 44*
	3 18	3 21
	3 00	2 99
	2 54	2 82
	2 27	2 66
	2 12	2 21
	1 77	1 83
	1 53	1 55
Short	1 09	1 05

Chromosome behaviour during meiosis —Chiasmata are distributed at random at diplotene. Some of the bivalents at this stage are found to be attached terminally by very fine threads which are Feulgen positive (Fig 3). Whether these associations are between heterochromatic segments of different chromosomes as claimed by Slack (1938) in Corixidae, Schrader (1941) in pentatomids and Thomas and Revell (1946) in *cicer* is not known.

The sex chromosome at metaphase frequently forms an accessory plate like other grasshoppers and shows irregular staining behaviour previously noted in the other species of grasshoppers (Ray Chaudhuri and Dutt, 1947) A large number of bivalents at this stage show an understained segment in only one of the chromosomes of a homologous pair (Fig 4) These are probably undercharged heterochromatic segments (Darlington and LaCour, 1940, Callan, 1942)

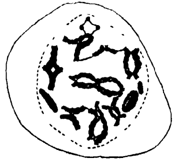


FIG 3 Diplotene of *Phloeoba* sp showing interbivalent connections × 2062

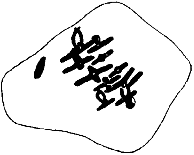


FIG 4 First meiotic metaphase of *Phloeoba* sp, showing heterochromatic segments × 1375

The first division anaphase is quite normal except for the occurrence of a bridge without a fragment (Fig 5) This may be due to stickiness of the chromosomes

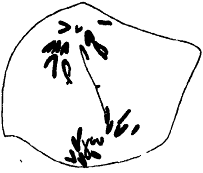


FIG 5 First division anaphase in *Phloeoba* sp., showing stickiness of Chromosome × 1335

caused by an earlier action of the centromere before the lapsing of attraction between the chromatids (Klingstedt, 1938).

The orientation of the chromosomes on the metaphase plate of second division is often peculiar The bodies of the chromosomes in these cases lie within the spindle substance with the attachment region, as usual, on the edge (Fig 6). The daughter chromatids at this stage remain in most cases closely apposed just like those of somatic mitoses, but occasionally in one or two chromosomes of a complement, the chromatids are fully separated so as to lie in a straight line (Fig 6). This may be due to attraction between the chromatids developing after the orientation of the chromosomes on the spindle and therefore, the chromatids which are by chance far apart cannot be pulled together when the force come into play The chromatids at this stage often show an external sign of a spiral structure

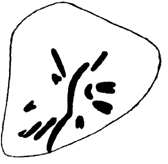


FIG 6 Second division metaphase plate in *Phloeoba* sp × 1780

Chiasma frequency —The frequency distribution of chiasmata per nucleus is shown in Table II The frequency ranges between 14 and 23 at diplotene, between 14 and 20 at diakinesis and metaphase The chiasma frequency per nucleus shows a progressive reduction from diplotene to metaphase (Table II) The differences are, however, not statistically significant

TABLE II

Frequency distribution of chiasmata in three stages of meiosis

Stage of meiosis	14	15	16	17	18	19	20	21	22	23	No of nuclei	Mean No of Xta per nucleus	Significance of difference
Dip			1	1	6	4	3	6	2	2	25	19 72 ± 1 85 T_1	$T_1 - T_3 = 2\ 8 \pm 2\ 5$
Dia	2	1	2	5	7	5	2		1		25	17 72 ± 1 85 T_2	
Met	1	6	6	2	4	4	2				25	16 88 ± 1 76 T_3	

An analysis of the chiasma frequency in the three different types of bivalents classified as long and medium were undertaken and the data are shown in Table III. Short bivalents regularly form only one chiasma and are therefore not included in the Table.

TABLE III

Chiasma frequencies in the long and medium bivalents

Stage of meiosis	No of nuclei	Percentage of bivalents						
		Long type				Medium type		
		1 Xma	2 Xta	3 Xta	4 Xta	1 Xma	2 Xta	3 Xta
Dip	25	0 0	36 0	57 3	6 6	50 3	45 7	4 0
Dia	25	8 0	53 3	37 4	1 3	61 7	36 5	1 7
Met	25	2 6	56 0	41 3	0 0	71 4	28 5	0 0

Relationships of chiasma frequency with the length of the chromosomes are shown in Table IV The chiasma frequency is not found to be directly proportional to the length of the chromosomes

TABLE IV

Length and chiasma frequency relationship in the long, medium and short bivalents

Types of chromosomes	Mean length in micron at second div metaphase	Xma frequency per biv at diplotene
Long	4 44	2 70
Medium	2 59	1 53
Short	1 05	1 00

Terminalisation —The terminalisation coefficient for three different types of bivalents at three different stages of meiosis has been shown in Table V The process of terminalisation either results in a reduction in the number of chiasmata from diplotene to metaphase due to one or more than one chiasma fusing

TABLE V

Terminalisation coefficients in the three types of bivalents at different stages of meiosis

Chromosome type.	No. of nuclei	Stage of meiosis	Total Xma	Term. Xma	Term. coeff	Significance of difference
Long	25	Dip	203	21	$10 \pm 02T_1$	$T_2 - T_1 = 15 \pm 03$
	25	Dia	174	32	$18 \pm 03T_2$	$T_3 - T_2 = 07 \pm 04$
	25	Met	179	45	$25 \pm 03T_3$	$T_3 - T_1 = 03 \pm 03$
Medium	25	Dip	269	70	$26 \pm 03T_1$	$T_1 - T_2 = 04 \pm \cdot 03$
	25	Dia	245	56	$22 \pm 03T_2$	$T_3 - T_1 = \cdot 02 \pm \cdot 04$
	25	Met	225	65	$28 \pm 03T_3$	$T_3 - T_2 = 06 \pm \cdot 03$
Short ..	25	Dip	25	21	$84 \pm \cdot 07T_1$	$T_1 - T_3 = 20 \pm \cdot 12$
	25	Dia	25	20	$80 \pm 08T_2$	$T_1 - T_2 = \cdot 04 \pm \cdot 10$
	25	Met.	25	16	$64 \pm 09T_3$	$T_2 - T_3 = 16 \pm 12$

at the end of the chromosome, or in an increase in the number of terminal chiasmata without actually reducing the total chiasma frequency. In the present study, however, it is seen from the data on significance of difference shown in Tables II and V, that none of the two facts holds good so far as the medium and the short types of bivalents are concerned The long types of bivalents, however, show a statistically significant increase in terminalisation coefficient It is to be concluded, therefore, that the chiasmata remain more or less stationary between diplotene and metaphase in the medium and short types of bivalents A statistically significant increase in terminalisation coefficients from diplotene to metaphase although quite rare amongst the grasshoppers was also found in *Attractomorpha* sp (Ray Chaudhuri and Bose, 1948)

DISCUSSION

A careful measurement of the chromosomes of form 'B' shows that the chromosomes of both the forms are almost identical in size (Table I) The sex chromosome in the Acrididae is largely made up of heterochromatin and is therefore potentially capable of surviving alterations of size in phylogeny than the euchromatic autosomes A difference in the sizes of the sex chromosome was, however, not found The X chromosome is not only fourth in the series according to size in both the forms but also gives an almost identical measurement at the second spermatocyte metaphase stage The nucleination cycle of the heterochromatin in the various stages of mitosis and meiosis in different sub-families of Acrididae appears to be similar (Ray Chaudhuri and Dutt, 1947, Ray Chaudhuri and Bose, unpublished, Ray Chaudhuri and Manna, unpublished), and, therefore, a difference in this respect in the forms compared is hardly expected, none was found either

Turning to our studies on chiasma frequency in the two forms, we find that the total number of chiasmata per nucleus is slightly higher at diplotene in form 'B' and slightly less in diakinesis and metaphase Table VI shows the relevant data

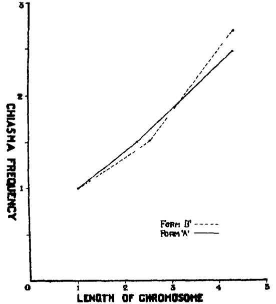
TABLE VI

Comparison of total chiasma frequencies at different stages in the two forms

Stage of meiosis	Mean No of Xta per nucleus	
	Form 'B'	Form 'A'
Dip	19 72±1 85	18 95±1 48
Dia.	17 72±1 85	18 06±1 69
Met	16 88±1 76	16 52±0 96

Since we expect changes in chiasma frequency at different temperatures (White, 1934), significance of differences obtained in the two forms reported above were not calculated statistically, because the studies on the two forms were done on material collected at different times and therefore are hardly comparable.

Our analysis of the chiasma frequency in the long and medium type of bivalents shows a real and significant difference In form 'B' 6 6% of long bivalents have a chiasma frequency of 4, whereas the maximum number of chiasmata in form 'A' in their longest bivalents is 3. Similarly, in the medium type of bivalents form 'B' has a much higher percentage (4 0%) of bivalents with 3 Xta as compared with that of form 'A' (0 6%) In order to avoid error of comparing chiasma frequencies under different conditions, both the forms were again collected and fixed on the same day and the frequencies of chiasmata for long and medium bivalents at diplotene were determined. A large number of nuclei were examined and the results agree with our previous observations, when the studies of the two forms were done separately (Table VII) A difference in the nature of the length-frequency curve in the two forms as is shown in graph 1, is the result of the above facts, because for the same length of the chromosome, form 'B' has a definitely higher chiasma frequency for long and medium bivalents .



GRAPH 1 Showing chromosome length and chiasma frequency relationship in the two forms of *Phloeoba* sp

TABLE VII

Chiasma frequencies in the long and medium types of bivalents at diplotene in forms 'B' and 'A'.

Forms	Long type.				Medium type		
	1 Xma	2 Xta	3 Xta	4 Xta	1 Xma	2 Xta	3 Xta
'B'	0 0	36 0	57 3	6 6	50 3	45 7	4 0
'A'	2 9	49 3	47 8		49 1	50 3	0 6

A difference in the terminalisation coefficients between the two forms is also quite clear. The long bivalents in form 'B' show a statistically significant difference (15± 03) between the terminalisation coefficients at diplotene and metaphase, whereas form 'A' does not show any such difference

How far chiasma frequency can be taken as meiotic constant for a particular species we do not know as yet A large amount of work on closely related species and varieties occurring in the same locality is needed before we pass any final judgement on the point. Similarity of chiasma frequency does not of course show phylogenetic relationship but whether such differences as we have noted above can occur in two varieties of the same species, we cannot say just now. **At least here is a case** where two closely related forms have revealed a constant cytological difference in their chiasma frequencies where accurate metrical studies of the chromosomes have failed to show any such difference

SUMMARY

1 Two closely related forms of grasshoppers belonging to the genus *Phloeoba* were discovered and a study on the chromosomes behaviour was undertaken in order to determine, if possible, any constant cytological difference between them correlated with their morphological difference

2 Number, size and the morphological features of the chromosomes in the two forms are almost identical

3 The long and medium bivalents of forms 'B' definitely show a higher chiasma frequency even when the forms are studied under similar condition of temperature

4 It has been pointed out that without extensive studies between closely related forms of grasshoppers, the full implications of the above findings cannot be realized

ACKNOWLEDGEMENTS

The author is indebted to Dr S P Ray Chaudhuri, Calcutta University, for the suggestion of the problem and for his help and guidance throughout the course of the investigation Thanks are also due to Dr P N Bhaduri, Calcutta University, for helpful criticisms and to Prof H K Mookerjee, Calcutta University, for providing laboratory facilities in his department

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ON THE SPAWNING HABITS AND EARLY DEVELOPMENT OF THE COPPER MAHSEER,[1] *BARBUS* (*LISSOCHILUS*) *HEXAGONOLEPIS* McCLELLAND[2]

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(Communicated by Dr S L Hora, F N I)

(*Received September 10, read November 7, 1947*)

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INTRODUCTION

Like other Mahseers, the Copper Mahseer, *Barbus* (*Lissochilus*) *hexagonolepis* McClelland is a popular game fish and according to Shaw and Shebbeare (1938, p. 38) weight for weight, there is nothing to choose between this and the deep-bodied Mahseer, *Barbus* (*Tor*) *tor* (Hamilton) The Copper Mahseer, during breeding season is known to go to the higher reaches for spawning purposes but the recent researches (Hora and Ahmad, 1946) have shown that this species can be made to breed in tanks also and can also be stripped Like Trout, the Copper Mahseer can be stripped and like Mirror Carp it can be induced to breed in tanks If the fish is not stripped in time, it deposits its ova in suitable place in the tank. Theoretically, once a tank is properly stocked with this fish and suitable conditions for its breeding provided, one needs only to thin out his stock from time to time

Although the fish is well known for its sporting qualities, nothing is known about its development Recent articles of Hora and Nair (1943), Hora (1944), Langdale Smith (1944) and Hora and Ahmad (1946) have, however, thrown some light on its breeding habits

In the present article a short account of the early development of the fish, based on the study of material obtained as the result of artificial fecundation, is given

My thanks are due to Rai Bahadur Dr S L Hora for giving me facilities for carrying out this important piece of research and to Messrs S K Chakraborty and Reza-ud-din Khan of the Directorate of Fisheries, Bengal, for supplying me some developmental stages and also information on certain points

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[1] Though popularly known as a variety of Mahseer by anglers, it is not a true Mahseer as its labial groove is interrupted in the middle It is a fish of the Barbel type

[2] Published with permission of Director of Fisheries, East Bengal.

MATERIAL.

The material for the present study was obtained as the result of artificial fertilization, except the larva described as stage No 15, which was collected from the terraced pond at Kalimpong The first 12 stages described in this paper were collected from Runglee Runglıot ın 1945, while the stages Nos 13–15, were obtained from Kalımpong ın 1946

In 1945, the fertilized ova (Hora and Ahmad, p 6) were transferred to hatching trays, which were kept in a shallow pool with a gentle, continuous flow of water At Kalimpong two types of hatching trays were used, *i e*, (i) ordinary trays, with fine wire-gauze as their bottom, and (ii) trays with wooden bottom instead of wire-gauze The first category of trays were internally lined with mosquito curtain so that the ova by coming into direct contact with the wire-gauze may not get injured and also that silt may not get into trays and produce unhygienic conditions In the trays with wooden bottom pebbles of various sizes were spread in order to provide, as far as possible, natural environment for the development of ova and larvae Both types of trays were in turn placed in a big wooden trough and a continuous current of water was set in it Near the entrance of the trough a break plate was fixed so as to break the force of the current as soon as it entered the trough The trough was water-tight so that some water always remained in it and at no stage was there any chance of its drying up At the bottom of the trough near its outlet an opening was provided for cleaning the trough, this opening could be controlled and regulated by a wooden plug

SPAWNING SEASON

The factors favouring spawning have already been enumerated in an earlier article (Hora and Ahmad, p 5) It was observed (*loc cit*, p 7) that its breeding season had been recorded to be May June by some and August-September by others But the recent observations made at Kalimpong reveal that the breeding season of the fish extends from April with interruptions to October

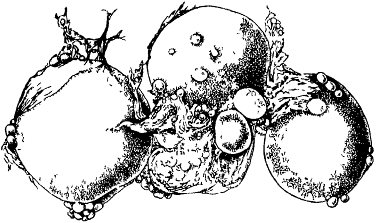
The tank at Kalimpong is situated at an altitude of about 3,500 feet The first sign of sexual excitement of the fish in this tank was noticed on the 28th of April 1946 and the fish were caught and stripped on the 30th April The maximum atmospheric temperature on the 30th April was 81°F and the minimum was 66°F while that of water ranged from 77°F to 67°F during that day There was no rain on the 28th and the 29th April while local observatory recorded 0 53″ rainfall on the 30th April After the 30th April the fish did not show any sign of sexual activity for about four months but it was renewed again in September and that month was found to be the peak period in the spawning of *Katli* at Kalimpong as shown in the table below —

Date of Stripping.	Time of Stripping	Number of males with size	Number of females with size	Temperature of water on the day		Atmospheric temperature on the day		Rain fall
				8 a m	5 p m	Max	Min	
8 9 1946	3 40 p m	Two, 10″ and 1[illegible]″	One, 20″	75°F	80°F	81°F	71 5°F	Nil
20-9 1946	8 a m 1 p m	One, 7″ Two, 12″ and 15″	One, 12″ Two, 2[illegible]″ and 21″	73 5°F	75 8°F	80°F	72°F	Nil
24 9 1946	12 30 noon	One, 12″	One, 16″	71°F	75°F	78°F	70°F	0 40″
14 10 1946	8 a m	Two, 8″ and 10″	One, 17″	70 5°F	72 5°F	77 5°F	66 5°F	Nil

On the other hand, at Runglee Runghot, as has been stated in the previous article (Hora and Ahmad, p 6), the fish were stripped on the 8th, 9th and 24th August and 1st and 10th of September It follows from the observations recorded during the last two years that although the peak period in the spawning of this fish is August and September, it begins to shed ova from the month of April Further observations at different altitudes will show whether the breeding season extends right from April to October or during some of the months there is no sexual activity

OVARIES AND OVARIAN OVA

During the breeding season ovaries contain innumerable ova in various stages of development A *Kathi* caught on the 11th August, 1945, from the terraced ponds at Runglee Rangliot (Dist Darjeeling), measured 21¼" in length, weighed 3½ lbs and possessed ovaries, each measuring 4 5 inches in length and weighing 4 tolas (approximately 1 7 oz) Another *Kathi* secured from the same pond and on the same day, had ovaries weighing 2¾ tolas (approximately 1 oz) and 4 inches in length



TEXT FIG 1 —Ovarian ova × 16⅔

The ova obtained from these ovaries were of various sizes (Text-fig 1) suggesting that all the ova in an ovary do not become ripe just at the same time.

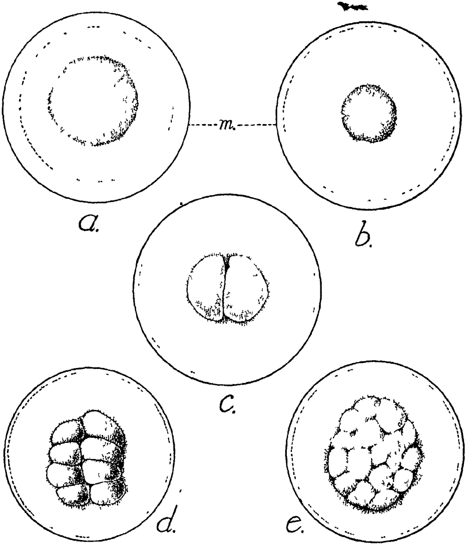
RIPE UNFERTILIZED OVUM

At the time of shedding, the ova are almost translucent and yellowish in colour These are spherical in outline and measure from 2 3 to 2 5 mm in diameter They are demersal and settle down at the bottom, when shed in comparatively still water The yolk is devoid of oil-globules and the ovum is closely surrounded by a single thick egg-membrane

EMBRYONIC DEVELOPMENT

Stage 1 —Fifteen minutes after fertilization The ovum collected fifteen minutes after fertilization shows a thin blastodisc at the animal pole (Text-fig 2*a*) The periphery of the disc is thinner than the central portion and from its study it appears that cytoplasm in the yolk has concentrated to form this mass It is spherical in form and measures 1 2 mm in diameter in the specimen under report. The egg-membrane is separate from the egg proper and the small perivitelline space is full of imbibed water.

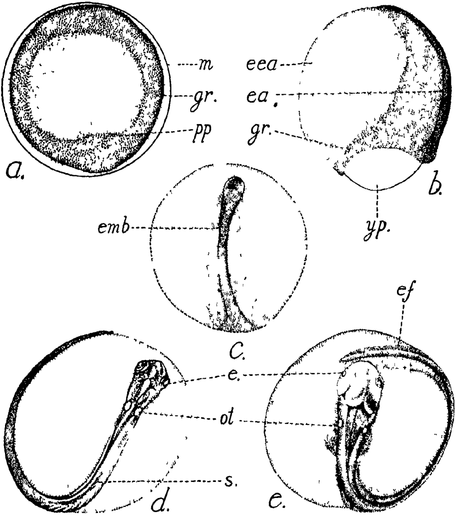
Stage 2 —Two hours and fifteen minutes after fertilization (Text-fig 2*b*). The blastodisc is more condensed and prominent It is shorter in diameter than in the earlier stage



Text fig 2 —Early embryonic development of *Barbus* (*Lissochilus*) *hexagonolepis* McClelland. ×20
(*a*) Egg fifteen minutes after fertilization
(*b*) Egg with fully formed blastodisc.
(*c*) Egg with two blastomeres.
(*d*) Egg with eight blastomeres.
(*e*) Egg with seventeen blastomeres
m —Egg-membrane.

Stage 3 —Three and a half hours after fertilization (Text-fig 2*c*). The germinal disc has divided into almost equal halves Each blastomere has rounded outer and straight inner margins The cleavage of the blastodisc appears to have been almost complete. The free edges of the blastomeres are quite prominent At the bases of the blastomeres, there is a layer of thin protoplasm

Stage 4 —Five and a half hours after fertilization (Text-fig. 2*d*) The blastoderm consists of 8 blastomeres From the study of a number of ova of this stage, it appears that before the blastoderm divides into 4 cells, third set of furrows makes its appearance with the result that the ovum comes to consist of 8 blastomeres



TEXT-FIG 3 —Embryo formation in *Barbus* (*Lissochilus*) *hexagonolepis* McClelland. ×20

(*a*) Egg showing formation of germ-ring.
(*b*) Egg showing differentiation of embryonic shield
(*c*) Further stage in the formation of embryo
(*d*) Embryo showing otocysts, eyes, somites, etc
(*e*) Embryo 62½ hours before hatching.

e.—Eye, *ea*.—Embryonic area; *eea*.—Extra embryonic area, *ef*—Embryonic fin-fold; *emb*.—Embryo, *gr*.—Germ ring, *m*.—Egg membrane, *ot*.—Otocyst; *pp*.—Posterior pole of blastoderm, *s*.—Somites, *yp*.—Yolk-plug

The protoplasm surrounding the cells is reduced and the cells stand out more prominently than in the last stage The blastomeres are much smaller than those in *Stage* 3 In some cases, it is noticed that 4 blastomeres resulting from the division of one of the blastomeres of the two-celled stage, remain quite separate from those of the other

Stage 5 —Seven hours after fertilization (Text-fig. 2*e*) There is no regular arrangement of blastomeres Eggs with 12, 17 and 18 blastomeres are fairly common although a few possess 16 blastomeres also There is not much trace of the protoplasm surrounding the blastomeres

Stage 6 —Twenty-five and a half hours after fertilization The blastomeres have divided and subdivided forming a mass of smaller cells, which have covered a greater part of the yolk than in earlier stages

Stage 7 —Forty-two and a half hours after fertilization (Text-fig 3*a*) The blastoderm has covered almost half of the yolk The free margin of the blastoderm has become thickened to form a band-like thickening, the germ-ring (*gr*) At one point the germ-ring is thickened and broader, this represents the posterior pole (*pp*) of the blastoderm

Stage 8 —Forty-eight hours after fertilization (Text-fig 3*b*) The blastoderm cells have covered more of the yolk than in earlier stages At this stage only about one-fifth of the yolk remains exposed The embryonic shield is well developed, it is triangular in outline and is distinguishable into two parts, (1) a thickened ridge running antero-posteriorly, representing the axis (*ea*) of the embryo, and (2) a thin sheet of protoplasm representing extra embryonic area (*eea*)

Stage 9 —Fifty-seven and a half hours after fertilization (Text-fig 3*c*) The blastoderm has completely grown round the yolk mass and the blastopore is closed The embryonic axis is more developed and it extends about two-thirds along the circumference of the yolk The region of the closed blastopore has thick mass of tissue while anteriorly the embryonic area becomes narrow and ends bluntly

Stage 10 —Eighty-one and a half hours after fertilization (Text-fig 3*d*) The embryo is well defined and is closely attached to the yolk The rudiment of eye (*e*) is present but so far there is no pigment in it Otocysts (*ot*) are present. Somites (*s*) are distinguishable in the middle of body

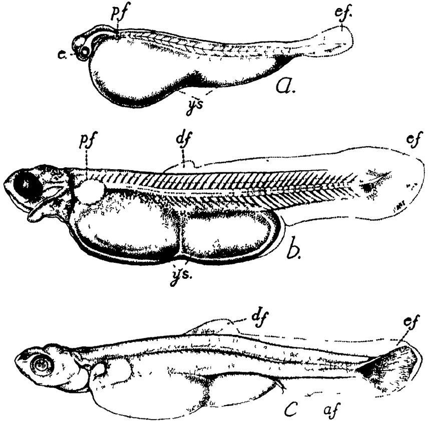
Stage 11 —Ninety hours after fertilization The embryo is slightly more elongated than in the last stage Head cavities are quite prominent The embryonic fin-fold is present in the form of a narrow fold surrounding the tail and extending forward both along the dorsal and the ventral sides of the body

Stage 12 —One hundred and twenty hours after fertilization (Text-fig 3*e*) The lens of the eye can be seen The head cavities are not so prominent as in the last stage The fin-fold is more developed Behind each otocyst, there is seen a rudiment of a gill-slit Pectoral fin appears as a small bud behind each gill-slit

LARVAL DEVELOPMENT

Stage 13 —Newly hatched larva Six days after fertilization (Text-fig. 4*a*). The newly hatched out larva is almost colourless It possesses deflected head, ventral mouth, elongated yolk-sac, bud-like pectoral fins (*pf*), otocysts and rudimentary gill-slits It measures about 6 2 mm in length The first batch of larvae hatched out at 103½ hours and the last one at 190 hours after fertilization of ova

Stage 14 —Six days old larva (Text-fig 4*b*) The larva has grown in length to 9 3 mm The continuous fin-fold (*ef*) is quite wide and is much more developed than in the early stage The anterior-most part of the fin-fold on the dorsal side has become enlarged to form the rudiment of the dorsal fin (*df*) The posterior end of the notochord has bent upwards In the caudal region, rudimentary rays have made their appearance in the fin-fold The eyes have developed pigment Pigment is also distributed on the head and along the body, more so immediately below the notochord than above it



TEXT FIG 4—Larval development of *Barbus* (*Lissochilus*) *hexagonolepis* McClelland

(*a*) Newly hatched out larva ×12½
(*b*) Six days old larva ×12½
(*c*) About one month old larva. ×9½

af—Anal fin fold, *df*—Dorsal fin rudiment, *e.*—Eye, *ef.*—Embryonic fin fold, *pf.*—Pectoral fin, *ys*—Yolk sac

Stage 15—Larva measures 1 ⅓ cms in length (Text-fig 4*c*) The eyes and gills are well developed, dorsal fin (*df*) is more prominent, yolk sac is reduced and the anal fin (*af*) has made its appearance

The above specimen was collected last year from a tank at Kalimpong and according to the statement of the owner of the tank, it is about a month old Since there is no definite evidence to prove the statement, nothing much can be said on this point

SUMMARY

Barbus (*Lissochilus*) *hexagonolepis* McClelland can be stripped like Trout and by providing suitable conditions can be induced to breed in tanks like Mirror Carp

The fish breeds from April to October but the peak period reaches in August and September.

Sometimes ripe females yielded relatively few ova at a time by stripping although innumerable ova in various stages of development were found in the ovaries. It follows that all the ova in an ovary do not become mature at the same time

Ova are typical like those of other carps Blastodisc appears 15 minutes after fertilization, 2 celled stage is formed 3½ hours after, 8-celled stage 2 hours later and the cells become an irregular mass 7 hours after fertilization Germ ring makes its appearance when the ovum is about 42½ hours old At this stage the blastoderm has invested about half of the yolk Forty-eight hours after fertilization, only ¼ of the yolk remains exposed and the embryonic shield is well developed and is distinguishable into an embryonic and an extra-embryonic area

The embryo becomes well-defined when it attains an age of 81½ hours At this stage the rudiment of eyes, otocysts and somites are clearly visible Embryonic fin-fold appears about 90 hours after fertilization and lens of eye as well as rudiment of gill slits appear 30 hours later

Incubation period was found to vary from 103½ to 190 hours in different cases. Newly hatched out larva is almost colourless, possesses deflected head, elongated yolk sac, bud-like pectoral fins, gill slits and otocysts

The anteriormost part of the fin fold becomes enlarged to form the rudiment of the dorsal fin, posterior part of the notochord bends upwards, rudimentary rays make their appearance and eyes develop pigment when the larva becomes 6 days old

In about a month old larva, eyes, gills, dorsal fin and rudimentary anal fin are clearly seen.

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ON SLOW HOMOLOGOUS CONTRACTION OF STARS *

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(Communicated by Prof N R Sen, F N I)

(*Received June 6; read August 1, 1947*)

ABSTRACT

This work examines the possibility of slow homologous contraction of stars under quite general physical conditions, and obtains the restrictions necessary on the relation between opacity and sub-atomic energy generation, as also on the density temperature field Some properties of the stellar models that satisfy these conditions have been investigated, and a test with the conditions shows the impossibility of such contraction by a strictly Cowling model

§ 1 INTRODUCTION

In the current theories of evolution of stars the slow homologous contraction of stellar configuration plays an important rôle The different stages of evolution are supposed to be attained by a stellar mass through contraction of this nature

The possibility of such change was examined by L H Thomas (1930) several years ago He came to the conclusion that slow homologous contraction of stars, in which there is no sub-atomic energy generation can take place only when the law of opacity satisfies certain condition, Kramer's opacity formula conforming to this condition Further, the density and temperature fields should be subjected to additional restrictions To all this must also be added the condition for the stability of the entire configuration In view of the definite knowledge about the generation of sub-atomic energy within a star, which we now have at present, it is necessary to examine the question of homologous contraction from this point of view again The object of this paper is a discussion of this problem The addition of a law of generation of energy complicates the original problem Nevertheless, it is shown that under very plausible assumptions (regarding the opacity of stellar matter and generation of sub-atomic energy), quite definite conclusions can be reached. We have not, however, made any attempt to consider the stability problem here

§ 2 EQUATIONS OF HOMOLOGOUS CHANGE

Thomas's discussion of the problem is based on the idea that the acceleration is negligible and the instantaneous configuration of the star is one of statical equilibrium, so that the stellar mass at any time is subjected virtually to the equations of mechanical equilibrium The infinitely slow disturbance of the mechanical equilibrium is supposed to be brought about by thermodynamic causes It is the thermodynamic equations which contain the time dependent terms whose variation, however, is taken to be extremely slow. We adhere to the same concept in our present discussion and write the general equations of the mechanical motion (radial) as follows

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* Some results of this paper were reported at the Thirty-fourth Annual Session of the Indian Science Congress and an abstract appeared in the *Proceedings* (Section—Physics, Sub-section—Astrophysics).

$$r = -4\pi r^2 \frac{\partial P}{\partial m} - \frac{\gamma m}{r^2} \tag{1}$$

$$4\pi r^2 \rho \frac{\partial r}{\partial m} = 1 \tag{2}$$

m being the mass enclosed within a shell of radius r

The equation of energy is

$$T \quad S = -\frac{\partial F}{\partial m} + \epsilon \tag{3}$$

S being the entropy F the rate of flux of energy and ϵ the rate of generation of sub atomic energy per unit mass With Thomas we neglect r in equation (1) The total pressure is given by

$$P = R\rho T + aT^4 \tag{4}$$

The left hand side of equation (3) implies the rate of increase of heat energy within a spherical shell of thickness dr while the right hand side stands for the increase of energy in the shell due to net flux of radiation across the boundaries and the sub atomic generation of energy

To these we add the following thermodynamic equations

$$T \quad ds = dU + Pd\left(\frac{1}{\rho}\right) \tag{5}$$

$$U = C_v \quad T + \frac{1}{\rho} \quad aT^4 \tag{6}$$

Also F is given by

$$F = -\frac{16\pi^2 ac}{3} \frac{r^4}{k} \cdot \frac{\partial T^4}{\partial m} \tag{7}$$

The seven equations (1) to (7) are sufficient to determine the seven quantities r P ρ T S F U as functions of m and t (ϵ and k being supposed known functions of ρ and T) under given initial conditions It is evident that a further restriction on motion will make the problem over determined and consequently solutions can exist only under special conditions

The condition of homologous contraction (or expansion) can be introduced by the equation

$$r(m \quad t) = r_0(m) \quad f(t) \tag{8}$$

$$f(0) = 1$$

We shall now investigate what special conditions should be satisfied in order that equations (1) to (7) may have solutions of the type (8)

Thomas proved that equations (1) (2) (4) (5) (6) will be consistent with (8) only when the following conditions are satisfied

$$\rho = \frac{\rho_0(m)}{f^3} \tag{8A}$$

$$T = \frac{T_0(m)}{f} \tag{8B}$$

$$P = \frac{P_0(m)}{f^4} \tag{8C}$$

$$S = (3R - C_v) \frac{f}{f} \tag{8D}$$

where the suffix 0 indicates value at $t = 0$ and f' signifies df/dt The equations of this section were all given by Thomas

§ 3 HOMOLOGOUS CHANGE UNDER SUB-ATOMIC ENERGY GENERATION

In order to facilitate our discussion we assume that k and ϵ follow power laws in ρ and T (the assumption being a plausible one), which we write thus

$$k = K\left(\frac{\rho}{T^3}\right)^a T^{-\nu} \tag{9A}$$

$$\epsilon = C\left(\frac{\rho}{T^3}\right)^b T^{\mu} \tag{9B}$$

In what follows we shall write

$$K\left(\frac{\rho_0}{T_0{}^3}\right)^a T_0^{-\nu} = k_0$$

and

$$C\left(\frac{\rho_0}{T_0{}^3}\right)^b T_0{}^{\mu} = \epsilon_0$$

Eliminating F between (3) and (7), substituting for k and ϵ from (9), and using relations (8), (8A), (8B), (8D), which still remain valid, we obtain

$$T_0 \,.\, (3R - C_v)\,\frac{f'}{f} = \frac{1}{f^{\nu}}\,\frac{d}{dm}\left(\frac{16\pi^2 ac}{3}\,\frac{r_0{}^4}{k_0}\,\frac{dT_0{}^4}{dm}\right) + \frac{\epsilon_0}{f^{\mu}} \tag{10}$$

To draw further conclusion from this we use the following

Lemma If

$$\phi_1(m)\; f_1(t) = \phi_2(m)\; f_2(t) + \phi_3(m)\; f_3(t) \tag{11}$$

where ϕ's and f's are (continuous and differentiable) functions of m and t respectively and neither of them vanishes, then *either*,

Case I.

$$f_1(t) = A\; f_2(t) = B\; f_3(t)$$

and

$$\phi_1(m) = \frac{\phi_2(m)}{A} + \frac{\phi_3(m)}{B},$$

or

Case II

$$\phi_1(m) = A\;\; \phi_2(m) = B\;\; \phi_3(m)$$

and

$$f_1(t) = \frac{f_2(t)}{A} + \frac{f_3(t)}{B}$$

A and B being constants

The result is almost evident and is easily proved as follows

Dividing (11) by ϕ_1, and differentiating with respect to m we obtain

$$\frac{d}{dm}\left(\frac{\phi_2}{\phi_1}\right) f_2 + \frac{d}{dm}\left(\frac{\phi_3}{\phi_1}\right) f_3 = 0 \tag{11A}$$

Case I If

$$\frac{d}{dm}\left(\frac{\phi_2}{\phi_1}\right) \neq 0$$

then

$$\frac{f_2}{f_3} = -\frac{\dfrac{d}{dm}\left(\dfrac{\phi_3}{\phi_1}\right)}{\dfrac{d}{dm}\left(\dfrac{\phi_2}{\phi_1}\right)} \tag{12}$$

The left-hand side being function of t only, and right-hand side being function of m only, both members must be constant Hence

$$f_3 = \text{const } f_2 = \frac{A}{B} \; f_2 \text{ (say)}, \tag{13}$$

substituting in (11) and dividing by $f_2 \;\; \phi_1$, we have

$$\frac{f_1}{f_2} = \frac{\phi_2 + \dfrac{A}{B}\phi_3}{\phi_1} = A \text{ (say)}, \tag{14}$$

whence

$$f_1 = Af_2 = Bf_3, \tag{15}$$

and

$$\phi_1 = \frac{\phi_2}{A} + \frac{\phi_3}{B} \tag{16}$$

Case II is identical with case I, only the ϕ functions now play the rôle of the functions f of Case I Analytically, it corresponds to the condition

$$\frac{d}{dm}\left(\frac{\phi_2}{\phi_1}\right) = 0$$

in (11A) Our lemma is thus proved

Now coming back to our equation (10) and assuming $3R \neq C_v$, this being maintained throughout this paper, we note that it is of the form (11), *provided* $\epsilon \neq 0$ According to the lemma proved above we must have, *either*,

$$\text{(I)} \qquad \frac{f'}{f^2} = \frac{A_0}{f^\nu} = \frac{B_0}{f^\mu} \tag{17}$$

and

$$T_0(3R - C_v) = \frac{1}{A_0} \; \frac{d}{dm}\left(\frac{16\pi^2 ac}{3} \; \frac{r_0^4}{k_0} \; \frac{dT_0^4}{dm}\right) + \frac{\epsilon_0}{B_0}, \tag{18}$$

or,

$$\text{(II)} \qquad T_0(3R - C_v) = \frac{1}{A_1} \cdot \frac{16\pi^2 ac}{3} \; \frac{d}{dm}\left(\frac{r_0^4}{k_0} \frac{dT_0^4}{dm}\right) = \frac{\epsilon_0}{B_1} \quad .. \tag{19}$$

and

$$\frac{f'}{f^2} = \frac{A_1}{f^\nu} + \frac{B_1}{f^\mu} \quad . \tag{20}$$

In Case I we have from (17)

$$A_0 = B_0, \quad \mu = \nu$$

and

$$\frac{f'}{f^2} = \frac{A_0}{f^\nu}, \qquad .. \qquad (17A)$$

$\mu = \nu$ implies k_e must be some power of ρ/T^3

From (17A) we obtain, using $f(0) = 1$

$$f'' = A_0^2 \ f^{3-2\nu} \times (2-\nu) \qquad (17B)$$

$$f' = A_0 \ f^{2-\nu} \qquad (17C)$$

$$f = [(\nu-1)A_0 t + 1]^{\frac{1}{\nu-1}} \qquad (17D)$$

We can now confirm that our assumption of $\ddot{r}$ being a small quantity of second order is legitimate If A_0 is a small quantity of the first order, f' will also be so, while f will differ only slightly from unity, so long as t is small compared with $1/A_0$ In that case $\ddot{r}$, which is proportional to f'', will involve A_0^2, and will be of the second order

Continuing our discussion of Case I, we note that Equation (18) imposes a relation between T_0, ρ_0 (the values of T, and ρ, at $t = 0$) To obtain T_0, ρ_0 as functions of m completely we have to use equations (1), (2), and (4) with P_0 written for P, and similarly for other variables The equations in P_0, ρ_0, T_0, r_0 will then be

$$4\pi r_0^4 \frac{dP_0}{dm} + r \ m = 0 \qquad (21)$$

$$4\pi r_0^2 \rho_0 \frac{dr_0}{dm} = 1 \qquad (22)$$

$$P_0 = R\rho_0 T_0 + \frac{1}{3} a T_0^4 \qquad (23)$$

$$A_0 \ T_0 \ (3R - C_v) = \frac{d}{dm}\left(\frac{16\pi^2 ac}{3} \ \frac{T_0^4}{k_0} \cdot \frac{dT_0^4}{dm}\right) + \epsilon_0 \qquad (18A)$$

These equations are co-variant with regard to transformations (8), (8A), (8B), (8C) In case of (18A), A_0, is transformed as

$$A = \frac{A_0}{f^{\nu-1}}$$

Hence the mass, density and temperature fields have the same structure at all times (which is only a property of the homologous change), only the parameter, A, changes from epoch to epoch

In Case II, we have

$$(3R - C_v) \ T_0 = \frac{\epsilon_0}{B_1} = \frac{C}{B_1} \left(\frac{\rho_0}{T_0^3}\right)^b \ T_0^\mu \qquad (24)$$

which if $b \neq 0$, implies a polytropic relation between ρ_0, and T_0 Moreover, the initial distribution will then be determined by (21), (22), (23), and the first equality of (19) which we rewrite here as

$$A_1 \ (3R - C_v) \ T_0 = \frac{16\pi^2 ac}{3} \ \frac{d}{dm}\left(\frac{r_0^4}{k_0} \ \frac{dT_0^4}{dm}\right) \qquad (25)$$

Hence the Case II for $b \neq 0$ requires that (21), (22), (23), and (25) should hold good for a polytropic relation between ρ_0 and T_0

It is proved in the appendix that these conditions cannot be satisfied, and so the case $b \neq 0$, has to be ruled out We are then left with the case $b = 0$ (24) then gives

$$\mu = 1, \qquad B_1 = \frac{C}{3R - C_v}$$

$b = 0$, $\mu = 1$ indicates generation of sub atomic energy proportional to the temperature Equations (21), (22), (23), (25) then determine the initial configuration while f is obtained from (20), putting therein $\mu = 1$ The case $\epsilon = 0$ has not been included in our discussion The case was considered by Thomas without assuming a definite law of opacity, like (9A) He showed that if $\epsilon = 0$, then for homologous motion to be possible

$$k = f\left(\frac{\rho}{T^3}\right) T^{\nu} \tag{9C}$$

This includes (9A) as a particular case Further, the initial distribution at any epoch should be determined by equations (21), (22), (23) and (25), only k_0 in (25) should then mean $f(\rho_0/T_0^3)T_0^{-\nu}$ The form of $f(t)$ is determined by Equation (17A) Equations (21), (22), (23), (25) require four boundary conditions to determine the configuration completely, and we can take the following as the boundary conditions

$$r = 0, \; \frac{dT}{dm} = 0 \;\text{ when }\; m = 0, \;\text{ and }\; T = 0, \;\; \rho = 0 \;\text{ when }\; m = M \;\text{ (the total mass)}$$

Thus the knowledge of M and A_1 determines the configuration uniquely

It may now be noted that the case $\epsilon \propto T$, can be treated exactly in the same manner By transposing ϵ ($= B T$) to the left hand side in Equation (10) we find that k must necessarily be of the same form as (9C) The same differential equations and the same boundary conditions as before determine the configuration here also, only $f(t)$ is now given by

$$\frac{f}{f^2} = \frac{A_0}{f^{\nu}} + \frac{B}{f}$$

instead of by (17A)

By comparing the results stated in the above two paragraphs we may conclude *that two stars of the same mass and having the same law of opacity, but with no generation of energy in one, and generation* $\epsilon \propto T$ *in the other, when contracting homologously will pass through the same set of configurations though at different rates*

In what follows we summarise all the other results obtained in this section *With restrictions (9A), and (9B), a necessary condition for slow homologous contraction of a star composed of perfect gas is*

Firstly, any one of the following conditions should be satisfied

(1) $k\epsilon = \left(\frac{\rho}{T^3}\right)^{\delta}$ *δ being any number*,

(2) $\epsilon \propto T$,

(3) $\epsilon = 0$

Secondly, corresponding to every k *and* ϵ *conforming to any of the above conditions a definite distribution of mass, temperature and density should prevail, viz, those distributions given by (21), (22), (23) and (18A) for case (1), and by (21), (22), (23) and (25) for cases (2) and (3)*

§ 4 CHANGE IN LUMINOSITY UNDER HOMOLOGOUS CONTRACTION

We can calculate the change in the luminosity of a star which is undergoing homologous change It has been previously shown that in such a star for the case

$$k\epsilon = \left(\frac{\rho}{I^{3}}\right)^{\delta}$$

from Equation (18) we can obtain the relation

$$A_0 \;\; T_0 \;\; (3R-C_v) = \frac{d}{dm}\left(\frac{16\pi^2 ac}{3} \;\; \frac{r^4}{k_0} \;\; \frac{dT_0^4}{dm}\right)+\epsilon_0 \tag{26}$$

Substituting from relations (8) (8A) (8C) (17A) and then integrating from $m = 0$ to $m = M$ (the total mass of the star) we obtain

$$\frac{A_0}{f^{\nu \;\; 1}} \;\; (3R-C_v) \int_0^M I \;\; dm = \left[\frac{16\pi^2 ac}{3} \;\; \frac{r^4}{k} \;\; \frac{dI^4}{dm}\right]_M + \int_0^M \epsilon \;\; dm$$

The first term on the right hand side is $-F_M$ where F_M means the rate of total flux of radiation at the surface and represents the luminosity of the star So we obtain from the above equation

$$F_M = (\text{Total sub atomic generation of energy}) - \frac{A_0}{f^{\nu \;\; 1}} \;\; (3R-C_v) \;\; \overline{T} \;\; M \tag{27}$$

$\overline{T}$ being the mean temperature of the star but by (17A)

$$\frac{A_0}{f^{\nu \;\; 1}} = \frac{f}{f} = \frac{r}{r} = A \tag{27A}$$

where A could mean the velocity of expansion at unit distance from the centre *Hence we conclude that the Luminosity of a star undergoing slow homologous contraction exceeds the total sub atomic energy generation by* $-A \;\; (3R-C_v) \;\; \overline{T} \;\; m$ We can also directly verify that this last quantity is equal to the rate of decrease of total internal (i e thermal as well as gravitational) energy E of the star as follows

Expressing the mean temperature $\overline{T}$ in terms of the gravitational energy $-\Omega$ (Chandrashekhar 1938) and replacing R by (C_p-C_v) we obtain

$$A \;\; (3R-C_v) \;\; \overline{T} \;\; M = \frac{A}{3} \;\; \frac{3C_p-4C_v}{C_p-C_v} \;\; \Omega \tag{28}$$

Now
E (total energy) $= -\Omega$ (gravitational energy) $+ U$ (internal thermal energy)

$$= -\Omega\, \frac{3C_p-4C_v}{3(C_p-C_v)}$$

Hence

$$\frac{dE}{dt} = -\frac{3C_p-4C_v}{3(C_p-C_v)} \int_0^M \frac{\gamma \;\; m \;\; dm}{r_0 f} \;\; \frac{f}{f}$$

$$= -\frac{3C_p-4C_v}{3(C_p-C_v)} \;\; \Omega \;\; \frac{A_0}{f^{\nu-1}}$$

$$= -\frac{A}{3} \;\; \frac{3C_p-4C_v}{C_p-C_v} \;\; \Omega \tag{29}$$

which taken along with (28) verifies our result

§ 5 A PROPERTY OF HOMOLOGOUS CHANGE

Integrating (26) from $m = 0$, to $m = m$ we obtain

$$A(3R-C_v)\ \overline{T}(m)\ m = \frac{16\pi^2 ac}{3}\ \frac{r^4}{k}\ \frac{dT^4}{dm} + \int_0^m \epsilon\ dm \tag{30}$$

The corresponding equation in the case of equilibrium is obtained by replacing the left-hand side by zero The right-hand side represents the excess of net flow of radiation, across a sphere enclosing a mass m, over the sub-atomic energy generation within this mass The left-hand side can be interpreted as increase (or decrease) in equilibrium flux brought about by slow homologous contraction (or expansion)

This is an extension of a result enunciated by Milne (1930), viz , that every element of mass of a star with no internal energy generation, while contracting homologously gains an amount of heat proportional to the temperature of the element This generation of heat, however, is to be sharply differentiated from the internal (or sub-atomic) generation of energy represented by ϵ
Now putting

$$\int_0^m \epsilon\ dm = E(m)^* = \frac{E_0(m)}{f^\nu} \tag{30A}$$

we write

$$\eta(m) = \frac{E(m)}{m}\ \frac{M}{E} = \frac{E_0(m)/f^\nu}{m}\ \frac{M}{E_0/f^\nu} = \eta_0(m) \tag{30B}$$

[where $E = E(M)$], an obvious result which we shall use presently.
Now, substituting

$$\int_0^m \epsilon\ dm = \frac{E^*}{M}\ \eta(m)\ m$$

in Equation (30) and dividing by (1), (remembering that $\dot{r} = 0$) we get

$$-A \cdot \frac{3R-C_v}{\gamma}\ \overline{T}(m) = \frac{4\pi ac}{3}\ \frac{1}{k}\ \frac{dT^4}{dP} - \frac{E}{M\gamma}\ \eta \tag{31}$$

here η means $\eta(m)$ (31) together with (1), (2), (4) may now be taken as equations defining the configuration of a star (with given k, ϵ, A_0, M) undergoing slow homologous motion It should be noted that A and E in (31) are parameters varying from epoch to epoch We write Equation (31) in the form

$$\frac{dT^4}{dP} = B\ k\ \eta - \zeta\ k\overline{T}(m) \tag{32}$$

where

$$\left.\begin{aligned} B\dagger &= \frac{3E}{4\pi ac\gamma}\ \frac{1}{M} \\ \zeta &= \frac{3(3R-C_v)}{4\pi ac\gamma} \cdot A \end{aligned}\right\} \tag{32A}$$

---

* These E's are different from the one occurring in the previous article.

† This B is entirely different from B occurring in § 3

ζ, which involves A, is a small quantity, hence $\overline{T}(m)$ in (32) may be taken to be the equilibrium value of this quantity B and ζ here are parameters depending on the epoch t

We shall utilise Equation (32) to deduce some result in the next article

§ 6 A HOMOLOGOUSLY CONTRACTING MODEL IN THE NEIGHBOURHOOD OF THE STANDARD MODEL

Let us find the configuration which can undergo homologous contraction under the condition $k =$ const , and $\epsilon =$ const (i e $\eta = 1$)

In this case the equilibrium configuration (corresponding to $\zeta = 0$ in (32)) is known to be the polytrop $n = 3$ Let us therefore seek an approximate first order solution of (32) in the form

$$\rho = \lambda T^3 + \zeta\,\frac{\phi(T)}{T} \tag{33}$$

where λ and ζ may vary with time We shall obtain λ in terms of B and other parameters and also determine the form of $\phi(T)$

To the first approximation, Equation (32) can be written as (since $\eta = 1$)

$$\frac{dP}{dT^4} = \frac{1}{B\,k} + \frac{\zeta}{B^2\,k}\,\overline{T}(m) \tag{34}$$

Substituting for P and ρ from (4) and (33) respectively, and then equating coefficients of ζ, we get

$$R\,\frac{\phi'(T)}{T^3} = \frac{4}{B^2k}\,\overline{T}(m) \qquad .. \text{(35A)}$$

and

$$R\lambda + \frac{1}{3}\,a = \frac{1}{Bk} \tag{35B}$$

(35B) determines λ, and $\overline{T}(m)$ being known (as it corresponds to the equilibrium polytrop $n = 3$) (35A) determines $\phi'(T)$ Therefore $\phi(T)$ also becomes determined in form Thus the solution of (32) for the case $k =$ const , and $\epsilon =$ const becomes known Further in this case $\nu = 0$, so by relation (30A) $E = E_0$, and B is a time-independent constant, so also is λ as is evident from (35B) Further from (27A) and (32A) we obtain

$$\zeta = \zeta_0\ f$$

where

$$\zeta_0 = \frac{3(3R - C_v)}{4\pi ac\gamma}\,A_0$$

and therefore ζ_0 is time-independent Thus in the ρ, T relation (33), λ is an absolute constant, while ζ slowly varies with time We thus conclude that homologous configurations in the neighbourhood of the standard model have their ρ, T relations governed by equations (33) and (35)

§ 7. ON A COMPARISON BETWEEN THE HOMOLOGOUSLY CONTRACTING MODEL AND A SPECIAL SET OF EQUILIBRIUM MODELS

We have seen that given k and ϵ, the equilibrium configuration and the homologously varying configuration are given (in addition to a common set of equations) respectively by:

$$\frac{1}{k}\cdot\frac{dT^4}{dP} = \frac{3}{4\pi ac\gamma}\cdot\frac{E}{M}\,\eta \qquad \text{(36E)}$$

and

$$\frac{1}{k}\ \frac{dT^4}{dP}=\frac{3}{4\pi ac\gamma}\cdot\frac{E}{M}\eta-\frac{3(3R-C_v)}{4\pi ac\gamma}\ A\ \bar{T}(m) \qquad . \quad (36H)$$

(36H) has only a small additional term on the right-hand side over that in (36E) The quantity E in (36H) and (36E) involves the same constant C, occurring in the expression (9) for E Suppose now we think C to be different in the above two equations In (36H) we consider the fixed value of C, appropriate to the physical law In this equation E, as also A, will change from epoch to epoch ($E=E_0/f^{\nu}$, $A=A_0/f^{\nu-1}$) In (36E), however, we shall suppose that for the moment E is being calculated from (9) in which C is now a variable parameter For different values of this parameter, E in (36E) will thus be different We now ask the question if in (36E) there exists a value E' of E corresponding to the value C' of (the variable parameter) C, such that this equation may lead to the same solution as (36H) at a definite epoch at which E of (36H) is calculated from the fixed value of C appropriate to the physical law, stated before E and A of (36H) varying slowly with time, our question means, if by changing E in (36E) (keeping the form of energy generation formula (9B) the same, but varying only the constant factor C in the formula) it is possible that the equilibrium equation (36E) may give the same configuration as (36H) in which the right-hand side is slowly varying with time A positive answer to this question will mean that the successive configurations of a homologously contracting star will be the same as a set of equilibrium configurations obtained by quickening up or slowing down the process of energy generation *uniformly* throughout the mass (by only slightly changing the constant term C in the energy generation formula)

This would require

$$E'\eta=E\eta-(3R-C_v)\times\text{const}\times\bar{T}$$

as a necessary condition

From this it follows

$$\eta\propto\bar{T}$$

which leads to

$$\epsilon\propto T$$

or

$$\left(\frac{\rho}{T^3}\right)^b T^{\mu}\propto T$$

This for $b\neq 0$ implies a polytropic relation Writing const $\times T$ for ϵ in (18) we are led to Equation (25) (with T written for T_0, etc) Hence Equations (21), (22), (23), (25) (with noughts dropped) will have a polytropic solution, but this has been shown to be impossible in the appendix

For $b=0$, $\mu=1$ leading to

$$\epsilon=\text{const}\ \times T$$

as a necessary condition

To prove the sufficiency of this condition let us put·

$$\epsilon=C\,.\,T$$

in (36H) and

$$\epsilon=C'\ T$$

In (36E). Then the two equations will be identical if

$$C'=C-(3R-C_v)\ A. \qquad (37)$$

thus to every value of A, corresponding to an epoch, there exists a value of C'_n which makes the Equations (36E) and (36H) identical and since the boundary conditions are the same the equations will have identical solutions

Hence we conclude that *under restrictions of energy generation and opacity represented by (9), it is only when $\epsilon \propto T$, that the successive configurations of a homologously contracting star will be identifiable with the equilibrium configurations obtained by only uniformly speeding up energy generation throughout the star in a definite manner*

§ 8 The point source model

We have so far dealt only with stars having continuous generation of energy We shall now consider the case of the point source model i e a star in which $\epsilon = 0$ everywhere except at $m = 0$, and

$$\left[\frac{16\pi^2 ac}{3} \; \frac{r^4}{k} \; \frac{\partial T^4}{\partial m}\right]_{m=0} = -E$$

Substituting from (8) (8A) (8B), etc , we find that $E \; f^\nu$ is independent of time, also as

$$k = k_0 f^\nu$$

it follows

$$kE = k_0 E_0 \tag{38}$$

which is a necessary condition for a point source model being suitable for homologous motion

§9 Homologous contraction not possible for the Cowling model

We now come to the discussion of the possibility of homologous contraction of the Cowling model with the law of opacity and energy generation given by Kramer's and Bethe's law respectively

Inside the core which is in convective equilibrium there may be transfer of mass from one portion of the star to another If we neglect the kinetic energy of this mass motion as being of small order compared to other energies we can write the same equation as (3) of §2, but we are to remember that here the value of F cannot be substituted from (7), as F is here not due to radiation alone but includes energy of convection also

The distribution of mass, density, and temperature within the core is assumed to be given by that of a polytrop $n = \frac{3}{2}$, for which

$$T \propto \rho^{\frac{2}{3}} \tag{39}$$

and further we have the gas law (neglecting radiation pressure for the Cowling model)

$$P = R\rho T \tag{4A}$$

this last equation replacing (4) of § 2

Equations (1), (2), (39), together with (4A) determine P, ρ, T, r within the core, and when these are obtained S, U can be determined from (5) and (6)

If homologous contraction takes place we can deduce from (1), (2), (4A), (5), and (6) the relations (8A), (8B), (8C), (8D) (the same results will be obtained if we use (4) in place of (4A), i e if we do not neglect radiation pressure)

Now substituting from (8A), etc , in (3) we get

$$(3R - C_v) \;\; T \; \frac{f}{f} = -\frac{\partial F}{\partial m} + \epsilon$$

integrating from $m = 0$ to the interface (denoted by ι)

$$F_\iota = \int_0^\iota \epsilon \; dm - (3R - C_v) \; \frac{f'}{f} \int_0^\iota T \; dm$$

$$= \frac{1}{f^\mu} \int_0^\iota \epsilon_0 \; dm - (3R - C_v) \; \frac{f'}{f^2} \int_0^\iota T_0 \; dm \tag{40}$$

F_ι, denoting the flux at the interface which should be the same whether the interface is approached from inside or outside

We assume the energy production to be entirely confined within the convective core Outside the core where the equilibrium is radiative we have

$$\frac{\partial}{\partial m}\left(\frac{16\pi^2 ac}{3} \; \frac{r^4}{k} \; \frac{\partial T^4}{\partial m}\right) = (3R - C_v) \; T \; \frac{f'}{f}$$

Integrating this outside from interface ι to m, and writing, $T = T_0/f$ etc , we obtain

$$\frac{1}{f^\nu}\left(\frac{16\pi^2 ac}{3} \; \frac{r_0{}^4}{k_0} \; \frac{dT_0{}^4}{dm}\right) + F_\iota = (3R - C_v) \; \frac{f'}{f^2} \int_\iota^m T_0 \; dm \tag{41}$$

Eliminating F_ι between (40) and (41), we get

$$\frac{16\pi^2 ac}{3} \; \frac{r_0{}^4}{k_0} \; \frac{dT_0{}^4}{dm} \; \frac{1}{f^\nu} = \frac{1}{f^\mu} \int_0^i \epsilon_0 \; dm + (3R - C_v) \; \frac{f}{f^2} \int_0^m T_0 \; dm \tag{42}$$

By the lemma proved in §2 we must have either of the following as a necessary condition

(1) $\mu = \nu$

(2) $\int_0^m T_0 \; dm = \text{const} \times \int_0^\iota E_0 \; dm$

(2) is obviously absurd as the upper limit on the left is arbitrary, and (1) cannot also hold for the following reasons For Kramer's law

$$k \propto \frac{\rho}{T^{3\ 5}}$$

we have to put

$$\nu = \tfrac{1}{2} ,$$

so that in order to satisfy condition (1) we must have

$$\epsilon \propto \left(\frac{\rho}{T^3}\right)^b T^{\frac{1}{2}}$$

(it being assumed that the generation of energy takes place only within the convective core) This is absolutely inconsistent with Bethe's law of energy generation, which can well be represented by the law

$$\epsilon \propto \rho T^x$$

where x may range from 15 to 19, we have to put $b = 1$ It is moreover absurd, as for $b > \frac{1}{2}$ the generation would decrease with rise of temperature

When the generation overflows the core we have Equations (3) and (7) holding for a portion outside the core, whence the conclusion $\mu = \nu$ can be drawn as in §3, or $\epsilon \propto T$, both of which are excluded by the above arguments

Thus *a Cowling model is not suitable for homologous contraction irrespective of whether the generation of energy is entirely confined within the core or overflows it.*

In conclusion I desire to express my grateful thanks to my professor, Dr N R Sen, for suggesting the problem and also for his constant interest and many helpful discussions

Appendix

We shall prove that the equations

$$\rho_0 = C \quad T_0^n \qquad \cdot\cdot \tag{1}$$

$$A_0 \quad (3R-C_v) \quad T_0 = \frac{16\pi^2 ac}{3} \quad \frac{d}{dm}\left(\frac{r_0^4}{k_0} \quad \frac{dT_0^4}{dm}\right) \tag{2}$$

$$\frac{dP_0}{dm} \quad 4\pi r_0^2 + \gamma \quad m \tag{3}$$

$$4\pi r_0^2 \, \rho_0 dr_0 = dm \tag{4}$$

where P_0, ρ_0, T_0 are connected by the relation

$$P_0 = R\rho_0 T_0 + \tfrac{1}{3} \, aT_0^4 \tag{5}$$

cannot have a physically possible common solution

Proof Dropping noughts we get from (3), (4), (1) and (5)

$$\frac{1}{\xi^2} \quad \frac{d}{d\xi}\left\{\xi^2\left[(n+1)+\frac{4A}{3}\,T^{3-n}\right]\frac{dT}{d\xi}\right\} = -T^n \tag{6}$$

where

$$\xi = \sqrt{B}\,.\,r$$

$$B = \frac{4\pi rc}{R} \quad B>0$$

$$A = \frac{a}{Rc} \quad A>0$$

From (1), (5), (2) and (4)

$$\frac{1}{\xi^2} \quad \frac{d}{d\xi}\left(\xi^2 T^\lambda \frac{dT}{d\xi}\right) = b\,.\,T^{n+1} \qquad \cdot\cdot \qquad \cdot\cdot \tag{7}$$

where

$$b = A_0(3R-c_v) \quad \frac{3K}{ac} \quad \frac{C^{\alpha+2}}{4B}$$

$$\lambda = 3-n-n\alpha+3\alpha+\nu$$

α and ν being as given in formula (9) of § 2

Eliminating $d^2T/d\xi^2$ between (6) and (7) we get

$$\left(\frac{dT}{d\xi}\right)^2 = \frac{(n+1) \quad b \quad T^{n+2-\lambda} + \frac{4A}{5} b \quad T^{5-\lambda} + T^{n+1}}{(n+1)\lambda + \frac{4A}{3}\,(\lambda-3+n) \quad T^{3-n}} \qquad . \qquad . \tag{8}$$

At this stage let us note the following as necessary conditions for a possible solution

(i) As from (3) and (4)

$$\gamma\ m = -\frac{r^2}{\rho}\ \frac{dP}{dr} = -r^2\left\{R(n+1)+\frac{4a}{3C}\ T^{3-n}\right\}\frac{dT}{dr}$$

but as $(m)_{T=0}$ must be finite (being the total mass of the star)

$$\left[r^2\left\{R(n+1)+\frac{4a}{3C}\ T^{3\ n}\right\}\ \frac{dT}{dr}\right]_{T=0}$$

is finite so that for $n < 3$

$$\left(\frac{dT}{dr}\right)_{T=0}$$

must be finite that is

$$\left(\frac{dT}{d\xi}\right)^2_{T=0}$$

must be positive and finite

For $n > 3$

$$\left[\frac{\left(\frac{dT}{d\xi}\right)^2}{T^{2n\ 6}}\right]_{T=0}$$

must be positive and finite

(ii) $(dT/d\xi)^2 = 0$ for some finite positive value $(T = T_c)$ of T We shall now show that (8) cannot satisfy these conditions We shall prove that in order to satisfy (i) $b > 0$ so that all terms in the numerator of the R H S of (8) become positive so that no positive value of T can make $dT/d\xi$ vanish so that (ii) cannot be satisfied Proof of $b > 0$ Case I $n < 3$

To satisfy (i)

either (1) $\lambda = n+2$ and $\lambda < 5$

or (2) $\lambda < n+2$ and $\lambda = 5$

or (3) $\lambda = n+2$ and $\lambda = 5$

(2) gives $n > 3$ and (3) gives $n = 3$ So they are ruled out For (1) we get

$$\left(\frac{dT}{d\xi}\right)^2_{T=0} = \frac{b}{\lambda} = \frac{b}{n+2} > 0$$

$$b > 0$$

Case II $n = 3$

To satisfy (i)

$$\lambda = 5$$

so that

$$\left(\frac{dT}{d\xi}\right)^2_{T=0} = \frac{b}{5} > 0$$

$$b > 0$$

Case III $n > 3$

$$\frac{\left(\frac{dT}{d\xi}\right)^2}{T^{2n\ 6}} = \frac{(n+1)b\ \ T^{5\ \ \lambda}+\frac{4A}{3}b\ \ T^{8\ \ \lambda\ \ n}+T^4}{(n+1)\lambda T^{n-3}+\frac{4A}{3}(\lambda-3+n)}$$

here to satisfy (i)

either (1) $8-\lambda-n=0$ and $\lambda<5$
or (2) $\lambda=5$ and $8-\lambda-n>0$
or (3) $\lambda=5$ and $8-\lambda-n=0$

Cases (2) and (3) are ruled out as they give $n<3$ and $n=3$ respectively For (1)

$$\left\{\frac{1}{T^{2n-6}}\left(\frac{dT}{d\xi}\right)^2\right\}_{T=0}=\frac{b}{\lambda-3+n}=\frac{b}{5}>0$$

Thus proving that

$$b>0$$

in all cases and thereby establishing our theorem

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PARALLEL DISPLACEMENT AND SCALAR PRODUCT OF VECTORS

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(Communicated by Dr N R Sen, D Sc, F N I)

(*Received February 24, read August 1, 1947*)

ABSTRACT

In this paper it is shown initially that the consideration of an arbitrary parallel displacement of a vector in a Riemannian space gives rise to an associated parallelism depending on the change in the scalar product of vectors in a particular manner and some properties involving the two kinds of parallel displacements of vectors round an infinitesimal closed circuit are deduced The general form of a parallelism with symmetric connection is then established from which Weyl's parallelism is easily deduced On imposing certain condition a connection between this parallelism and the Levi Civita parallelism is finally given through the introduction of an orthogonal ennuple in the space

1 Let us consider an n dimensional space with the hypotheses that the metric of the space is given a quadratic differential form

$$ds^2 = g_{ij}\,dx^i dx^j \tag{1 1}$$

as in Riemannian geometry, and that there is, in the space, a law of parallel displacement of a contravariant vector defined by

$$dV^l + \Gamma^l_{pq} V^p dx^q = 0 \tag{1 2}$$

as in geometry with affine connection The covariant derivative of a tensor with respect to the Γ's or rather with respect to the parallelism (1 2) which we shall denote by the notation , (comma), is constructed in the usual manner, as for example, for a mixed tensor T^i_{jk} of the third order,

$$T^i_{jk,l} = \frac{\partial T^i_{jk}}{\partial x^l} + T^t_{jk}\Gamma^i_{tl} - T^i_{tk}\Gamma^t_{jl} - T^i_{jt}\Gamma^t_{kl}$$

so that the covariant differentiation of the sum, difference, outer and inner product of tensors obey the same rules as ordinary differentiation

The convention of summation with respect to indices, when they occur once above and once below, is followed throughout, but other Σ's are retained

On account of the existence of the metric (1 1), the covariant and contravariant components of a vector V are, as usual, derived from one another by

$$V_i = g_{ij}V^j, \quad V^i = g^{ij}V_j$$

and the scalar product of two vectors U and V is given by

$$U^iV_i = U_iV^i = g_{ij}U^iV^j = g^{ij}U_iV_j$$

It therefore follows from (1.2) that

$$dV_i = d(g_{il}V^l) = \left[\frac{\partial g_{il}}{\partial x^q} - g_{it}\Gamma^t_{lq}\right] V^l dx^q = \left[g^{lt}g_{il,q} + \Gamma^t_{iq}\right] V_t\, dx^q$$

Accordingly,

$$dV_i = \bar{\Gamma}^t_{iq} V_t \, dx^q \tag{1 2'}$$

where

$$\bar{\Gamma}^j_{pq} = \Gamma^j_{pq} + g^{jt} g_{pt,q} \tag{1 3}$$

The equations (1 2) and (1 2′) give the increments of the contravariant and covariant components respectively of a vector V due to the parallel displacement (1 2) Since we have identically

$$g^{pq}{}_{,r} = -g^{pi} g^{qj} g_{ij,r}$$

the parallelism (1 2) can as well be defined by (1 2′)

With the parallel displacement (1 2) we can associate another parallel displacement defined by

$$dV^j + \bar{\Gamma}^j_{pq} V^p dx^q = 0 \quad . \tag{1 4}$$

where $\bar{\Gamma}^j_{pq}$ is defined by (1 3). For the parallel displacement (1 4) we have

$$dV_i = \Gamma^t_{iq} V_t \, dx^q \quad . \quad . \tag{1 4'}$$

giving the increment of the covariant component The parallelism (1 4) can as well be defined by (1 4′)

Using the notation ; (semi-colon) for the covariant derivative with respect to (1 4), it can be easily seen that

$$g_{pq,r} + g_{pq;r} = 0 \quad . \quad . \tag{1 5}$$

Therefore (1 3) can be written as

$$\Gamma^j_{pq} = \bar{\Gamma}^j_{pq} + g^{jt} g_{pt;q} \quad . \quad . \quad . \tag{1 3'}$$

From (1 5) it is seen that the change in the scalar product of two arbitrary vectors, when the vectors are given the parallel transport (1 2) along any direction, is equal but opposite in sign to that when given the parallel transport (1 4) along the same direction Obviously, when the scalar product remains unaltered for either parallelism, the two parallelism become one and the same The square of the length of a vector is of course a particular case of the scalar product

It may be seen that the scalar product of two vectors remains unaltered when one of the vectors is given the parallel transport (1 2) and the other its associate (1 4)

The relation between the second covariant derivatives of the g_{pq}'s with respect to (1 2) and (1 4) can easily be obtained from (1 3) and (1 5) It is seen that

$$g_{pq,rs} + g_{pq;rs} = g^{ij}\{g_{ip,r} g_{jq,s} + g_{ip,s} g_{jq,r} + g_{pq,i} g_{jr,s}\}.$$

Therefore

$$\begin{aligned}(g_{pq,rs} - g_{pq,sr}) + (g_{pq;rs} - g_{pq;sr}) &= g^{ij} g_{pq,i}(g_{jr,s} - g_{js,r}) \\ &= g^{ij} g_{pq;i}(g_{jr;s} - g_{js;r}). \quad .. \end{aligned} \tag{1 6}$$

2. Let

$$L^i_{tjh} = \frac{\partial \Gamma^i_{th}}{\partial x^j} - \frac{\partial \Gamma^i_{tj}}{\partial x^h} + \Gamma^i_{sj}\Gamma^s_{th} - \Gamma^i_{sh}\Gamma^s_{tj} \quad . \quad . \quad . \tag{2.1}$$

be the curvature tensor formed with respect to (1 2) Similarly, let $\bar{L}^t_{ijk}$ be the curvature tensor formed with respect to (1 4) It can then be seen from (1 3), (1 3′) and (1 6) that

$$L^t_{ijk}-\bar{L}^t_{ijk}=g^{st}\{g_{is,\,jk}-g_{is,\,kj}+g_{is,\,m}(\Gamma^m_{jk}-\Gamma^m_{kj})\} \tag{2 2}$$

$$\bar{L}^t_{ijk}-L^t_{ijk}=g^{st}\{g_{is,\,jk}-g_{is,\,kj}+g_{is,\,m}(\bar{\Gamma}^m_{jk}-\bar{\Gamma}^m_{kj})\} \tag{2 2′}$$

Now if we put

$$d_1\xi^t=L^t_{ijk}\xi^i dx^j\delta x^k,\quad d_2\xi^t=\bar{L}^t_{ijk}\xi^i dx^j\delta x^k,$$

then it is well known that $d_1\xi^t$ is the change in the contravariant component of a vector ξ when the vector is carried by parallel displacement (1 2) round the infinitesimal closed circuit $(dx,\,\delta x)$ Similarly, $d_2\xi^t$ is the change of the vector when taken by (1 4) round the same circuit

Also, from the nature of the two parallel displacements,

$$d_1\xi_t=-\bar{L}^i_{tjk}\xi_i dx^j\delta x^k,\quad d_2\xi_t=-L^i_{tjk}\xi_i dx^j\delta x^k$$

It therefore follows that, for an arbitrary vector η,

$$\eta_t(d_1-d_2)\xi^t=\xi^t(d_1-d_2)\eta_t \tag{2 3}$$

And from (2 2) it follows that

$$\eta_t(d_1-d_2)\xi^t=\{g_{st,\,jk}-g_{st,\,kj}+g_{st,\,m}(\Gamma^m_{jk}-\Gamma^m_{kj})\}\xi^s\eta^t dx^j\delta x^k \tag{2 4}$$

The geometrical significance of (2 4) can be seen as follows

$$g_{st,\,j}\xi^s\eta^t dx^j$$

is the change of $g_{st}\xi^s\eta^t$, i e, of the scalar product of the vectors ξ and η, when the vectors are given the parallel displacement (1 2) along dx. And

$$(g_{st,\,jk}+g_{st,\,m}\Gamma^m_{jk})\xi^s\eta^t dx^j\delta x^k=\left(\frac{\partial g_{st,\,j}}{\partial x^k}-g_{mt,\,j}\Gamma^m_{sk}-g_{sm,\,j}\Gamma^m_{tk}\right)\xi^s\eta^t dx^j\delta x^k$$

is the change in the scalar product of the vectors ξ and η when the vectors are given the parallel displacement (1 2) first along dx and then along δx Therefore, the right-hand side of (2 4) gives the change in the scalar product of the vectors ξ and η when the vectors are given the parallel displacement (1 2) round the closed circuit $(dx,\,\delta x)$

Hence, when a vector ξ is given one of the parallel displacements (1 2) and (1 4) round a closed circuit $(dx,\,\delta x)$ in one sense and then the other in the opposite sense, the scalar product of the change of ξ on account of such displacements and an arbitrary vector η is equal to, within sign, the increment in the scalar product of the vectors ξ and η when the vectors are transported simultaneously round the circuit once, and this increment is numerically the same whether it is calculated with reference to (1.2) or to (1 4)

This result is exactly what we could expect from (1 5) and (2 3)

Again let

$$dV^l+\Delta^l_{pq}V^p dx^q=0 \quad \ldots \quad \ldots \tag{2 5}$$

be any other parallel displacement, and R^t_{ijk} be the curvature tensor formed with respect to this parallelism

Put

$$\Gamma^l_{pq}-\Delta^l_{pq}=T^l_{pq},\quad R^t_{ijk}-L^t_{ijk}=S^t_{ijk}.$$

Then

$$S^t_{ijk} = T^t_{ij,k} - T^t_{ik,j} + T^t_{sj}T^s_{ik} - T^t_{sk}T^s_{ij} - T^t_{is}\left(\Gamma^s_{jk} - \Gamma^s_{kj}\right) \quad . \qquad . \quad (2.6)$$

A geometrical significance of S^t_{ijk} may be seen as follows Consider the scalar product

$$\xi_t \eta^t$$

and let the covariant and contravariant vectors be given respectively the parallel displacements (1 4) (i e, the associate of (1 2)) and (2 5) firstly along dx and then along δx So, for the general increment of the scalar product over these displacements, we have firstly

$$\left[\Gamma^t_{ij} - \Delta^t_{ij}\right]\xi_t \eta^i dx^j = T^t_{ij}\xi_t \eta^i dx^j$$

And then

$$\left[\frac{\partial T^t_{ij}}{\partial x^k} + T^s_{ij}\Gamma^t_{sk} - T^t_{sj}\Delta^s_{ik}\right]\xi_t \eta^i dx^j \delta x^k = \left[T^t_{ij,k} + T^t_{sj}T^s_{ik} + T^t_{is}\Gamma^s_{jk}\right]\xi_t \eta^i dx^j \delta x^k .$$

Interchanging j, k and subtracting, by which the vectors are brought back to the starting point round the infinitesimal parallelogram (dx, δx), we get by (2 6)

$$S^t_{ijk}\xi_t \eta^i dx^j \delta x^k$$

3 Let

$$\nabla^l_{pq} = \tfrac{1}{2}\left(\Gamma^l_{pq} + \Gamma^l_{qp}\right) \quad . \qquad . \quad (3\ 1)$$

be the symmetric part of Γ^l_{pq} Similarly, let ∇'^l_{pq} be the symmetric part of $\bar{\Gamma}^l_{pq}$ Then from (1 3)

$$\nabla'^l_{pq} = \nabla^l_{pq} + \tfrac{1}{2} g^{lt}(g_{pt,q} + g_{qt,p}) \quad . \quad (3\ 2)$$

Now, consider the parallel displacement of a contravariant vector defined by

$$dV^l + \nabla^l_{pq} V^p dx^q = 0 \qquad (3\ 3)$$

and denote the covariant derivative with respect to (3 3) by the notation ( ) with a subscript It can then be seen that

$$g_{pt,q} + g_{qt,p} = 2\left\{(g_{pt})_q + (g_{qt})_p\right\} - \left\{\frac{\partial g_{pt}}{\partial x^q} + \frac{\partial g_{qt}}{\partial x^p} - \frac{\partial g_{pq}}{\partial x^t} + g_{pq,t} - 2g_{ts}\nabla^s_{pq}\right\}$$

$$\therefore \quad \tfrac{1}{2} g^{lt}(g_{pt,q} + g_{qt,p}) = \nabla^l_{pq} - \begin{Bmatrix} l \\ pq \end{Bmatrix} + g^{lt}\left\{(g_{pt})_q + (g_{qt})_p - \tfrac{1}{2} g_{pq,t}\right\},$$

where $\begin{Bmatrix} l \\ pq \end{Bmatrix}$ is the Christoffel symbol. Therefore from (3 2) we obtain

$$\nabla'^l_{pq} - \begin{Bmatrix} l \\ pq \end{Bmatrix} = \tfrac{1}{2} g^{lt}\left[\left\{g_{pt,q} + g_{qt,p} + g_{pq,t}\right\} - 2\left\{(g_{pt})_q + (g_{qt})_p\right\}\right]$$

But from (3 1)

$$g_{pt,q} + g_{qt,p} + g_{pq,t} = (g_{pt})_q + (g_{qt})_p + (g_{pq})_t \quad . \qquad . \quad (3\ 4)$$

Hence finally

$$\nabla'^l_{pq} = \begin{Bmatrix} l \\ pq \end{Bmatrix} + \tfrac{1}{2} g^{lt}\left[(g_{pq})_t - (g_{pt})_q - (g_{qt})_p\right] \quad .. \quad (3.5)$$

This is the general form of relationship of the parallelism (3 3) with the Levi-Civita parallelism in our space. It is of course evident that (3.3) reduces to the Levi-Civita parallelism on imposing the condition that the scalar product of two vectors remains unaltered when the vectors are given the parallel displacement (3 3).

It is at once recognised that Weyl's parallelism is obtained from (3 3) by writing down his characteristic equation (Weyl, 1921), namely

$$(g_{pq})_r+g_{pq}\omega_r=0$$

and thus getting from (3.5)

$$\nabla^l_{pq}=\left\{ {l \atop pq} \right\}+\frac{1}{2}\left[\delta^l_p\omega_q+\delta^l_q\omega_p-g_{pq}\omega\right]$$

For the geometry of Weyl's space, as is well known, we have to impose the condition

$$dS+S\,d\omega=0$$

where S is the scalar product of two vectors and dS its change due to parallel displacement (3 3) of the vectors

We now suppose that the parallelism (3 3) possesses the property that the length of a vector remains unaltered when the vector is given this parallel displacement *in the direction of the vector* Then (3 4) vanishes, that is,

$$\begin{aligned}&(g_{pq})_r+(g_{qr})_p+(g_{rp})_q\\&=\frac{\partial g_{pq}}{\partial x^r}+\frac{\partial g_{qr}}{\partial x^p}+\frac{\partial g_{rp}}{\partial x^q}-2\left(g_{ps}\nabla^s_{qr}+g_{qs}\nabla^s_{rp}+g_{rs}\nabla^s_{pq}\right)=0\end{aligned} \qquad (3\ 6)$$

Accordingly (3 5) reduces to

$$\nabla^l_{pq}=\left\{ {l \atop pq} \right\}+g^{lt}(g_{pq})_t \qquad . \qquad (3\ 7)$$

The equations (3.6) and (3 7) can easily be deduced from one another

4. We now propose to study the parallelism (3 3) defined by the property (3.6) from a different standpoint

Let us specialise an arbitrary vector field h_t in the space by laying down the condition that the scalar product h_tV^t remains unaltered when the arbitrary vector V is given the parallel displacement (3 3) in the direction of V while h_t is given the local displacement in the same direction Then

$$\left(\frac{\partial h_p}{\partial x^q}-h_t\nabla^t_{pq}\right)V^pV^q=0,\quad \text{or,}\quad (h_p)_qV^pV^q=0$$

$$\therefore\qquad (h_p)_q+(h_q)_p=0 \qquad . \qquad . \qquad (4\ 1)$$

In order to obtain an explicit expression for the ∇^l_{pq}, let ${}^\alpha h_t$ be n^2 functions of the x's specifying n linearly independent vector fields and satisfying the condition (4 1), namely

$$({}^\alpha h_p)_q+({}^\alpha h_q)_p=0$$

or

$$\frac{\partial\,{}^\alpha h_p}{\partial x^q}+\frac{\partial\,{}^\alpha h_q}{\partial x^p}-2\,{}^\alpha h_r\nabla^r_{pq}=0 \qquad .. \qquad .. \qquad .. \ (4\ 2)$$

If ${}_{\alpha}h^{i}$ denotes the cofactor of ${}^{\alpha}h_{i}$ in the determinant $|{}^{\alpha}h_{i}|$ divided by this determinant, then (4 2) gives

$$\nabla^{l}_{pq} = \tfrac{1}{2}\, {}_{\alpha}h^{l}\left(\frac{\partial\, {}^{\alpha}h_{p}}{\partial x^{q}} + \frac{\partial\, {}^{\alpha}h_{q}}{\partial x^{p}}\right) \qquad \ldots \quad \ldots \quad \ldots (4.3)$$

It is evident that this expression of ∇^{l}_{pq} is so far independent of any metric of the space

Among the possibilities of the expression of Γ^{l}_{pq} consistent with (3 1) and (4 3), we may mention the following

$$(1)\ \Gamma^{l}_{pq} = {}_{\alpha}h^{l}\,\frac{\partial\, {}^{\alpha}h_{p}}{\partial x^{q}}, \qquad (2)\ \Gamma^{l}_{pq} = {}_{\alpha}h^{l}\,\frac{\partial\, {}^{\alpha}h_{p}}{\partial x^{q}} + a^{l}_{pq},$$

$$(3)\ \Gamma^{l}_{pq} = {}_{\alpha}h^{l}\,\frac{\partial\, {}^{\alpha}h_{q}}{\partial x^{p}} \text{ and } (4)\ \Gamma^{l}_{pq} = {}_{\alpha}h^{l}\,\frac{\partial\, {}^{\alpha}h_{q}}{\partial x^{p}} + a^{l}_{pq}$$

where a^{l}_{pq} is an arbitrary tensor which is skew in the indices p and q The auto-parallels are of course the same in all these cases as in the case (4.3)

When the Γ's are expressed as in the case (1), the parallelism (1.2) is the well-known distant parallelism or teleparallelism (Weitzenbock, 1923) for which

$${}^{\alpha}h_{i,j} = 0, \quad L^{t}_{ijk} = 0$$

where L^{t}_{ijk} is defined by (2 1). The differential equations (1 2) possess, in this case, solutions which are linear in the V's Also $L^{t}_{ijk} = 0$ is the condition of compatability of the partial differential equations

$$g_{ij,k} = 0 \qquad . \qquad (4\ 4)$$

We can therefore choose the solutions of (4 4) as the fundamental metric tensor. Obviously in this case $\Gamma^{l}_{pq} = \Gamma^{l}_{pq}$ and the scalar product of two vectors remains unaltered for this parallel transport

When the parallelism (1 2) is defined by either of the cases (2), (3) or (4), we have

$${}^{\alpha}h_{i,j} + {}^{\alpha}h_{j,i} = 0$$

and, if the g_{ij}'s are chosen as the solutions of (4 4),

$$g_{ij,k} + g_{jk,i} + g_{ki,j} = 0,$$

and the length of a vector remains unaltered when the vector is given any one of these parallel transports in the direction of the vector

5. Having obtained (4 3) we can find all the metrices compatible with (3 6).

A particular solution of (3 6) is

$$g_{pq} = {}^{\alpha}h_{p}\,{}^{\beta}h_{q} + {}^{\beta}h_{p}\,{}^{\alpha}h_{q}, \qquad . \quad \ldots \quad \ldots (5.1)$$

where α and β are any two of the numbers 1, , n

For, differentiating (5 1) and using the equations (4 2) and (5 1), it is seen that (3 6) is satisfied.

On account of the linearity of the equations (5 1), the sum of any number of particular solutions, each multiplied by an arbitrary constant, is a solution. Thus the most general solution of (3 6) is, the c's being constants,

$$g_{pq} = c_{\alpha\beta}\,{}^{\alpha}h_{p}\,{}^{\beta}h_{q} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots (5.2)$$

The result (5.2) may also be obtained from the following consideration.

When (4.1) is satisfied, the differential equations

$$\frac{d^2x^i}{ds^2} + \nabla^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0 \tag{5.3}$$

of the autoparallel with respect to (3.3) admit the following homogeneous linear first integral (Eisenhart, 1927)

$$h_i\frac{dx^i}{ds} = \text{constant} \tag{5.4}$$

For, differentiating (5.4) covariantly with respect to x^k, multiplying by dx^k/ds and summing for k, we have, by virtue of (5.3),

$$(h_i)_k\frac{dx^i}{ds}\frac{dx^k}{ds} = 0$$

Thus (4.1) is satisfied. In a similar way, when (3.6) is satisfied, the differential equations (5.3) admit the homogeneous quadratic integral

$$g_{pq}\frac{dx^p}{ds}\frac{dx^q}{ds} = \text{constant} \tag{5.5}$$

The constant is here equal to unity, since s is the arc of the curve.

As in the last article, let ${}^\alpha h_i$ with their reciprocals ${}_\alpha h^i$, be n^2 functions which satisfy (5.4), namely

$$\frac{dx^p}{ds} = a^\alpha\,{}_\alpha h^p,$$

where a^α is an arbitrary constant.

Putting $a^\alpha a^\beta = b^{\alpha\beta}$, and substituting in (5.5) we obtain

$$g_{pq}\,b^{\alpha\beta}\,{}_\alpha h^p\,{}_\alpha h^q = 1$$

This can be satisfied when g_{pq} has the form (5.2).

6. It was seen in the last article that when the integrals of (5.3) are

$${}^\alpha h_i\frac{dx^i}{ds} = \text{constant}, \qquad \alpha = 1, \ldots, n,$$

the most general metric satisfying (3.6) is given by

$$ds^2 = C_{\alpha\beta}X^\alpha X^\beta, \quad \text{where} \quad X^\alpha = {}^\alpha h_i\,dx^i$$

If we select one of these metrices, we can replace the X^α's by linear combinations of them so as to secure

$$ds^2 = \sum_\alpha (X^\alpha)^2$$

Therefore, when the parallelism (3.3) is defined by (4.3) and (3.6) and is compatible with a given metric (1.1), we must have

$$ds^2 = g_{pq}dx^p dx^q = \sum_\alpha ({}^\alpha h_i\,dx^i)^2.$$

Hence

$$g_{pq} = \sum_{\alpha} {}^{\alpha}h_p {}^{\alpha}h_q \qquad \ldots \qquad \ldots \quad (6.1)$$

This shows that ${}^{\alpha}h_i$'s form an orthogonal ennuple

The parallelism (1 2) considered under case (1) of § 4 together with (6 1) is Einstein's teleparallelism (Einstein, 1928)

The parallelism (3 3) defined by (4 3) and (6 1) has been studied by the author (Sen, 1946) We may mention here a connection between this and the Levi-Civita parallelism

If f^l and f_l are the contravariant and covariant components of a vector f, its components referred to the orthogonal ennuple, i e to the local system, are

$$f_\alpha = {}^{\alpha}h_l f^l = {}_{\alpha}h^l f_l$$

$$f^l = \sum_{\alpha} {}_{\alpha}h^l f_\alpha, \quad f_l = {}^{\alpha}h_l f_\alpha$$

Let

$$f_\alpha = (g_{ij})_k V^i dx^j {}_{\alpha}h^k \qquad . \qquad . \qquad . \qquad (6\ 2)$$

Then, if the notations δ and d be used to denote respectively the increments with reference to the Levi-Civita parallelism and the parallelism considered here along an elementary path dx^i, we have in consequence of (3 7) and (1.2),

$$\delta V^l = dV^l + f^l, \quad \delta V_l = dV_l + f_l \qquad . \qquad \ldots \quad (6\ 3)$$

The equations (6 3) give the required connection, where, by (6 2), the components f_α of the vector f measure the rate of change, with respect to the arc, of the scalar product of the vectors V and dx when these vectors are given the parallel displacement under consideration in the directions of the vectors of the ennuple.

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NON-STATIC ELECTROMAGNETIC FIELDS WITH SPHERICAL SYMMETRY

By V V NARLIKAR *and* P C VAIDYA

(*Received March 3, read March 7, 1947*)

ABSTRACT

Of the field equations of electromagnetism in general relativity there is only one solution known of spherical symmetry, viz., the static solution for an electron. A new solution of spherical symmetry, which is non-static in character, is given here.

We consider here the usual field equations (Eddington, 1930)

$$G^{\gamma}_{\mu}-\tfrac{1}{2}Gg^{\gamma}_{\mu}=-8\pi T^{\gamma}_{\mu}, \qquad (1)$$

where

$$T^{\gamma}_{\mu}=-F^{\gamma\sigma}F_{\mu\sigma}+\tfrac{1}{4}\delta^{\gamma}_{\mu}F^{\alpha\beta}F_{\alpha\beta}, \qquad (2)$$

$$F_{\mu\gamma}=(K_{\mu})_{\gamma}-(K_{\gamma})_{\mu}, \qquad (3)$$

$$(F^{\mu\gamma})_{\gamma}=J^{\mu}, \qquad (4)$$

and

$$(F_{\mu\gamma})_{\sigma}+(F_{\gamma\sigma})_{\mu}+(F_{\sigma\mu})_{\gamma}=0 \qquad (5)$$

K_{μ} is the potential four-vector, J^{μ} is the charge-and-current vector and $F_{\mu\gamma}$, the skew-symmetrical field tensor We proceed to obtain a non-static solution of the form,

$$ds^2=-e^{\lambda}\,dr^2-r^2\,(d\theta^2+\sin^2\theta\,d\phi^2)+e^{\gamma}\,dt^2, \qquad (6)$$

$$\lambda=\lambda\,(r,t)\,,\ \gamma=\gamma\,(r,t)$$

A new solution satisfying the equations (1) is found to be given by

$$e^{\lambda}=\left(1-\frac{2m}{r}\right)^{-1},\ e^{\gamma}=m^2\left(1-\frac{2m}{r}\right)\Big/f^2 \quad . \qquad (7)$$

where f is an arbitrary function of m such that

$$f\,(m)=m'\left(1-\frac{2m}{r}\right) \qquad (8)$$

In the above and throughout what follows an overhead dot denotes a differentiation with regard to t and an overhead dash denotes a differentiation with regard to r. The surviving components of T^{γ}_{μ} are given by

$$-T_1{}^1=T_4{}^4=\frac{m'}{4\pi r^2}\,,\ T_1{}^4=\frac{m'^2}{4\pi m r^2}\,,\ T_4{}^1=-\frac{m}{4\pi r^2}\,. \qquad (9)$$

It is obvious from (7) that if

$$\frac{d}{d\tau} = e^{-\lambda/2}\frac{\partial}{\partial r} + e^{-\lambda/2}\frac{\partial}{\partial t} \qquad (10)$$

$$\frac{dm}{d\tau} = 0, \qquad (11)$$

m being considered, for the sake of definiteness, negative

Since $m \neq 0$ and all other components of T^{γ}_{μ} except the four given in (9) vanish it follows from (2) that

$$F_{23} = 0,\ F_{14} = 0,\ F_{12} = e^{(\lambda-\gamma)/2}F_{24},\ F_{13} = e^{(\lambda-\gamma)/2}F_{34} \qquad (12)$$

Thus all the components of $F_{\mu\gamma}$ become known if F_{12} and F_{13} are determined For the latter we have from (2) and (5),

$$(F_{12})^2+(F_{13})^2\operatorname{cosec}^2\theta = \left(1-\frac{2m^{-1}}{r}\right)m'/4\pi, \qquad (13)$$

$$\frac{\partial F_{12}}{\partial\phi} = \frac{\partial F_{13}}{\partial\theta}, \qquad (14)$$

$$\frac{d}{d\tau}\left[F_{12}\left(1-\frac{2m}{r}\right)\right] = 0, \qquad (15)$$

$$\frac{d}{d\tau}\left[F_{13}\left(1-\frac{2m}{r}\right)\right] = 0 \qquad (16)$$

A complete solution of (3) and the last four equations is presented by

$$K_1 = 0,\ K_2 = L\psi_2,\ K_3 = L\psi_3,\ K_4 = 0, \qquad (17)$$

where

$$L = -\int\frac{dm}{(4\pi f)^{\frac{1}{2}}}, \qquad (18)$$

$$\psi_2 = \frac{\cos\theta\sin(\phi+\beta)}{[1-\sin^2\theta\sin^2(\phi+\beta)]^{\frac{1}{2}}},\ \psi_3 = \frac{\sin\theta\cos(\phi+\beta)}{[1-\sin^2\theta\sin^2(\phi+\beta)]^{\frac{1}{2}}}, \qquad (19)$$

and

$$F_{12} = -\frac{m'}{m}F_{24} = \frac{m'}{(4\pi f)^{\frac{1}{2}}}\psi_2, \qquad (20)$$

$$F_{13} = -\frac{m'}{m}F_{34} = \frac{m'}{(4\pi f)^{\frac{1}{2}}}\psi_3 \qquad (21)$$

β is an arbitrary constant in (19) J^{μ} is now given by (4) A particular solution has been already published by the authors (1947)

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PROCEEDINGS
OF THE
NATIONAL INSTITUTE OF SCIENCES OF INDIA

FEBRUARY, 1948

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PUBLISHED
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A REVISION OF THE GENUS PHYLLOSTICTA IN INDIA.

By G. C. da Costa *and* B. B. Mundkur

(*Read February 7, 1947.*)

Thirty-nine species of *Phyllosticta* are recorded for India (excluding one species from Burma) by Butler and Bisby (1931) and six more by Mundkur (1938). Of the latter, *Phyllosticta solitaria* on leaves of apple is stated to be a doubtful record by Uppal in a personal communication. Since then three more species, *Phyllosticta bacilliformis*, *Phyllosticta sulata* and *Phyllosticta zingiberis*, have been added to the Indian fungous flora, bringing the total, excluding the doubtful and the Burman records, to 47 species. In the collections at the *Herb. Crypt. Ind. Orient.*, there were several undetermined specimens of which seven, on determination, were found to be new records and six to be so far undescribed species, which are therefore proposed as new.

The genus is represented by 1,640 species in Saccardo's *Sylloge Fungorum* of which 500 are considered valid by Ainsworth and Bisby (1945). Species are usually proposed as new in this genus on the assumption that there is host-specificity, but we have, before proposing the species described in this paper as new, taken care to see that they did not match with the descriptions of any species reported on the host-family or order. The total number now recorded for India is sixty.

The genus has undergone considerable emendation since it was first proposed by Persoon. Several species of *Phyllosticta*, for example, have been transferred to *Ascochyta* or *Phyllostictina* or other genera. The species of this genus are parasitic on leaves and more rarely on stems, giving rise to spots of variable size and form. Pycnidia are formed as minute, black, usually globose, or more rarely lens-shaped, hemispherical or subconical, bodies immersed in the host tissue. They are thus subepidermal and erumpent, later slightly projecting above the leaf surface. They may be with or without ostioles and are usually scattered over the spot or occasionally congested into a stroma. Spores are produced on inconspicuous sporophores within the pycnidia, they are simple, minute, oval, oblong to ellipsoid, hyaline or subhyaline and one-celled. Fifty spores and ten to fifteen pycnidia, both as to length and breadth, have been measured under a fluorite oil immersion lens in order to precisely determine their range and identity.

Of the 60 species recorded in this paper, we have actually examined only 56. Among these, seven were without pycnidia or spores. Some of these latter are type specimens and it is very much to be regretted that at present they are without these essential fructifications. New records are indicated by an asterisk (*).

PHYLLOSTICTA Persoon *in Champ. Comest.* IV, p. 147, 1818 ex Fries *Syst. Mycol.* ii 527.

1. **Phyllosticta ambrosioides** Thuemen *in Instituto* XXVIII (45), 1881; Saccardo, *Syll. Fung.* iii, p. 55, 1884, Butler and Bisby, *Sci. Monogr.* i, p. 160, 1931.

On leaves of *Chenopodium album* L., Mussoorie (U. P.), 6 viii 1905, coll. S. N. Mitra.

Linlithgow Library.

2 **Phyllosticta bacilliformis** Padwick and Mehr *in Mycol Pap Imp Mycol Inst* vii, p 4, 1943

On leaves of *Chenopodium album* L Karnal (Punjab) 13 ii 1941, coll G Watts Padwick (*Type*)

3 ***Phyllosticta barleriae*** da Costa and Mundkur sp nov

Spots circular, dirty white, amphigenous, pale brown underneath, reaching a diameter of 2 mm Pycnidia oval, hypophyllous, brown, 62–93 μ Spores hyaline, unicellular, oblong or elliptical, 3 5 μ in length and 2 to 3 μ in breadth

On leaves of *Barleria* sp , Dehra Dun (U P), 17 xi 1903

Maculae circulares, sordide albidae, amphigenae, subtus pallide brunneae, diam usque ad 2 mm Pycnidia ovalia, hypophylla, brunnea, diam usque ad 62–93 μ Sporae hyalinae, unicellulares, oblongae verl ellipticae, 3 5 μ longae, 2 3 μ latae

In foliis Barleriae cuiusdam speciei, Dehra Dun (U P), 17 xi 1903

*4 **Phyllosticta bauhiniae** Cooke *in Grevillea* xii, p 26, 1883, Saccardo, *Syll Fung* iii, p 11, 1884

On leaves of *Bauhinia purpurea* L , Pusa (Bihar), 21 ix 1908, coll E J Butler, Bandra (Bombay), 15 i 1912, coll J F Dastur

5 ***Phyllosticta bischofiae*** da Costa and Mundkur sp nov

Spots circular, numerous, amphigenous, dark brown, with a slightly raised margin, light brown on under-surface with a raised margin, and dark brown halo, gradually blending into the colour of the leaf, with a diameter of 8 mm forming shot holes Pycnidia epiphyllous, oval golden brown, ostiolate, 60 to 90 μ in diameter Spores hyaline, unicellular, elongate, elliptical, 5–8 μ in length and 2-3 μ in breadth

On leaves of *Bischofia* sp Coota Munda, Wyanad (Madras), 22 xi 1909, coll E J Butler

Maculae circulares, frequentes, amphigenae, fusce brunneae plus minusve eiusdem coloris as folia, margine tenuiter elevato, subtus maculae sunt tenuiter brunneae, margine elevato atque corona brunnea ornatae, haec vero corona gradatim transit in foliorum colorem Pycnidia epiphylla, ovalia, auree lutea, ostiolata, magnitudinis 60–90 μ Sporae hyalinae, unicellulares, elongatae ad ellipticae, 5–8 μ longae, 2-3 μ latae

In foliis Bischofiae sp Coota Munda, Wyanad (Madras), 22 xi 1909, leg E J Butler

6 **Phyllosticta buddleiae** Sydow *apud* Sydow and Butler *in Ann Mycol* xiv, p 183, 1916, Saccardo, *Syll Fung* xxv, p 54, 1931, Butler and Bisby, *Sci Monogr* i, p 160, 1930

On leaves of *Buddleia* sp Nalapani, Dehra Dun (U P), 8 vii 1905, coll E J Butler This is the *type specimen* but does not at present have any fructifications

7 **Phyllosticta butae** Sydow *apud* Sydow and Butler *in Ann Mycol* xiv, p 180, 1916, Saccardo, *Syll Fung* xxv, p 46, 1931, Butler and Bisby, *Sci Monogr* i, p 160, 1931

On leaves of *Butea frondosa* Roxb , Pusa (Bihar), 10 iii 1911, coll L S Subramaniam (*Type*), Dehra Dun (U P), 9 iii 1937, coll Azmatullah Khan

8 **Phyllosticta cajani** Sydow *apud* Sydow and Butler *in Ann Mycol* xiv, p 178, 1916, Saccardo, *Syll Fung* xxv, p 47, 1931, Butler and Bisby, *Sci Monogr* i, p 160, 1931

On leaves of *Cajanus cajan* (L.) Millsp , Muzaffarpur (Bihar), 2.x 1911, coll E J Butler (*Type*), Pusa (Bihar), 3 viii 1935, coll Azmatullah Khan

9 **Phyllosticta capparidicola** Spegazzini in *An Mus Bac B Aires* xx, p 331, 1910, Saccardo, *Syll Fung* xxii, p 824, Butler and Bisby, *Sci. Monogr* i, p 160, 1931

On leaves of *Capparis* sp , Burdwan (Bengal), 6 vii 1907 This specimen identified by Butler is at present without any fructifications

10 ***Phyllosticta capparidis-heyneanae*** da Costa and Mundkur sp nov

Spots circular to subcircular, greyish brown, with a dark brown margin, 3 mm in diameter Pycnidia globose, large, chocolate brown, epiphyllous, immersed in host tissue, 46 to 265 µ in length and 31 to 228 µ in breadth Spores hyaline, unicellular, minute, elliptical, 2 to 4 µ long and 1 to 2 µ broad

On leaves of *Capparis heyneana* Wall , Karwar (Bombay), x 1919, coll. L J Sedgwick (*Type*)

Maculae circulares ad subcirculares, griseo-brunneae, margine irregulari fusco brunneo, diam usque ad 3 mm Pycnidia globosa, ampla, badia, epiphylla, immersa in textura plantae hospitae, magnitudine 46–265 µ longa atque 31–228 µ lata Sporae hyalinae, unicellulares, minutae, ellipticae, 2–4 µ longae, 1-2 µ latae

In foliis Capparidis heyneanae Wall , Karwar (Bombay), x 1919, leg L J Sedgwick, *Typus*

Three species of *Phyllosticta* have been described on the genus *Capparis* but this species is characterised by extremely large pycnidia and very small spores

*11 **Phyllosticta caricae-papayae** Allescher *apud* Hennings, P *in Hedwigia* xxxiv, p 114, 1895, Saccardo, *Syll Fung* xi, p 475, 1895

On leaves of *Carica papaya* L , Pusa (Bihar), 12 i 1936, coll T B Lal

*12 **Phyllosticta carissae** Kalch and Cooke *in Grevillea* ix, p 29, 1880, Saccardo, *Syll Fung* iii, p 36, 1884

On leaves of *Carissa spinarum* A DC , Dehra Dun (U P), 8 viii 1905, coll E J Butler

13 **Phyllosticta chrysanthemi** Ell and Dearness *in Canadian Rec Sci* v, p 267, 1893, Saccardo, *Syll Fung* xi, p 472, 1892, Butler and Bisby, *Sci Monogr* i, p 160, 1931

On leaves of *Chrysanthemum* sp , Pusa (Bihar), 28 x 1910, coll E J Butler.

14 **Phyllosticta clerodendri** Sydow and Butler *in Ann Mycol* xiv, p 177, 1916, Saccardo, *Syll. Fung* xxv, p 77, 1931, Butler and Bisby, *Sci Monogr* i, p 160, 1931

On leaves of *Clerodendron* sp , Nadiad (Bombay), 12 xi 1905, coll E J Butler (*Type*)

15 **Phyllosticta cocos** Cooke *in Grevillea* viii, p 94, 1880, Saccardo, *Syll Fung* iii, p 59, 1884, Butler and Bisby, *Sci Monogr* i, p 160, 1931

On leaves of *Cocos nucifera* L , Belgaum (Bombay), coll Hobson (*Type*) (We have not seen this specimen) On leaves of *Caryota* sp., Dacca (Bengal), 28 viii 1910, coll A L Som

16 **Phyllosticta codalaeicola** Diedicke *apud* Sydow and Butler *in Ann Mycol* xiv, p 184, 1916, Saccardo, *Syll Fung* xxv, p 35, 1931, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Codiaeum* sp , Ganeshkhind, Poona (Bombay), 23 x 1905, coll S N Mitra This is the *type* specimen but there at present are no spores in the pycnidia

17 **Phyllosticta coffeicola** Spegazzini *in Rev Facult Agron Veter La Plata*, 1896, p 345, Saccardo, *Syll Fung* xiv, p 857, 1899, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Coffea liberica* L , Vayatri, Malabar (Madras), 1 x 1904, coll E J Butler, Koppa (Mysore), 15 ix 1903, coll E J Butler Both the specimens are now without fructifications and Butler's identification has been relied upon

*18 **Phyllosticta combreticola** P Hennings *in Verh Bot Ver Prov Brandenburg* xxx, p 161, 1898, Saccardo, *Syll Fung* xvi, p 832, 1902

On leaves of *Combretum ovalifolium* Roxb , Dharwar (Bombay), 1918, coll L J Sedgwick

*19 **Phyllosticta cucurbitacearum** Saccardo *in Michelia* i, p 145, 1879, *Syll Fung* iii, p 52, 1884

On leaves of *Cucurbita* sp , Hyderabad (Deccan), 1945, coll S Vaheeduddin

20 **Phyllosticta cycadina** Passerini *in Rend R Accad Lincei Roma*, 4 Ser , iv, p 64, 1888, Saccardo, *Syll Fung* x, p 124, 1892, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Cycas* sp , Pusa (Bihar), 14 viii 1916, Poona (Bombay), 6 vii 1908, coll H M Chibber

21 **Phyllosticta desmodiicola** Diedicke *apud* Sydow and Butler *in Ann Mycol* xiv, p 178, 1916, Saccardo, *Syll Fung* xxv, p 47, 1931, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Desmodium* sp , Mussoorie (U P), 5 vii 1905, coll S N Mitra (*Type*), *Desmodium tiliaefolium* L , Ranikhet and Chakratha (U P), 1932 and 1935, coll K D Bagchee

*22 **Phyllosticta dioscoreae** Cooke *in Grevillea* viii, p 136, Saccardo, *Syll Fung* iii, p 58, 1884

On leaves of *Dioscorea* sp , Panora, Malabar (Madras), 15 xi 1909, coll W McRae, Surat (Bombay), 8 x 1912, coll E J Butler

23 **Phyllosticta diospyri** Sydow and Butler *in Ann Mycol* xiv, p 183, 1916, Saccardo, *Syll Fung* xxv, p 34, 1931, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Diospyros embryopteris* Pers , Pusa (Bihar), 23 i 1911, coll E J. Butler (*Type*)

24 **Phyllosticta dolichi** Brunaud *in Bull Soc Bot Fr* xi, p 221, 1893, Saccardo, *Syll Fung* xi, p 478, 1895, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Dolichos biflorus* L , Pusa (Bihar), 1912, coll E J Butler

25 **Phyllosticta eriodendri** Diedicke *apud* Sydow and Butler *in Ann Mycol* xiv, p 179, 1916, Saccardo, *Syll Fung* xxv, p 26, 1931, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Eriodendron anfractuosum* DC , Ganeshkhind, Poona (Bombay), 23 x 1905, coll S N Mitra (*Type*)

26 **Phyllosticta exigua** Sydow *apud* Sydow and Butler *in Ann Mycol* xiv, p 183, 1916, Saccardo, *Syll Fung* xxv, p 31, 1931, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Quercus* sp , Kumaon (U P), 15 vi 1907, coll E J Butler (*Type*)

27 **Phyllosticta glycines** Thuemen in *Jour Sci Math Phys Publ Acad R Sci Lisboa* xxiv, p 236, 1878, Saccardo, *Syll Fung* iii, p 11, 1884, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Glycine hispida* Max, Vernag (Kashmir), 7 ix 1908, coll E J Butler

28 **Phyllosticta glycosmidis** Sydow and Butler in *Ann Mycol* xiv, p 177, 1916, Saccardo, *Syll Fung* xxv, p 60, 1931, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Glycosmis pentaphylla* Correa, Dehra Dun (U P), 22 vi 1904, coll E J Butler (*Type*), Khulna (Bengal), 12 v 1907, coll P N Bhattacharjee, Wahjain (Assam), 5 iv 1910, coll A L Som, Pullyanur (Travancore), 8 x 1907, coll E J Butler

29 **Phyllosticta grewiae** Diedicke *apud* Sydow and Butler in *Ann Mycol* xiv, p 181, 1916, Saccardo, *Syll Fung* xxv, p 75, 1931, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Grewia* sp, Dehra Dun (U P), 17 xi 1903, coll E J Butler (*Type*)

30 **Phyllosticta hederae** Saccardo and Roumeguere in *Michelia* ii, p 620, 1882, Saccardo, *Syll Fung* iii, p 20, 1884, Mundkur, *Sci Monogr* xii, p 40, 1938

On leaves of *Hedera helix* L, Mussoorie (U P), 17 x 1931, coll J H Mitter, Kasauli (Punjab), 3 x 1908, coll S N Mitra

31 **Phyllosticta hibiscina** Ellis and Everhart in *Jour Mycol* iv, p 9, 1888, Saccardo *Syll Fung* x, p 103, 1892

Syn *Phyllosticta hibisci* Peck in *Ann Rep N Y State Mus* xiii, p 125, 1889, *Syll Fung* x, p 103, 1892, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Hibiscus cannabinus* L, Cuttack (Orissa), vi 1907, coll E J Butler

32 **Phyllosticta hortorum** Spegazzini in *Atti Soc. Critt Ital* iii, p 67, 1881, Saccardo, *Syll Fung* iii, p 49, 1884, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Solanum melongena* L, Achibal (Kashmir), 4 ix 1908, coll Inayatullah Khan, Surat (Bombay), 15 x 1902, coll E J Butler Both the specimens are at present without pycnidia and Butler's identification is given here

33 **Phyllosticta butleri** da Costa and Mundkur nom nov

Syn *Phyllosticta hoyae* Diedicke *apud* Sydow and Butler in *Ann Mycol* xiv, p 180, 1916, Saccardo, *Syll Fung* xxv, p 24, 1931, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Hoya* sp, Pusa (Bihar), 17 x 1906, coll E J Butler (*Type*), on leaves of *Hoya wightii* Hook f, Khandala (Bombay), 27 xii 1944, coll H Santapau

The name given by Diedicke is a later homonym of *Phyllosticta hoyae* Allescher, 1901, spores of which are 1 5 to 2 μ long and 1 to 1 5 μ broad as against 7 to 10 μ long and 2 5 to 3 5 μ broad of the Bihar specimen

34 **Phyllosticta humuli** Saccardo and Spegazzini in *Michelia* i, p 144, 1879, Saccardo, *Syll Fung* iii, p 53, 1884, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Humulus lupulus* L, Dubgaon (Kashmir), 18 ix 1908, coll. E J. Butler Material is at present without fructifications and Butler's identification is given.

35 **Phyllosticta ingae-dulcis** Diedicke *apud* Sydow and Butler *in Ann Mycol* xiv, p 178, 1916, Saccardo, *Syll Fung* xxv, p 48, 1931, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Inga dulcis* Willd, Coconada (Madras), 11 viii 1905, coll E J Butler (*Type*)

36 **Phyllosticta ipomoeae** Ellis and Kellerman *in Jour Mycol* iii, p 102, 1887, Saccardo, *Syll Fung* x, p 126, 1892, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Ipomoea* sp, Kirkee, Poona (Bombay), 21 x 1905, coll S N Mitra

37 **Phyllosticta marmorata** Cooke *in Grevillea* ix, p 13, 1880, Saccardo, *Syll Fung* iii, p 36, 1884, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Mallotus philippinensis* Muell, Nalapani, Dehra Dun (U P), 2 viii 1905, coll E J Butler

38 **Phyllosticta miurai** Miyake *in Jour Agric Tokyo*, ii, p 253, 1910, Saccardo, *Syll Fung* xxii, p 864, 1913, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves and culms of *Oryza sativa* L, Meharpur (Bengal), 7 ix 1905

*39 **Phyllosticta morifolia** Passerini *in Rend R Accad Lincei Roma*, 4 Ser, iv, 2 sem 10, 99, 1888, Saccardo, *Syll Fung* x, p 120, 1892

On leaves of *Morus alba* L, Achibal (Kashmir), 4 ix 1908, coll E J Butler

40 ***Phyllosticta moringicola*** da Costa and Mundkur sp nov

Spots are minute, dirty white, with a pale green discoloured area underneath, reaching a diameter of 2 to 3 mm Pycnidia very scarce, reddish brown, epiphyllous, 46 to 140 μ in length and 46 to 109 μ in breadth Spores are hyaline, unicellular, elliptical, slightly tapering at one end, measuring 3–6 μ in length and 1·2 μ in breadth

On leaves of *Moringa* sp, Savanur (Bombay), 4 x 1904 (*Type*)

Maculae minutae, sordide albidae, area quadam viridi discolorata subtus, magnitudinis usque ad 2-3 mm Pycnia rarissima, rubro-brunnea, epiphylla, magnitudine usque ad 46–140 μ longa, 46–109 μ lata Sporae hyalinae, unicellulares, ellipticae, tenuiter acuminata uno apice, 3–6 μ longae, 1·2 μ latae

In foliis Moringae sp, Savanur (Bombay), 4 x 1904, *Type*

41 ***Phyllosticta myroxyli*** da Costa and Mundkur sp nov

Spots subcircular to irregular, yellowish brown with a fine rust coloured margin, equally visible on both sides of leaf, 5 to 10 mm in diameter Pycnidia dark brown, epiphyllous, subglobose, measuring 109–184 μ in length and 109–156 μ in breadth Spores hyaline, unicellular to elliptical, measuring 3–7 μ in length and 2–4 μ in breadth

On leaves of *Myroxylon toluiferum* L, Poona (Bombay), 6 vii 1908, coll H M Chibber (*Type*)

Maculae subcirculares ad irregulares, luteo brunneae, tenui, rubiginoso margine, aeque visibiles in utraque pagina folii, diam usque 5–10 mm Pycnidia fusco brunnea, epiphylla, subglobosa, 109–184 μ longa, 109–156 μ lata Sporae hyalinae, unicellulares, ovatae ad ellipticas, 3–7 μ longae, 2-4 μ latae

In foliis Myroxili toluiferi L, Poona (Bombay), 6 vii 1908, leg H M Chibber, *Type*

42 **Phyllosticta persicae** Saccardo *in Michelia* i, p 147, 1879, *Syll Fung* iii, p 48, 1884, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Pyrus malus* L, Tukvar (Bengal), 31 viii 1909, coll W McRae.

43 **Phyllosticta pirina** Saccardo *in Michelia* i, p 134, 1879, *Syll Fung* iii, p 7, 1884, Butler and Bisby, *Sci Monogr* i, p 161, 1931

On leaves of *Pyrus communis* L , Chaubhattia (U P), 25 x 1934, coll U B Singh, Lyallpur (Punjab), 28 ix 1908, coll E J Butler , Darjeeling (Bengal), 7 ix 1909, coll W McRae, on leaves of *Pyrus malus* L , Chaubhattia (U P), 29 x 1934, coll U B Singh

44 **Phyllosticta pongamiae** Sydow *apud* Sydow and Butler *in Ann Mycol.* xiv, p 178, 1916, Saccardo, *Syll Fung* xxv, p 50, 1931, Butler and Bisby, *Sci Monogr* i, p 162, 1931

On leaves of *Pongamia glabra* Vent , Dacca (Bengal), 5 iv 1910, coll A L Som (*Type*)

45 **Phyllosticta prunicola** (Opiz ?) Saccardo *in Michelia* i, p 157, 1879, *Syll Fung* iii, p 4, 1884, Butler and Bisby, *Sci Monogr* i, p 162, 1931

On leaves of *Prunus armeniaca* L , Achibal (Kashmir), 2 ix 1918, coll E J Butler, Kothian, Chakrata (U P), 15 x 1935, coll K Bagchee, on leaves of *Prunus puddum* Roxb , Vernag (Kashmir), 8 ix 1908, coll E J Butler, on leaves of *Prunus persica* Benth and Hooker f , Pusa (Bihar), 16 x 1916, coll R Sen, Quetta (Baluchistan), 19 viii 1932, coll K F Kheswalla , Achibal (Kashmir), 28 viii 1908, coll E J Butler, on leaves of *Amygdalus communis* L , Quetta (Baluchistan), 17 viii 1932, coll K F Kheswalla

46 **Phyllosticta religiosa** Sydow *apud* Sydow and Butler *in Ann Mycol* xiv, p 183, 1916, Saccardo, *Syll Fung* xxv, p 56, 1931 , Butler and Bisby, *Sci Monogr* i, p 162, 1931

On leaves of *Ficus religiosa* L , Poona (Bombay), 6 vii 1908, coll H M Chibber (*Type*), Gwalior, August 1946, coll G S Kulkarni

47 ***Phyllosticta sedgwickii*** da Costa and Mundkur sp nov

Spots circular, salmon coloured, surrounded by a chocolate brown diffuse halo, 2 to 3 mm in diameter Pycnidia golden to dark brown, globose, immersed, epiphyllous, 62 to 109 μ in length and 62–78 μ in breadth Spores hyaline, unicellular, oblong, oval, elliptical, measuring 2–4 μ in length and 1·2 μ in breadth

On leaves of *Grewia tiliaefolia* Vahl , Dharwar (Bombay), July 1918, coll L J Sedgwick (*Type*), Karwar (Bombay), October 1919, coll L J Sedgwick

Maculae circulares, salmoneae, obsitae corona diffusa atropurpurea, magnitudinis usque ad 2-3 mm Pycnidia globosa, aureae ad fusce brunnea, immersa, epiphylla, 62–109 μ longa, 62–78 μ lata Sporae hyalinae, unicellulares, oblongae, ovatae, ellipticae, 2–4 μ longae, 1·2 μ latae

In foliis Grewiae tiliaefoliae Vahl , Dharwar (Bombay), Julio 1918, coll L J Sedgwick *Typus*, Karwar (Bombay), Octobri 1919, leg L J Sedgwick

48 **Phyllosticta sesbaniae** Sydow *apud* Sydow and Butler *in Ann Mycol* xiv, p 179, 1916, Saccardo, *Syll Fung* xxv, p 50, 1931 , Butler and Bisby, *Sci Monogr* i, p 162, 1931

On leaves of *Sesbania* sp , Pusa (Bihar), 1 ix 1913, coll E J Butler (*Type*)

49 **Phyllosticta sissoo** Diedicke *apud* Sydow and Butler *in Ann Mycol* xiv, p 179, 1916, Saccardo, *Syll Fung* xxv, p 47, 1931 , Butler and Bisby, *Sci. Monogr* i, p 162, 1931

On leaves of *Dalbergia sissoo* Roxb , Pusa (Bihar), 31 i 1910, coll E J Butler (*Type*).

50 **Phyllosticta sorghina** Saccardo *in Michelia* i, p 140, 1879, *Syll Fung* iii, p 61, Butler and Bisby, *Sci Monogr* i, p 162, 1931

On leaves of *Sorghum vulgare* Pers , Manaparai (Madras), 25 [illegible] 1904, coll E J Butler, Hyderabad (Deccan), May 1942, coll S Vaheeduddin

51 **Phyllosticta sulata** Chowdhury *in Indian J Agric Sci* xiv, p 397, 1944

On leaves of *Carica papaya* L , Halflong (Assam), 7 xi 1941, coll S Chowdhury (*Type*)

52 **Phyllosticta symploci** Sydow *apud* Sydow and Butler *in Ann Mycol* xiv, p 180, 1916, Saccardo, *Syll Fung* xxv, p 75, 1931 Butler and Bisby, *Sci Monogr* i, p 162, 1931

On leaves of *Symplocos* sp , Coorg, Nilgiris (Madras), 12 x 1904, coll E J Butler (*Type*)

53 **Phyllosticta tectonae** Sydow and Butler *in Ann Mycol* xiv, p 181, 1914 Saccardo, *Syll Fung* xxv, p 78, 1931, Butler and Bisby, *Sci Monogr* i, p 162, 1931

On leaves of *Tectona grandis* L , Puttimari (Assam), 3 iii 1912, coll Md Taslim (*Type*)

54 **Phyllosticta tricoloris** Sydow and Butler *in Ann Mycol* xiv, p 182, 1916, Butler and Bisby, *Sci Monogr* i, p 162, 1931

Syn *Phyllosticta violae* Desm var *violae-tricoloris* Saccardo *in Michelia* i, p 143, 1879 *Syll Fung* iii, p 38, 1884

On leaves of *Viola odorata* L , Ganeshkhind (Bombay), 24 x 1905, coll E J Butler (*Type*)

55 **Phyllosticta violae** Desmazières *in Ann Sci Nat Bot* , 2 Ser , xiv, p 29, 1840, Saccardo, *Syll Fung* iii, p 38, 1884, Mundkur, *Sci Monogr* xii, p 41, 1938

On leaves of *Viola odorata* L , Pusa (Bihar), 12 xi 1914, coll T B Fletcher, Allahabad (U P), 12 ii 1931, coll J H Mitter

56 **Phyllosticta zingiberis** Ramakrishnan *in Proc Indian Acad Sci* B, xv, p 170, 1942

On leaves of *Zingiber officinale* L , Madras Prov No date or other details on the specimen

SPECIES NOT SEEN

57 **Phyllosticta cocculi** Thuemen *in Rev Mycol* 2 36, 1880, Saccardo, *Syll Fung* iii, p 29, 1884

This species was collected by Keck in Kanara on *Anamirta cocculus* W and A who sent it to Thuemen Specimen of this fungus, not reported from any other country, is not available in India

58 **Phyllosticta confertissima** Ellis and Everhart *in Proc Acad Phila* 1893, p 455, Saccardo, *Syll Fung* xi, p 476, 1895, Mundkur, *Sci Monogr* xii, p 40, 1938

Recorded on leaves of *Ulmus integrifolia* Roxb at Allahabad (U P) by Mitter

59 **Phyllosticta dracaenae** Griff and Maubl in *Bull Soc Mycol Fr* xxv, p 27, 1909, Saccardo, *Syll Fung* xx, p 384, 1911, Mundkur, *Sci Monogr* xii, p 40, 1938

Recorded on leaves of *Dracaena lindii* L , Allahabad (U P), by Mitter, 60 **Phyllosticta mortoni** Fairman in *Mycologia* 5 p 247, 1913, Saccardo, *Syll Fung* xxv, p 20, 1931

Recorded on leaves of *Mangifera indica* L at Sholapur (Bombay) by Uppal

The following species are also in the *Herb Crypt Ind Orient* *Phyllosticta briardo* Sacc on *Pyrus malus* from Kashmir, *Phyllosticta conchonae* Koord on *Cinchona ledgeriana* from Valparai (South India), *Phyllosticta roumeguerii* Saccardo on *Virurnum* sp from Kashmir, *Phyllosticta ruborum* Saccardo on *Rubus* sp from Darjeeling, and *Phyllosticta sacchari* Speg on *Saccharum officinarum* L from Samalkot (Madras) None of them have any fructifications at present and there is no means of checking their identification It is not known who identified them either We have not therefore included them in this paper

SUMMARY

This paper records sixty species of *Phyllosticta* for India Of these, seven are new records and six are proposed as new species The name of one species has been changed, as it was found to be a later homonym The Latin diagnoses were kindly prepared by Rev Father H Santapau Head of Department of Biology, St Xavier's College, Bombay, to whom we hereby express our deep debt of gratitude

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New Delhi

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THE OSTEOLOGY OF INDIAN CYPRINODONTS

PART I —COMPARATIVE STUDY OF THE *Head* SKELETON OF *Aplocheilus*, *Oryzias* AND *Horaichthys*

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(*Received January 13*, *read April 4*, *1947*)

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INTRODUCTION

The discovery of *Horaichthys setnai* Kulk possessing a specialised gonopodium in the male and representing a new family Horaichthyidae allied to the equally unique, but widely separated, family of Tomeuridae in South America led to a study of the skeletal structures of the gonopodium of the former (Kulkarni, 1940) A comparative account of the anatomical structures (*loc cit* , p 398) revealed that the gonopodium of *H setnai* had not only advanced in the same direction as the specialised viviparous Poecillids of America but had become even a more complicated and specialised structure than the gonopodium in any member of the Poecillidae The occurrence of *H setnai* in India among other comparatively simple cyprinodonts, such as *Aplocheilus* and *Oryzias*, illustrates how remarkable a turn evolution may take in what would otherwise appear to be a very ordinary fish The author has already indicated (*loc cit*, p 383) that *H setnai* must have evolved directly from *Oryzias melastigma* (McClelland) Dr C L Hubbs, University of Michigan, Ann Arbor and Dr G S Myers, Stanford University, California, also expressed this view (Hubbs, 1941)

The aforesaid relationships of these fishes were based on only their superficial resemblance, but this could not be conclusive without a comparison of their skeletal structures Fishes appearing superficially to belong to a particular group have been found to differ taxonomically when the details of their bony structures were investigated Considerable changes have been made particularly in the taxonomy of cyprinodonts after a closer study of their osteology Starks (1904*a*), Regan (1911), Hubbs (1924), Myers (1928) and Chapman (1934) investigated the skeletal structures of these fishes on different occasions In the light of their investigations, Berg (1940) separated the members of the group once listed under the single order Haplomi into three different orders, viz Esociformes, Cyprinodontiformes and Phallostethiformes

The determination of the precise relationship of *H setnai* to other members of the cyprinodont group postulated a comparative study of its endoskeleton No detailed osteological investigation of this group of Indian fishes has, so far, been carried out except for the study of their jaws and teeth undertaken by Sundara Raj (1916) Studies of the osteology of oviparous cyprinodonts occurring in other countries are also not sufficiently detailed Further, Dr Myers and Dr S L Hora also suggested that a detailed study of the skeletal structures of these fishes would be invaluable The present study was undertaken in order to have a fairly exhaustive account of the skeletal features of the Indian cyprinodonts, particularly with a view to establishing their inter-relationships Three different genera, viz *Aplocheilus*, *Oryzias* and *Horaichthys*, have been selected for the study *A lineatus*, being the commonest form along the western coast of India and being also slightly larger among these small fishes, was selected for a detailed account of its osteology Other genera are discussed in general terms for comparison of their important skeletal structures An American genus is also compared

ACKNOWLEDGEMENT

The author takes this opportunity of expressing his deep sense of gratitude to Dr S B Setna, the Director of Fisheries, Bombay, for his valuable guidance and constant encouragement during the course of the investigation

METHODS AND TECHNIQUE

Preparation of dry skeletons by maceration of the fleshy tissue or by any other method was practically impossible in view of the extremely small size of the fishes studied Staining in bulk of the entire skeletons by Benda's Alizarine KOH method (Lee, 1928) was, therefore, employed for each fish after the specimens had been fixed in alcohol and also in formalin on different occasions

HEAD SKELETON OF *Aplocheilus lineatus* (Cuv and Val)

The head skeleton of *Aplocheilus lineatus* C & V, as is common with most fishes, is made up of the original cartilage bones, the secondarily developed investing bones, and a certain number of cartilaginous structures which remain unossified even in the adult condition The head skeleton consists of the skull proper, the jaws and the visceral skeleton, including the bones of the opercular apparatus The *head of an average well built specimen is about 11 or 12 mm in length from the tip of the jaws to the posterior margin of the skull It is dorsally flat and is fairly* broad as compared with its length and the size of the fish The jaws, being wide and laterally flattened, do not make the skull wedge-shaped or pointed as in many other fishes, but considerably broad at the anterior end The breadth of the skull in the postorbital region, is contained about 1 5 times in the total length of the head The jaws are obtusely rounded and their breadth at the corner of the mouth is contained twice in the total length of the head skeleton

(A) *Skull* (Figs 1, 2 and 3)

The skull proper or the cranium for all practical purposes, may be considered to represent that complex of bones and cartilage which is situated around the brain and is not very readily separable from it This includes all those bony structures developed out of the original chondrocranium and the secondary bones attached to it, including the vomer and parasphenoid

The skull as defined above is almost flat on the dorsal surface except for the slight crests on the surface of the frontals corresponding with the regions marked on them There is no supra-occipital crest, and the elongated posterior processes of the supra-occipital bone are situated at the dorsal level of the skull The postero lateral corner of the skull and the surface on either side of the supra-occipital process, sinks below the general level, the depression in the living condition being filled up by connective tissue On the postero-dorsal margin, the epiotic is situated on the same level as the supra-occipital, but on account of the adjoining area being low appears like an epiotic crest

On the ventral side, the anterior two-thirds of the skull is excavated by the large orbits with a narrow ridge of the parasphenoid in between Right from the vomer to the basi-occipital, the mid ventral line is almost on the same level Small depressions are present in the postero-lateral corners Two small elevations are also formed due to the presence of large sacculiths, one on either side of the basi-occipital Most of the bones which are performed in cartilage are separated from each other by thin interspaces of cartilage between their margins Another peculiarity of these bones is that between the inner and outer surfaces of each bone, there is a thin layer of cartilage which remains unstained in alizarine preparations

When the membrane bones are removed, the brain case is insufficiently provided by bony structures in its anterior region There is only a small ethmoid cartilage in front The orbitosphenoids and the basisphenoid are absent Alisphenoids are very small, posteriorly situated and do not meet in the median line The only connecting link between the alisphenoids is a thin strand of cartilage which runs between the distal end of the supra-occipital and the alisphenoid of each side A thin membrane from the latter extends to the posterior membranes of the ethmoid region and establishes a connection with the anterior region of the brain case

Different methods of describing skull are followed by different authors, some deal with the replacing (cartilage) bones and investing bones separately, while others follow the method of describing the bones region by region The latter method appears to be preferable and it has been followed in this paper as far as possible

The skull is thus divided into four different regions

(1) the ethmoidal,
(2) orbito-temporal,
(3) otic or auditory, and
(4) occipital region

The details of these regions are given as follow —

(1) *Ethmoid Region*

The ethmoid region (fig 4) —The anterior-most part of the cranium is formed by bones and cartilages arising out of the original ethmoid cartilage and also by those bones which subsequently attach themselves to it superficially Dorsally, it is partially covered by the nasals and the anterior portion of the frontals, while ventrally, it is supported by the vomer, which is so much anteriorly disposed as to be attached to this region only by its posterior portion In the same region the parasphenoid is also ventrally attached by its anterior end which overlaps the posterior extremity of the vomer Anteriorly, the region is in contact with the pre-maxillaries, the posterior median processes of which actually project into the anterior concavity of this region The whole region, therefore, comprises the median ethmoid cartilage, the paired lateral ethmoids, a median mes-ethmoid, paired nasals and a vomer (see Fig 4)

The median ethmoid cartilage is a prominent bar-like structure situated transversely and forming the actual anterior margin of the brain case Its lateral extremities are covered by well-ossified secondary bones called the lateral or ectethmoids In the posterior direction, the cartilage is basally produced into a thin horizontal membrane which forms the anterior floor of the brain case A similarly directed narrow membrane with a median triangular projection also arises from the dorsal margin of the cartilage These two membranes, along with the posterior face of the ethmoid cartilage, and a part of the lateral ethmoids which are all concave internally, form the anterior concavity of the brain chamber The other face (anterior) of the cartilage is excavated in the middle into a specious conical recess and is also basally produced in the anterior direction into another extensive thin membrane similar to the one directed posteriorly The conical recess of the cartilage lodges the posterior end of the mesethmoid

Mesethmoid —Mesethmoid is a small, median, heart-shaped piece of cartilage resembling a heart-shaped locket It is situated in the anterior concavity of the ethmoid cartilage with its (mesethmoid) broad end facing anteriorly The posterior extremities of the pre-maxillary processes which converge medially, rest on this cartilage and are closely attached to it by muscular tissue It is also attached by a few muscle strands to the ethmoid cartilage and also to the articulating surface of the maxillae With the movement of the pre-maxillary processes the mesethmoid also slides backwards and forwards The attachment of the mesethmoid to all these bones help in co-ordinating the movements of the jaws

Similar type of mesethmoid is found in both *A panchax* and *A blochii*

Lateral ethmoid —Starks (1904, *b*) records that Owen originally designated this bone as pre-frontal and Parkar (W K) ecto-ethmoid or lateral ethmoid Starks himself has retained Owen's terminology in his several writings Gregory (1933) mentions par-ethmoid as synonymous with lateral ethmoid and states clearly that it is wrongly called pre-frontal Starks (1926) describes pre-frontal in some forms as being 'composed of surface bone only containing unossified cartilage, that is, continuation of the median mass of cartilage of the ethmoid region' This description shows that Starks refers to the lateral ossification on the ethmoid region Parkar's self-explanatory term, viz lateral ethmoid, should, therefore, be preferable By calling the bone pre-frontal, its association with the frontal is shown to be much closer than it is in reality Actually, its relation with the ethmoid region is much closer as it has actually grown on, or applied itself to, that cartilage Again, the bone is not on the same level as the frontal, and it is distinctly below the frontal in *Aplocheilus* In *Dallia, Fundulus, Umbra* and in many other forms as shown by Starks (1926) the bone is mainly below the frontal although a portion of it extends beyond the margin of the former To call the bone pre-frontal thus appears rather misleading Parker's terminology, which is also accepted by Regan and others, has accordingly been followed in this paper also

The lateral ethmoids are paired bones ossified, as has already been stated, on the lateral extremities of the ethmoid cartilage They are very irregular in shape and present different aspects dorsally and ventrally Each lateral ethmoid is situated in the preorbital region and is dorsally covered by the anterior and posterior margins of the frontal and the nasal respectively Laterally, it is in contact with the lachrymal

Ventrally viewed (fig 2), each ossification appears to be composed of two portions, one an antero-posteriorly disposed medial part and the other a more or less vertically directed transverse twisted plate The latter is obliquely attached from above to the posterior half of the former The medial portion, although it appears like a horizontal membrane of bone when seen from above, is, in fact, a hoodlike bent membrane which fits like a cap on the lateral extremity of the ethmoid cartilage The distal outer expansion of the twisted plate, being also ventrally disposed to some extent, gives the appearance of a lateral wing of the bone and forms a wide and shallow concavity opening posteriorly The margins of the plates are sinuous and smooth except in the medial part, where its external lateral margin is thickened and gives attachment to the anterior extremity of the palatopterygoid element

In the dorsal aspect (fig 4), the lateral ethmoid appears quite different in shape when the covering bones (frontal and nasal) are removed Instead of giving the appearance of twisted and curved plates, the lateral ethmoid roughly resembles a butterfly, with the medial and lateral parts representing the wings, and the median antero-lateral stem the body of the butterfly Its surface is irregular and thickened at places for articulation with other bones The articular surfaces between the bone and the palatopterygoid as also the twisted articular surface of the maxilla are evident in dorsal view In the antero-lateral corner the medial and lateral parts are slightly curved and enclose between them a large space which, along with the nasal on the top and the portions of maxilla and pre-maxilla antero-laterally and the lachrymal on the external side, forms the olfactory cavity of the fish The olfactory nerve opens into this cavity by a prominent foramen situated in the antero-lateral corner of the brain case and passing through the central portion or the body of the lateral ethmoid bone

Nasals —The nasals (fig 1) are a pair of almost circular discs of membrane bones situated in front of the frontals towards the lateral margin of the skull Each nasal lies over the medial wing of the lateral ethmoid and possesses an indistinct ridge on its ventral surface It partially covers olfactory area and is attached by its anterior edge to the articular region (twisted portion) of the maxilla The palatine head and medial wing of the lateral ethmoid are also attached to it from below like the maxilla

Vomer —The vomer (fig 2) is a median comparatively well-sized flat membrane bone, attached ventrally to the skull and forming the roof of the buccal cavity situated below The bone is roughly ogee shaped (a term used in drawing), with its posterior end much drawn out and pointed By its pointed portion it is attached to the parasphenoid, and passing over the ventral surface of the ethmoidal region extends sufficiently forward so as to project beyond the mesial extremities of the maxillae and the posterior processes of the pre-maxillaries Its major portion is in front of the ethmoid cartilage and laterally extends only below the anterior extremities of the lateral ethmoid The anterior margin has a single row of short conical teeth directed ventrally Numbering about 40 to 50, they cover almost the whole of the anterior margin At some places the row is double and consequently the total number of teeth on the bone is larger At the extreme anterior end of the bone, instead of a single or a double row, there is a small group of teeth which forms a sort of crown of teeth on the bone

Dr Myers, diagnosing the characters of the tribe Rivulini of 1931 (Aplocheilini of later years) which includes *Aplocheilus* (*Panchax*), states (1931, p 10) that 'the vomerine teeth are in a rounded patch and usually reduced in number' In a few *Panchax* (*Aplocheilus*) he found them to be even missing The teeth in *Aplocheilus lineatus* (*P lineatus*), as the present investigations showed, were quite numerous and disposed on the anterior margin as described above Sundara Raj (1916) has figured a vomer (plate XXV) showing about four rows or almost a band of teeth Such a band, however, was never observed during the present study

(2) *Orbito-temporal Region*

The orbito-temporal region, although comparatively extensive, is rather imperfectly developed and, unlike in many other fishes (*Otolithus*, *Labeo rohita*, etc.) cannot be divided clearly into orbital and temporal or sphenoidal regions There is neither the basisphenoid on the ventral side nor the orbitosphenoid on the lateral sides The alisphenoid (fig 5), though present, is rather inconspicuous and lies in only a corner between the sphenotic and orbital portions of the frontal The lateral walls of the brain chamber in this region has no bony supports for its protection, but are covered by the large eye-balls which occupy the entire lateral orbital space The orbital space is bounded on the anterior side by the lachrymal and the lateral ethmoid, and posteriorly by the alisphenoid, sphenotic and the dermosphenotic The parietals also are comparatively small and so much posteriorly disposed that they almost lie over the auditory capsules The frontals are, however, remarkably well developed and compensate for the scanty growth of other bones They not only cover the entire orbito-temporal region but extend anteriorly on the ethmoidal region, which also they cover considerably On the ventral side the orbito-temporal region is supported by the elongated parasphenoid only This region although it covers a major portion of the skull represents only a small number of bones, viz the paired frontals and parietals on the dorsal side, the parasphenoid on the ventral side and the bones of the circum-orbital series and alisphenoids on the lateral side Of these only the last (alisphenoid) which, in fact, is the smallest of all, is the true bone, the rest being membrane bones attached secondarily to the cranium Details of these bones are as follows —

Frontals —The frontals (fig 1) are the largest of the bones of *Aplocheilus* and cover a considerable part of the skull on the dorsal side Anteriorly, they overlie the anterior margin of the ethmoidal region and reach the auditory region posteriorly They are elongated, broad, flat ossified plates overlapping the margins of each other medially in the inter-orbital space After covering the inter-orbital space the proximal parts of the frontals diverge laterally and the median supra-occipital is wedged in between them

Superficially, the dorsal surface of the frontal appears to be divided into three different regions, viz the inter-orbital, supra-orbital and sphenoidal regions owing to the corresponding curvatures on its dorsal surface Moreover, on the ventral surface there is a distant triradiate ridge which is also visible from above, and helps to separate these regions more clearly The inter-orbital area is slightly convex dorsally The supra-orbital area is also convex dorsally and its posterior curvatures are so developed as to afford protection to the rounded eye-ball As this part projects somewhat more laterally, it also appears like an orbital wing Posterior to these two regions is the sphenoidal area This area is situated between the inter-orbital area anteriorly and the supra-occipital and the auditory areas posteriorly, and may be, therefore, rightly considered to correspond with the sphenoidal area of other fishes (there being no basi-sphenoid in *Aplocheilus*) The area is also laterally supported by the alisphenoid, which runs obliquely upwards and meets the frontal in this area

Parietals —The parietals (fig 1) are paired thin membrane bones situated obliquely on the either side of the supra-occipital As compared with the expansive frontals, each parietal is a small bone attached to the postero-lateral margin of the frontal Antero-laterally, it is attached to the sphenotic, laterally to the pterotic, and postero-laterally to the epiotic and supra-occipital It serves as a covering bone and protects the unossified area between the epiotic, pterotic and sphenotic

Parasphenoid —The parasphenoid (fig 2) is a prominent median bone on the ventral side situated posterior to the vomer It is elongated and shaped like a dagger with the anterior pointed end being somewhat obtuse and broadened It is ventrally applied to the skull and forms a narrow floor for the brain case almost along its entire length Anteriorly, it is attached from below to the posterior membranous extension of the ethmoidal cartilage and overlaps the posterior end of vomer Posteriorly it is attached to the mesial portions of the pro-otics and finally ends on the anterior half of the basi-occipital There are two pairs of very short lateral extensions of the bone, the posterior one on the pro-otic and the anterior in front of the alisphenoid

On the ventral or outer surface of the bone there is a pair of very thin elongated ridges, forming narrow median groove which disappears at both ends The posterior end of the above groove is broad and breaks up into a number of fine ridges as it disappears finally

Internally the bone is somewhat concave, the inner surface of its posterior region being produced into a pair of small posteriorly opening pockets below the lateral extensions on the pro otic In the same area there is a median round depression or pit exactly below the posterior wider part of the ventral groove of the bone This depression corresponds with the position of the hypophysis which is situated above it

There is no myodome or the so called eye muscle canal which Allis (1919) describes in *Hyodon*, *Scomber*, etc The superior and inferior recti of the eye are attached to the external or lateral edge of the parasphenoid just in front of the antero lateral extension of the bone The oblique muscles which Dharmrajan (1936) describes as forming the anterior myodome are attached to the posterior face of the lateral ethmoid

Alisphenoid —The alisphenoid (fig 5) is a somewhat obliquely situated plate of bone which forms a part of the lateral wall of the brain case immediately behind the orbit of the eye It is attached to the bones of the auditory capsule postero-laterally and proceeds obliquely upwards and forwards to meet the orbital part of the frontal from below Being oblique in form and situated on the lateral wall, the bone appears triangular in a dorsal view Basally it is composed of two plates forming an inverted V with its apex produced anteriorly into a plate twice the length of the arms, the intervening space between the arms being filled by clear cartilage By the outer plate the bone is attached to the sphenotic and by the inner to the vertically disposed columnar structure on the pro-otic The bone thus has a wide and firm base which tapers upwards into a thin twisted plate to support the frontal from beneath

Circum-orbital series —In common with such fishes as *Fundulus*, *Goodea*, *Poecilia*, etc , the bones of the circum orbital series are very few in *Aplocheilus* also They are represented by only two bones, a prominent lachrymal in the pre-orbital region and dermosphenotic in the post-orbital area, there being no supra- or sub-orbitals In place of the supra-orbitals, a lateral extension of the frontal known as the supra orbital wing serves as a roof for the orbit and protects the eye from above, the orbital wing itself being covered by three or four prominent scales Details of the existing circum orbitals are as follows —

Lachrymal —The lachrymal (fig 7) is a purely dermal bone situated in the pre-orbital region of the head It is loosely embedded in the tissue and can be easily pulled out from outside It has a very irregular channeled structure with curved plates produced in different planes A major portion of the lachrymal bone has a channel-like appearance extending vertically Its distal portion is twisted and bent so as to resemble a narrow neck where another twisted plate-like portion joins it from behind Two short channeled pieces are also grafted on the outer edge of this region on the orbital side There is a small central aperture in the neck region for the passage of the sensory nerve The channeled nature of the bone is intended for the lodgment of the sensory canal system.

Dermosphenotic —Dermosphenotic (fig 8) is another bone of the circum orbital series situated anteriorly to the sphenotic process in the post-orbital region Like the lachrymal, it is also loosely sunk in the dermal tissue as the name of the bone suggests and is shaped like an elongated scoop It is not attached to any bone except that it touches the post-orbital process of the frontal from where it starts and extends downwards to form the post orbital margin of the eye Posteriorly it is closely attached to the sphenotic spine

Besides these two bones, a most unusual peculiarity is the appearance of two independent patches of bony elements noticeable in the sclerotic coat of the eye of *Aplocheilus* With alizarine dye these patches stain red just as other bony elements This fact gives ground for the view that they must be bony elements which have developed in that area The patches are situated in the anterior and posterior corners of the eye ball opposite the lachrymal and the dermosphenotic respectively They are very thin, small and somewhat oval in shape Ramaswami also (1945) has recently recorded the occurrence of broad cup-shaped sclerotic bones in the sclerotic coat in the eye of *Gambusia*

The fact that ossification has taken place in the sclerotic coat of this fish in close proximity of the dermosphenotic and lachrymal ossifications only, bears out an apropòs observation of Ridewood (1904, p 56) that 'the endosteal ossification is set up in sympathy with the ossification taking place in the dermal tissue The process of ossification is infectious if one may employ such a term in this connection '

(3) *Otic Region*

The otic or the auditory region is composed of bones forming the auditory capsules, situated as usual in the postero-lateral corners of the skull The bones of the capsule originate during development as separate ossifications in the original cartilaginous capsule, which gets secondarily attached to the brain case on either side in its hind region Normally, there are five separate bones forming the auditory capsule on each side, but in *Aplocheilus* and its allies only four elements, namely, the epi-otic, pro-otic, pterotic and sphenotic are present, the fifth, opisthotic, being undeveloped or fused with the ex-occipital which thus takes some part in the formation of the auditory capsule A space which remains unossified between the epiotic, pterotic and sphenotic, is covered by the parietals leaving a small fontanelle at the junction of the epiotic, pterotic and parietal Below the parietal, the anterior strip of cartilage which stretches between the supra-occipital, and the other otic bones on the dorsal side, is thickened into a band and appears different from the other cartilage Chapman (1934, p 384) met with similar elements in *Novumbra hubbsi* and called them supra otics He described them as 'highly ossified ridges running parallel and just posterior to the posterior edge of the frontals' In *Dallia* and *Umbra*, however, the same structure remains unossified and resembles the cartilaginous band occurring in *A lineatus* (See fig 6a)

The otic bones are most irregular in shape all throughout, and are curved in different ways so as to form the walls of an irregularly shaped capsule Internally, the inner laminae of some of these bones are ossified in such a way as to give rise to canals or tunnel-like passages which form a sort of bony labyrinth for the lodgement of the semi-circular canals of the membraneous labyrinth Concavities are also formed on the inner surfaces of the bones for the ampullae and otoliths Columnar bony supports running from the floor to the ceiling of the capsule are present in some places in order to strengthen the auditory capsule A few more details of these bones and their locations are as follow —

Pro-otic —The pro-otic (fig 2) is a ventrally situated bone which forms a major portion of the floor of the auditory capsule It is the most anteriorly disposed element of the group and although concave internally is comparatively flat on its outer ventral surface It is somewhat oblong in form and is disposed obliquely so as to meet the basi-occipital postero laterally, the ex-occipital posteriorly and the pterotic and sphenotic laterally and antero-laterally respectively The pro-otic in its antero-lateral direction also supports the alisphenoid and the sphenotic by a dorsally directed buttress like structure originating from the inner surface of the bone From the base of the buttress like support (fig 7a) two or three small ridges radiate sideways and enclose between them concavities on the floor of the bone which, along with the portions contributed by the basi- and ex-occipitals, form the recesses for the sacculith (auditory ossicles) and for the ampulla of the posterior and the anterior semi circular canals, the epiotic forming only a part of the dorsal cover Basally, the pro otic protrudes a short distance beyond the margin of the capsule and bears one large and another small foramen on its surface

Epiotic —The epiotic (fig 1 and 5a) occupies the postero-dorsal corner of the auditory capsule and extends somewhat downwards It forms an epiotic crest on the dorsal side It is somewhat boat shaped in form, with its concavity facing anteriorly and the keel forming the postero-dorsal margin of the auditory sac Medially, the epiotic is in contact with the supra occipitals and ex occipitals and laterally with the pterotic Internally, its lamina is rolled up to form a bony canal for the passage of the posterior semi circular canal Ventrally it shares with the ex-occipital of its side a recess for the reception of the ampullae of the posterior and horizontal semi-circular canals of the membranous labyrinth

Pterotic —The pterotic (fig 5b) occupies the postero-lateral corner of the skull and is conspicuously irregular in shape It extends both on the dorsal and ventral surfaces of the skull and forms the lateral margin of the skull Medially, it is in contact with the epiotic, ventrally, with the pro-otic and antero-laterally with the sphenotic Like the epiotic, the pterotic also is excavated or tunnelled internally for the passage of the horizontal semi-circular canal and is depressed dorso-ventrally towards the outer margin Laterally, it develops two blade-like extensions or wings, the upper or the dorsal one being broader than the ventral Behind these two extensions there is a third small projection directed postero-laterally, while the fourth one is posteriorly directed The junction of the third and the first two projections form a comparatively large ventrally disposed facet for the reception of the posterior condylar head of the hyomandibular

Sphenotic —Like the pterotic, the sphenotic (figs 5b and c) also extends on the dorsal and ventral surfaces and occupies the antero lateral corner of the auditory capsule In addition to its contact with the pro otic and the pterotic the bone is also attached to the small alisphenoid antero-dorsally Internally it (sphenotic) is excavated into a short tunnel for the anterior semi circular canal and shares with the pro otic and pterotic, a recess for the lodging of the ampulla between that canal and the horizontal one On the dorsal side of the bone there is a posteriorly directed blade-like expansion or wing which faces an anteriorly directed similar expansion of the pterotic The ventral margin of this wing is considerably thickened and projects out posteriorly as a thick prominent spine known as the sphenotic process Below this spine and situated on the ventral surface there is an articular facet for the anterior condylar head of the hyomandibular

A semi-transparent (lightly stained) quadrilateral area extending from the pterotic to epiotic and giving the appearance of a large fontanelle is visible, in stained preparations, both on the dorsal and ventral surfaces of the auditory capsule, but in fact, there is no fontanelle of that type Its apparent presence is merely due to the margins of the inner canals of the bony labyrinth which bound the area on all sides, there being no canals or any other thickening in the quadrilateral areas excepting the thin layers of the pterotic and the epiotic, which, in contrast with the surrounding thickened area, appears semi-transparent

Out of the three otoliths the sacculith (sagitta) is the largest It is situated in a depression formed by the basi-occipital, ex-occipital and pro-otic on the ventral side, and is roofed over by the supra-occipital and epiotic It is a laterally flattened piece almost round in outline and being situated vertically appears oblong in the dorsal or ventral view Utriculith (lapillas) is a much smaller piece and is situated in front of the sacculith in the antero-lateral direction It is almost a spherical piece and is situated on the ampulla between the horizontal and anterior semicircular canals The lagenalith (astericus) lying in the lagena or the rudimentary cochlea is situated immediately behind the sacculith It is slightly longer than the utriculith The three otoliths together form an arc which faces the postero-lateral corner of the skull

(4) *Occipital Region*

The occipital region is composed of four replacing bones, the supra occipital basi-occipital and the two ex-occipitals situated on the dorsal, ventral and lateral sides of the skull respectively The bones originate from the primary chondrocranium as independent ossifications and form the hinder region of the brain case, which articulates posteriorly with the vertebral column, the articulating surfaces being situated on the median basi-occipital and the two ex-occipitals The foramen magnum is formed jointly by two ex-occipitals on the lateral sides and the basi-occipital on the ventral side Laterally, the occipital region is in contact with the auditory capsules and anteriorly with the frontal on the dorsal sides and the parasphenoid on the ventral, there being no basisphenoid and the parietals, although present, being very small and laterally disposed All the bones of the occipital region are comparatively large and prominent in *Aplocheilus* and may perhaps compensate for the absence of basisphenoid and the reduced parietals (Figs 2 and 3)

Supra-occipital (fig 6a) —The supra-occipital is situated as usual in the hinder region of the skull, but unlike in many other forms does not form the hindmost margin of the foramen magnum of the skull The bone is roughly paw-shaped (fig 6a) and concave ventrally (internally) and is provided with a pair of long processes trailing behind The processes, instead of starting from the hind margin of the bone, originate from its convex upper surface and extend posteriorly beyond the margin of the skull on the same level They are in the form of a pair of vertical laminae, which, in the proximal portion, fuse ventrally so as to form a sort of keel After the formation of the keel the laminae again separate and protrude posteriorly as occipital processes to which the dorsal muscle fibres are attached

The supra occipital in *Aplocheilus* is prominently disposed anteriorly in comparison with other Teleosts It not only touches the frontals but is also wedged between the posterior portions of the bones and extends even below their margins It is attached by a narrow surface to the parietals laterally and comes in contact with the two ex-occipitals by its posterior margin which is directed ventrally It does not figure in the formation of the occipital condyles

The frontals and parietals, being membrane bones developed secondarily on the cranium, overlap the margins of the supra-occipital on both sides and reduce its open surface In the distal portion of the bone, there is a triangular area which is lightly stained in all specimens The area has a thin sheet of cartilage on its inner side which divides into a pair of narrow bands at the distal margin of the bone and proceeds laterally on each side to meet the distal extremity of the alisphenoid

Basi-occipital —The basi-occipital (fig 2) is another prominent bone on the mid-ventral side of the skull corresponding to the supra-occipital on the dorsal surface, but the basi-occipital, unlike the supra occipital contributes materially towards the formation of the posterior margin of the skull Anteriorly it is in contact with the mesial portions of the pro otics and laterally and dorsally with the ex-occipitals Its anterior end is overlapped ventrally by the posterior extremity of the parasphenoid The bone is somewhat rhomboid in shape (fig 6b) and has a large circular concavity which forms an articulating surface with the centrum of the first vertebra Its ventral outer surface is flat, but internally there are two bony plates running vertically upwards and backwards from its inner surface and connecting the bone dorso-laterally with the ex-occipitals on each side The bony plates, starting from the posterior articular concavity of the bone, are disposed longitudinally and cut off the lateral corners of the rhomboidal bone They thus form a narrow channel in which the posterior portion of the medulla oblongata is lodged The lateral corners of the bone are somewhat excavated and form a part of the recess for the auditory ossicles The plates thus separate the brain case from the auditory capsules and also strengthen the occipital region internally There is a large foramen in front of the vagus foramen of the ex-occipital

Ex occipitals —The ex-occipitals (fig 2) are paired bones, situated posteriorly in the occipital region They take part in the formation of the otic capsule in place of the opisthotic and this become the auto-occipital, but the term ex-occipital is retained here for convenience They form the dorsal and lateral margin of the foramen magnum and also give rise to a pair of occipital condyles for the articulation of the skull with the vertebral column The articulating surfaces of the condyles are large and flat and situated on the ventro-lateral corner of the foramen magnum Each ex-occipital is in contact ventrally with the basi occipital and the pro otic, laterally with the pterotic and dorsally and dorso laterally with the supra occipital and epiotic respectively The ex-occipital figures in the formation of a major portion of the dorsal surface of the foramen magnum, in addition to the lateral walls, the share of the supra occipital in its formation being almost negligible After bordering the foramen magnum, the ex occipitals extend laterally and ventrally to join the basi-occipital and pterotic to form a large recess for the ampulla and the ear ossicles of the auditory capsule Being produced in different directions and planes, the shape of the bone (figs 6c and d) is very irregular The origin of the vertical supports or plates joining the basi- and ex-occipitals internally is not yet quite certain, but they appear to issue more probably from the latter than the former

Each ex-occipital has three or four foramina on its surface Three medium-shaped foramina for the passage of the occipito-spinal nerves are situated in a row on the lateral wall above the occipital condyle A large vagus foramen exists on the ventro-posterior margin of the skull and forms an exit for the glassopharyngeal and vagus nerves Another equally large foramen occurs as stated earlier, on the inner vertical plate through which the glassopharyngeal and vagus emerge and immediately leave the skull through the vagus foramen

(B) *Visceral Skeleton*

The visceral skeleton consists of the (1) Mandibular arch or the suspensorium of the jaws, (2) the hyoid arch forming the hyoid cornu supporting the tongue, and (3) the skeleton of the branchial arches The entire skeleton is made up of seven arches The first two arches are more specialised than the rest and being close to the chondrocranium are associated with the latter in their early development The first or the mandibular arch gives rise to the palato-quadrate and mandibular bars which support the upper and lower jaws respectively From the second or the hyoid arch, the hyomandibular is developed and gives attachment to both the palato-quadrate and hyoid arch or the hyoid cornu The next four arches support the gill filaments and the seventh arch is reduced in size and forms a tooth bearing inferior pharyngeal bone for masticatory purpose

(1) *Mandibular Arch*

In *A. lineatus*, the mandibular arch (fig 10) has the usual palato-quadrate and mandibular bars The former develops three independent ossifications, viz the metapterygoid, quadrate and palato-pterygoid A thin secondary bone, the mesopterygoid (entopterygoid) is attached to the above three from behind The metapterygoid which forms the posteriormost limit of the arch is attached to the antero-ventral edge of the hyomandibular The quadrate which is in front of and somewhat below the metapterygoid is also in contact with the hyomandibular through a large symplectic, to the distal end of which the quadrate is attached The distal piece, the palatopterygoid or autopalatine is a fused element and lies in front of the distal end of the quadrate It ascends upwards and forwards and articulates with the lateral ethmoid of the cranium and also the maxillae The palato quadrate has therefore connection on the posterior side with the otic region of the cranium through the hyomandibular and on the anterior side with the ethmoid region directly through the palatine itself The mandibular part of the arch, which represents the Meckel's cartilage gives rise to the articular bone of the lower jaw and persists as a slender rod of cartilage even in the adult The tooth-bearing dentary is grown around the distal part of the Meckel s cartilage and forms the biting part of the lower jaw A small angular and a sesamoid articular are also developed on the inner portion of the jaw The lower jaw is connected with the palato-quadrate arch through the condylar head of the quadrate which articulates with the articular of the jaw The upper jaw is formed of independently developed paired dermal bones, the pre maxillaries and the maxillaries The pre maxillae of both sides together form the entire gape of the mouth The maxillae are closely associated with them and further articulate with the distal ends of the palatine and the lateral ethmoid The different bones of the mondibular arch are further discribed as under —

Metapterygoid —It is a flat, somewhat chisel-shaped bone (fig 10a) with a broad anterior end At the posterior extremity it has a number of pointed splinter-like processes which fit into corresponding sutural notches in the antero ventral corner of the hyomandibular Its upper margin is entire but slightly concave and forms a part of the upper edge of the palato quadrate arch The lower margin is straight and is applied all along its length to the upper margin of the symplectic without the intervention of any cartilage At the anterior margin of the bone, there is a thin layer of cartilage by which it is connected with the broad portion of the quadrate

The presence of a distinct metapterygoid in *A lineatus* is quite remarkable Starks (1904, *a*), Regan (1911) and Myers (1931) dealing with the distinguishing characters even of the sub-order Poecilloidea, to which the family Cyprinodontidae belongs, have stated that the metapterygoid is absent in this group Contrary to their observations it is found to be in its normal position in *A lineatus* It is a well-ossified bone with an endosteal element in it and its associations with the hyomandibular, symplectic, quadrate and the mesopterygoid confirm its identity Moreover, its shape and position are typically those of the metapterygoid in the Haplomous fish *Novumbra hubbsi* (Chapman, 1934) and others

In *A panchax* and *A blochii* (parvus—Sundara Raj) too, this bone is present just as in *A lineatus* In *A blochii*, however, it is rather more elongated than in *A lineatus*

The other bones of the mandibular arch in *A blochii* are similar to those in *A lineatus* but somewhat more elongated The mesopterygoid is, however, more anteriorly placed and is not quite in contact with the metapterygoid in this fish

Quadrate —In front of but slightly below the metapterygoid is a large quadrate (fig 10a) which too is connected with the apex of the symplectic It has two distinct portions, viz a large vertical one and the other narrow and somewhat horizontal in position Both the portions are fused at the middle of the ventral margin of the vertical portion where they have a fused condylar head for articulation with the mandible The horizontal portion is narrow, straight and extends posteriorly from the condylar portion to end in a point Being narrow and obliquely situated, the portion appears like a thick rod in a lateral view The vertical portion is thin and more expansive It is connected with the metapterygoid behind and the palatine in front by a thin sheet of cartilage at either end Its anterior portion is slightly tapering upwards and has a posteriorly directed part of the palatine attached to its ventral edge

Mesopterygoid —The mesopterygoid (fig 10c) is a thin flat bone, slightly canoe-shaped in outline It adheres internally to the quadrate and is attached to the metapterygoid by its posterior end Anteriorly it extends to the dorsal portion of the palatine to which it is firmly attached from behind Its upper concave margin is slightly thickened and forms the dorsal margin of the palato-quadrate arch

Palatine —The palatine or autopalatine which is a fused element of palatopterygoid, forms the anteriormost bone of the palato quadrate arch It is divided into dorsal and ventral portions by a wavy line running along its long axis The ventral portion is somewhat lanceolate in shape, smooth and toothless Its posterior end is pointed and lies free on the upper margin of the palato-quadrate arch The dorsal portion is thickened and irregular in outline Its distal portion bears two thick processes or heads, ethmopalatine and rostropalatine for articulation with the lateral ethmoid and the maxillae respectively

Dentary —The dentary which forms the ramus of the lower jaw is a curved bone (fig 11a) with its anterior thick portion bearing teeth on its broader upper surface and the posterior thinner portion devoid of them The anterior portion is circular in outline, slightly narrowing in front and along with its fellow (ramus) on the other side which it meets in the middle, forms the entire semi circular tooth-bearing part of the lower jaw The anterior ends of the two rami instead of being fused into a symphysis are attached to each other in the middle of the jaw by a short ligament The posterior portion of the dentary in edentulous, thin and vertically disposed, with a small horizontal part extending medially from the ventral margin below the angle of the bone The vertically disposed posterior portion is excavated internally in which the anterior pointed end of the flat articular bone as also the distal part of the Meckel's cartilage are lodged

The teeth on the dentary are disposed into a broad band consisting of four or five rows The anterior row is composed of conical teeth, slightly curved inside, while the innermost has slightly smaller but erect teeth In between these two rows there are two or three rows of close-set smaller but erect conical teeth In the angle of the dentary or posterior dentigerous part of the bone, two or three teeth of the outer row are slightly thicker and longer than the rest of the outer row The inner smaller teeth are also more numerous in this area

Articular —The articular which immediately precedes the dentary forms the ventro-lateral margin of the jaw on the posterior side It (fig 11a) has two distinct wings—a ventrally disposed internal one and the other vertically disposed external or lateral The external wing is broad in first one third and then narrows anteriorly The narrow tapering end is applied internally to the dentary, resulting in only the broad posterior portion of the articular being visible in the side-view of the entire mandible, the rest being covered by the dentary The ventrally disposed wing is thin flat, somewhat linear obovate in outline and helps to form the floor of the jaw The external and inner margins of the two wings meet posteriorly and fuse into a ridge which finally dilates at the extreme posterior end into a specious double articular facet for the double condylar head of the quadrate

Sesamoid articular —On the inner side of the vertical wing of the articular is situated a small oblong tendon bone, the sesamoid articular (fig 11a) It is superficially attached to the proximal part of the rod-like Meckel's cartilage and marks the insertion of a part of adductor mandibulae muscles to the articular Sesamoid articular of ***Fundulus*** is large and of a different shape, but that of a Percoid fish ***Labrax lupus*** (Starks 1916, p 30) is strikingly similar in shape and position to that in ***Aplocheilus***

Angular —Angular (fig 11a) is a very small triangular bone which forms the inner angle of the ventral wing of the articular It is close to the articular facet but does not take part in the actual articulation of the jaw

Pre-maxillary —The pre-maxillaries are prominent tooth-bearing bones which on both sides together form the entire upper jaw They share equally in the formation of the jaw and meet anteriorly in the middle but do not fuse into a symphysis Each side after bordering the jaw protrudes backwards and downwards as a stout arm to end into a thick prominent spine below the angle of the mouth The arm is slender at the angle of the jaw, but flattens posteriorly and finally terminates into the aforesaid spine Further, the teeth-bearing portion of the bones have flat triangular processes of considerable size extending posteriorly along the dorsal level of the skull The processes on both sides approach each other medially, but remain slightly apart all throughout The teeth on the upper jaw are almost of the same type as on the dentary except that in the former there are more rows of smaller teeth between the larger outer row and the smaller inner one All the rows together form a broad band of conical teeth A few teeth in the corner of the jaw are also thicker and longer corresponding to similar teeth on the dentary The descending arm of the pre-maxillary, too, has one or two rows of smaller teeth within the gape of the mouth which are directed anteriorly The remaining terminal portion of the pre-maxilla is devoid of teeth

Maxilla —The maxillae or the maxillary bones of both sides do not take part in the formation of the upper jaw but are situated immediately behind it and in front of the lachrymal Each maxilla is a curved, rod-like bone which starts externally to the terminal process (spine) of the pre-maxilla, which it overlaps, and ascends medially upwards, describing a slight curvature As it reaches below the nasal bone, it twists over itself and changes its curvature and proceeds medially to reach below the distal portion of the posteriorly directed triangular processes of the pre-maxillaries At the twisted portion, the bone is considerably thickened and possesses articular surfaces Of these a prominent articular facet is situated in the ventro-posterior direction for the terminal rostropalatine head of the palatopterygoid The lateral ethmoid and the nasal are also in contact with this surface The distal curvature is shorter and the bone slenderer than the proximal portion The lower end of the maxilla is in contact with the posterior spine like portion of the pre maxilla, and upper extremity is attached ventrally to the posterior process of the same bone Thus, with the help of the articular surfaces on the palatine both the ends of the maxilla may be easily turned to push the pre-maxilla outwards and forwards

(2) *Hyoid Arch*

The second or the hyoid arch has two portions, the dorsal including the hyomandibular and symplectic, and the ventral forming the hyoid cornu The hyomandibular is one of the most important bones of the visceral arches It gives attachment to the jaws, the hyoid cornu and the opercular bones The palato-quadrate arch is suspended from the skull through the intervention of this bone, giving the hyostylic suspensorium to the jaws The details of these bones are as follows —

The hyomandibular (fig 10a) is a somewhat quadrilateral flat bone, vertically disposed in the post-orbital region with several short processes, condyles and facets on its surface for attachment to other bones There are two prominent condylar heads on its upper edge by which it is articulated with the pterotic and sphenotic of the otic capsule, corresponding facets for this articulation being situated on the respective bones The third prominent condyle is situated on the posterior edge of the bone and serves for the articulation of the operculum The two upper condylar heads are at the extremities of two distinct rod-like stems on the body of the bone which appear to meet in the middle of the posterior half of the bone From the upper half of the bone a flange-like ridge passes obliquely downwards to terminate into a point on the posterior edge A passage for the hyomandibular nerve is present but not quite apparent in the lateral view

At the lower extremity of the hyomandibular three separate bones, viz inter-hyal, symplectic and metapterygoid are attached to it At the postero-ventral corner a small inter-hyal is attached through a short interspace of cartilage To the same cartilage but above the antero ventral corner the metapterygoid is sutured The hyomandibular thus forms an important link through which the mandibular arch as well as the entire branchial skeleton are attached to the cranium

Symplectic —Unlike in many other fishes the symplectic in *Aplocheilus* is quite large and well developed (fig 10a) It is roughly plough-like in shape, with its base towards the hyomandibular and apex anteriorly directed The inner or the upper margin is bent inwards, making a small angle There appear to be two different regions located on this bone, namely a main pointed rod-like portion and a thin membranous wing below and in front of it The rod like portion is fusiform, like the symplectic of many other fishes, and is in contact with the lower articular end of the hyomandibular, with a portion of cartilage covering both the articular surfaces The wing-like portion forms the lower and the front border of the bone, including the pointed apex The latter has a small ridge on it running to its extremity, and the whole is firmly wedged in the posterior notch of the quadrate

The hyoid cornu —The hyoid cornu or the hyobranchial skeleton is fully developed with all its usual characteristic structures It consists of two lateral arches on two sides of the buccal cavity, starting from the respective hyomandibula, and sloping downwards and forwards to meet the corresponding bone from the other side in the mid ventral line below the floor of the same cavity Each half of the arch is made up of four segments, the inter hyal epi hyal, cerato hyal and a double hypo-hyal on each side These are attached to a median basi-hyal on the ventral floor, which, in fact, forms the anteriormost part of the arch and supports the tongue The last element is further supported by a median urohyal from below These segments are connected with one another by means of a small layer of cartilage and form a strong supporting structure to the body wall Being closely associated with the branchial arches which are in close proximity on the posterior side, this arch is also called the hyobranchial skeleton

The dorsalmost segment of the arch is a small inter-hyal or the stylo-hyal which is dorsally attached to cartilaginous interspace below the postero-ventral end of the hyomandibular 'It has sometimes been compared to the epi-branchial but probably is a new formation (Goodrich, 1930) ' The thickened dorsal head of the bone is lodged in the cartilaginous interspace, being immediately behind the symplectic, which is also attached to the same cartilage but lies in a different direction The inter-hyal is a small hour-glass shaped bone (fig 9a) extending obliquely forward from its origin and furnishing means of attachment to the next piece of the arch, viz the epi-hyal, with a pad of cartilage in between To the same cartilage, the dorsal edges of the preopercle and interopercle are attached by ligaments and help to synchronise the joint action of the opercular apparatus and that of the branchiostegals which are placed on this arch

The epi hyal (fig 9a) is a roughly triangular piece hanging from the inter-hyal and proceeding in the antero-ventral direction Its posterior margin is circular (convex) and the apex, which is directed posteriorly is slightly bent and thickens to form a cup which fits exactly on the cartilage at the lower end of the inter-hyal The base of the triangular bone is broad, points anteriorly and touches the following segment of the arch, the cerato-hyal with a thin interspace of cartilage between them The dorsal side, however, is without any cartilaginous interspace owing to a secondary thickening of the bones, which form a sort of long ridge running along the entire dorsal edge of the bone right from its posterior end and continuing over the dorsal edge of the cerato-hyal to its anterior extremity Such connections on the corresponding bones but in the shape of very small splints have also been recorded in *O ruber* (Dharmarajan, 1936) The ridge found in *Aplocheilus* is of a similar type, but comparatively pronounced and long, and appears to exist merely to strengthen the bones Appearance of such strengthening ridges on the particular bone in fishes phylogenetically far apart is significant in view of an opinion which states that the epi-hyal and the cerato-hyal of modern Teleost have been a single element

But for the aforesaid ridge and the posterior articulating head, the epi-hyal is thin and its surface plain The first or the posteriormost branchiostegal ray is attached to this bone from outside The next ray is only partially attached to this bone as a portion of its attaching surface lies on the cartilaginous interspace between this bone and the cerato-hyal There is no depression on the bone for the attachment of either of these rays as recorded in *O ruber*

The cerato hyal (fig 9a) which immediately follows the epi-hyal and is connected with it along the dorsal edge, has a broad proximal portion which immediately narrows into a bar in the middle of the bone, eventually bifurcating into two rod-like extremities The extremities are tipped with cartilaginous heads for attachment to the two hypo-hyals The bony membrane connecting them is not complete and towards the distal ends they have a quantity of cartilage between them also The third and the fourth branchiostegal rays are attached to the broad proximal portion of the bone from outside, while the remaining two branchiostegals are attached to the anterior bifurcating part of the bone from inside

The hypo-hyals (fig 9a) which are in close contact with the cerato-hyals, are double pieces of bone on either side of the arch They are small nodule like structures placed one over the other as dorsal and ventral pieces with a thin layer of cartilage in between They correspond with the anterior dorsal and ventral extremities of the cerato-hyal, the terminal cartilaginous heads fitting closely in the small posterior concavities of the hypo-hyals One of these bones is somewhat large in some specimens, but in *Aplocheilus* both hypo-hyals appear to be of almost equal size and enclose a good amount of cartilage in them, the ossifications occurring in only the superficial layers of the bones

The hypo-hyals are fronted by a median, large cartilaginous plate, the basi-hyal, which extends forward from the hypo-hyals and forms the support for the tongue It is a flat triangular piece, with the base facing anteriorly and the apex lying above the hypo hyals to which it is attached by fibrous tissue The apex of the bone is ossified and develops an articular surface by which it is attached to the anterior head of the first basibranchial Though the basi-hyal is mainly cartilaginous, ossifications, nevertheless, appear in different places and specially, to a greater extent at the posterior portion near the hypo-hyals and the articulating apex of the bone

Description of the hyoid cornu would be incomplete without mention of two other elements, although they do not belong to this arch Those elements are the branchiostegal rays, attached directly to the segments of the hyoid cornu, and the uro hyal or the basi-branchiostegal attached to the cornu only through short ligaments

The number of branchiostegals is six They are attached to the hyoid cornu anteriorly while they are free posteriorly In the living condition they are connected all throughout by a thin fold of skin between them and are folded fan-like below the opercular elements They are grouped according to their attachments, the posterior group consisting of the first (posterior) four rays and the anterior of the remaining two rays

The rays (fig 9a) of the first group are broad, sabre-shaped and attached, as already stated, to the outer surfaces of the porterior margin of the epi-hyal and the cerato-hyal The attaching surfaces are broad and flat and adhere to the respective bones without any ligament The rays are curved, pointed posteriorly and broader towards their anterior extremities They are thin and flattened laterally, but in the middle third of their length they thicken slightly on their dorsal edge Further on, the thickening develops into a central ridge on the outer surface of the rays as they approach the attaching surfaces The first (posterior) ray is the broadest of all and is also the longest, reaching even the posterior extremity of the opercle The other rays that follow become gradually shorter and narrower as they extend towards the anterior end All the rays fold below the opercular apparatus, but only the first two are covered by the lower margins of the opercular bones, behind which they fold

The rays of the second group, namely, the 5th and 6th rays although similarly disposed and folded, are shorter than the rays of the posterior group There is an appreciable distance between the points of attachment of the rays of the two groups Moreover, the rays of the anterior group are not so modified in different regions as they are in case of the posterior group Another important point of difference is that, unlike the rays of the posterior group, the 5th and 6th rays are attached to the inner surface of the narrow anterior portion of the cerato hyal This attachment being through a short ligament is firmer than the posterior group of rays which having no ligament can be easily detached

Hubbs (1920) pays considerable attention to the form and arrangement of the branchiostegal rays in different groups of fishes His own view is that in higher groups of teleosts there is 'a peculiarly constant arrangement of the branchiostegals' and that the arrangement found in Microcyprini is similar to that in Acanthopteri The arrangement of branchiostegals in *A lineatus* studied here also agrees with the description given by Hubbs for Microcyprini and, thus justifies for it a place near the Acanthopteri (Perciformes) which Hubbs, too, has allotted to this order (Microcyprini)

The uro-hyal or the basi-branchiostegal which was also styled 'sternum' by Allis, is a thin, vertically situated plate of bone lying very close to, or as if bisecting, the angle formed by the mesially meeting halves of the hyoid cornu below the floor of the throat The uro hyal is, somewhat triangular (acute angled) in shape, with its apex near the angle of the cornu and the rest of the body trailing behind At the apex the uro hyal thickens into a round tuberosity from which two strong, short ligaments proceed to the mesial surfaces of the lower hypo-hyals on each side To the lateral surfaces of the posterior part of the bone similar ligaments arising from the antero-ventral extremities of the pectoral arch are attached The shape of the bone slightly varies, however, in some specimens in which on the dorsal side there is either a dorsally or postero-dorsally directed process just behind the anterior tuberosity The thickness and the length of the process also vary according to specimens

The opercle (fig 8) is a prominent bone on the lateral side of the skull It is almost triangular in shape, the triangle being equilateral with its base parallel to the dorsal profile, and the apex pointing below At the anterior corner there is a prominent spine, at the base of which lies a facet for the articulation of the opercle, with a large condyle on the posterior edge of the hyomandibular The inner surface has two or three ridges radiating from the articular facet The outer surface is smooth and slightly convex

The pre-opercle is peculiarly shaped It has two arms one posterior and the other ventral and anteriorly directed with a membranous connecting lamina between them The posterior arm is almost vertical and originates near the hyomandibular articulation of the opercle It extends round the angle of the bone and the ventral arm proceeds anteriorly to meet the quadrate Both arms are channelled and form, in fact, a single continuous canal along the entire posterior and ventral margin of the bone The canal is comparatively wide and opens outwards (laterally) along the vertical arm and then slowly twists at the angle to open ventrally along the lower arm This ventral arm, terminating near the quadrate, is disposed in such a way that the articular condyle of the latter is in the same curvature as that of the former, and, superficially examined, appears continuous

The sub-opercle is another thin membranous bone bordering the opercle posteriorly as well as below The inner edge has a wide notch wherein fits the lower extremity of the opercle The bone terminates in a truncated edge near the angle of the pre-opercle Immediately in front of the truncated edge is another membranous bone, the inter-opercle, which together with the sub-opercle forms the posterior and ventral margin of the operculum The inter-opercle is also somewhat triangular in form with a short base at the truncated edge of the sub-opercle and two long sides directed anteriorly The bone runs below the margin of the pre opercle and ends along with the latter near the articular surface of the quadrate A ligament joins the inter-opercle to the angular of the lower jaw

(3) *Branchial Arches*

Immediately behind the hyoid cornu are situated five branchial arches, which together form the branchial basket or the skeleton supporting the gills in the pharyngeal wall Of these only the anterior four bear gills on their posterior faces The fifth is thickened and modified to bear the lower pharyngeal teeth for mastication Each arch, is, as usual, made up of two lateral halves in the pharyngeal wall and unite in the mid-ventral line Unlike in the hyoid cornu, the halves of the branchial arches meet in the mid-dorsal line also Each half of the arch consists of almost the same number of segments as in hyoid cornu with certain parts modified and other missing according to the specialised functions of the parts Generally, each half is made up of the pharyngo-branchial and epi-branchial on the dorsal side, the cerato branchial on the lateral and ventral side, and the small hypo-branchial near the mid-ventral line In addition to these four there is a median basi-branchial, corresponding to basihyal on the hyoid cornu and is attached to both hypo-branchials as a common piece to both halves

The higher degree of specialisation expected in the branchial skeleton has led to different modifications of its parts Nevertheless, corresponding segments of different arches have almost similar modifications or special structures on them Hence, instead of each arch being described separately, the corresponding segments from different arches are treated serially group by group in the following account

Pharyngo-branchials —The pharyngo-branchials are originally the dorsal segments of the branchial arches In the embryonic stages they are cartilaginous and rod-like, but as they develop their structure changes completely They become thicker, flattened dorso-ventrally and are ultimately designated as pharyngeal bones In addition to these changes, the bones undergo further modification to bear prominent teeth on their ventral surfaces which project inside the throat from the roof of the pharynx The bones from both sides meet in the mid-dorsal line, but do not fuse, there being some muscular tissue between them They, thus, form two patches of strong tooth-bearing bones on the dorsal side The ventrally directed teeth on these bones work against similar but dorsally pointed teeth on the fifth arch on the floor of the throat and help in mastication

The pharyngo-branchial bones on each side are three irregularly shaped pieces of bones (fig 9c) which together form the pharyngeal bone on that side The middle piece is nearly double the size of the other two and represents fused pharyngo-branchials All the pieces, though separate in themselves, are imbricated and very closely set together The bone can, however, be split up on maceration

The first pharyngo-branchial is thickened triangular piece of bone which has a single row of teeth on its posterior margin The last tooth on the mesial side is bigger in size, the others behind being smaller In the anterior corner of the triangular bone there is a short articular head covered with a thin layer of cartilage To the back of the head a cartilaginous rod is attached and it points laterally in place of the respective epi-branchial

The second and third pharyngo-branchials are fused into the large middle piece of the pharyngeal bone It is a broad quadrangular piece of bone, bearing on its ventral surface prominent conical teeth slightly curved at their tips They are thick at their bases, two or three of them on the anterior margin being particularly thickened and larger than the rest The lateral edge of the bone has two small articular heads—one anterior and the other posterior—for the articulation of the second and third epi-branchials A third very prominent articular head is situated on the posterior edge of the bone, with which the fourth thickened epi-branchial is articulated

The third toothed piece which represents the 4th pharyngo-branchial, is more thickened than the first bone and has on it a larger number of teeth of different sizes

Epi-branchials —Among several ossifications in this area the first epi-branchial, remarkably enough, remains quite cartilaginous and rod like throughout the adult condition It is attached, as stated previously, below the anterior articulating head of the first pharyngo-branchial and extends towards the lateral wall of the pharynx to meet the cerato-branchial At the junction of and in the angle of these two bones is a rod-like ossification firmly attached to the cartilaginous joint In form, it is similar to the third and fourth epi branchials, although much smaller in size Its position suggests that it must be either a super-epi-branchial ossification similar to the supra-pharyngo-branchial in some forms like *Scomber*, *Otolithus*, etc (Dharmarajan) or that the posteriormost gill-raker of the first cerato branchial must have developed abnormally and attached itself to the cartilaginous end of the first epi-branchial

The second epi-branchial is rod-like towards the pharyngeal bone, but at its contact with the cerato branchial it has a hammer shapped double head It is attached by fibrous tissue to the first pharyngo-branchial, but its real articulating surface is on the second bone as stated previously

The third epi-branchial is also rod-like towards the pharyngeal bones and hammer-like at the other end, but is shorter in length and slightly thicker than the previous piece The hammer-like portion is considerably larger, and the outer head is also longer The bone articulates mesially with the second pharyngeal bone at the antero-lateral corner of the latter

The fourth or the last epi-branchial is the most thickened and well-developed piece amongst the epi-branchials It is slightly flattened and bent outwards behind the spot where it attaches itself to the pharyngo-branchial bone The outer surface of the bent portion has a thickened head projecting outwards Unlike in the former two epi-branchials there is no hammer shaped thickening or any other tuberosity at the other end of the bone, the cartilage at the tip of the corresponding cerato-branchial being attached only to the inner surface of its extremity The bone is attached to the second pharyngo-branchial bone by a prominent articulating surface on the posterior face of the latter and to the fourth pharyngo-branchial by a fibrous connection Thus, the fourth epi branchial is, by far, the most prominent and best developed bone of the group and on it must be falling the task of mastication through the movement of the pharyngeal bones (See figs 9c and d)

Cerato-branchials —The cerato-branchials are the most prominent as also the longest parts of the branchial skeleton Their structure, position and points of attachment are almost similar They are four elongated and slightly rod-like structures on the lateral wall and the floor of the pharynx Dorsally, they have cartilaginous tips which are attached to the epi-branchials at the lateral margin of the pharynx Immediately behind this point of attachment they bend sharply inwards and converge from both sides mesially, ventrally and anteriorly for a considerable distance from their origin, to meet the corresponding hypo-branchials near the mid-ventral line The first three cerato-branchials are alike in this respect, because their anterior ends are capped with the cartilage and are firmly united with the hypo-branchials The fourth pair is, however, different from the rest as the cerato-branchials fuse with hypo-branchials and the fused elements from either side are attached to the median cartilaginous basi-branchial (Fig 12)

The inner and outer faces of the cerato-branchials have gill-rakers of different forms in different places Unlike *O ruber* and other species in which gill-rakers are found on all the three pieces, namely the epi-, cerato- and hypo-branchials, they (gill-rakers) are usually found only on the cerato-branchials in *A lineatus*

On the first cerato-branchial, the first gill-raker (fig 12) on the outer side is a small triangular bunch of minute conical teeth attached to the outer face of the branchial bar The second and succeeding gill-rakers on the outer face of the first cerato-branchial are peculiarly modified The second gill-raker (fig 9b) is a linear lanceolate flat piece, on the postero-mesial edge of which grow, inwardly directed, minute conical teeth on the distal two thirds of its length The teeth are in a single row, but towards the proximal portion there are a few double rows also The succeeding gill-rakers on the outer face are of the same shape as the second, but longer posteriorly The fifth and sixth, or at times the seventh also are of the maximum size, while the rest are shorter posteriorly The lanceolate gill-rakers are generally eight or ten, and are attached obliquely to the face of the branchial bar, so that the row of teeth points towards the throat The last two or three gill-rakers are small and of the same type as the first raker The posteriormost, however, is large, dorsally placed and modified as a rod like piece of bone on the proximal portion of the epi branchial

The inner row of gill rakers on the first cerato-branchial consists of teeth (fig 9b) almost of the same type as the first gill-raker on the outer side, but the row of teeth in each group is slightly semi circular, instead of triangular, and attached from the inner side Each group forms a single gill-raker and the total number is almost the same as on the outer side

The gill-rakers of the inner and outer rows of the second and third, and outer row of the fourth are similar and of the same type as on the inner row of the first There are no gill-rakers on the inner side of the fourth, but on the inner surface of the bar itself there is a narrow band of conical teeth along its entire length On the outer surface of the reduced fifth arch also there is a row of vestigeal gill-rakers attached only to the middle portion of the bone

Hypo-branchials —The hypo-branchials (fig 12) are present only in the first three arches They are thick nodule like structures attached to the distal end of the cerato-branchials on the one side and to the basi-branchials in the median line, cartilaginous interspaces intervening between all attachments On the dorsal surface of these hypo-branchials there are groups of erect conical teeth projecting into the throat On the first hypo-branchial the group of teeth is rather small, but increases in extent on the second and is widest on the third, on which the teeth also are bigger and more prominent

There is no separate and distinct hypo-branchial segment in the fourth arch just as in the Haplomous fishes, but the terminal portions of the fourth cerato-branchials have fused with the hypo-branchials These portions bear prominent teeth just as in case of the third hypo-branchials, the patch of teeth, however, being continuous with small teeth on the inner surface of the bone Chapman (1934) also subscribes to this view in respect of *Novumbra*, wherein the same segment is found to be absent

Basi-branchials —The four functional arches are supported by four basi-branchials (fig 12) Being the median segments of the arches the basi-branchials are situated in the mid-ventral line, and the hypo-branchials from both sides are attached to them laterally in the horizontal level The four basi branchials, along with the three hypo-branchials, a basihyal and the cartilage between all these pieces produce a strong and firm floor for the buccal cavity As the basi-branchials in the adult form are considerably displaced from their premordial position between the hypo-branchial segments of the respective arches, their attachments are not strictly limited to the corresponding segments only but are mutually adjusted according to their positions, so as to perfect a firm supporting floor.

Only the first three out of the four basi-branchials (fig 12) are ossified and elongated, while the fourth, which is cartilaginous throughout, is short and rhomboid in shape Curiously enough, the same fourth basi-branchial is small and unossified both in *O ruber* and *Scomber* (Dharmarajan, p 43) This indicates that some definite significance is attached to the cartilaginous nature of this segment, which remains unossified among the assemblage of so many other bony structures Perpetually mobile condition of the branchial apparatus for respiration and for food requires a firm binding quality and at the same time certain amount of elasticity for the central uniting piece Those qualities are better afforded by cartilage, and that appears to be the reason why the fourth basi-branchial remains cartilaginous in *A lineatus* as also in other forms

Fifth arch —The fifth arch, which forms the posteriormost element of the branchial skeleton, is a considerably reduced and much modified piece of this skeleton Instead of forming part of the lateral wall, the arch remains entirely on the floor of the pharynx and is developed into two pharyngeal bones The posterior halves of the bones are somewhat tapering, bent postero-laterally and thus diverge from each other The anterior parts are straight and lie close and parallel to each other near the mid-ventral line The anterior extremities approach the fourth cerato branchials from behind and are attached to the fourth basi branchial The pharyngeal bones are considerably thickened in the middle angle of their bodies and bear thick prominent teeth of different sizes on their dorsal surfaces The teeth project into the throat from the floor of the pharynx, which is formed by the bones themselves The posterior row of teeth is very large and thick, particularly two or three teeth towards the median line are remarkably large and are the biggest of the whole lot They are of the same type as the large teeth occurring on the upper pharyngeal bones The tips of the ventral teeth are bent anteriorly and those of the dorsal posteriorly Both these types of teeth work on each other with the help of the strong muscles attached to the respective bones and masticate the food The other teeth on the lower pharyngeals become smaller towards the anterior end and are of the same type as the small teeth on the upper pharyngeals

Post-temporal —The post-temporal (fig 8) is a purely dermal bone through which the pectoral arch on each side is connected with the skull It has a rod-like anterior portion which rests on the epiotic end and a broad slightly channelled posterior portion which is applied to the outer surface of the cleithrum below its spine The channelled portion faces anteriorly and the lower side of the channel in that position is broader and extends more anteriorly than the other side which remains narrow and small The sides of the channel represent the upper and lower wings of the complicated post-temporal bone, developed in Poecilhdae Although the distal end of the bone rests on the dorsal crest of the epiotic, there is no ligamentous connection between them The bone merely lies in the dermal tissue and may be easily dislodged with the flesh The real contact of the post-temporal with the skull, however, is through a ligament which arises on the inner anterior surface of the bone in front of the posterior broad portion and attaches itself to the posterior ventral margin of the epiotic In some specimens the ligament is ossified at its insertion on the skull while in others it is ossified at the other end on the post-temporal The ligament when ossified in the post-temporal region appears like a small pointed process which originates in front of the broad portion, almost in the middle of the bone The distal ossification of the ligament is disc-like and is applied to the outer surface of the epiotic Some specimens have both ossifications The post-temporal fork found in many fishes is absent in *A lineatus*, but since its place is taken up by the ligament which has a tendency to ossify, the ligament may be said to represent the lacking fork, as has been conjectured by Gilbert (1895, p 403) in regard to Haplomous fishes

HEAD SKELETON OF *Oryzias melastigma* (McCLELLAND)

The head skeleton of *O melastigma* (fig 13) comprising the skull proper and the visceral skeleton, is hardly 6 mm in length and less than half as long as that of *Aplocheilus lineatus* Although the skull is not very broad and dorsally flattened like that of *Aplocheilus*, it is nevertheless, moderately flat and slightly tapering anteriorly, on account of the jaws being comparatively smaller in size Moreover, the jaws not being much prolonged anteriorly, the pre-orbital region is also rather short, the orbits, however, being proportionately larger than in *A lineatus* The dorsal surface of the skull is smooth, with the characteristic curvatures of the frontals and has a transverse oblong depression in the pre-ethmoidal region The depression is not so evident in *Aplocheilus* The visceral arches are situated mainly below the posterior portion of the skull and contribute towards the height of the head skeleton which is greatest in this region The arches extend anteriorly also and give a tapering character to the ventral profile The dorsal profile is almost straight except in the pre-ethmoidal region, where there is a slight depression The height and the breadth of the head skeleton in the posterior region are almost equal, and are contained about 1 5 times in the total length of the skull with the jaws

(1) *The Skull* (Figs 13 and 14)

The skull proper is limited anteriorly by a thin mesethmoid, as the usual vomer is absent in *O melastigma* On the posterior side the processes of the supra-occipital are shorter than in *A lineatus* Another remarkable difference in this region is that the epiotics bear a pair of long posteriorly directed processes on their outer surfaces These are not found in *Aplocheilus* The epiotic crest is also not so prominent as in the latter The skull is depressed in the postero-lateral corner and the pterotic bone representing this area is placed on a lower level than that of the adjoining frontal and the supra occipital On the inclined portion of these two regions is a small cartilaginous area between the pterotic and the frontal, which remains uncovered due to the absence of parietals The cartilaginous area, being on the inclined portion, is not quite visible in the dorsal view, but is evident only if the skull is tilted slightly There is another prominent quadrilateral area in the postero-lateral corner, which is very lightly stained, but just as in *A lineatus*, the area is apparent only because there are no thickened structures below it in contrast with the adjoining area which possesses them

The ventral surface of the skull is excavated by the orbits as usual, and the inter-orbital space being uncovered by any skeleton structures, is wider than in *A lineatus* owing to narrower parasphenoids The ventral level of the skull is also not so uniform The ethmoid region being low in height and there being no vomer, the parasphenoid which occupies the mid-ventral position ascends upwards in the anterior region to meet the dorsally situated mesethmoid

The basi-occipital (fig 14) is on a slightly higher level than the pro-otic and the parasphenoid, which proceeds to meet the former, leaves a small gap between itself and the ventral surface of the pro-otic The ventral surfaces of both basi-occipital and the pro-otics are inclined on the lateral sides, and with the narrow parasphenoid in the median line form a low keel on the ventral side At the posterior end of the skull there is the articulatory concavity of the basi-occipital, with a wide constriction behind it On the lower level of this concavity are the ex-occipital condyles on either side of the foramen magnum The postero lateral corner of the skull is flattened just as on the dorsal side, but the depression on the medial portion of the pterotic is deeper than in *A lineatus* and resembles the pterotic recess of other fishes

Just as in *A lineatus* there are no cartilaginous areas on the ventral surface of the skull of this fish, but the replacing bones of the skull are separated by thin layer of cartilage between them, the layer being somewhat wider between the pterotic and other bones The other peculiarity in common with that of *A lineatus* is that the replacing bones have a thin layer of cartilage even between the inner and outer surfaces or laminae of each bone

After the removal of the frontal bone, which covers a major portion of the brain case, the anterior region is almost as equally ill-provided with skeletal structures as in *A lineatus* The orbito sphenoid and basi sphenoid are absent and the ethmoid remains cartilaginous The only improvement in *O melastigma* is that the cartilaginous strand at the distal extremity of the supra-occipital, joining the alisphenoids on either side, is wider and more extensive Moreover, the alisphenoids are also wider and more strongly built in this fish

The skull is divided as usual, in the four regions, namely, the ethmoid, orbito-temporal, otic and occipital

Ethmoid region—The ethmoid region of *O melastigma* differs from that of *A. lineatus* in certain remarkable features It has a prominent mesethmoid, which is both large and well ossified, and has no vomer developed on its ventral surface It has the same position in relation with other bones as in *A lineatus* and forms the anterior margin of the brain cavity The median transverse portion of the ethmoid region remains cartilaginous The lateral extremities are covered by the lateral ethmoids On the ventral surface, the parasphenoid extends beyond the median cartilage and meets the mesethmoid from below The posterior face of the ethmoid cartilage extends posteriorly, on both the dorsal and ventral levels into the membranous projections, almost in the same way as in *A lineatus*, but the membrane is narrower, and the triangular extremity of the dorsal entirely absent The ethmoid cartilage is produced in the anterior direction just as in *A lineatus*, but, unlike in the latter, the membrane is on the dorsal level of the cartilage, and is ossified into a mesethmoid bone in this fish

Mesethmoid—The mesethmoid (fig 13) is a median, circular scale-like bone in front of the ethmoid cartilage It has the nasals on its either side, the lateral ethmoid in the postero-lateral direction and the frontals behind In front, the extremities of the posterior processes of both pre-maxillaries rest on the anterior portion of the bone The ventral surface has a shallow groove in the posterior half of the bone in which the anterior extremity of the parasphenoid is lodged

As stated above, the mesethmoid in *O melastigma* is an ossification of the anterior prolongation of the ethmoid cartilage, and according to some authors it is termed as ethmoid bone A narrow strip of cartilage continues to remain unossified on the anterior and lateral margins of the bone even in the adult condition Between the upper and lower surfaces of the bone, is a layer of cartilage which is continuous with the ethmoid cartilage behind and divides the bone into upper and lower laminae The cartilaginous layer is thin anteriorly and thicker in the posterior portion, so as to make the two laminae more evident only on the posterior side

The mesethmoid in *O melastigma* agrees in certain respects with the description of that bone in *Fundulus* (Starks, 1926) as well as in Poeciliidae in general Starks, however, describes the bone in *Fundulus* as having a double laminae only in the posterior part with some cartilage between In *Belonesox belizanus* also (Starks, *op cit*) 'the mesethmoid is a thin disc, scarcely ossified, though it is easily separable from the cartilage under it In some of the Haplomous fishes the mesethmoid is actually in the form of two bony discs, with a short column of cartilage between ' A double laminar structure with cartilage in between thus seems to be a common feature of mesethmoid in many fishes

Starks (*loc cit*) believes that mesethmoid is generally of dermal origin and in some cases (Percoids) of a 'dual origin, its surface being ectosteal and its interior endosteal though often of cartilage, with spicules of bone scattered through it' Elsewhere he observes that 'very often', even in most specialised fishes, 'mesethmoid is a thin shell of surface bone with the interior filled with cartilage' The association of mesethmoid with a certain amount of cartilage, thus, appears to be constant factor in most fishes and may indicate that the mesethmoid may be of cartilaginous origin, the cartilage having a tendency to ossify only on the upper and lower surfaces and to assume different shapes In *O melastigma*, at least, the mesethmoid seems to have originated in cartilage, ossifying only in a circular area on the upper and lower surfaces Even on its anterior and lateral sides there is still a thin strip of cartilage persisting Moreover, ossification on either surfaces, with cartilage in between, is a condition prevalent in all the replacing bones of this fish, including even the complicated otic bones A similar condition in the mesethmoid also strengthens its claim to be replacing bone originating in cartilage

The mesethmoid in *A lineatus*, although entirely cartilaginous (*vide infra*, p 4) is different from the type of mesethmoid described above, and is independent of the ethmoid cartilage

Lateral ethmoid —The lateral ethmoid (fig 14) is almost similar to that of *A lineatus*, except that the margins of the bone in *O melastigma* are less thickened Moreover, a small postero lateral lobe is not covered by ossification The posterior transverse portion of the bone lies below the anterior margins of the frontals The nasals are attached from above and roof the olfactory area which is situated in the antero lateral corner of the bone The area is bounded by the anterior and transverse portions of the bone on two sides and the lachrymal on the external side The olfactory nerve opens, as usual, through the olfactory foramen, which passes through the antero-lateral corner of the ethmoid cartilage Unlike in *A lineatus*, neither the palatine nor the maxilla comes in contact with the margins of the lateral ethmoid

Nasals —The paired nasals (fig 13) are not thin, scale-like circular as in *A lineatus* They are comparatively thicker, concave ventrally and somewhat bean shaped in outline The hilum side of the bean-shaped nasal, which is attached to the anterior end of the lateral ethmoid, is situated medially and is thicker than the outer thin margin of the bone It is more prominent than the nasal in *A lineatus*

Orbito-temporal Region

Just as in *A lineatus*, the orbito-temporal region of *O melastigma* is quite extensive and contains a small number of bones as compared with the expanse of the region In addition to the orbito-sphenoid and basi sphenoid, the parietals too, are absent in *O melastigma* The frontals cover the entire dorsal surface of the region The alisphenoids are somewhat bigger than in *A lineatus*, but the parasphenoid is narrower and leaves more space uncovered by bones on the ventral surface of the skull The circum-orbital series are represented by the dermosphenotic and the lachrymal only

Frontals —The frontals (fig 13) are very similar to those of *A lineatus* both in shape and position They have also the conspicuous triradiate ridge on the ventral surface, which is clearly visible dorsally and separates the three different regions of the frontal, viz the supra orbital, inter-orbital and the sphenoidal The supra-orbital portion is rather more extensive than in *A lineatus* and also descends a bit more ventrally on the posterior side for the protection of the eye On the dorsal surface the frontal has the typical curvatures corresponding to the three regions of the bone The mesial margins of the frontals overlap in the median line, and the distal extremity of the supra-occipital is wedged in between their posterior portion just as in *A lineatus*

Parasphenoid —The parasphenoid (fig 14) differs from that in *A lineatus* in several respects It is bent at two ends, has a different shape with additional processes, and is slender than in *A lineatus* It extends from near the base of the basi-occipital to the middle of the mesethmoid, and is perhaps the longest of the bones of this fish

The bone is a slender, elongated structure with a broad bridge in the middle, on either side of which it bends slightly The bridge is situated in front of the anterior margins of both pro-otics and is formed of a pair of lateral plates arising from the lower side of the parasphenoid, and proceeding dorso-laterally to meet the mesially directed plates of the pro otics The portion of the bone in front of the bridge gradually broadens out anteriorly and ends into an obtuse point Along the median line of this portion there is a flange-like blade of bone which is at right angles to the broad outer surface for about two thirds of its length from the bridge The posterior portion is slender and slightly flattened There is no flange like support in this region, except for a few indistinct ridges on its outer surface

At the base of the dorso-lateral plates there is a pair of small foramina for the oculomotor nerves of the eye muscles Behind the foramina and almost from the summit of the bone a pair of short arm-like processes extends postero-laterally They meet on each side the anterior edge of the pro otic, while the dorso-lateral plates of the bone meet the dorso-medial plates of the pro-otics on each side They thus, enclose between them a large concavity which opens anteriorly at the base of the eye There are no such plates in relation with the parasphenoid or pro-otics of *A lineatus*, and consequently no such foramen is formed in that fish The foramen and the plates forming the bridge may perhaps indicate a formation of an opening of the trigemino-facialis chamber or a posterior myodome

Behind the bridge the parasphenoid runs posteriorly over the ventral surface of the pro otics leaving a small space between the pro-otics and the bone itself, and is firmly attached to the basi occipital Anteriorly it ascends upwards to meet the mesethmoid At both these ends there are grooves on the attaching surfaces, giving more firmness to the contact Such grooves are absent in *A lineatus* The parasphenoid in *O melastigma* is, thus, more specialised both in structure and attachments and shows a distinct advance over *A lineatus*

Alisphenoid —The alisphenoid (fig 14) is similar in structure position and attachment to that in *A lineatus*, but extends more mesially and is thicker in build than in the latter At the distal end, too, it is broader and thicker and is almost rectangular in shape Its structure with respect to the laminae and the intervening cartilage is the same as in *A lineatus* From its distal end a band of cartilage extends mesially and meets the distal cartilage of the supra occipital On the ventral side of the distal end it is connected by a ligament to the distal end of the dorso-lateral plate of the parasphenoid The alisphenoid, thus plays better part than in *A lineatus* in affording lateral protection to the brain case and appears to be a step in advance towards the Percoid skull where the alisphenoids are more developed

Circum-orbital series —Just as in *A lineatus*, the circum-orbital bones are represented by only two bones, namely the lachrymal and dermosphenotic

Lachrymal —This bone occupies the same pre-orbital position as in *A lineatus* and possesses a similar tubular structure with other curved plates attached to its distal end In *O melastigma* the tubular portion is narrower and there is an additional curved plate in its proximal portion (fig 15)

Dermosphenotic —This is almost similar to that in *A lineatus* but is not so bent or concave as in the latter Another difference (fig 13) is in the position of the bone In *A lineatus* it is placed anterior to the sphenotic process and forms the posterior boundary of the orbit, whereas in *O melastigma* it is attached to the posterior face of the sphenotic process

The sclerotic bones found in *A lineatus* are absent in this fish

Otic Region

The otic region in *O melastigma* consists of the four usual ossifications, pro-otic, epiotic, pterotic, and sphenotic, just as in *A lineatus*, the fifth opisthotic being characteristically absent in both The otic-bones in *O melastigma* are more complex and specialised than in the latter They are irregular in shape and are either compressed or enlarged in different ways, so as to perform their main function of sheltering the auditory organ The otic capsule is not a complete bony case On the dorsal side a thin cartilaginous membrane remains unossified It extends from the lateral extremity of the supra-occipital to the base of the alisphenoid in front, the sphenotic and pterotic on the side, and the epiotic behind The cartilage is partially covered by the frontal leaving only a small open area on the posterior side Similar cartilage is found in *A lineatus* also, but the anterior margin of this cartilage, which remains distinct from the other cartilage in that fish, is not evident in *O melastigma*

In common with the other replacing bones of the fish, the otic bones also have outer and inner laminae with a thin layer of cartilage in between The laminae are ossified and rolled up in such a way as to give rise to tunnel like bony passages for semi-circular canals Larger spaces are also provided for the ampullae of the canals and the otoliths

Pro otic —The pro otic (fig 14) as usual, forms the floor of the auditory capsule on the anterior side and has the concavities on its floor for the ampullae and the utriculith, just as in *A lineatus*, but, besides these structures, it has developed a vertical plate on its anterior margin The plate originates in the lateral corner and proceeds mesially upwards to meet the dorso-lateral plate of the parasphenoid The dorsal edge of the base of the plate is in contact with the base of the alisphenoid and, as in *Novumbra hubbsi* (Chapman, page 385), forms the postero-mesial wall of the orbit The antero lateral corner of the bone (pro-otic) has a number of foramina for the branches of the facial and the trigeminal nerves

Behind the foramina there is a buttress-like support to the auditory capsule, which arises from the inner surface of the pro-otic There is a similar support in *A lineatus*, but that in *O melastigma* proceeds more dorsally and develops into a dorsal process, just as in *N hubbsi* The dorsal portion of this process curves inwards at the dorsal level and supports from inside the anterior margin of the dorsal cartilaginous membrane of the capsule On the dorso-lateral side the anterior lamina of the process is in contact with the base of the alisphenoid, while the posterior lamina connects with one of the laminae of the sphenotic As the bases of these three bones meet at the anterior lateral corner of the skull, the cartilage between the laminae of their bases fuses together and forms a cartilaginous column to strengthen the region

Epiotic —The bone (fig 13) occupies the normal postero-dorsal position on the skull, but is not so much compressed laterally as to form the epiotic crest as in *A lineatus* The bone is, nevertheless, remarkable for its possession of a long slender process (fig 13) on the outer surface, somewhat like the epiotic process of *Labeo rohita* It is a thin laterally compressed membranous prolongation which extends posteriorly from the outer surface of the bone It is about twice the length of the supra-occipital processes and is more prominent than the latter A process from each epiotic proceeds somewhat mesially and the two processes connect with the neural spines of the vertebral column by fibrous tissue

Pterotic —The pterotic (fig 13) is comparatively a simpler and less complicated bone than in *A lineatus* It has only a single curved wing along its outer margin and does not bear the articulating facet for the hyomandibular on its ventral surface The anterior edge of the wing, however, is slightly thickened at its end and helps in articulation of the hyomandibular The bone is tunneled as usual for the passage of the horizontal semi-circular canal

There is a lightly stained quadrilateral area visible in the postero-lateral corner of the skull just as in *A lineatus*, and, as in the latter, is apparent only because the other surrounding portion is more dense on account of the presence of bony passages for the semi-circular canals within those bones

Sphenotic.—This bone (fig 13) also has a shape different from that in *A lineatus*. It is pushed in the antero-lateral corner of the capsule and takes but a minor part in sheltering the auditory organs Only a portion of the anterior semi-circular canal is lodged in the cavity of the bone without any tunnel being formed The semi-circular canal, therefore, could be pulled out intact without any part of bone being damaged In *A lineatus* the semi-circular canal cannot be removed without cutting a portion of the lamina of the bone which forms a small tunnel inside

A portion of the bone which forms the sphenotic spine in *A lineatus* is flattened antero posteriorly with a slight concavity in front and forms the posterior margin of the orbit The articular facet for the hyomandibular situated on the ventral side is elongated instead of being round as in the other fish Its association with other bones is identical with that in *A lineatus*

The otoliths are almost of the same type as in *A lineatus*, except that they are situated in a straight line, instead of forming an arc just as in the latter

Occipital Region

The occipital region is almost similar in composition to that of *A lineatus* except in details of shape of the bones, etc The foramen magnum is dorsally and laterally bordered by the ex-occipital which has also the occipital condyles for articulation with the vertebral column The articular concavities of the basi-occipital and also the ex occipital condyles are comparatively smaller than in *A lineatus* The latter are somewhat mesially directed and have concave articular surfaces The basi-occipital and the ex-occipital have internal plates running between them to support the region just as in *A lineatus* while the ex-occipital has an additional plate developed for the support of its own curved region

Supra occipital —This bone (fig 13) differs in shape from that in *A lineatus* but the difference is due only to the modifications of structures found in the latter Both the distal and lateral processes which are quite short in *A lineatus*, are drawn out and elongated, and the body of the bone is made slenderer in this fish The lateral prolongations extend antero-laterally towards the sphenotic and compensate to some extent for the absence of the parietals The posterior processes of the bone are shorter and slenderer and do not extend beyond the posterior margin of the skull The lightly stained wedge like area seen in the distal portion of this bone in *A lineatus* is noticeable in this fish, too, in the same position but the cartilage extending from below the area in the anterior direction is distinctly broader and more extensive in *O melastigma* Consequently, the lateral bands of cartilage proceeding towards the alisphenoids also originate away from the distal margin of the supra occipital As stated previously, a sheet of cartilage extends from the lateral prolongation of this bone to join the alisphenoid, pterotic, sphenotic, and epiotic Just as in *A lineatus*, the supra-occipital in this fish, too, is wedged between the posterior parts of the frontals, but unlike in the former there are no parietals on the lateral side of this bone

Basi occipital —Although similar in shape, the basi-occipital in *O melastigma* (fig 14) is relatively larger in extent than in *A lineatus* Its posterior articulating concavity, a cone-shaped centrum, is smaller and more constricted behind The outer surface of the bone has a slight curvature and in the median line there is a groove extending about two-thirds of the distal portion of the bone, in which the posterior end of the parasphenoid is lodged The bone has internal plates running from its inner side to the inner side of the ex-occipital just as in *A lineatus*, but these are narrower and arise from the middle third of the bone They are lengthwise on the basi-occipital, but as they approach the ex-occipitals they twist sideways so as to support the extension of ex-occipitals

Ex-occipitals —These paired bones in *O melastigma* are even more complicated in structure than in *A lineatus* Each ex-occipital (fig 13) has a medial portion antero posterior in direction and a lateral portion. The medial portion forms the dorsal and lateral margin of the foramen magnum and has a flat condyle for the articulation of the skull with the vertebral column The lateral portion starting from the foramen magnum forms a portion of the posterior skull in the antero-lateral direction The epiotic is attached from above to the lateral portion of the bone and shares a space for the ampulla between the horizontal and posterior semicircular canals, with the basi-occipital it shares a concavity for the legenalith

The ex-occipital condyles (fig 14) are flat and broadly oval in shape They are directed slightly mesially on either side of the foramen magnum and meet similar articular surface of the first vertebra The foramina for the glosso-pharyngeal, vagus and occipito-spinal nerves are as usual present on the bone The large foramen on the inner plate between the basi- and ex-occipital of *A lineatus* is absent in this fish The plate being narrow, the pharyngeal and vagus nerves open out directly through the vagus foramen

(2) *Visceral Skeleton*

Considerable similarity exists between the visceral skeleton of this fish and that of *A lineatus*, except for the palato-quadrate arch, which does not contain a metapterygoid, and for the possession of a different type of pharyngeal bones The palato-quadrate arch, compared with that of *A lineatus*, is slender and more elongated, and resembles the typical Poecilid type The suspensorium is much anteriorly disposed and the quadrate extends considerably beyond the anterior margin of the orbit for the articulation of the jaws Other minor differences regarding shapes and sizes also exist to a certain extent and are described below

Hyomandibular —The hyomandibular (fig 17b) is more elongated, but comparatively less complicated, than in *A lineatus* It appears like a narrow plate which twists in the middle, so that the lower half, which is narrower at its lower end, makes a small angle with the upper half Thin, flange like bony plate is also found on the outer surface of the bone Instead of the two condylar heads of *A lineatus* the dorsal edge itself is turned into an elongated condylar ridge which articulates with a corresponding concavity on the ventral surface of the sphenotic There is no condylar head for the operculum, but the latter articulates with the posterior thickened edge of the hyomandibular A foramen for the passage of the hyomandibular nerve is situated in the middle but somewhat on the curved side of the bone

Symplectic —Attached to the lower end of the hyomandibular is a slender symplectic (fig 17b) with a cartilaginous interspace between the two bones Compared with the large and broad symplectic in *A lineatus*, this bone is considerably more elongated and slender It is almost twice as long as hyomandibular and curves upwards in the middle Its posterior half is directed antero ventrally, while the anterior half is directed antero-dorsally The bone thus, forms a small angle which delimits the ventral margin of the orbital cavity The bone, posteriorly is slightly broad and gradually tapers into a point which reaches from inside almost on the middle of the quadrate The metapterygoid, situated above this bone in *A lineatus*, is missing in *O melastigma*

Quadrate —The quadrate (fig 17b) is roughly similar to that in *A lineatus*, but is considerably more elongated just as the symplectic Its posteriorly directed narrow portion is rod-like and almost reaches the angle of the symplectic Its anterior portion is broad and extending upwards, while its antero-dorsal margin is truncated The articular head is double, just as in *A lineatus*

Mesopterygoid —Mesopterygoid (fig 17b) is roughly rectangular, but has its upper margin slightly concave, just as in *A lineatus* It is elongated in shape and has slender curved ridges on its outer surface There being no metapterygoid, the bone is free at its posterior end and is only feebly attached to the dorsal portion of the quadrate from behind and merely touches the posterior margin of the fused palatine by its anterior edge

Palatine—Compared with the palatine of *A lineatus*, the bone in this fish (fig 17b) is considerably smaller and is of a different shape too It also appears reduced when compared with the elongated symplectic and quadrate of the same fish It is attached to the antero dorsal edge of the quadrate through a cartilaginous interspace and after extending along the antero dorsal direction for a short distance ends in a thick articular head which has an anteriorly directed concavity for articulation with the maxilla Unlike in *A lineatus*, the articular head is more anteriorly placed and is not in contact with the lateral ethmoid or the nasal It is connected, however, with the ethmoid region through its contact with a small circular notch on the cartilage on the lateral margin of the mesethmoid

Lower jaw—The lower jaw (fig 15a) consists of the same elements as in *A lineatus*, but their shapes differ The articular is comparatively small in size and roughly triangular in shape with a blunt apex, while at its posterior thickened edge, it bears a large facet for the double condylar head of the quadrate Unlike in *A lineatus*, the bone has no inner and outer portions The inner side of the apex of the articular bone has a distinct sesamoid articular bone This bone is small in size, but compared with the size of articular and the jaw, it is quite well developed The angular (fig 15) is almost of the same size as the sesamoid articular, and occupies the usual position on the jaw

The extremity of the ligament joining the inter-opercle to the angular is also slightly ossified at the point where it meets the latter The dentary which is more prominent and well developed than in *A lineatus*, does not narrow down towards the median line, but forms a uniform transverse bar which approaches its fellow (ramus) in the median line The two rami are much further apart in the median line than in *A lineatus* and are connected by a short ligament Posteriorly, each dentary broadens out dorso-ventrally and extending up to the articulating surface of the articular ends in a postero-dorsally directed broad lobe-like process The latter is somewhat narrow in young specimens In a few old specimens the anterior margin of the broad process is slightly dentate

The transverse bar of the dentary has a double row of conical teeth The outer row has large teeth, which are slightly curved inwards at their apices The teeth on the inner row are small and erect In old specimens the teeth on the outer row are not finely pointed A few smaller teeth are also found at the base of the second row in some specimens They are particularly more numerous at the corner of the jaw (fig 16b)

Male specimens are remarkable for their possession of additional special teeth (fig 16b) on the anterior face of the dentary at the corner of the jaw They, too, are conical, but thicker and larger than those on the jaw and are closely set in a vertical row Their number varies from three to seven They are anteriorly directed and curved slightly downwards The female is devoid of any such special teeth on the dentary surface, which is plain and smooth

Meckel's cartilage is unossified for a short distance in front of the articular, but at its extremity it is slightly ossified on its outer surface Close to the point of this ossification there is a thin dermal ossification adhering to the corner of the dentary from inside The ossification probably represents the splenial or the infradentary which is not found in *A lineatus*

Upper jaw—The upper jaw is formed entirely by the pre-maxillaries, the maxillaries being behind and guiding the movements of the former The shape of the pre-maxillaries (fig 16a) is roughly similar to that in *A lineatus* except for the additional teeth which vary accordingly to sex The pre-maxillaries are thicker and shorter in structure than in *A lineatus*, and have a transverse portion which forms the actual transverse part of the jaw and a ventrally directed descending portion which bounds the jaw on the sides The descending portion does not terminate in the spine as in *A lineatus*, but has a comparatively broader and shorter ventral extremity The posterior triangular process of the transverse portion of the jaw near its mid-dorsal region is shorter and more rounded at its distal end than the prominent triangular process in *A lineatus* In some specimens the process is rather small

The teeth on the transverse portion are similar to those on the lower jaw, but the dentition varies on the descending portion In males the descending portion bears a row of three to five thick and large antero-laterally directed conical teeth They are similar in build to those found in males on the anterior face of the dentary and are slightly curved downwards Some forms which have only three such teeth, have them equally developed while in others which have more teeth, the uppermost tooth is the smallest and the others have progressively larger ventrally Females have a single large antero-laterally directed tooth situated somewhat above the ventral extremity of the bone In a few specimens even this tooth is absent

Sundara Raj (1916, plate XXV, fig 3) has figured a female pre-maxilla ending in a bifid tooth, which appears in fact, like two teeth In some specimens examined during the present investigation there was only one large antero-laterally directed tooth on the anterior face of the ventrally directed extremity of the bone, but in other specimens even this tooth was missing, its place being taken by a short crenate margin The absence of the tooth may be accounted for by its having been accidentally dislodged The male pre-maxilla also does not terminate in four to six teeth as shown by Sundara Raj The teeth are borne on the anterior face of the descending portion of the arm, leaving the extremity separate as in fig 16c

Opercular bones —The operculum (fig 17a) is not triangular and so much pointed as in *A lineatus* The ventral extremity being rather broad, it appears quadrilateral in shape Its articular surface is wider and glides over an undefined thickened posterior edge of the hyomandibular

The sub operculum (fig 17a) is similar in shape and attachment to that in *A lineatus*, but the former is more thickened and larger than in the latter The ventral margin of the opercle is closely attached to, and extends over, the dorsal notched portion of the bone, so that the two bones appear like one piece The sub-opercle extends almost up to the dorsal margin of the opercle from behind and forms the entire posterior and postero ventral margin of the opercular apparatus

The inter-opercle (fig 17a) is somewhat differently shaped from that in *A lineatus* The posterior one-third is broad, the remaining part being narrow, and elongated and slightly pointed The bone runs almost ventral to the palato-quadrate arch, its anterior third being attached to the ventral edge of the quadrate The distal extremity is connected with the angular through a short ligament

The pre-operculum (fig 17a) has the familiar vertical channeled portion but the horizontal portion has the channel branched ventrally The whole bone is smaller and different in shape compared with the pre-operculum of *A lineatus* At the dorsal end, it is attached to the hyomandibular behind the outer flange on the bone and possesses small openings in the channeled portion, suggesting its origin as a sensory canal bone The thin membrane in the inner angle of the two arms is attached by its dorsal margin to the ventral edge of the symplectic Unlike in *A lineatus*, this horizontal arm of the bone is not elongated and extends only up to two-thirds of the symplectic

Hyoid cornu —The inter-hyal of the hyoid cornu is not present in this fish and the epi-hyal is directly attached to the cartilaginous interspace between the hyomandibular and symplectic The epi-hyal and the cerato-hyal (fig 17c) are almost of the same shape as in *A lineatus*, except that the distal part of the latter is not bifurcated and contains large amount of cartilage The bony ridge continuous on the dorsal margins of both the bones in *A lineatus* is present in this fish also The hypo hyals are absent The basi-hyal is relatively extensive Its narrow posterior half is ossified and the wider anterior half remains cartilaginous

The branchiostegals (fig 17c) are five in number, the posterior two are more curved and flattened than the rest They are not excessively thickened and except in the posterior first do not possess the upper ridge like thickening at the attaching surface found in *A lineatus* The posterior four rays are attached as usual to the outer surface of the epi-hyal and the cerato-hyal, but the fifth instead of being attached to the inner surface of the cerato hyal as in most fishes (Hubbs, 1921) is attached to the outer surface near the base of the previous ray At times, the fifth ray is attached to the anterior edge of the broad portion of the cerato-hyal In one specimen a rudiment of a sixth branchiostegal was also found but it was only on one side It was thin and slender, and lay in the same level and in the same direction as the other rays The slightly flattened anterior extremity was, however, considerably away from the attaching surface of the cerato-hyal In another specimen the fourth and the fifth rays were almost fused together

The attachment of the fifth branchiostegal ray to the outer surface of the cerato-hyal in *O melastigma* is rather exceptional in Teleosts according to Hubbs (1921), but in a dozen specimens examined during the present investigation its character was quite constant, except for the slight variation mentioned above

Branchial skeleton —There are four functional branchial arches having gill filaments on their cerato-branchials and the usual hypo and basi-branchials forming the floor of the branchial apparatus The fifth branchial arch is reduced and modified into teeth-bearing, inferior pharyngeal bones The fourth basi-branchial is reduced almost to a cartilaginous vestige Just as in *A lineatus*, the last gill raker of the first arch is excessively thickened and enlarged and appears like a first epi-branchial, which is undeveloped in this fish The second epi-branchial is represented by a minute nodule of bone, the third being undeveloped The fourth (fig 18a) is remarkably well developed Its proximal portion is rod like, the distal region being bent and flattened into a thin plate

The pharyngo-branchial elements on each side fuse into a single triangular pharyngeal bone (fig 15b) with its apex directed posteriorly On the ventral surface of the bone there are 12 to 14 transverse rows of fine acicular teeth (fig 15b) which project into the throat Each individual row is again made up of a double row of closely set teeth The floor of the bone, instead of being of a compact structure, is made up of a spongy network of bony lamellae on which the teeth are borne The bone is concave on its dorsal surface and there is a narrow bridge over the concavity joining the longer sides of the bone The whole structure, therefore, exactly resembles a bath brush, with the teeth representing the bristles, the bony network, the surface on which the bristles are fixed and the narrow bridge connecting the sides of the bone, the strap at the back of the bath brush

By the side of the apex of the pharyngeal bone there is another bony structure (fig 15b) with a row of minute teeth on its ventral edge and a thick, short shaft directed vertically upwards The shaft has a slight cartilage at its tip and articulates with the cranium above The bone may represent either the supra-pharyngeal element present in some Haplomous fishes or the first pharyngo branchial which, instead of fusing with other similar segments, remains separately ossified and has its articular extremity turned upwards

The pharyngeal bones on each side lie side by side in the roof of the pharynx and their teeth work on similar teeth of the inferior pharyngeal bones on the floor of the pharynx The latter bones are derived from the fifth branchial arches which are completely modified to give rise to these teeth-bearing bones These structures are identical in all respects with the pharyngo-branchial bones on the upper side

The gill rakers (fig 18b) are found, as in *A lineatus*, in a double row on each cerato-branchial of the four gill arches The rakers on the outer row of the first cerato-branchial are somewhat similar to those in *A lineatus* They are, however, pointed much more distally than in the latter and do not possess the minute teeth on their inner surfaces They are somewhat awl shaped in appearance Those in the middle of the arch are the longest becoming shorter at either extremity The inner row, also, has similar gill rakers, but of a shorter type, which is found on both the inner and outer rows of the following three cerato branchials

The median uro-hyal (fig 18c) on the ventral side is a peculiarly shaped structure Its anterior portion, which is close to the distal extremities of the cerato-hyal, is small and triangular with its base in front and its apex behind The apex is produced posteriorly and divides into thin, lateral membranes, which extending upwards, fuse in the median line After extending a short distance, the membranes end in five short processes At the apex of the anterior triangular portion, there is a dorsally directed spur at right angle to the length of the bone Anteriorly, the uro hyal is attached to the basi-hyal and cerato hyal, and posteriorly to the pectoral arch, just as in *A lineatus*

The post-temporal (fig 18d) is comparatively much reduced in size and appears like a narrow splint of bone It is more firmly attached, however, to the epiotic and the cleithrum than in *A lineatus*

HEAD SKELETON OF *Horaichthys setnai* KULKARNI

The head skeleton of *Horaichthys setnai* (fig 19) which is hardly 3 to 4 mm in length, differs slightly according to sex, the female having a larger head than the male The length of the skull of an average full grown female specimen measures about 3 8 mm with the jaw, and that of males 3 3 mm The breadth is 2 5 mm and 2 2 mm and the inter-orbital space 9 mm and 6 mm respectively The dorsal surface of the skull and its tapering nature are almost identical in these particulars with *O melastigma* A slight crest formed by the frontals in the inter-orbital area is more prominent, however, in *H setnai* The dorsal profile is straight, but the ventral one at the lower jaw rises up suddenly to reach the dorsal level The proportion of the depth and breadth of the skull to its length is also the same as in *O melastigma*

(1) *Skull* (Figs 19 and 20)

The skull proper is limited anteriorly by a prominent mesethmoid owing to the absence of the vomer The posterior process of the supra-occipital is even much smaller than in *O melastigma* The frontals cover a large part of the skull Most of the other bones are feebly ossified and particularly the bones of the otic region are so very thin and feebly ossified that they appear almost transparent The parietals are absent A small cartilaginous area exists behind the posterior edge of the frontals, but the lightly stained area found both in *A lineatus* and *O melastigma* is not very apparent on account of the absence of thicker parts The postero-lateral corners of the skull are flattened, but the margin of the skull of either side of the foramen magnum is not so depressed as to give rise to the epiotic crest

The ventral surface of the skull represented by the occipital and otic regions is flat as in *A lineatus*, but the anterior region resembles that of *O melastigma* owing to the slender, ascending parasphenoid meeting the mesethmoid On the posterior margin the articular concavity of the basi-occipital and the ex-occipital condyles are present

As in the other two cyprinodonts, namely, *A lineatus* and *O melastigma*, replacing cartilage bones in *H setnai* are also separated by a thin layer of cartilage, and each bone, however complicated in shape, has a thin layer of cartilage between its upper and lower laminae

Ethmoid Region

This region is almost similar to that in *O melastigma* except for the better developed mesethmoid There is the bar-like cartilaginous ethmoid to which other bones are attached A remarkable feature of the mesethmoid in *H setnai* (fig 19) is that the development of the bone differs according to sex, being quite pronounced in males as compared with the size of the fish The bone takes the form of a transversely extending median plate in front of the ethmoid cartilage The position of the nasals, lateral ethmoids, lachrymals and frontals is the same as in *O melastigma* but none of these bones overlap the mesethmoid Even the frontals, which Starks considered (1926) as almost invariably covering at least some portion of the mesethmoid by their anterior margins, are quite behind the bone in this fish

The antero lateral corners of the mesethmoid are circular in outline and have thickened edges covered by a granulated surface The median anterior margin is concave, but not so thick as at the corners The posterior side is slightly truncate, but is also rounded in some specimens Where the margin is truncated the postero lateral corners of the bone have small notches The dorsal surface has a low concavity bounded by the antero lateral thickened edge The ventral surface has a very low median crest, on the posterior region of which there is a slight groove in which the anterior extremity of the parasphenoid is lodged (fig 22)

The mesethmoid has upper and lower laminae with cartilage between, as in *O melastigma* As in the latter the laminae are more apart in the posterior region and the intervening cartilage is continuous with the ethmoid cartilage The laminae fuse on the anterior and lateral margin except for a few small gaps on the antero lateral corner

In the females the mesethmoid (fig 19) is less developed and its osseous portion is smaller in size than in the males The thickenings of the antero lateral margins and the concavities on the dorsal and ventral surfaces are also absent The upper and the lower laminae meet in only the middle of the anterior margin, and have wide cartilaginous border all round as an extension of the inner cartilage between the laminae

The lateral ethmoids are simple in structure Each bone is an antero-posteriorly flattened vertically situated plate at either end of the ethmoid cartilage It is made up of two laminae with cartilage between It comes in contact with the frontal and nasal on its dorsal side, and with the palatine and lachrymal by the mesial and outer margin of its anterior face respectively The olfactory area is bounded by the lateral ethmoid, mesethmoid and lachrymal and roofed over by the nasal The olfactory nerve passes into the area through the cartilage between the lateral ethmoid and the mesethmoid

The nasals (fig 19) are semi-lunate in shape and, being attached to the lateral ethmoids and the frontals behind, reach the articular area of the pre-maxillaries by their apices

Orbito-temporal Region

The frontals (fig 19) cover, as usual, the entire dorsal surface of the region The triradiate ridge on the ventral surface of the bone, which was clearly visible on even the dorsal side both in *A lineatus* and *O melastigma*, is missing in this fish The bone is, nevertheless, divided into three regions, viz supra orbital, inter orbital and sphenoidal, just as in the other two cyprinodonts, the divisions being apparent from the respective curvatures, which in *H setnai* are particularly more prominent on the surface of the bone Another difference in *H setnai* is the presence of small pores of the sensory canal system on the bone These are not aparent in the other two forms Further their posterior portions of the frontals do not diverge so much from each other as in *O melastigma* or *A lineatus* Consequently the supra-occipital does not seem to be much embedded between them, but its distal portion remains merely covered by the latter

The parasphenoid (fig 20) is slender and bent, as in *O melastigma*, but is shorter than in the latter It has a narrow anterior portion forming about two thirds of the bone, a broader middle portion and a short proximal flat portion The anterior portion is similar to that in *O melastigma* in all respects The broader portion has two large foramina on either side of the median line The entire bone is bent in the middle of this rounded portion, so that the posterior half of the broader portion is horizontal and the anterior portion is inclined in front The foramina are situated on the inclined portion and face anteriorly, so as to resemble the openings of the trigemino facialis chamber or a rudimentary myodome found, but differently shaped, in *O melastigma* The proximal portion is short, slightly thickened in the median line and hardly reaches the extremity of the basi occipital It is attached only to the mesial margins of the two pro-otics

The alisphenoid in this fish (fig 20) is slightly different in shape, being somewhat more dorsally situated than in the other two cyprinodonts and roofing the dorso-lateral corner of the brain case behind the orbit The base is moderately broad and pyramidal in shape, but its anterior part for two thirds of its extent, instead of being flattened as in *O melastigma* is narrow and slender Being more mesially directed, the alisphenoids approach the distal end of the supra-occipital closer than in the other cyprinodonts and connect with the bone by a strand of cartilage The distal end of the alisphenoid is also connected by thin membranes with the broader portion of the parasphenoid on the ventral side and also with the ethmoid region in the anterior direction

The lachrymal does not possess the tubular structure found in other fishes It is a very thin mesially bent lamina to which another curved lamina is attached at its upper half The bone is perforated by the sensory canal system

The dermosphenotic also is a very thin, slightly bent, scale like bone, with a distinct perforation in the middle, and is attached posteriorly to the sphenotic process as in *O melastigma*

Otic Region

The otic region has the usual four ossifications as in *O melastigma* The region is much less complicated than in even *A lineatus* The tunnel like passages giving rise to the bony labyrinth are not well developed, while the cartilaginous areas on the capsule are present as in *O melastigma*

The pro otic is a simple bone and has a single but large foramen instead of two as in *A lineatus* The buttress like support to the wall of the auditory capsule and its connections with other bones resemble similar features in *A lineatus* On the sutural margin between each pro-otic and the rounded portion of the parasphenoid in the antero mesial direction, there is an opening left on either side of the para sphenoid and the two openings appear like another pair of foramina behind the previously described pair on the parasphenoid

The epiotic is comparatively small and more dorsally situated than in the other cyprinodonts The bony passage on the inner side is not completely developed In some specimens a very small process arises from the dorsal surfaces of the bone A ligament starting from this bone proceeds in the same direction and is attached to the neural spines in the same way as the epiotic process of *O melastigma* This shows probably that the small process in *H setnai* may be an occasional ossification of the base of the ligament

The pterotic does not possess the lateral wings on its margin nor the articular facet for the hyomandibular epiphysis It accommodates in its cavity only the horizontal semi-circular canal The sphenotic is similar to that in *A lineatus* The articular head of the hyomandibular is attached to the cartilage between the margins of the sphenotic, pterotic and pro-otic

Occipital Region

The occipital region is quite extensive as in *A lineatus* The supra-occipital (fig 19) is almost similar in shape and position to that in *A lineatus* except on the posterior margin, where it develops on its upper surface three low crests which terminate in short processes The middle of the three crests is median in position, its process, which is a single flat membrane, arises from the dorsal surface of the bone as does the posterior process of *A lineatus* The other two crests have their processes similar but shorter than that of the median one The lateral projections of the bone have a thin cartilaginous membrane connecting the base of alisphenoid, sphenotic, pterotic and epiotic, as in *O melastigma* At its distal end, the supra occipital has the typical wedge-shaped area as in the other two cyprinodonts

The basi-occipital (fig 20) is identical with that in *A lineatus* in most respects The contact of the internal supporting plates with this bone is not so firm as in *A lineatus* or *O melastigma* and shows that the plates belong to the ex-occipital element The parasphenoid is not attached to the outer surface of the bone The ex-occipitals are more extensive than in the other two cyprinodonts and extend more on the ventral side They have internal supporting plates between them and the basi-occipital In addition to these plates each ex-occipital has another supporting bridge as in *O melastigma* The ex-occipitals behave just as in the other two cyprinodonts in respect of the formation of the occipital condyles and the foramen magnum, as also in other details

(2) *Visceral Skeleton*

The visceral skeleton of *H setnai* is almost identical with that of *O melastigma* except for the pharyngeal bones, which resemble those of *A lineatus* The hyomandibular arch is of the Poeciliid type (fig 21b) and the suspensorium (fig 21b) is anteriorly disposed as in *O melastigma*, the metapterygoid is missing *H setnai* is remarkable in that it does not have the maxilla developed in the pre-orbital region The pre-maxillaries forming the upper jaw (fig 21a) articulate directly with the palatine

The hyomandibular (fig 21b) is strikingly similar to that in *O melastigma*, but is relatively shorter The articular head is elongated and, as stated previously, articulates with the cartilage between the pterotic, sphenotic and pro otic on the ventral surface of the otic capsule The condylar head of the opercle is rounded and prominent, while the flange on the outer surface is present on only the lower half The foramen for the hyomandibular nerve pierces the lower half of the bone from behind downwards, but the foramen being between the upper and lower laminae of the bone is not visible laterally

The symplectic and quadrate (fig 21b) are also shorter in length and thicker than in *O melastigma* The mesopterygoid is missing The palatine (fig 21b) is almost identical with that in *O melastigma*, except that it articulates directly with the pre-maxillary and at its distal articular head there is a small postero mesially directed nodule which articulates with the lateral cartilaginous margin of the mesethmoid

The dentary of the lower jaw (fig 21a) is well developed and constitutes almost the entire jaw The transverse portion of the jaw bears only a single row of small, conical teeth, the special teeth on the anterior face of the dentary in the male of *O melastigma* being not developed in *H setnai*

The articular (fig 21b) is elongated and adheres to the dentary from inside The angular (fig 21a) is comparatively large Meckel's cartilage persists as an inner core of the dentary and extends right up to the extremity of the dentary on the median line

The upper jaw (fig 21a) is formed by the pre-maxillaries only The transverse portion of the jaw is slender and bears a single row of conical teeth The descending portion is flat and thick and bears stout thickened teeth as in the male of *O melastigma*, but more numerous than the latter There is no difference in the dentition of either sex

The operculum and sub-operculum (fig 21a) closely adhere as in *O melastigma*, and the two together form a thin quadrilateral plate The operculum possesses, however, a short process above the articular head as in *A lineatus* and the articular cavity, too, is deep and well defined The inter-opercular is somewhat similar to that in *O melastigma* but its anterior slender portion is more elongated and approaches the articular surface of the lower jaw

The pre-operculum differs slightly from that in *O melastigma* It has the same general outline, but does not possess the characteristic channeled appearance The upper half of the vertical portion of the bone has a somewhat ill-defined canal and possesses the perforations of the sensory canal system The ventral and the anteriorly directed portion is thin and smooth The attachment of the bone behind the upper flange of the hyomandibular is present as usual, but, in addition, the inner margin of the bone is firmly in contact with the cartilage of the lower end of the hyomandibular and is difficult to dislodge

The hyoid cornu consists of the epi hyal and cerato hyal as in *O melastigma*, but the dorsal ridge on these bones is much more thickened and enlarged The articular facet is also wider than in the former The hypo-hyals are absent and basi-hyals are present in the normal manner

The branchiostegal rays (fig 23a) are only four in number and are shaped just as in *O melastigma* The posterior three of them are attached to the outer side of the epi-phyal and cerato-hyal, but the anteriormost is not attached to any bony element and remains free in the dermal tissue on the outer side of the cerato-hyal and on the same level as the other rays

The lower pharyngeal bone resembles similar bone in *A lineatus*, but with fewer and more erect teeth The upper pharyngeals also possess thickened and erect teeth as in the latter, but are disposed on two pharyngeal bones as in *O melastigma* The anterior pharyngeal is comparatively small and has unlike in *O melastigma*, a laterally direct shaft and represents the first pharyngo-branchial element The other pharyngo-branchials fuse together to form the main enlarged upper pharyngeal bone, which bears most of the thickened teeth

There are only two epi branchials to support the dentigerous pharyngeal plates The first is very small and probably represents the first epi-branchial element and the second is enlarged structure and may represent the 2nd and 3rd and 4th epi-branchial elements It is slender proximally and broader in its distal half, where it bears a cartilaginous margin for attachment to the major pharyngeal bone The general plan of the development of these structures is, however, quite similar to that form in *O melastigma*, there being two pharyngo branchials and two epi-branchials, one large and one small in each case

The post temporal is small and rod like as in *O melastigma*, but at the point of its contact with the cleithral spine there is a short process at the extremity of the bone which works like a hook and strengthens its grip on the spine (fig 19)

HEAD SKELETON OF *Mollienisia*

The head skeleton is flat dorsally as in the Indian cyprinodonts, but considerably tapers in the anterior direction The jaws are small and their suspensorium is anteriorly disposed The lateral ethmoids extend more medially than in the Indian cyprinodonts, but do not meet in the median line The mesethmoid is much less developed than in the male of *H setnai*, but resembles that of *O melastigma* or the female *H setnai* Vomer is present with a few teeth on it The frontals are extensive and divided into three regions as in the Indian forms Parietals, orbito-sphenoids and basi-sphenoid are absent

Alisphenoids are somewhat larger than in *O melastigma* Each has a mesially directed plate, which meets the lateral plate of the parasphenoid The anteriorly directed plate of the pro otic also meets the alisphenoid to form a larger foramen opening anteriorly The parasphenoid is somewhat similar to that in *O melastigma*, but differs from it in the length of its lateral plates extending sufficiently to meet the mesially directed plates of the alisphenoids The parasphenoid is firmly attached to the basi-occipital but while it passes over the pro-otics, it leaves a narrow space between the anterior ventral surface of the pro-otic and the bone itself The space indicates the formation of a myodome in these fishes The pro-otics have all the plates and processes found in *O melastigma* The supra- and basi-occipitals have the same structure as in the Indian cyprinodonts The ex-occipitals being comparatively small, the supra-occipital unlike the Indian cyprinodonts takes part in the formation of the foramen magnum The internal supporting plates of the basi-occipital support the ex occipitals, but the latter have no condyles for articulation with the vertebral column

The structure of the hyomandibular, symplectic and quadrate is as found in *O melastigma*, but the autopalatine is rather complicated It has a thick articular head at its dorsal extremity and a concavity on the mesial side, both for articulation with the curved maxilla On the lateral margin of the bone there is another thickened head for articulation with the lateral ethmoid

The pre-maxilla is very small as compared with the bone in the Indian cyprinodonts It has no posterior processes on its transverse portion The teeth are in two rows, those on the outer row being slender, elongated and curved inwards, and those on the inner row being short, erect and situated at the basis of the former The dentary is also small and has similar type of teeth The maxilla is extremely curved and twisted The hyoid cornu is normal as in the other cyprinodonts Branchiostegals are five in number and are attached in the same manner as in *O melastigma* The gill rakers and the pharyngeal bones and their teeth are strikingly similar to those in *O melastigma*

The post-temporal bone is quite different, however, from that in any Indian cyprinodont It is straight and slender in the anterior portion but is forked in the posterior region

COMPARATIVE SUMMARY

The salient features of the head skeleton of *A lineatus*, *O melastigma* and *H setnai* described above may be summarised as follows —

Chondrification of the skull is incomplete, brain case being inadequately protected by bones Small cartilaginous areas remain unossified at places Cartilaginous interspaces are visible between the margins of replacing bones, and also between their outer and inner laminae

Most of the ethmoid region in all the three genera is unossified except the lateral-ethmoid area

Orbito sphenoid and basi sphnoid are universally absent

Large and extensive frontals are developed in all the three genera

Mesethmoid (or ethmoid of other authors) is cartilaginous and detached from the ethmoid cartilage in *A lineatus* In *O melastigma*, however, it is an ossification of the anterior part of the median ethmoid mass of cartilage and consists of double laminae with cartilage in between The bone in *H setnai* has the same origin and structure as in *O melastigma*, but is further specialised to manifest sexual dimorphism both in shape and size, and it articulates with the palatine (autopalatine)

Alisphenoids are comparatively small in *A lineatus* and slightly more enlarged in the other two genera

Supra-occipital does not take part in the formation of the foramen magnum in the three genera

Ex-occipitals, in view of their share in the formation of the otic capsule, in place of opisthotic, become auto-occipitals in all the three genera

Basi-occipitals and ex-occipitals afford condylar surfaces for articulation of the skull with the first vertebra

Small parietals are present in *A lineatus*, but are absent in the other two genera

A distinct vomer with rows of teeth on it is present in *A lineatus*, but the bone is absent in the other two genera

Opisthotic is absent in all the three genera

There are no epiotic processes in *A lineatus* and *H setnai* whereas they are present in *O melastigma*

The pro otic is comparatively more developed and complicated in *O melastigma* and *H setnai*

An elongated parasphenoid is attached ventrally to the vomer in *A lineatus* and to mesethmoid in the other two genera in which it is more complicated

A lineatus has no myodome, but a rudimentary concavity of this type is observable in the other two genera

The lachrymal and dermosphenotic, the only bones of the circum-orbital series, are present in all three genera Dermosphenotic is attached, however, to the posterior face of the sphenotic process in both *O melastigma* and *H setnai*

Metapterygoid bone is present in *A lineatus*, but is undeveloped in the other two genera

Mesopterygoid element is present in *A lineatus* and *O melastigma*, but absent in *H setnai*

In *A lineatus* the palatine (or autopalatine) of the palato-quadrate arch has a double condylar head for articulation with the ethmoid and the maxilla, whereas it is more anteriorly disposed in *O melastigma*, articulating with only the maxilla on each side In *H setnai*, however, it articulates directly with the pre-maxillaries and partially with the mesethmoid The suspensorium of the jaws is, thus, progressively disposed in an anterior direction

Maxillae are present in *A lineatus* and *O melastigma*, but not in *H setnai*

The pre-maxillaries are well developed in all the three genera, but have, in addition, posterior processes developed in *A lineatus* and *O melastigma*, in which also there is sexual dimorphism in the dentition on these bones

Hyomandibular in *A lineatus* has two articular heads for articulation with the skull, whereas the other two genera have a single elongated ridge

Sclerotic bones are present in *A lineatus*, but not in the other two genera

A wedge shaped cartilaginous area at the extremity of the supra-occipital connecting it to other bones by cartilaginous strands occurs in all the three genera

Symplectic is well developed in *A lineatus*, but only less developed in the other two genera

Dentary, articular and angular occur in all the genera, but are well developed in only *A lineatus* Dentition on dentary shows sexual dimorphism in *O melastigma*

There is a distinct condylar head on the hyomandibular for articulation of the operculum in *A lineatus* and *H setnai* Such a head is absent in *O melastigma*, the operculum hinging on the thickened edge of the hyomandibular

Hypo-hyals are separate in *A lineatus*, but fused with the cerato-hyals in the other two genera

In *A lineatus* the last two branchiostegal rays, viz fifth and sixth are attached to the cerato-hyal from inside, as is found in the higher groups, such as Perciformes, etc The attachment of the last branchiostegal rays is, however, exceptional in the other two genera, the last rays being attached to the outer surface of the cerato-hyals

Fourth basi-branchial is reduced in size and cartilaginous in *A lineatus* and *O melastigma*, but absent in *H setnai*

The structure of the lower and upper pharyngeal bones is similar in *A lineatus* and *H setnai*, but different in *O melastigma*

In *A lineatus* there are three pharyngeal bones (four pharyngo-branchials) and four epi-branchs In *O melastigma* and *H setnai* there are two pharyngeal bones and two epi-branchs, the first pharyngo-branch and epi-branch representing the bones of the first arch, and the second pharyngeal bone and second epi-branch representing fused bones of respective elements of second, third, and fourth arches

A strengthening ridge between the epi-hyal and carato-hyal, characteristic of some higher groups, is found in all the three genera

Gill rakers are of two different shapes on the first cerato-branchials in *A lineatus* Those in *O melastigma* and *H setnai* bear a greater resemblance to each other than those in *A lineatus*

Post-temporal is unforked in all the three genera

This comparison of the skeletal structures reveals that *O melastigma* is more closely related to *H setnai* than to *A lineatus* *H setnai*, appears, however, to have further progressed and shows structural similarities to *Gambusia* and *Mollienisia* (Poeciliidae) under the following heads —

(1) Suspensorium of the jaws disposed considerably in an anterior direction,
(2) Pre-maxillaries having no posterior processes,
(3) Presence of a typical mesethmoid,
(4) Absence of metapterygoid,
(5) Presence of rudimentary myodome,
(6) Absence of basi-sphenoid and orbito-sphenoid,
(7) Development of typical gonopodium and its suspensorium (Kulkarni, 1940)

CONCLUDING REMARKS

Detailed study of the osteology of *A lineatus* emphasises the need for a revision of the present distinguishing characters of the sub order Poeciloidea, of the order Microcyprini to which the cyprinodonts belong The skeletal features so far considered to be characteristic of the aforementioned sub-order are not universally applicable to all representatives of this group Starks (1904*a*) listed the presence of a circular, scale-like ethmoid (mesethmoid) and the absence of a metapterygoid in the palato-quadrate arch as the first two diagnostic features of the super-family Poecilioidea Regan (1911), revising the classification of the order Microcyprini (Cyprinodonts), considered the absence of the metapterygoid as one of the distinguishing features of the sub-order Poecilioidea Myers, too, in 1931 accepted this view

The present investigation reveals, however, that the absence of the metapterygoid cannot be regarded as a distinguishing character of the group, since the metapterygoid is invariably present in *A lineatus*, *A blochii* (*A parvus* of Sundara Raj) and *A panchax*, all of which are members of the family Cyprinodontidae of the sub-order Poecilioidea The metapterygoid is absent, however, in *O melastigma* and *H setnai*

Further, the structure of the mesethmoid in the three species of *Aplocheilus* examined, is peculiar In these forms the element is not a thin and scale-like bone but an independent piece of cartilage The typical mesethmoid characteristic of other Cyprinodonts is, however, present in *O melastigma* and *H setnai* This indicates that the genus *Aplocheilus* may be somewhat differently organised in the Cyprinodont group, both in possession of a distinct metapterygoid piece and in having a peculiar cartilaginous mesethmoid

Regan (1911) has characterised the post-temporal of the order Microcyprini as being forked, but this bone in all the three Indian genera (sub-order Poeciloidea) is not forked

A comparison of the various skeletal features of the three Indian Cyprinodonts studied reveals greater affinity between *H setnai* and *O melastigma* than between *A lineatus* and *O melastigma* and finally substantiates the view held previously by the author that *H setnai* must have evolved directly from *O melastigma* Moreover, the slight sexual dimorphism apparent in *O melastigma* in respect of the modification of a few anal fin rays of the male is further accentuated in *H setnai* (Kulkarni, 1940) so as to give rise to a complicated gonopodium—a feature that necessitated the inclusion of the latter in a separate family Horaichthyidae In view of this, *H. setnai* marks a definitely higher stage of specialization

The skeletal features of the Indian forms *H setnai* and *O. melastigma* have also disclosed a striking similarity with those of the American viviparous Poeciliids, *Mollienisia* and *Gambusia* suggesting convergent affinities in forms, geographically distributed so widely apart

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EXPLANATION OF ABBREVIATIONS

a lat eth	anterior process of lateral ethmoid
al sph	alisphenoid
ant lat eth	anterior process of lateral ethmoid
ant t	anterior teeth
ang	angular
ant th	anterior thickening
art	articular
art bas oc	articular surface of basi occipital
bas con	basi occipital condyle
bas oc	basi occipital
b br	basi branchial
b hl	basi hyal
br st 1–6	branchiostegal rays 1–6
b s	buttress like support
b s r	branchiostegal ray
cart a	cartilaginous area
cer br 3	cerato branchial (third)
cer br. 1	cerato branchial (first)
cer hl	cerato hyal
clt	cleithrum
den	dentary
der sph	dermosphenotic
ep	epiotic
ep br 1–4	epi branchials 1 to 4
ep hl	epi hyal
ep pr	epiotic process
eth ctl	ethmoidal cartilage
ex con	ex occipital condyle
ex oc	ex occipital
fr	frontal
fr mag	foramen magnum
g fil	gill filament
hym	hyomandibular
hyp pl	hypo hyal
in gl r	inner gill raker
int hl	inter hyal
in op	inter opercular
i phr b	inferior pharyngeal bone
lat eth	lateral ethmoid
lc	lachrymal
Mec	Meckel's cartilage
mes	mesethmoid
ms pt	mesopterygoid
mt pt	metapterygoid
mx	maxilla
na	nasal
oc con	occipital condyle
o gl r	outer gill raker
par sph	parasphenoid
phr b 1-2	pharyngeal bones 1 to 2
phr br	pharyngo branchial
pal	(auto) palatine
phr t	pharyngeal teeth

p max	pre maxilla
p max pr	posterior pre-maxillary process
p pro	pro otic process
pro	pro otic
pr op	pre-opercular.
pr ot.	pro otic.
pr par sph	parasphenoid process
pt clt	post-cleithrum
pto	pterotic
pt temp	post temporal
pt tp	post temporal
qu	quadrate
sc pl	sclerotic plate
ses	sesamoid articular
sm c	part of semi-circular canal
s op	sub operculum
sph	sphenotic
sph pr	sphenotic process
sub. op	sub-operculum.
sup oc	supra occipital bone
sup pr	supra occipital process
sy	symplectic
tr lat eth	transverse process of lateral ethmoid
ur hl	uro hyal
vo	vomer

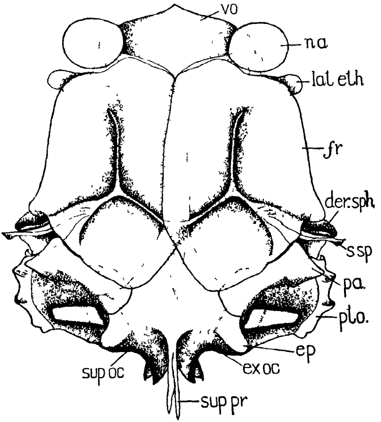

Fig 1 —The dorsal view of the skull of *Aplocheilus lineatus* (C & V.) ×10

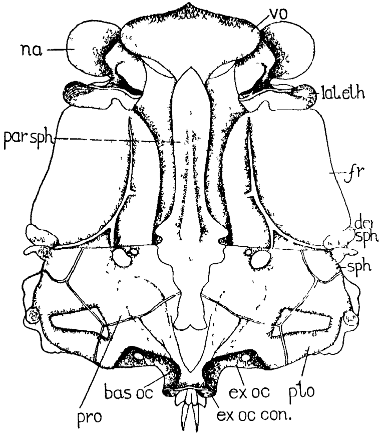

FIG 2 —The ventral aspect of the skull of *Aplocheilus lineatus* (C & V) × 10

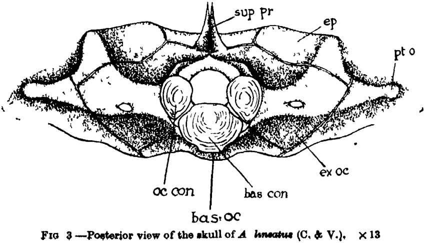

FIG 3 —Posterior view of the skull of *A lineatus* (C. & V.). × 13

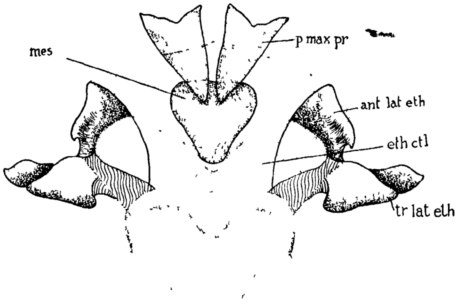

FIG 4 —Ethmoid region of *A lineatus* (C & V) × 13

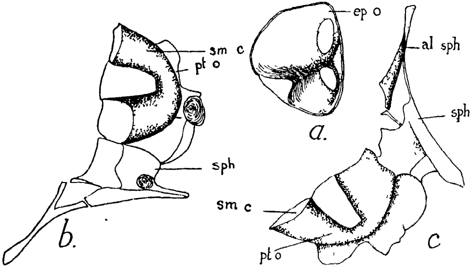

FIG 5 —The bones of the auditory capsule of *A lineatus* (C & V) ×18

(a) Inner view of epiotic
(b) Ventral aspect of the right side pterotic
(c) Dorsal aspect of the left side pterotic

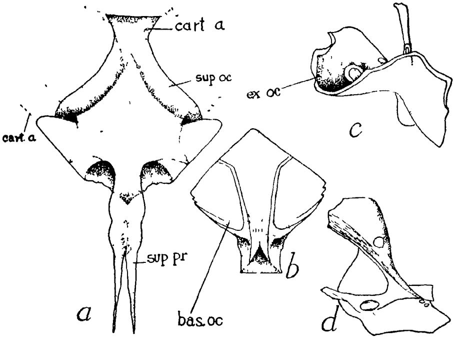

FIG 6 —The occipital bones of *A lineatus* (C & V) ×18.

(a) Supra occipital
(b) Inside aspect of Basi occipital
(c) Ex-occipital (outer view)
(d) Ex occipital (inner view)

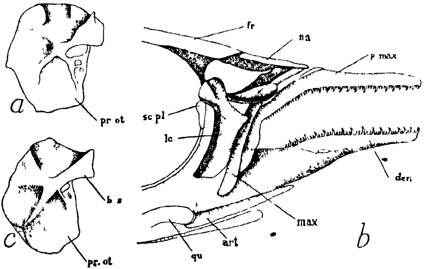

FIG 7 —(a & c) Internal aspect of the Pro otic bone of *A lineatus* (C & V) ×14
(b) The lateral view of the pre-orbital region of the skull of *A. lineatus* ×10

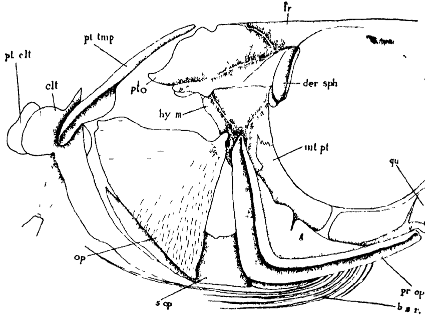

FIG 8 —The lateral view of the post-orbital portion of the skull of *A lineatus* (C & V) ×10

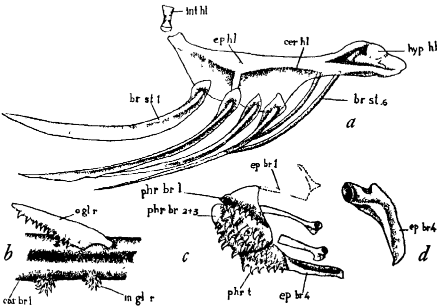

FIG 9 —Bones of the visceral skeleton of *A lineatus* (C & V)
(a) Lateral view of the hyoid cornu ×10.
(b) Gill rakers ×22
(c) Pharyngeal bone ×10
(d) Fourth epi-branchial ×14.

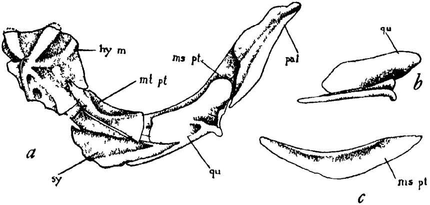

Fig 10 —(a) Hyomandibular arch of *A lineatus* ×10
(b) Inner aspect of quadrate ×10
(c) Lateral aspect of mesopterygoid ×10

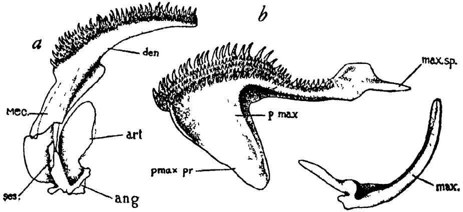

Fig 11 —Jaw bones of *A lineatus* (C & V) ×10
(a) Internal aspect of left lower jaw
(b) Inner view of left pre-maxilla
(c) Inner view of left maxilla

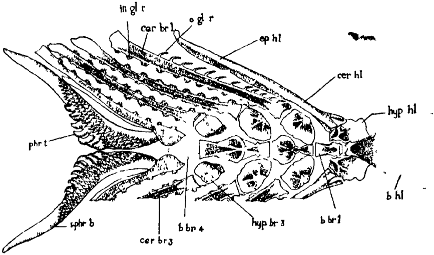

Fig 12 —Pharyngo branchial skeleton of *A lineatus* (C & V) ×10

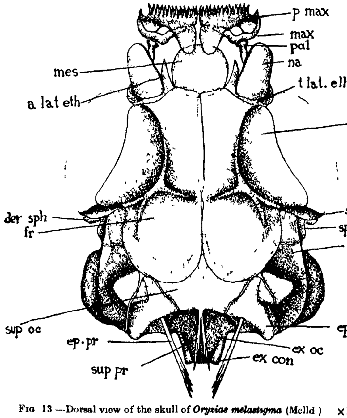

Fig 13 —Dorsal view of the skull of *Oryzias melastigma* (Mclld) ×16

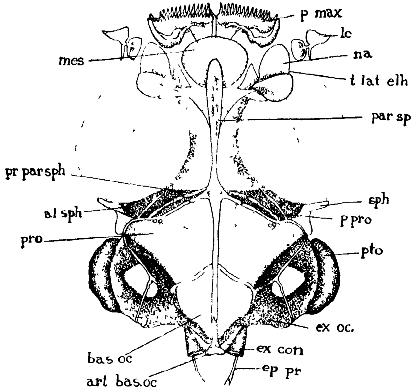

FIG 14 —Ventral view of the skull of *O melastigma* (Mclld) ×16

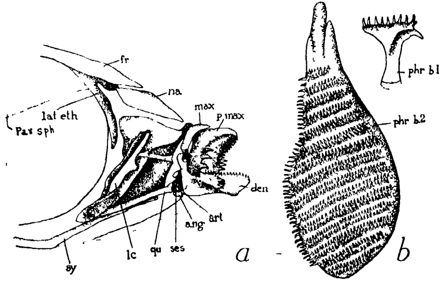

FIG 15 —(a) Lateral view of the skull of *O melastigma* ×22.
(b) Pharyngeal bones of *O melastigma* (Mclld) ×27.

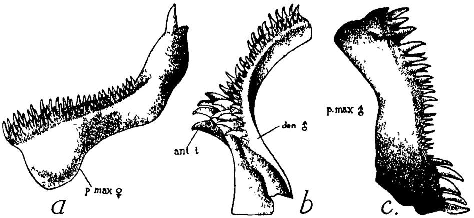

Fig 16 —Jaw bones of *O melastigma* (Mclld) ×35
(a) Right pre maxilla of female
(b) Left dentary of male
(c) Right pre maxilla of male.

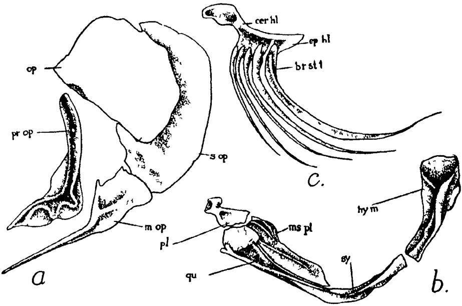

Fig 17 —Bones of visceral skeleton of *O melastigma* (Mclld).
(a) Opercular bones ×26
(b) Hyomandibular arch ×26
(c) Hyoid cornu ×20

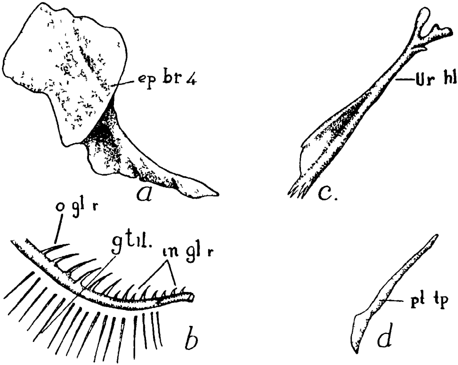

FIG 18 —Bones from visceral skeleton of *O melastigma* (McClell)
(a) Fourth epi branchial ×35
(b) Lateral view of first cerato branchial ×35
(c) Ventral aspect of uro hyal ×35
(d) Post-temporal bone ×12

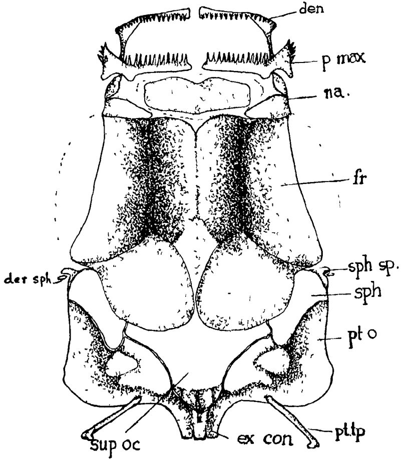

FIG 19 —Dorsal view of the skull of *Horaichthys setnai* Kulk ♀ ×26

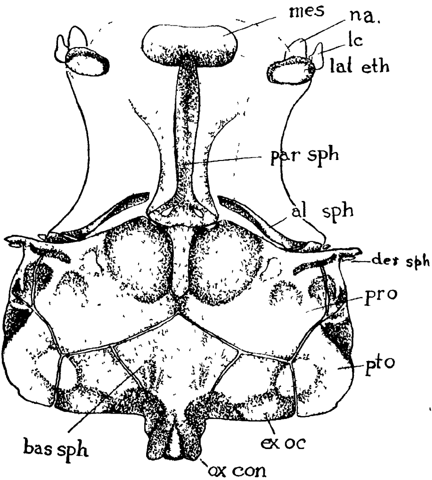

FIG 20 —Ventral view of the skull of *H setnai* Kulk ♀ ×26

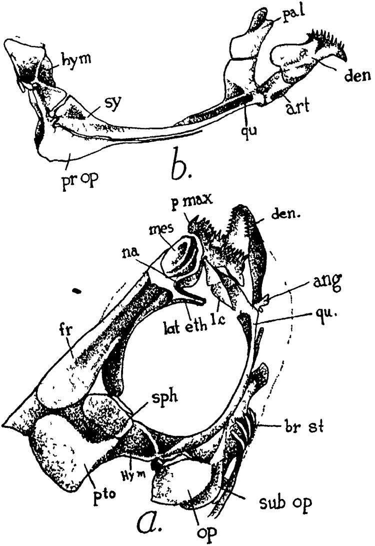

FIG 21 —(a) Lateral view of the skull of *H setnai* Kulk ♂ ×29
(b) Hyomandibular arch of *H setnai* Kulk ×29

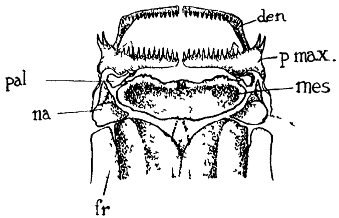

FIG 22 Dorsal view of the pre orbital region of a male *H setnai* 33

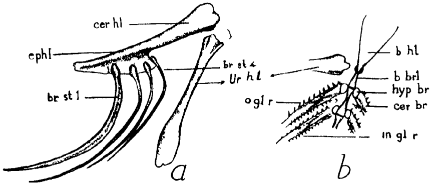

FIG 23 (a) Hyoid cornu of *H setnai* Kulk × 26
(b) A part of branchial skeleton of *H setnai* Kulk × 26

ON A GRAVITATIONAL INVARIANT

By V V **Narlikar** *and* K P **Singh**, *Benares Hindu University*

(*Read March 7, 1947*)

Abstract

The gravitational invariant $B_{\epsilon\mu\gamma\sigma}B^{\epsilon\mu\gamma\sigma}$ is evaluated for several Riemannian metrics. It is found to be positive everywhere in certain well known gravitational fields. A metric satisfying the gravitational field equations for empty space is given for which the invariant is negative everywhere.

Eddington (1930) has discussed the invariant

$$K = B_{\epsilon\mu\gamma\sigma}B^{\epsilon\mu\gamma\sigma}\,, \tag{1}$$

in connection with the third crucial test and also with reference to the possibility of constructing alternative field equations and representing the propagation of gravitational waves. The invariant must be distinguished from

$$\text{(i) } B_{\epsilon\gamma\sigma\mu}B^{\epsilon\mu\gamma\sigma} \text{ and (ii) } B_{\epsilon\sigma\mu\gamma}B^{\epsilon\mu\gamma\sigma}$$

either of which can be easily shown to be $-\frac{1}{2}K$. The importance of the invariant lies in the fact that the condition of the existence or non-existence of a gravitational field at a point is expressed by

$$B_{\epsilon\mu\gamma\sigma} \neq 0 \text{ or } B_{\epsilon\mu\gamma\sigma} = 0,$$

respectively, at the point and instant concerned.

The 20 independent components which do not identically vanish are

$$B_{1212} = x_1,\ B_{1313} = x_2,\ B_{2323} = x_3,\ B_{1414} = x_4,\ B_{2424} = x_5,$$

$$B_{3434} = x_6,\ B_{1123} = x_7,\ B_{1124} = x_8,\ B_{1134} = x_9,\ B_{2213} = x_{10},$$

$$B_{2241} = x_{11},\ B_{2234} = x_{12},\ B_{3312} = x_{13},\ B_{3314} = x_{14},\ B_{3324} = x_{15},$$

$$B_{4412} = x_{16},\ B_{4413} = x_{17},\ B_{4423} = x_{18},\ B_{1234} = x_{19},\ B_{1342} = x_{20},$$

with

$$B_{1423} = x_{21} \text{ and } x_{19}+x_{20}+x_{21} = 0$$

For a *weak* gravitational field for which

$$ds^2 = g_{\mu\gamma}dx^\mu dx^\gamma = (\delta_{\mu\gamma}+h_{\mu\gamma})dx^\mu dx^\gamma\,, \tag{2}$$

in the usual notation, if $G_{\mu\gamma} \neq 0$,

$$\begin{aligned} K = {} & 4\left(x_1^2+x_2^2+x_3^2+x_4^2+x_5^2+x_6^2\right) \\ & +8\left(x_7^2+x_{10}^2+x_{13}^2+x_{16}^2+x_{17}^2+x_{18}^2\right) \\ & -8\left(x_8^2+x_9^2+x_{11}^2+x_{12}^2+x_{14}^2+x_{15}^2\right) \\ & -8\left(x_{19}^2+x_{20}^2+x_{21}^2\right). \end{aligned} \tag{3}$$

to the second order in h's If $G_{\mu\gamma} = 0$,

$$\begin{gathered} x_1 = -x_6,\ x_2 = -x_5,\ x_3 = -x_4,\ x_7 = x_{18}, \\ x_{10} = x_{17},\ x_{18} = x_{16},\ x_8 = -x_{15},\ x_9 = -x_{12}, \\ x_{11} = -x_{14},\ x_1+x_2+x_3 = 0\,, \end{gathered} \tag{4}$$

and

$$\begin{aligned} K = {} & 8\left(x_1^2+x_2^2+x_3^2\right)+16\left(x_7^2+x_{10}^2+x_{13}^2-x_8^2-x_9^2-x_{11}^2\right) \\ & -8\left(x_{19}^2+x_{20}^2+x_{21}^2\right), \end{aligned} \tag{5}$$

to the same order of approximation If

$$\frac{\delta g_{\mu\gamma}}{\delta x^4} = 0,\ g_{14} = g_{24} = g_{34} = 0, \tag{6}$$

the invariant expressions (3) and (5) become sums of squares Our calculation of K for the two-body metric given by Einstein, Infeld and Hoffmann (1938) shows how the positive and negative contributions to K arise and the relative magnitudes of the two

$$\begin{gathered} h_{11} = h_{22} = h_{33} = h_{44} = \lambda^2\left(-\frac{2m_1}{r_1}-\frac{2m_2}{r_2}\right)+O(\lambda^4), \\ h_{\alpha 4} = O(\lambda^3),\ h_{\alpha\beta} = O(\lambda^4)\,,\ \alpha, \beta = 1, 2, 3 \\ r_1^2 = (x^1-\xi^1)^2+(x^2-\eta^1)^2+(x^3-\zeta^1)^2, \\ r_2^2 = (x^1-\xi^2)^2+(x^2-\eta^2)^2+(x^3-\zeta^2)^2 \end{gathered}$$

m_1 and m_2 are mass constants associated with the points (ξ^1, η^1, ζ^1) and (ξ^2, η^2, ζ^2) respectively λ is a parameter of expansion of small but definite magnitude K is given by

$$\begin{aligned} \frac{K}{8\lambda^4} = {} & 6\left(\frac{m_1^2}{r_1^6}-\frac{m_1m_2}{r_1^3r_2^3}+\frac{m_2^2}{r_2^6}\right)+18\,\frac{m_1m_2}{r_1^5r_2^5}\,[\Sigma(x^1-\xi^1)(x^1-\xi^2)]^2 \\ & -2\left[\frac{m_1^2u_1^2}{r_1^6}+\frac{m_2^2u_2^2}{r_2^6}+\frac{3m_1^2r_1^2}{r_1^6}+\frac{3m_2^2r_2^2}{r_2^6}+2\,\frac{m_1m_2}{r_1^3r_2^3}\,\Sigma\xi^1\xi^2\right] \\ & -12\,\frac{m_1m_2}{r_1^3r_2^3}\left[\frac{r_2}{r_2}\,\Sigma(x^1-\xi^2)\xi^1+\frac{r_1}{r_1}\,\Sigma(x^1-\xi^1)\xi^2\right] \\ & -36\,\frac{m_1m_2}{r_1^4r_2^4}\,r_1r_2\Sigma(x^1-\xi^1)(x^1-\xi^2), \end{aligned} \tag{7}$$

where

$$u_1^2 = (\xi^1)^2+(\eta^1)^2+(\zeta^1)^2,\quad u_2^2 = (\xi^2)^2+(\dot\eta^2)^2+(\zeta^2)^2$$

The first two terms in the expression for K are clearly positive All the remaining terms involve velocities In view of the smallness of the velocities in natural co-ordinates the invariant is always found to be positive Incidentally, it may be noted that the overhead dot is used in the above to denote a differentiation with regard to x^4

If x^1, x^2, x^3 and x^4 are represented as x, y, z and t respectively and if we take all h's zero except

$$\begin{aligned} h_{44} &= ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2lxt+2myt+2nzt+dt^2, \\ h_{14} &= axt+hyt+gzt+\lambda_1 x^2+\lambda_2 y^2+\lambda_3 z^2, \\ h_{24} &= hxt+byt+fzt+\mu_1 x^2+\mu_2 y^2+\mu_3 z^2, \\ h_{34} &= gxt+fyt+czt+\gamma_1 x^2+\gamma_2 y^2+\gamma_3 z^2, \end{aligned} \tag{8}$$

we have, curiously enough,

$$B_{\epsilon\mu\gamma\sigma} \neq 0, \ G_{\mu\gamma} = 0, \ K = -16\left(\lambda_2^2+\mu_3^2+\gamma_1^2\right), \tag{9}$$

provided all the coefficients, a, b, c, etc , are taken as constants

For the metric (Narlikar and Karmarkar, 1946),

$$ds^2 = -(1+kt)^p dx^2-(1+kt)^q dy^2-(1+kt)^r dz^2+dt^2, \tag{10}$$

for which p, q, r are constants satisfying

$$p+q+r = 2, \ pq+qr+rp = 0 \tag{11}$$

and $G_{\mu\gamma} = 0$, the *exact* value of the invariant is found to be

$$\frac{k^4}{2(1+kt)^4}(p^2q^2+q^2r^2+r^2p^2) \tag{12}$$

which is always positive

For a perfectly general line element of spherical symmetry, viz ,

$$ds^2 = -e^{\lambda}\,dr^2 - r^2\,(d\theta^2+\sin^2\theta\,d\phi^2) + e^{\gamma}\,dt^2, \tag{13}$$

$$\lambda = \lambda(r, t), \gamma = \gamma(r, t),$$

we find the exact value of the invariant to be

$$\begin{aligned} K = \frac{2}{r^2}\Bigg[& e^{-2\lambda}\,(\lambda'^2+\gamma'^2)+2\,(e^{-\lambda}-1)/r^2 \\ & +2r^2 e^{-2\lambda-2\gamma}\left\{e^{\gamma}\left(\frac{\lambda'\gamma'}{4}-\frac{\gamma'^2}{4}-\frac{\gamma''}{2}\right)-e^{\lambda}\left(\frac{\lambda\gamma}{4}-\frac{\dot{\lambda}^2}{4}-\frac{\lambda}{2}\right)\right\}^2\Bigg] \\ & -4e^{-\lambda-\gamma}\,\dot{\lambda}^2/r^2, \end{aligned} \tag{14}$$

where an overhead dash denotes a differentiation with regard to r and an overhead dot a differentiation with regard to t Our detailed investigation shows that the negative term either disappears or is weak in the particular cases which are known to be physically significant as gravitational fields

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ON THE MAGNETIC BEHAVIOUR OF FREE ELECTRONS

By K S Singwi *

(Communicated by Dr D S Kothari, F N I)

(*Read April 18, 1947*)

Abstract

A relativistic version of the quantum mechanical theory of Klein Lindhard of a free electron gas in a magnetic field is given, and an expression for the susceptibility of a relativistic degenerate electron gas is derived It is shown that the ratio of the paramagnetic to the diamagnetic susceptibility is 3 even in the general case

Introduction

In a recent paper O Klein (1945) has discussed the problem of a system of electrons subject to a weak magnetic field where account is also taken of the field generated by the electrons themselves The method in the general case leads to a linear integero-differential equation for the field, and not only readily leads to an expression for magnetic susceptibility but it can also deal with problems where the concept of susceptibility is inapplicable Following Klein, Lindhard (1946) has treated in detail the case of a free electron assembly subject to magnetic and electric fields Lindhard's treatment is throughout non-relativistic

In the present paper which is confined to the case of free electrons subject to a magnetic field the treatment is made relativistic In the non-relativistic theory of Klein and Lindhard the diamagnetic susceptibility is derived from the current density vector arising on account of the perturbation of the translational motion of the electrons, and the paramagnetic susceptibility is obtained from the magnetic moment arising on account of the perturbation of the spin motion of the electrons The relativistic treatment, on the other hand, has the advantage that the general expression for the perturbed current-density vector consists of two parts, one part corresponding to the perturbation of the translational motion and the other part corresponding to the perturbation of the spin motion of the electrons, the former giving rise to diamagnetism and the latter to paramagnetism The two currents enter together as is to be expected in a relativistic theory

Derivation of the Current-Density Vector

We consider an assembly of free electrons occupying a volume V The wave function of a free electron in the absence of field is

$$\psi_k(r) = \frac{U_k}{V^{\frac{1}{2}}} e^{i(k\cdot r)} , \tag{1}$$

where U_k is the Dirac amplitude, k the wave vector of the electron ($k = 2\pi/\lambda$, λ being the wavelength) and r the co-ordinate vector

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The Hamiltonian of an electron with charge $-e$ in a field defined by the vector potential A_s is

$$H=\sum_s \alpha_s \ (p+e\,A_s)+\beta\mu, \tag{2}$$

$s = 1, 2, 3$

where α_s and β are the Dirac matrices, p the momentum vector (in energy units) and $\mu = mc^2$ the rest energy of the electron

In the presence of a magnetic field the wave functions are perturbed Denoting by $\psi(r)$ the perturbed wave function corresponding to the unperturbed wave function $\psi_{k_0}(r)$ we have

$$\psi(r)=\psi_{k_0}(r)+\sum_{k\neq k_0} a_{k_0k}\psi_k(r), \tag{3}$$

where the coefficients a_{k_0k} are given by

$$a_{k_0k}=\frac{H'_{kk_0}}{E_{k_0}-E_k}, \quad k\neq k_0 \tag{4}$$

$$\text{and } H'_{kk_0}=\int \psi_k^* H'\psi_{k_0}d\tau' = \frac{e}{V}\int \left(U_k^*\,\alpha_s\ A_s\,U_{k_0}\right)e^{i(k_0-k)\ r}\,d\tau', \tag{5}$$

$d\tau'$ being the volume element and $H' = e\alpha_s\ A_s$, the perturbing term in the Hamiltonian The current-vector is

$$i_s = -ce\,(\psi^*\alpha_s\psi) \tag{6}$$

The perturbed current vector of the electron originally in the state k_0 is to a first approximation

$$i_{k_0s} = -ce\left[(\psi^*_{k_0}\alpha_s\psi_{k_0})+\sum_{k\neq k_0} a^*_{k_0k}(\psi^*_k\alpha_s\psi_{k_0})+\psi^*_{k_0}\alpha_s\sum_{k\neq k_0}a_{k_0k}\psi_k\right], \tag{7}$$

and hence the modification in the current due to the perturbation is

$$\delta i_{k_0s}=\frac{-ce}{V}\left[\sum{}' a^*_{k_0k}\left(U^*_k\alpha_sU_{k_0}\right)e^{i(k_0-k)\ r}+\sum{}' a_{k_0k}\left(U^*_{k_0}\alpha_sU_k\right)e^{-i(k_0-k)\ r}\right] \tag{8}$$

Substituting for a_{k_0k} from (4) in (8) we have for the X-component of the current

$$\delta i_{k_0x}=-\frac{ce^2}{V^2}\left[\int \tfrac{1}{2}S_0S\sum{}'\frac{1}{(E^2_{k_0}-E^2_k)}\left(U^*_k\alpha\ \ AU_{k_0}\right)^*\left(U^*_k\alpha_xU_{k_0}\right)e^{i(k_0-k)\ (r-r')}d\tau'\right.$$

$$\left.+\int \tfrac{1}{2}S_0S\sum\frac{1}{(E^2_{k_0}-E^2_k)}\left(U^*_k\alpha\ \ AU_{k_0}\right)\left(U^*_{k_0}\alpha_xU_k\right)e^{-i(k_0-k)\ (r-r')}\,d\tau'\right], \tag{9}$$

where S* denotes the summation over both signs of energy and spin for states of given k and S_0 is the summation over both signs of spins for states with momentum k_0 and positive energy

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* It may be noted here that if the summation S is performed over states with positive energy only, an extra term occurs in the current expression as can be seen by evaluating the spurs after introducing the annihilation operator $H_k+E_k/2\,|E_k|$ The extra term has no physical significance

Introducing the annihilation operator $H_{k_0}+E_{k_0}/2|E_{k_0}|$ and evaluating the spurs we have

$$\tfrac{1}{2}S_0S\,\frac{1}{\left(E_{k_0}^2-E_k^2\right)}\left(U_k^*\alpha\cdot AU_{k_0}\right)^*\left(U_k^*\alpha_xU_{k_0}\right)=\frac{c^2\hbar^2A_xk_0^2}{E_{k_0}\left(E_{k_0}^2-E_k^2\right)}+\frac{c^2\hbar^2}{E_{k_0}\left(E_{k_0}^2-E_k^2\right)}$$
$$\left[A_x(k_xk_{0x}-k_yk_{0y}-k_zk_{0z})+A_y(k_xk_{0y}+k_yk_{0x})+A_z(k_xk_{0z}+k_zk_{0x})\right]\tag{10}$$

Similarly

$$\tfrac{1}{2}S_0S\,\frac{1}{\left(E_{k_0}^2-E_k^2\right)}\left(U_k^*\alpha\cdot AU_{k_0}\right)^*\left(U_k^*\alpha_yU_{k_0}\right)=\frac{c^2\hbar^2A_yk_0^2}{E_{k_0}\left(E_{k_0}^2-E_k^2\right)}+\frac{c^2\hbar^2}{E_{k_0}\left(E_{k_0}^2-E_k^2\right)}$$
$$[A_x(k_xk_{0y}+k_yk_{0x})+A_y(k_yk_{0y}-k_xk_{0x}-k_zk_{0z})+A_z(k_yk_{0z}+k_zk_{0y})],\tag{11}$$

and

$$\tfrac{1}{2}S_0S\,\frac{1}{\left(E_{k_0}^2-E_k^2\right)}\left(U_k^*\alpha\cdot AU_{k_0}\right)^*\left(U_k^*\alpha_zU_{k_0}\right)=\frac{c^2\hbar^2A_zk_0^2}{E_{k_0}\left(E_{k_0}^2-E_k^2\right)}+\frac{c^2\hbar^2}{E_{k_0}\left(E_{k_0}^2-E_k^2\right)}$$
$$[A_x(k_xk_{0z}+k_zk_{0x})+A_y(k_yk_{0z}+k_zk_{0y})+A_z(k_zk_{0z}-k_xk_{0x}-k_yk_{0y})]\tag{12}$$

The perturbed current, therefore, is

$$\overrightarrow{\delta i_{k_0}}=\frac{-c^3e^2\hbar^2}{V^2}\left[\sum\int\{k_0^2A(r)+(A(r)\cdot k_0)k+(A(r)\cdot k)k_0-A(r)(k\cdot k_0)\}\right.$$
$$\frac{e^{i(k_0-k)\cdot(r-r')}}{E_{k_0}\left(E_{k_0}^2-E_k^2\right)}d\tau'$$
$$\left.+\sum{}'\int\{k_0^2A(r)+(A(r)\cdot k_0)k+(A(r)\cdot k)k_0-A(r)(k\cdot k_0)\}\frac{e^{-i(k_0-k)\cdot(r-r')}}{E_{k_0}(E_{k_0}^2-E_k^2)}d\tau'\right]\tag{13}$$

Introducing in (13) the Fourier expansion

$$\left.\begin{aligned}A(r)&=\frac{(2\pi)^{\frac{3}{2}}}{V}\sum_k A(k)e^{ik\cdot r}\\ \text{or}\qquad A(k)&=\frac{1}{(2\pi)^{\frac{3}{2}}}\int A(r)e^{-ik\cdot r}d\tau\end{aligned}\right\},\tag{14}$$

where $d\tau$ stands for $dx\;dy\;dz$, we have

$$\overrightarrow{\delta i_{k_0}}=-\frac{c^3\hbar^2e^2}{V(2\pi)^{\frac{3}{2}}}\int\Big[\Big\{k_0^2A(k_0-k)+\big(A(k_0-k)\cdot k_0\big)k+\big(A(k_0-k)\cdot k\big)k_0$$
$$-A(k_0-k)(k\cdot k_0)\Big\}e^{i(k_0-k)\cdot r}+\Big\{k_0^2A(k-k_0)+\big(A(k-k_0)\cdot k_0\big)k+\big(A(k-k_0)\cdot k\big)k_0$$
$$-A(k-k_0)(k\cdot k_0)\Big\}e^{i(k-k_0)\cdot r}\Big]\frac{dk}{E_{k_0}\left(E_{k_0}^2-E_k^2\right)},\tag{15}$$

where dk stands for $dk_x\;dk_y\;dk_z$ the volume element in k-space

The above equation in terms of the variable $k' = k_0 - k$, assuming $\operatorname{div} A(r) = 0$, i.e. $A(k') \cdot k' = 0$, reduces to

$$\overrightarrow{\delta\imath_{k_0}} = \frac{ce^2}{(2\pi)^{\frac{3}{2}}V} \int dk e^{ik\cdot r} \frac{1}{\sqrt{k_0^2c^2\hbar^2+\mu^2}} \left[\left\{ \frac{2k_0-k}{k^2-2k_0\cdot k} + \frac{2k_0+k}{k^2+2k_0\cdot k} \right\} (A(k)\cdot k_0) + \left\{ \frac{k\cdot k_0}{k^2-2k_0\cdot k} - \frac{k\cdot k_0}{k^2+2k_0\cdot k} \right\} A(k) \right], \tag{16}$$

where in (16) we have dropped the dash on k.

We shall now average $\overrightarrow{\delta\imath_{k_0}}$ for all directions of k_0. As we have assumed $\operatorname{div} A(r) = 0$ the vectors k and $A(k)$ are perpendicular to each other. Let k be chosen along the axis of z and let the direction of k_0 be given by θ and ϕ, the former being the angle between k_0 and k and the latter the angle between the plane through k and k_0 and the plane through k and the direction of $A(k)$. Integrating first with respect to ϕ we see that the contribution to the integral will arise only from that part of the integrand which is parallel to $A(k)$. Thus we get

$$\overrightarrow{\delta\imath_{k_0}} = -\frac{ce^2}{V(2\pi)^{\frac{3}{2}}} \int \frac{dk e^{ik\cdot r} A(k)}{2\sqrt{k_0^2c^2\hbar^2+\mu^2}} f(\xi) + \frac{ce^2}{V(2\pi)^{\frac{3}{2}}} \int \frac{dk e^{ik\cdot r} A(k)}{\sqrt{k_0^2c^2\hbar^2+\mu^2}} F(\xi), \tag{17}$$

where

$$\left. \begin{aligned} f(\xi) &= 1-\frac{1}{2}\left(\xi-\frac{1}{\xi}\right)\log\frac{1+\xi}{|1-\xi|} \\ F(\xi) &= \frac{1}{2\xi}\log\frac{1+\xi}{|1-\xi|} \end{aligned} \right\}, \tag{18}$$

and

$$\xi = \frac{2|k_0|}{|k|}$$

Since by definition ξ is always positive, we shall be concerned with the behaviour of the functions $f(\xi)$ and $F(\xi)$ in the interval $0 < \xi < \infty$. For large values of $\xi (\xi \gg 1)$ the two functions $f(\xi)$ and $F(\xi)$ have respectively the asymptotic forms $\frac{2}{3\xi^2}$ and $\frac{1}{\xi^2}$.

For $\xi = 1$ $\qquad f(\xi) = 1$

$\qquad F(\xi) = \infty$

and for $\xi = 0$ $\qquad f(\xi) = 2$

$\qquad F(\xi) = 1$

The two integrands in (17) are always positive. We see from expression (17) of the current that it consists of two parts, the negative part giving rise to diamagnetism and the positive part to spin paramagnetism. In contrast to the non-relativistic theory of Klein and Lindhard, the two currents here enter together.

In the non-relativistic case (17) reduces to

$$\overrightarrow{\delta\imath_{k_0}} = -\frac{e^2}{2(2\pi)^{\frac{3}{2}}mcV} \int dk e^{ik\cdot r} A(k) f(\xi) + \frac{e^2}{(2\pi)^{\frac{3}{2}}mcV} \int dk e^{ik\cdot r} A(k) F(\xi) \tag{19}$$

The first part of (19) is identical with that deduced by Klein, whereas the second part when transformed in terms of magnetic moment reduces to Lindhard's expression.

CALCULATION OF THE INDUCED FIELDS

The vector potential $\delta A(r)$ of the field due to the current $\delta i(r)$ is given by

$$\nabla^2 \delta A(r) = -\frac{4\pi}{c}\,\delta i(r) \tag{20}$$

From (14) and (20) we have

$$\frac{1}{(2\pi)^{\frac{3}{2}}}\int \delta A(k) k^2 e^{ik\cdot r} dk = \frac{4\pi}{c}\,\delta i(r) \tag{21}$$

Comparing (21) with (19) we have for the induced field corresponding to the first part of (19)

$$\delta A_c(k) = -\frac{4\pi n(k_0) e^2 A(k)}{4k_0^2\sqrt{k_0^2c^2\hbar^2+\mu^2}}\,\frac{\xi^2}{2} f(\xi), \tag{22}$$

and corresponding to the second part

$$\delta A_s(k) = \frac{4\pi e^2 n(k_0) A(k)}{4k_0^2\sqrt{k_0^2c^2\hbar^2+\mu^2}}\,\xi^2 F(\xi), \tag{23}$$

where $\delta A_c(k)$ denotes the current induced field and $\delta A_s(k)$ the spin induced field. $n(k_0)$ is the number of electrons per unit volume with wave number k_0 and per unit range about it. Equations (22) and (23) are fundamental equations of the present theory. For $\xi \gg 1$ (22) and (23) respectively reduce to

$$\delta A_c(k) = -\frac{1}{3}\,\frac{4\pi e^2 n(k_0) A(k)}{4k_0^2\sqrt{k_0^2c^2\hbar^2+\mu^2}}, \tag{24}$$

and

$$\delta A_s(k) = \frac{4\pi e^2 n(k_0)}{4k_0^2\sqrt{k_0^2c^2\hbar^2+\mu^2}}\,A(k) \tag{25}$$

Following Lindhard we determine the susceptibility in the following manner. If the ratio of the induced field $\delta A_t(k)$ to the total field $A(k)$ is independent of k, except perhaps for a negligible surface layer at the gas boundary the susceptibility is given by $\delta A_t(k)/4\pi A(k)$. Such an independence of k is evidently indicated by (24) and (25) but is not found in the more general case represented by (22) and (23). For a fuller discussion of the problem of the existence of susceptibility we refer to Lindhard's paper. It is interesting to note that in the general case represented by (24) and (25) where account is taken of the relativistic mechanics the ratio of the two induced fields is $-\frac{1}{3}$, which is in agreement with Landau's result for the non-relativistic case.

For $\xi \ll 1$, i.e. $|k|$ large, $\dfrac{A_c(k)}{A(k)}$ and $\dfrac{A_s(k)}{A(k)}$ both tend to zero. The boundary conditions for the field demand that the induced field should vanish at the boundary. The vanishing of the induced field at the boundary of the gas volume is equivalent to the vanishing of the higher Fourier coefficients of this field. This decrease of the higher Fourier coefficients is a purely quantum theoretical phenomenon and has of course no connection whatever with Bohr's proof that the susceptibility of an electron gas, according to classical physics, is zero

SUSCEPTIBILITY OF A COMPLETELY DEGENERATE ELECTRON GAS

According to our definition the paramagnetic susceptibility in the general case is given by

$$\chi_p = \frac{\delta A_s(k)}{4\pi A(k)} = \frac{e^2}{4} \sum_{k_0} \frac{1}{k_0^2} \frac{n(k_0)}{\sqrt{k_0^2 c^2 \hbar^2 + \mu^2}}, \tag{26}$$

where the summation includes all values of k_0. The number of electrons per unit volume between k_0 and $k_0 + dk_0$ is given by,

$$n(k_0)dk_0 = \frac{4\pi g k_0^2 dk_0}{(2\pi)^3}, \tag{27}$$

where g is the weight factor for the electron ($g = 2$). The paramagnetic susceptibility is, therefore, given by

$$\chi_p = \frac{4\pi g e^2}{4(2\pi)^3} \int_0^{K_0} \frac{dk_0}{\sqrt{k_0^2 c^2 \hbar^2 + \mu^2}}, \tag{28}$$

where K_0 (the maximum value for k_0) is related to the electron concentration by

$$K_0 = 2\pi \left(\frac{3n}{4\pi g}\right)^{\frac{1}{3}} \tag{29}$$

Integrating (28) we have

$$\chi_p = \frac{\pi g e^2}{(2\pi)^3 c\hbar} \log_e \frac{2\pi \left(\frac{3n}{4\pi g}\right)^{\frac{1}{3}} + \left\{\left(\frac{3n}{4\pi g}\right)^{\frac{2}{3}} (2\pi)^2 + \frac{\mu^2}{c^2\hbar^2}\right\}^{\frac{1}{2}}}{\mu/c\hbar} \tag{30}$$

In the non-relativistic case ($K_0 c\hbar \ll \mu$) (30) reduces to

$$\chi_p \approx \frac{B^2}{\hbar^2} \frac{m 3^{\frac{1}{3}}}{\pi^{\frac{4}{3}}} n^{\frac{1}{3}}, \tag{31}$$

which is the well-known expression of Pauli

In the extremely relativistic case ($K_0 c\hbar \gg \mu$) we have

$$\chi_p \approx \frac{B^2 m^2 c}{\hbar^3 \pi^2} \log_e \frac{(24\pi^2)^{\frac{1}{3}} \hbar}{mc} n^{\frac{1}{3}}, \tag{32}$$

where B is the Bohr-magneton $\left(B = \frac{e\hbar}{2mc}\right)$

The diamagnetic susceptibility will be $-\frac{1}{3}$ of the above value. The occurrence of $\log n$ in (32) is interesting, and is due to the fact that the magnetic moment of the electron varies with the kinetic energy

In conclusion I have great pleasure in expressing my gratitude to Prof D S Kothari under whose guidance this work was carried out

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MARCH, 1948

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ON OPERATIONAL CALCULUS

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(Communicated by Dr Ram Behari, M A , Ph D , Sc.D , F N I)

(*Received April 4 , read May 2, 1947*)

Theorem I *If* $f(p) \doteqdot h(x)$, *then for* $\mu > 0$, $R(\lambda) > -1$,

$$p^{\mu-\lambda} f(p^{-\mu}) = F(x) \equiv x^{\lambda} \int_0^{\infty} h(s)\, J_{\lambda}^{\mu}(sx^{\mu})\, ds, \tag{0 1}$$

where *

$$J_{\lambda}^{\mu}(x) \equiv \sum_{r=0}^{\infty} \{(-x)^r / r!\,\Gamma(1+\lambda+\mu r)\},$$

provided that

(i) $$\text{as } s \to 0,\ h(s) = o\,(s^{-1+\epsilon}),$$

(ii) $$\text{as } s \to \infty,\ h(s) = o\,[s^{k(\lambda+\frac{1}{2})-1-\epsilon'} \exp\,(-s^{k+\epsilon''\cos \pi k} \cos \pi k)],$$

k *being equal to* $1/(\mu+1)$ *and* ϵ, ϵ', ϵ'' *being arbitrarily small positive numbers and*

(iii) $$\int_0^{\infty} e^{-px} F(x)\, dx$$

converges absolutely

Proof By the definition of operational relationship

$$x^{\lambda} \int_0^{\infty} h(s)\, J_{\lambda}^{\mu}(sx^{\mu})\, ds = p \int_0^{\infty} e^{-px}\, x^{\lambda}\, dx \int_0^{\infty} h(s)\, J_{\lambda}^{\mu}(sx^{\mu})\, ds$$

$$= p \int_0^{\infty} h(s)\, ds \int_0^{\infty} x^{\lambda} e^{-px} \sum_{r=0}^{\infty} \frac{(-sx^{\mu})^r}{r!\,\Gamma(1+\lambda+\mu r)}\, dx \tag{I$_1$}$$

$$= p \int_0^{\infty} h(s)\, ds \sum_{r=0}^{\infty} \frac{(-s)^r}{r!\,\Gamma(1+\lambda+\mu r)} \int_0^{\infty} x^{\lambda+\mu r} e^{-px}\, dx \quad . \tag{I$_2$}$$

$$= p \int_0^{\infty} h(s) \sum_{r=0}^{\infty} \frac{(-s)^r}{r!\, p^{1+\lambda+\mu r}}\, ds$$

$$= p^{\mu-\lambda}\, p^{-\mu} \int_0^{\infty} h(s) \exp\,(-p^{-\mu} s)\, ds$$

$$= p^{\mu-\lambda} f(p^{-\mu}), \text{ since } p \int_0^{\infty} h(x) e^{-px} dx = f(p)$$

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* This is Maitland's generalised Bessel function See Ref 18

The foregoing proof involves two inversions which have to be justified, viz. in steps (I_1) and (I_2)

The first inversion is justified, under the conditions stated, by virtue of De la Vallee Poussin's Theorem,[1] since both the x- and s- integrals converge absolutely and the double integral exists, for by the formula * B(1)

$$\text{as } x \to \infty,\ J_\lambda^\mu(x) = O\left[x^{-k(\lambda+\frac{1}{2})}\exp\{(\mu x)^k(\cos \pi k)/\mu k\}\right],\ k = 1/(1+\mu), \qquad (0\ 2)$$

and

$$\text{as } x \to 0,\ J_\lambda^\mu(x) = O\,(1)$$

The inversion in the step (I_2) is valid since each term of the series,

$$U_r(x) = (-sx^\mu)^r/r!\,\Gamma(1+\lambda+\mu r)$$

is continuous and

$$\sum_{r=0}^{\infty} |\,U_r(x)\,|$$

converges uniformly in the arbitrarily large interval $0<x<\alpha$, also $x^\lambda e^{-px}$ is bounded and integrable in $0<x<\alpha$ and

$$\sum_{r=0}^{\infty}\int_0^\infty x^\lambda e^{-px}\,|\,U_r(x)\,|\,dx \text{ converges to } p^{-\lambda-1}\exp\,(sp^{-\mu})$$

COROLLARIES 1 *If $f(p) \doteq h(x)$ and μ is a positive integer,*

$$p^{1+\mu-\mu k} f(p^{-\mu}) \doteq \frac{x^{\mu k-1}}{\Gamma(\mu k)}\int_0^\infty h(s)\ {}_0F_\mu\left\{k\,,\,k+\frac{1}{\mu}, \qquad ,\,k+1-\frac{1}{\mu},\ -s\left(\frac{x}{\mu}\right)^\mu\right\}ds$$

In particular, if $\mu = 1$, we obtain Tricomi's theorem[14], viz

If $f(p) = h(x)$, then provided that the integrals converge

$$p^{1-\nu}f(1/p) = \int_0^\infty (x/s)^{\frac{1}{2}\nu}J_\nu\{2\surd(sx)\}\,h(s)ds$$

2 Similarly, for $\mu = 2$ we have the result If $f(p) \doteq h(x)$, then under suitable conditions of convergence

$$p^{2-\lambda}f(1/p^2) = \surd\pi\,(\tfrac{1}{2}x)^{\frac{1}{2}(\lambda+1)}\int_0^\infty s^{\frac{1}{2}-\frac{1}{2}\lambda}J_{\frac{1}{2}\lambda\,,\,\frac{1}{2}\lambda-\frac{1}{2}}\{3(\tfrac{1}{4}sx^2)^{\frac{1}{3}}\}\,h(s)\,ds,$$

where $J_{m,n}(x)$ represents Humbert's function defined by F(10) This is a generalisation of the result given by Humbert[7] for $\lambda = 2$

To evaluate the integral in (0 1) we shall frequently require to change the order of summation and integration, we therefore set out the conditions under which it is valid Since the infinite series for $J_\lambda^\mu(sx^\mu)$ in the integrand is a series of continuous functions and is uniformly convergent in the arbitrary interval $(0, \alpha)$, where α may be taken as large as we please, the termwise integration is legitimate if

(i) $h(s)$ is bounded and integrable in $(0, \alpha)$, and

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* The number of the formula is written within the brackets ( ) preceded by the letter F or B appertaining to the table of formulae given at the end of the paper

(ii) $\int_0^\infty h(s)J_\lambda^\mu(-sx^\mu)ds$ converges From B (2) we note that as $s \to \infty$,

$$J_\lambda^\mu(-sx^\mu) = O\left[s^{-k(\lambda+\frac{1}{2})}\exp\left\{(\mu s x^\mu)^k/\mu k\right\}\right], k = 1/(1+\mu) \tag{0 3}$$

This set of conditions will be referred to as conditions A

Result 1 Applying the theorem to the operational relation [10]

$$f(p) \equiv p^{1-k}\exp(1/2p)W_{-k,m}(1/p) = 2x^{k-\frac{1}{2}}K_{2m}(2\sqrt{x})/\Gamma_2^{*}(\tfrac{1}{2}+k\pm m) \equiv h(x),$$
$$R(\tfrac{1}{2}+k\pm m)>0,$$

we get

$$p^{\mu k-\lambda}\exp(\tfrac{1}{2}p^\mu)W_{-k,m}(p^\mu) = \frac{2x^\lambda}{\Gamma_2(\frac{1}{2}+k\pm m)}\int_0^\infty s^{k-\frac{1}{2}}K_{2m}(2\sqrt{s})J_\lambda^\mu(sx^\mu)ds$$

$$= \sum_{r=0}^{\infty}\frac{\Gamma_2(r+k+\frac{1}{2}\pm m)\,(-)^r x^{\lambda+\mu r}}{\Gamma_2(k+\frac{1}{2}\pm m)r!\,\Gamma(1+\lambda+\mu r)} = F(x), \text{ say}, \quad R(\lambda)>-1, \tag{1}$$

on expanding J_λ^μ, changing the order of integration and summation and using the integral [16] [B F , 383 (3)]

$$\int_0^\infty s^{\lambda-1}K_{2m}(2\sqrt{s})ds = \tfrac{1}{2}\Gamma_2(\lambda\pm m), \; R(\lambda\pm m)>0 \tag{1 1}$$

Since [16] $K_{2m}(2\sqrt{s}) = O(s^{-\frac{1}{4}}e^{-2\sqrt{s}})$ as $s \to \infty$, [B F , 202 (1)] (1 2)

the conditions A are satisfied if $\mu>1$ and the inversion is therefore legitimate For $\mu = 1$ the function J_λ^μ reduces to the Bessel function and the same value of the integral is obtained by (1), § 13 45 [B F] as from (1) Further for $F(x)$ to be a valid original the integral

$$\int_0^\infty e^{-px}F(x)dx \tag{1 21}$$

should also converge

As for the behaviour of $F(x)$ as $x \to \infty$ we find by the formulae B(6), B(5) and B(4) respectively that

$$F(x) = O(x^\rho),\ \rho = \lambda-\mu(k+\tfrac{1}{2}-|m|) \text{ when } 1<\mu<3,$$

$$F(x) = O(x^\rho+x^\sigma),\ \sigma = \lambda+\mu(2k-\lambda-\tfrac{1}{2})/(\mu-1) \text{ when } \mu = 3,$$

and $$F(x) = O\{x^\sigma \exp(Ax^{\mu/(\mu-1)})\},\ A>0, \text{ when } \mu>3$$

Also for all values of μ, $F(x) = O(x^\lambda)$ as $x \to 0$

Hence the integral (1 21) converges if $\mu<3$ and $R(\lambda)>-1$ (1 22)

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* For the sake of brevity of expression, the symbol Γ_2 has been used to express the product of two Γ's Thus $\Gamma_2(a\pm\beta) \equiv \Gamma(a+\beta)\Gamma(a-\beta)$

Similarly, $\Gamma_4(\pm a\pm\beta) \equiv \Gamma(a+\beta)\Gamma(a-\beta)\Gamma(-a+\beta)\Gamma(-a-\beta)$

Thus both the sides of (1) are analytic under the conditions (1.22). Hence* by the principle of analytic continuation the relation (1) holds if $R(\lambda) > -1$ and $1 < \mu < 3$.

Particular Cases. (1) Taking $\mu = 1$ we find† from (1) that

$$p^{k-\lambda} e^{\frac{1}{2}p} W_{-k,m}(p) = x^{\lambda}\, {}_2F_1(k+\tfrac{1}{2}\pm m,\, 1+\lambda, -x)/\Gamma(1+\lambda),\quad R(\lambda) > -1, \tag{1.3}$$

the hypergeometric function being defined for $x > 1$ by its analytic continuation.[2]

By taking in succession (i) $\lambda = k+m-\frac{1}{2}$, (ii) and (iii)‡ $m = \frac{1}{4}$, $\lambda = 2k \mp \frac{1}{2}$, (iv) and (v) $m = \frac{1}{4}$, $\lambda = \mp\frac{1}{2}$, (vi) and (vii) $k = \frac{1}{4} \mp \frac{1}{4}$, $\lambda = \mp\frac{1}{2}$ and using respectively the formulae F(16) to F(22) we have on writing P for

$$e^{\frac{1}{2}p} W_{-k,\frac{1}{4}}(p),\ \text{i.e.}\ 2^{k}(2p)^{\frac{1}{4}} e^{\frac{1}{2}p} D_{-2k-\frac{1}{2}}\{\sqrt{(2p)}\},\ \text{by F (7)},$$

(i)[17] $$\Gamma(\tfrac{1}{2}+k+m)\, p^{\frac{1}{2}-m} e^{\frac{1}{2}p} W_{-k,m}(p) = x^{k+m-\frac{1}{2}}(1+x)^{-k+m-\frac{1}{2}},\quad R(m+k) > -\tfrac{1}{2},$$ [M. A., 340],

(ii) and (iii)† $$2^{-\frac{1}{2}}(2p)^{-k\pm\frac{1}{4}} P = \{\sqrt{(1+x)}-1\}^{2k\mp\frac{1}{2}}/\Gamma(2k\mp\tfrac{1}{2}+1)(1+x)^{\frac{1}{4}\pm\frac{1}{4}},\quad R(k) > -\tfrac{1}{2}\pm\tfrac{1}{4},$$

(iv) $$p^{k+\frac{1}{4}} P = \frac{\cos\{(2k+\frac{1}{2})\tan^{-1}\sqrt{x}\}}{\sqrt{(\pi x)}(1+x)^{k+\frac{1}{4}}},$$ (v) $$p^{k-\frac{1}{4}} P = \frac{\sin\{(2k-\frac{1}{2})\tan^{-1}\sqrt{x}\}}{\sqrt{\pi}(k-\frac{1}{4})(1+x)^{k-\frac{1}{4}}},$$

(vi) $$p e^{\frac{1}{2}p} K_m(\tfrac{1}{2}p) = \frac{\cosh(2m \sinh^{-1}\sqrt{x})}{\sqrt{\{x(1+x)\}}},$$

(vii) $$e^{\frac{1}{2}p} W_{-\frac{1}{2},m}(p) = \frac{\sinh(2m \sinh^{-1}\sqrt{x})}{m\sqrt{\{\pi(1+x)\}}}$$

(2) When $\mu = 2$, the relation (1) may be put in the form[6]

$$p^{2k-\lambda} e^{\frac{1}{2}p^2} W_{-k,m}(p^2) = \frac{x^{\lambda}}{\Gamma(1+\lambda)}\, {}_2F_2\!\left(\begin{matrix} k+\frac{1}{2}\pm m, \\ \frac{1}{2}\lambda+\frac{1}{2},\ \frac{1}{2}\lambda+1, \end{matrix}\ -\tfrac{1}{4}x^2\right),\quad R(\lambda) > -1 \tag{1.4}$$

The relations, obtained by taking $\lambda = 2k-1\pm 2m$ or $\lambda = 2k\pm 2m$ and using F(5), may be written down by taking $r = 0$ or 1 in the following. [Case $r = 1$ is due to Shastri[18]]

$$\Gamma(2m+2k+r)\, p^{1-2m-r} e^{\frac{1}{2}p^2} W_{-k,m}(p^2)$$
$$\doteq 2^{1+r}(2x)^{k+m-1} e^{-\frac{1}{8}x^2} M_{\frac{1}{4}(k-3m-r+1),\,\frac{1}{4}(k+m+r-1)}(\tfrac{1}{4}x^2)$$

(3) For $\mu = 3$ the relation (1) assumes the form

$$p^{3k-\lambda} e^{\frac{1}{2}p^3} W_{-k,m}(p^3) = \frac{x^{\lambda}}{\Gamma(1+\lambda)}\, {}_2F_3\!\left(\begin{matrix} k\pm m+\frac{1}{2}, \\ \frac{1}{3}(1+\lambda),\ \frac{1}{3}(2+\lambda),\ 1+\frac{1}{3}\lambda, \end{matrix}\ -\frac{x^3}{27}\right) \tag{1.5}$$

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* This is the line of argument that has been used to extend by the principle of analytic continuation the range of validity of the various formulae established henceforth, and for the sake of brevity the whole argument will be indicated by the abbreviation A.C.

† The ambiguous sign in the ${}_2F_1$ has been used to write the two parameters $k+\frac{1}{2}+m$ and $k+\frac{1}{2}-m$ in a compact form. Another use of the sign is given in the following note.

‡ Where the ambiguous sign may not be taken indifferently, it is to be implied that the lower signs taken together give one formula and the upper ones another. These uses (see foot note, p. 133, also) of the ambiguous sign save considerable space.

The relations, obtained from this by taking (i) $m = \frac{1}{6}$, $k = \frac{1}{3}\lambda$ or (ii) $m = \frac{1}{6}$, $k = \frac{1}{3}(\lambda+1)$ or (iii) $m = \frac{1}{3}$, $k = \frac{1}{3}\lambda+\frac{1}{6}$ and using **F**(1), may be written down from the following by taking $r = -1$, 1 or 0

$$p^{\frac{1}{2}+\frac{1}{2}r}e^{\frac{1}{2}p^3}W_{-k,\frac{1}{3}-\frac{1}{2}r^2}(p^3) = \frac{\Gamma(k+\frac{1}{2}-\frac{1}{2}r)}{\Gamma(3k+\frac{1}{2}-\frac{1}{2}r)}(3x)^{\frac{3}{2}(k-\frac{1}{2}-\frac{1}{2}r)}x^{1+r}J_{k-\frac{1}{2}r-\frac{1}{2}}\{2(\tfrac{1}{3}x)^{\frac{3}{2}}\}.$$

Result 2 On termwise interpretation, valid for $0<\beta<1$, we have

$$h(x) \equiv x^{a}\exp(-\tfrac{1}{2}x^{\beta})W_{k,m}(x^{\beta}),$$

which by **F**(16),

$$= \frac{\Gamma(-2m)}{\Gamma(\frac{1}{2}-m-k)}x^{a+\beta(m+\frac{1}{2})}{}_1F_1\begin{pmatrix} m+\frac{1}{2}+k, & -x^{\beta} \\ 2m+1, & \end{pmatrix} + \begin{matrix}\text{a similar expression}\\ \text{with} -m \text{ written for } m\end{matrix}$$

$$\doteq \sum_{r=0}^{\infty}\frac{\Gamma(-2m-r)(-)^r\Gamma\{1+\alpha+\beta(m+\frac{1}{2}+r)\}}{\Gamma(\frac{1}{2}-m-k-r)\; r!\; p^{\alpha+\beta(m+\frac{1}{2}+r)}} + \begin{matrix}\text{a similar expression}\\ \text{with} -m \text{ written for } m\end{matrix}$$

$$\equiv f(p), \text{ say}$$

Hence by Th I,

$$p^{\mu-\lambda}f(p^{-\mu}) = x^{\lambda}\int_0^{\infty}s^{a}\exp(-\tfrac{1}{2}s^{\beta})W_{k,m}(s^{\beta})\sum_{r=0}^{\infty}\frac{(-sx^{\mu})^r}{r!\,\Gamma(1+\lambda+\mu r)}\,ds$$

$$= \sum_{r=0}^{\infty}\frac{(-)^r x^{\lambda+\mu r}\Gamma_2\{\frac{1}{2}\pm m+(r+1+\alpha)/\beta\}}{\beta\quad r!\,\Gamma(1+\lambda+\mu r)\Gamma\{1-k+(r+1+\alpha)/\beta)\}},$$

$$R(\lambda)>-1,\; R\{\alpha+\beta(\tfrac{1}{2}\pm m)\}>-1, \quad (2)$$

on integrating term by term by means of the formula[4]

$$\int_0^{\infty}x^{l-1}e^{-\frac{1}{2}x}W_{k,m}(x)dx = \Gamma_2(l\pm m+\tfrac{1}{2})/\Gamma(l-k+1),\; R(l\pm m)>-\tfrac{1}{2} \qquad (2\ 1)$$

Since[17] as $x\to\infty$, $W_{k\ m}(x) = O(x^{k}e^{-\frac{1}{2}x})$, [M A, 343], the conditions A are satisfied if besides the restrictions mentioned above, $0<\mu+1-1/\beta$, and the inversion effected above is justified Further in order that the original in (2) may give a convergent integral of the type (1 21), we should have $\mu-1/\beta\leqslant 1$ Formula (2) is therefore proved to be valid if

$$R(\lambda)>-1<R\{\alpha+\beta(\tfrac{1}{2}-|m|)\},\; 0<\beta\leq 1,\; 0<\mu+1-1/\beta\leq 2,$$

and by **A C** the range can be extended to include the case $0 = \mu+1-1/\beta$

Particular Cases (i) Of the two series in the image function only one survives when $k = m+\frac{1}{2}$, and we obtain the relation

$$\sum_{r=0}^{\infty}\frac{(-)^r\Gamma(1+\nu+\beta r)}{r!\; p^{\lambda\ \mu(1+\nu+\beta r)}} = \sum_{r=0}^{\infty}\frac{(-)^r x^{\lambda+\mu r}\Gamma\{(\nu+r+1)/\beta\}}{\beta\quad r!\,\Gamma(1+\lambda+\mu r)}, \qquad (2\ 3)$$

$$R(\lambda)>-1<R(\nu),\; 0<\beta\leq 1,\; 0\leq\mu+1-1/\beta\leq 2,$$

where for simplicity we have used the parameter ν for $\alpha+\beta(m+\frac{1}{2})$

If we further take $\beta = \frac{1}{2}$ and use F(7), we have

$$p^{\mu(\nu+1)-\lambda} \exp(\tfrac{1}{2}p^{\mu})D_{-2\nu-2}\left(\frac{1}{\sqrt{2}}p^{\frac{1}{2}\mu}\right) \doteq \frac{2^{\nu+1}}{\Gamma(2\nu+2)} \sum_{r=0}^{\infty} \frac{(-)^r x^{\lambda+\mu r}\Gamma(2\nu+2+2r)}{r!\,\Gamma(1+\lambda+\mu r)},$$

$$1 \leq \mu \leq 3,\ R(\lambda) > -1 < R(\nu), \quad (2\,4)$$

which was given in the special case $\mu = 2$, $\lambda = 2\nu+1$ by Varma [15]

(ii) When $\beta = 1$, the image function in (2) is easily seen to be the A C of a single ${}_2F_1$ expansible in descending powers of p, which on termwise interpretation reproduces the original function

(iii) When μ and $1/\beta$ ($= n$, say) are both positive integers, the original in (2) may be expressed as a single hypge function and by choosing the parameters suitably this original may be made susceptible to the formulae F (1, 5, 11) and thus provide us with the images of a few more Bessel's and Kummer's functions, but the images do not lend themselves to expression in a closed and simplified form

Result 3 Take the fundamental operational relation

$$f(p) = p^{-\nu} = x^{\nu}/\Gamma(1+\nu) = h(x), \quad R(\nu) > -1$$

Then by Th I

$$p^{\mu-\lambda}f(p^{-\mu}) = x^{\lambda}\int_0^x J_{\lambda}^{\mu}(sx^{\mu})s^{\nu}ds/\Gamma(1+\nu)$$

But $p^{\mu-\lambda}f(p^{-\mu}) = p^{\mu-\lambda+\mu\nu} = x^{\lambda-\mu(\nu+1)}/\Gamma(1+\lambda-\mu-\mu\nu)$, $R(\lambda-\mu-\mu\nu) > -1$

Hence by Lerch's theorem [8], according to which every legitimate image function has a unique original function, we have

$$\int_0^{\infty} s^{\nu}J_{\lambda}^{\mu}(sw)ds = \Gamma(1+\nu)/\Gamma(1+\lambda-\mu-\mu\nu)w^{\nu+1}, \quad (3)$$

valid by A C when $\mu < 1$ and $R(\nu) > -1$, with an additional condition $R(\lambda-2\nu) > \frac{1}{2}$ in case $\mu = 1$

When $\mu = 1$, (3) reduces to the known integral [16] [B F, 391(1)]

$$\int_0^{\infty} t^{\nu-\frac{1}{2}\lambda}J_{\lambda}(2\sqrt{t})dt = \Gamma(1+\nu)/\Gamma(\lambda-\nu),\ R(\nu) > -1,\ R(\lambda-2\nu) > \tfrac{1}{2}$$

By differentiating with respect to ν, we may introduce powers of $\log s$ in the integrand of (3)

Result 4 Applying the theorem to the relation (1) and writing ρ for $1+\lambda_1+\mu_1(\mu k+\frac{1}{2}\mu-\lambda-1)$, we have the integral

$$x^{\lambda_1}\int_0^{\infty} J_{\lambda_1}^{\mu_1}(sx^{\mu_1})F(s)ds = \text{original of } p^{1-\rho+\frac{1}{2}\mu\mu_1}\exp(\tfrac{1}{2}p^{-\mu\mu_1})W_{-k,m}(p^{-\mu\mu_1})$$

$$= \sum_{r=0}^{\infty}\left[\frac{\Gamma(-2m)(\frac{1}{2}+m+k,r)x^{\rho+\mu\mu_1(r+m)-1}}{\Gamma(\frac{1}{2}-m+k)(2m+1,r)r!\,\Gamma\{\rho+\mu\mu_1(r+m)\}} + \begin{matrix}\text{a similar exp with}\\ -m \text{ written for } m\end{matrix}\right], \quad (4)$$

on using F (5, 6) to expand $W_{-k\,m}$ and interpreting term-by-term The integral (4) is convergent when $1<\mu<3$, $0<\mu_1<1$ and $R(\lambda)>-1$, with an additional condition $R\{\frac{1}{2}\lambda_1-\lambda+\mu(k+\frac{1}{2}\pm m)\}>\frac{1}{4}$ when $\mu_1=1$, and a further one $R(\lambda+\lambda_1-6k)>-1$ if also $\mu=3$

PARTICULAR CASES (i) Taking $\mu_1=1$ and writing ν for λ_1 we have

$$\int_0^\infty s^{\lambda-\frac{1}{2}\nu} J_\nu\{2\surd(sx)\}\,{}_2F_\mu\left[k+\tfrac{1}{2}\pm m,\ \frac{\lambda+1}{\mu},\quad ,\ 1+\frac{\lambda}{\mu},\ -\left(\frac{s}{\mu}\right)^\mu\right] ds$$

$$=\frac{\Gamma(\lambda+1)\Gamma(-2m)\,x^{\rho+\mu m-1-\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}-m+k)\Gamma(\rho+\mu m)}\,{}_1F_{\mu+1}\left[\tfrac{1}{2}+m+k,\ 2m+1,\ m+\frac{\rho}{\mu},\right.$$

$$\left. m+\frac{\rho+1}{\mu},\quad ,\ m+1+\frac{\rho-1}{\mu},\ \left(\frac{x}{\mu}\right)^\mu\right]$$

$+$ a similar expression with $-m$ written for m, (4 1)

where $\rho=\nu-\lambda+\mu(k+\frac{1}{2})$ and $\mu=1$, 2 or 3

It is interesting to note that the expression on the right of (4 1) lends itself to expression in a compact form as a single infinite series in case $m=\frac{1}{4}$ or $-\frac{1}{4}$, namely,

$$\frac{\Gamma(\lambda+1)x^{\rho-\frac{1}{2}\nu-\frac{1}{4}\mu-1}}{2^{\frac{1}{2}-2k}\Gamma(2k+\frac{1}{2})}\sum_{r=0}^{\infty}\frac{\Gamma(\frac{1}{2}r+k+\frac{1}{4})}{r!\,\Gamma(\rho+\frac{1}{2}\mu r-\frac{1}{4}\mu)}(-2x^{\frac{1}{2}\mu})^r$$

(ii) Taking $\mu=1$, the ${}_2F_1$ in (4 1) may be made susceptible to the formulae F(16) to F(22) The first of these gives a formula which may be verified[16] [by (1), § 13 6, B F , p 434] To apply F(17, 18) put $m=\frac{1}{4}$ and $2k=\lambda\pm\frac{1}{2}$ and we get

$$\int_0^\infty s^{-\frac{1}{2}\nu} J_\nu(2\sqrt{sx})\,\frac{\{\surd(1+s)-1\}^\lambda}{(1+s)^{\frac{1}{4}\pm\frac{1}{4}}}\,ds=\frac{x^{\frac{1}{2}(\nu-\lambda-\frac{1}{2})\pm\frac{1}{4}}}{(\frac{1}{2}\lambda)^{-\frac{1}{4}\pm\frac{1}{4}}}\sum_{r=0}^{\infty}\frac{\Gamma(\frac{1}{2}r+\frac{1}{2}\lambda+\frac{1}{4}\pm\frac{1}{4})(-2\surd x)^r}{r!\,\Gamma(\nu-\frac{1}{2}\lambda+\frac{1}{2}r+\frac{1}{4}\pm\frac{1}{4})},$$

$$R(\lambda-\nu)<\pm\tfrac{1}{2},\ R(\lambda)>-1$$

When $\nu=\lambda$ in the first of these, we find that

$$\int_0^\infty J_\lambda(2\sqrt{sx})\{\surd(1+s)-1\}^\lambda s^{-\frac{1}{2}\lambda}(1+s)^{-\frac{1}{2}}\,ds=x^{-\frac{1}{2}}e^{-2\surd x},\ R(\lambda)>-1$$

To apply F(19, 20) take $m=\frac{1}{4}$ and $\lambda=\mp\frac{1}{2}$, and we have

$$\int_0^\infty\frac{s^{-\frac{1}{2}\nu-\frac{1}{4}\mp\frac{1}{4}}}{(1+s)^{k+\frac{1}{4}}}J_\nu(2\sqrt{sx})\,{}^{\cos}_{\sin}\{(2k+\tfrac{1}{2})\tan^{-1}\surd s\}\,ds$$

$$=\frac{\surd\pi\,x^{\frac{1}{2}\nu+k-\frac{1}{4}\pm\frac{1}{4}}}{2^{\frac{1}{2}-2k}\Gamma(2k+\frac{1}{2})}\sum_{r=0}^{\infty}\frac{\Gamma(\frac{1}{2}r+k+\frac{1}{2}\mp\frac{1}{4})(-2\surd x)^r}{r!\,\Gamma(\frac{1}{2}r+\nu+k+\frac{1}{2}\pm\frac{1}{4})},\ R(k+\tfrac{1}{2}\nu)>-\tfrac{1}{4}\mp\tfrac{1}{4}$$

When $\nu = -\frac12$ in the first and $\nu = \frac12$ in the second of these integrals, we get

$$\int_0^\infty \begin{matrix}\cos\\ \sin\end{matrix}(2\sqrt{sx})\begin{matrix}\cos\\ \sin\end{matrix}\{(2k+\tfrac12)\tan^{-1}\sqrt{s}\}\frac{ds}{\sqrt{s}\,(1+s)^{k+\frac14}} = \frac{\pi(4x)^{k-\frac14}}{\Gamma(2k+\frac12)}e^{[illegible]}, \quad R(k) > -\tfrac14$$

To apply F(21, 22) we have to take $2k-\frac12 = \lambda = \mp\frac12$. These yield the values of

$$\int_0^\infty s^{-\frac14-\frac12\nu\mp\frac14}\frac{J_\nu(2\sqrt{sx})}{\sqrt{(1+s)}}\begin{matrix}\cosh\\ \sinh\end{matrix}(2m\sinh^{-1}\sqrt{s})\,ds$$

(iii) The integrals obtained by taking $\mu = 2$, $k-m = \frac12\lambda+\frac12$ or $\frac12\lambda$ and using F(5), may be written out by taking $r = 1$ or 0 in the integral

$$\int_0^\infty s^{\frac12(\lambda-\nu-2+r)}e^{-\frac14 s^2}J_\nu(2\sqrt{sx})\,M_{2k-\frac12\lambda+\frac12 r,\ \frac12\lambda-\frac12 r}(\tfrac14 s^2)\,ds$$

$$= \frac{\Gamma(\frac12\lambda-\frac12 r+1)\,2^{\frac12(\lambda+r)-1}\,\Gamma(\lambda-2k+r)}{\sqrt{\pi}\,\Gamma(4k+\nu-2\lambda-r+1)\,x^{-4k-\frac12\nu+2\lambda}}\ {}_1F_3\left(\begin{matrix}2k-\frac12(\lambda-1+r),\ 2k+1-\lambda-r \\ 2k-\lambda+\frac12\nu+\frac34\pm\frac14-\frac12 r,\end{matrix}\ \frac{x^2}{4}\right)$$

$$+ \frac{\Gamma(\lambda+1)\Gamma(2k-\lambda-r)\,x^{\frac12\nu+r}}{2^{\frac12\lambda+1-\frac12 r}\,\Gamma(2k-\frac12\lambda+\frac12-\frac12 r)\,\Gamma(\nu+r+1)}\ {}_1F_3\left(\begin{matrix}\frac12(1+\lambda+r),\ \lambda-2k+1+r, \\ \frac12(\nu+1+r),\ \frac12(\nu+r)+1,\end{matrix}\ \tfrac14 x^2\right), \qquad (4.2)$$

valid when $R(4k-2\lambda+\frac12\nu-\frac14-r) > -1 < R(\lambda)$

(iva) Take $\mu = 3$, and use F(1) when (1) $m = \frac16$, $\lambda = 3k$ or (2) $m = \frac16$, $\lambda = 3k-1$ or (3) $m = \frac13$, $\lambda = 3k-\frac12$. The integrals thus obtained may be written out at length by giving in succession to the set of parameters (A, B) the values (1, 2), (2, 3) and (1, 3) in the formula

$$\int_0^\infty J_\nu(2\sqrt{sx})\,J_{k-\frac13(A+B-3)}\left(\frac{2s^{\frac32}}{3\sqrt3}\right)s^{\frac32 k-1-\frac12\nu+\frac13(A+B)}\,ds = \frac{3^{\frac32 k+\frac12(A+B-1)}}{2\pi x^{1-\frac12\nu}}\left[\frac{\Gamma(\frac13 A-\frac13 B)}{\Gamma(\nu+B)}\times\right.$$

$$x^B\Gamma(k+\tfrac12+\tfrac16 B-\tfrac16 A)\,{}_1F_4\,k+\tfrac12+\tfrac16(B-A),\ 1+\tfrac13(B-A),\ \tfrac13(B+\nu),\ \tfrac13(B+\nu+1),$$

$$\left.\tfrac13(B+\nu+2),\ -\frac{1}{27}x^3\right] + \text{a similar expression with } A \text{ and } B \text{ interchanged}, \qquad (4.3)$$

valid when $R(6k+5) > B+A < 5-R(6k-2\nu)$

(ivb) Take $\mu = 3$ and use F(11) when (i) $\lambda = 3$, $k = \frac34$, $m = \frac14$, or (ii) $\lambda = 1$, $k = \frac14 = m$. The two integrals thus obtained are

$$\int_0^\infty J_\nu(2\sqrt{sx})\,J_{\frac13}\{(\tfrac13 s)^{\frac32}\}\,J_{\frac13\pm\frac13}\{(\tfrac13 s)^{\frac32}\}\,s^{\frac12-\frac12\nu\pm\frac12}\,ds$$

$$= \mp\frac{3^{\frac12}x^{\frac12\nu}(x\sqrt3)^{\pm\frac12}}{2\pi}\left[\frac{\sqrt{\pi}}{\Gamma(\nu+1\pm\frac12)}\,{}_0F_3\left(\frac{\nu+r\pm\frac12}{3},\ {}_{r=1,2,3},\ -\frac{x^3}{27}\right) - \frac{(2x^3)^{\mp\frac12}\sqrt2}{\Gamma(\nu+1\mp1)}\times\right.$$

$$\left.{}_1F_4\left(1,\ 1\mp\tfrac12,\ \tfrac13(\nu\mp1+r),\ {}_{r=1,2,3}\ -\frac{x^3}{27}\right)\right],\quad R(\nu) > \pm\tfrac12$$

Result 5 Consider the relation[11]

$$(1/2x)\exp(-1/2x)I_\nu(1/2x) = pI_\nu(\sqrt{p})K_\nu(\sqrt{p}) \doteq f(p), \text{ which by } \mathbf{F}(11a),$$

$$= \tfrac12 p\{\Gamma(1+\nu)\}^{-1}[\Gamma(-\nu)(\tfrac14 p)^{\nu}{}_1F_2(\tfrac12+\nu,\ 1+\nu, 1+2\nu, p)+\Gamma(\nu){}_1F_2(\tfrac12,\ 1\pm\nu, p)] \quad (5\ 0)$$

The original of $p^{\mu-\lambda}f(p^{-\mu})$ is easily deduced by interpreting term by term Applying Th I it is found that

$$\int_0^\infty J_\lambda^\mu(sw)\exp\left(-\frac{1}{2s}\right)I_\nu\left(\frac{1}{2s}\right)\frac{ds}{s}$$

$$=\frac{\Gamma(-\nu)}{2^{\nu}}\sum_{r=0}^{\infty}\frac{(\frac12+\nu,\ r)w^{\nu+r}}{\Gamma(1+\nu+r)(1+2\nu,\ r)\,r!\,\Gamma\{1+\lambda+\mu(\nu+r)\}}$$

$$+\frac{1}{\nu}\sum_{r=0}^{\infty}\frac{(\frac12,\ r)w^{r}}{(1+\nu,\ r)(1-\nu\ \ r)\,r!\,\Gamma(1+\lambda+\mu r)}, \tag{5}$$

provided that $0<\mu<1$ and $R(2\nu+\lambda)>-\frac32$ in case $\mu=1$, when the integral becomes

$$\int_0^\infty s^{-\frac12\lambda-1}J_\lambda(2\sqrt{sw})\exp\left(-\frac{1}{2s}\right)I_\nu\left(\frac{1}{2s}\right)ds=\frac{2^{-2\nu}\Gamma(-\nu)w^{\nu+\frac12\lambda}}{\Gamma(1+\nu)\Gamma(\nu+\lambda+1)}\times$$

$$\times\,{}_1F_3(\tfrac12+\nu\ \ 1+\nu, 1+2\nu\ \ 1+\nu+\lambda,\ w)+\frac{w^{\frac12\lambda}}{\nu\Gamma(\lambda+1)}\,{}_1F_3(\tfrac12,\ 1\pm\nu, 1+\lambda,\ w)$$

Result 6 Take now the relation[11]

$$x^{-1}J_\nu(x^{-1})=2pJ_\nu(\sqrt{2p})K_\nu(\sqrt{2p})\doteq f(p), \text{ which by } \mathbf{F}(2,\ 12,\ 13),$$

$$=-\sum_{r=0}^{\infty}\frac{\Gamma(\frac12\nu-\frac12 r)(-\frac12 p)^{r+1}}{\Gamma(1+\frac12\nu+\frac12 r)r!}+\frac{2\Gamma(-\nu)(\frac12 p)^{\nu+1}}{\Gamma(1+\nu)}\,{}_0F_3\left(\frac{1+\nu}{2}, 1+\tfrac12\nu, 1+\nu,\ -\frac{1}{16}p^2\right) \quad (6\ 0)$$

Proceeding as before we finally arrive at the formula

$$\int_0^\infty J_\lambda^\mu(sw)J_\nu\left(\frac1s\right)\frac{ds}{s}=\sum_{r=0}^{\infty}\frac{\Gamma(\frac12\nu-\frac12 r)(-\frac12)^r w^r}{2\Gamma(1+\frac12\nu+\frac12 r)r!\,\Gamma(1+\lambda+\mu r)}+\sum_{r=0}^{\infty}\frac{2^\nu\Gamma(-\nu)(\frac14 w)^{\nu+2r}(-)^r}{r!\,(\frac12+\frac12\nu,\ r)(1+\frac12\nu,\ r)}\times$$

$$\times\frac{1}{\Gamma\{1+\lambda+\mu(\nu+2r)\}\,\Gamma(1+\nu+r)}, \tag{6}$$

where $0<\mu<1$ and $R(2\nu+\lambda)>-\frac32$ in case $\mu=1$, when we have

$$\int_0^\infty J_\lambda(2\sqrt{sw})\,J_\nu\left(\frac1s\right)\frac{ds}{s^{1+\frac12\lambda}}=\frac{w^{\frac12\lambda}}{\nu\Gamma(\lambda+1)}\,{}_0F_5\left(\tfrac12\ \ 1\pm\tfrac12\nu, \frac{1+\lambda}{2}, 1+\tfrac12\lambda\ \ -\frac{1}{64}w^2\right)$$

$$+\frac{w^{1+\frac12\lambda}}{(1-\nu^2)\Gamma(\lambda+2)}\,{}_0F_5\left(\tfrac32,\ \tfrac12\pm\tfrac12\nu, \frac{\lambda+3}{2}, 1+\tfrac12\lambda,\ -\frac{w^2}{64}\right)$$

$$+\frac{\Gamma(-\nu)w^{\nu+\frac12\lambda}}{2^\nu\Gamma(1+\nu)\Gamma(1+\lambda+\nu)}\,{}_0F_5\left(\frac{1+\nu}{2}, 1+\tfrac12\nu\ \ 1+\nu, \frac{\lambda+\nu+2}{2}, \frac{\lambda+\nu+1}{2},\ -\frac{w^2}{64}\right) \quad (6\ 1)$$

Result 7 Let us now start with Goldstein's relation[4],

$$x^{-k}\exp(-1/2x)W_{k,m}(1/x) = 2p^{k+\frac{1}{2}}K_{2m}(2\sqrt{p}) = f(p) \tag{7·0}$$

Using F(4) to expand $p^{\mu-\lambda}f(p^{-\mu})$ and interpreting term by term we finally arrive at the formula

$$\int_0^\infty J_\lambda^\mu(sw)s^{-k}e^{-\frac{1}{2s}}W_{k,m}\left(\frac{1}{s}\right)ds = \sum_{r=0}^{\infty}\frac{\Gamma(-2m-r)w^{k-\frac{1}{2}+m+r}(-)^r}{r!\,\Gamma\{1+\lambda+\mu(k-\frac{1}{2}+m+r)\}}+$$

$$+\text{ a similar expression with } -m \text{ written for } m, \tag{7}$$

valid when $0<\mu<1$ and also $R(k+\frac{1}{2}\lambda\pm m)>-\frac{1}{4}$ in case $\mu=1$, when we have

$$\int_0^\infty s^{\frac{1}{2}\lambda-k}\exp\left(-\frac{1}{2s}\right)J_\lambda(2\sqrt{sw})\,W_{k,m}\left(\frac{1}{s}\right)ds = \frac{\Gamma(-2m)w^{\frac{1}{2}\lambda+m+k-\frac{1}{2}}}{\Gamma(\lambda+k+m+\frac{1}{2})}\times$$

$${}_0F_2(1+2m,\lambda+k+m+\tfrac{1}{2},w) + \text{ a similar expression with } -m \text{ written for } m$$

Theorem II. *If $f(p) \doteq \phi(x)$, then provided that the integrals involved converge and $\mu>0$,*

$$x^{-1+\lambda/\mu}\phi(x^{-1/\mu}) \doteq \mu p\int_0^\infty t^{\lambda-1}J_\lambda^\mu(pt^\mu)f(t)\,dt \tag{8·0}$$

Proof By termwise interpretation we have

$$p^{-\lambda}\exp\left(-\frac{a}{p^\mu}\right) = \sum_{r=0}^{\infty}\frac{(-a)^r}{r!\,p^{\lambda+\mu r}} \doteq \sum_{r=0}^{\infty}\frac{(-a)^r x^{\lambda+\mu r}}{r!\,\Gamma(1+\lambda+\mu r)} = x^\lambda J_\lambda^\mu(ax^\mu)$$

Applying Parseval-Goldstein theorem to this relation and the given one, viz $f(p)\doteq\phi(x)$, it follows that

$$\int^\infty y^{-\lambda-1}\exp\left(-\frac{a}{y^\mu}\right)\phi(y)\,dy = \int_0^\infty t^{\lambda-1}J_\lambda^\mu(at^\mu)f(t)\,dt,$$

which on putting $1/x$ for y^μ and p for a may be written as

$$p\int_0^\infty x^{-1+\lambda/\mu}e^{-px}\phi(x^{-1/\mu})dx = \mu p\int_0^\infty f(t)\,t^{\lambda-1}J_\lambda^\mu(pt^\mu)\,dt,$$

whence the theorem For the convergence problem we note that as $t\to0$, $J_\lambda^\mu = O(1)$ and as $t\to\infty$,

$$t^{\lambda-1}J_\lambda^\mu(pt^\mu) = O\left[t^{(\lambda+\frac{1}{2})k-\frac{1}{2}}\exp\left\{k^{-1}p^k\left(\frac{t}{\mu}\right)^{\mu k}\cos\pi k\right\}\right],\ k=1/(\mu+1) \tag{8·01}$$

Whenever it is permissible to evaluate the infinite integral in (8·0) by expanding the Bessel function and integrating term-by-term we get the original of an image function expansible in ascending integral powers of p The termwise integration is justified under the following set of conditions B —

(i) $f(t)$ is bounded and integrable in $(0, \alpha)$ where α may be taken as large as we please,

(ii) the integral $\int_0^\infty t^{\lambda-1} f(t)\, J_\lambda^\mu(-pt^\mu)\, dt$ converges, the asymptotic behaviour of the function J_λ^μ being given in (0 3)

Result 8 Applying the theorem to the relation (7 0), we have

$$x^{-1+(\lambda+k)/\mu} \exp\left(-\tfrac{1}{2}x^{1/\mu}\right) W_{k,m}\left(x^{1/\mu}\right) = 2\mu p \int_0^\infty K_{2m}(2\sqrt{t}) \sum_{r=0}^{\infty} \frac{(-p)^r t^{\mu r+\lambda+k-\frac{1}{2}}}{r!\ \Gamma(1+\lambda+\mu r)}\, dt$$

$$= \mu p \sum_{r=0}^{r} (-p)^r \Gamma_2(\lambda+\mu r+k+\tfrac{1}{2}\pm m)/r!\ \Gamma(1+\lambda+\mu r), \qquad (8)$$

on carrying out termwise integration by means of (1 1)

By virtue of the asymptotic behaviours (0 2) and (0 3) the conditions B are satisfied if $0<\mu<1$ and $R(\lambda+k-|m|)>-\frac{1}{2}$ By A C the result (8) is also valid when $\mu=1$, as may be easily verified [by (1) § 13 45, B F]

PARTICULAR CASES (i) When $\mu=1$ (8) reduces to a known relation

(ii) When $\mu=\frac{1}{2}$, we have from (8) the relation

$$2x^{2\nu-2} e^{-\frac{1}{2}x^2} W_{k,m}(x^2) = \frac{\Gamma_2(\nu\pm m)}{\Gamma(1+\lambda)}\, p\, {}_2F_2\left(\begin{matrix}\nu\pm m,\ \frac{p^2}{4}\\ \lambda+1,\ \frac{1}{2},\ \end{matrix}\right) - \frac{\Gamma_2(\nu\pm m+\frac{1}{2})}{\Gamma(\lambda+\frac{3}{2})}\, p^2 \times$$

$$\times\, {}_2F_2\left(\begin{matrix}\nu\pm m+\frac{1}{2},\ \frac{p^2}{4}\\ \frac{3}{2},\ \lambda+\frac{3}{2},\ \end{matrix}\right),\quad R(\nu\pm m)>0,$$

where $\nu=k+\lambda+\frac{1}{2}$ Taking $k=m+\frac{1}{2}$ and using F(7) this gives the known relation [15c]

$$x^{\lambda-1} \exp(-\tfrac{1}{2}x^2) = \Gamma(\lambda)\, p \exp(\tfrac{1}{4}p^2)\, D_{-\lambda}(p),\ R(\lambda)>0$$

Result 9 Applying the theorem to the relation [11]

$$2p\, J_\nu(2\sqrt{p})\, K_\nu(2\sqrt{p}) = (1/x)\, J_\nu(2/x)$$

we have

$$x^{-1+(\lambda+1)/\mu}\, J_\nu(2x^{1/\mu})$$

$$= 2\mu p \int_0^\infty J_\nu(2\sqrt{t})\, K_\nu(2\sqrt{t}) \left\{ \sum_{r=0}^{\infty} (-p)^r t^{\lambda+\mu r} / r!\ \Gamma(1+\lambda+\mu r) \right\} dt$$

$$= \tfrac{1}{4}\mu p \sum_{r=0}^{\infty} (-p)^r \Gamma\{\tfrac{1}{2}(\lambda+\nu+1+\mu r)\} / r!\ \Gamma\{\tfrac{1}{2}(\nu+1-\lambda-\mu r)\}, \qquad (9)$$

$$0<\mu<1,\ R(-\nu)<R(\lambda+1)>0,$$

on carrying out termwise integration by means of the formula

$$\int_0^\infty x^{l-1} J_\nu(2\sqrt{x}) K_\nu(2\sqrt{x})\, dx = \Gamma(l)\Gamma(\tfrac{1}{2}l+\tfrac{1}{2}\nu)/4\Gamma(\tfrac{1}{2}\nu-\tfrac{1}{2}l+1),\ 0<R(l)>R(-\nu), \qquad (9\ 1)$$

which may be deduced from (i) § 13 45, B F on using F(23)

PARTICULAR CASES (i) Taking $\mu = \frac12 = -\lambda$, we get Howell's relation[6]

$$\sqrt{\pi}\, J_\nu(\tfrac12 x^2) = \Gamma(\nu+\tfrac12)pD_{-\nu-\frac12}(pe^{\frac14\pi i})\, D_{-\nu-\frac12}(pe^{-\frac14\pi i}),\ R(\nu) > -\tfrac12$$

To show this we multiply out the expansions of the $D_{-\nu-\frac12}$'s as given in F(7) and express the product by F(15, 14) as a sum of four ${}_2F_3(\frac14 p^2)$'s This sum put in compact form as a single series assumes the form given in (9)

(ii) By taking $\mu = \frac23$ and $\nu-\lambda = \pm\frac13$ or -1, the image function can be expressed by a Whittaker function For example, taking $\mu = \frac23$ and $\nu=\lambda+\frac13$, the relation (9) becomes

$$x^{\frac12\nu}J_\nu(2x^{\frac12}) = \sum_{r=0}^{\infty}\frac{(-\frac13 p)^{3r+1}}{(\frac23, r)r!}\left[-\frac{\Gamma(\nu+\frac13+r)}{(\frac13, r)\Gamma(\frac23-r)}+\frac{\Gamma(\nu+r+\frac23)p}{\Gamma(\frac13-r)(\frac43, r)}\right] + \text{a series of vanishing terms}$$

$$= \frac{p}{2\pi\sqrt3}\left[\Gamma(\tfrac13)\Gamma(\nu+\tfrac13)\,{}_1F_1\left(\nu+\tfrac13,\, \tfrac23,\, \frac{1}{27}p^3\right)+\right.$$

$$\left.+\Gamma(\nu+\tfrac23)\Gamma(-\tfrac13)\tfrac13 p\,{}_1F_1\left(\nu+\tfrac23,\, \tfrac43,\, \frac{1}{27}p^3\right)\right]$$

$$= (\sqrt3/2\pi)\Gamma(\nu+\tfrac13)\Gamma(\nu+\tfrac23)W_{-\nu,\frac16}(\tfrac{2}{27}p^3)\exp(\tfrac{1}{54}p^3)$$

COROLLARY Take $\mu = 1$ in Th II, then we have

If $f(p) = \phi(x)$, then provided that the integrals involved converge,

$$x^{\lambda-1}\phi(1/x)=p^{1-\frac12\lambda}\int_0^\infty t^{\frac12\lambda-1}J_\lambda(2\sqrt{tp})\, f(t)\,dt \tag{10 0}$$

Result 10. Consider the operational[3] relation*

$$f(p) = \Gamma(\mu+\nu)p^{1-\nu+\rho}/(1+p)^{\rho+\mu} = x^{\nu+\mu-1}\,{}_1F_1(\rho+\mu,\, \mu+\nu,\, -x)$$

$$= x^{\frac12\mu+\frac12\nu-1}e^{-\frac12 x}M_{\rho+\frac12\mu-\frac12\nu,\ \frac12(\mu+\nu-1)}(x) \equiv \phi(x)$$

Applying the corollary we are led to the relation

$$x^{\nu-2}\phi(1/x) = p^{\frac32-\frac12\nu}\Gamma(\mu+\nu)\int_0^\infty t^{-\frac12\nu-\frac12+\rho}J_{\nu-1}(2\sqrt{tp})\,dt/(1+t)^{\rho+\mu}$$

Evaluating the integral by the formula (1) [B F, 434] we finally get

$$x^{-\frac12\mu+\frac12\nu-1}\exp(-1/2x)M_{\rho+\frac12\mu-\frac12\nu,\ \frac12(\mu+\nu-1)}(1/x) \equiv x^{-\mu-1}\,{}_1F_1(\rho+\mu,\, \mu+\nu,\, -1/x)$$

$$= \frac{\Gamma(\mu+\nu)\Gamma(\rho)\Gamma(\mu)}{\Gamma(\rho+\mu)\Gamma(\nu)}p\,{}_1F_2\left(\begin{matrix}\rho, \\ 1-\mu,\, \nu,\end{matrix}\, p\right)+p^{\mu+1}\Gamma(-\mu)\,{}_1F_2\left(\begin{matrix}\rho+\mu \\ 1+\mu,\, \mu+\nu,\end{matrix}\, p\right) \tag{10}$$

The integration is valid under the conditions $R(\rho)>0$ and $R(2\mu+\nu)>-\frac12$, of which the latter may be waived by A C

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* This relation as well as the similar ones in the next two results can easily be obtained for $p>1$ by expanding $f(p)$ in ascending powers of $1/p$ and interpreting term by term The range of validity is extended to $p>0$ by A C

Particular Cases (i) We first construct from (10) Goldstein's relation (7 0) For this we take $\rho = 1$ and put for convenience $k+m-\frac{1}{2}$ for μ and $m-k+\frac{3}{2}$ for ν, then the relation (10) after some simplification becomes

$$\frac{\Gamma(-2m)}{\Gamma(\frac{1}{2}-m-k)}x^{-k}e^{-1/2x}M_{k,m}\left(\frac{1}{x}\right) = \frac{\Gamma(2m)\Gamma(1-2m)}{\Gamma_2(\frac{1}{2}-k\pm m)}p\ {}_1F_2\left(\begin{matrix}1\,,\\ \frac{1}{2}-k\pm m\,,\end{matrix}\ p\right)+$$
$$+p^{k+m+\frac{1}{2}}\ \Gamma(-2m)\,{}_0F_1(2m+1\ ,\ p)$$

Now add to this a similar result obtained by writing $-m$ for m, then the first terms of the images being odd functions of m cancel out and the desired relation follows at once by F(2, 6)

(ii) If we suppose $\rho = \frac{1}{2}\nu$ and $\mu = \frac{1}{2}-\frac{1}{2}\nu$ or $\rho = \frac{1}{2}$ and $\mu = \nu-1$ and use F(8, 11a) we get

$$x^{-\frac{1}{2}-\frac{1}{2}\mu}e^{-1/2x}M_{\frac{1}{2}\mu,\,-\frac{1}{2}\mu}(1/x) = 2\Gamma(1-\mu)p^{1+\mu}I_{-\mu}(\sqrt{p})K_{\mu}(\sqrt{p}),\ R(\mu)<\tfrac{1}{2}\ ,$$

and[11] $\{1/2^{2\mu}\Gamma(\mu+1)\}\ x^{-\frac{1}{2}}e^{-1/2x}M_{0,\ \mu}(1/x) = (1/x)e^{-1/2x}I_{\mu}(1/2x) = 2pI_{\mu}(\sqrt{p})K_{\mu}(\sqrt{p})$

(iii) The relations obtained by taking * (1) $\rho = 1$, $\nu = \frac{1}{2}$, $\lambda = \frac{1}{2}\mu+\frac{1}{4}$ or (2) $\rho = 1$, $\mu = -\frac{1}{2}$, $\lambda = \frac{1}{2}\nu-\frac{3}{2}$ or (3) $\rho = \mu = \frac{1}{2}$, $\lambda = \frac{1}{2}\nu-\frac{1}{2}$ or (4) $\rho = 1-\mu$, $\nu = \frac{1}{2}-\mu$, $\lambda = \frac{1}{2}-\mu$ and using F(1, 3), may be put in the compact form

$$x^{-r\lambda-st}e^{-1/2x}M_{r\lambda+t,\ \lambda+t}(1/x) = \Gamma(s)\Gamma(2\lambda+2-s)\Gamma(\tfrac{1}{2}-\lambda-\lambda r)p^{2t+r\lambda+\frac{1}{2}}\times$$
$$\times\{I_{2\lambda}(2\sqrt{p})-L_{-2\lambda r}(2\sqrt{p})\},$$

where to the set of parameters $(r,\ s,\ t)$ we assign in consecutive succession the values $(1,\ 1,\ 0)$, $(-1,\ 1,\ 0)$, $(-1,\ \frac{1}{2},\ \frac{1}{2})$ and $(1,\ \frac{1}{2},\ \frac{1}{2}-\lambda)$ The last formula is valid only if $R(\lambda)>-\frac{1}{2}$

(iv) In the six cases, viz (1) and (2) $\nu=\rho+\frac{1}{2}$, $2\rho+\mu = 1$ or 2, (3) and (4) $\rho = \frac{1}{2}$, $\nu-\mu = 0$ or 1, (5) and (6) $\rho+\mu = \frac{1}{2}$ and $2\rho-\nu = 1$ or 0, we may use the following special cases of F(11) —

(i) ${}_1F_2(\lambda+\frac{1}{2}\ ,\ \lambda+1,\ 2\lambda+1\ ,\ z^2) = \{2^{\lambda}\Gamma(1+\lambda)z^{-\lambda}I_{\lambda}(z)\}^2$,

(ii) ${}_1F_2(\lambda+\frac{1}{2},\ \lambda+1,\ 2\lambda\ ,\ z^2) = \Gamma(\lambda)\Gamma(1+\lambda)(\frac{1}{2}z)^{1-2\lambda}I_{\lambda-1}(z)I_{\lambda}(z)$,

(iii) ${}_1F_2(\frac{1}{2}\ ,\ \lambda,\ 1-\lambda\ ,\ z^2) = -1+\Gamma(\lambda)\Gamma(1-\lambda)zI_{\lambda-1}(z)I_{\ \lambda}(z)$,

(iv) ${}_1F_2(\frac{1}{2}\ ,\ \lambda,\ 2-\lambda\ ,\ z^2) = -1+2\Gamma(\lambda)\Gamma(2-\lambda)I_{1-\lambda}(z)I_{\lambda-1}(z)$

The last two are obtained by writing out the ${}_2F_3$ in the expanded form and considering the limiting form it assumes when the parameters tend to zero

Result 11 Expanding $f(p)$ in ascending powers of $1/p$ and interpreting term by term we have

$$f(p) = \frac{p^{2\rho-4\nu+3}}{(1+p^2)^{\mu}} = \frac{x^{2\mu-2\rho+4\nu-3}}{\Gamma(2\mu-2\rho+4\nu-2)}\ {}_1F_2\,(\mu\,,\ \mu-\rho+2\nu-\tfrac{3}{4}\pm\tfrac{1}{4},-\tfrac{1}{4}x^2) = \phi(x),$$
$$R\,(\mu+2\nu-\rho)>1$$

Putting λ for $4\nu-2$ and applying the corollary we have

$$x^{4\nu-3}\,\phi\,(1/x) = p^{2-2\nu}\int_0^{\infty}t^{\,2\rho-2\nu+1}J_{4\nu-2}(2\sqrt{tp})\ dt/(1+t^2)^{\mu}$$

---

* The new parameter λ has been introduced for elegance of results

Evaluating the integral by a known formula [B F, p 435] we are led to the relation

$$
\begin{aligned}
&{}_1F_2\left(\mu,\ \mu-\rho+2\nu\pm\tfrac{1}{4}-\tfrac{3}{4},\ -1/4x^2\right)x^{2\rho-2\mu}\\
&=\frac{\Gamma(2\mu+4\nu-2\rho-2)}{2\Gamma(\mu)}\,p\left[\frac{\Gamma(\rho+\frac{1}{2})\Gamma(\mu-\rho-\frac{1}{2})}{\Gamma(4\nu-1)}\times\right.\\
&\qquad\qquad {}_1F_4\left(\rho+\tfrac{1}{2},\ \tfrac{1}{2},\ 2\nu-\tfrac{1}{2},\ 2\nu,\ \rho+\tfrac{3}{2}-\mu,\ -\tfrac{1}{16}p^2\right)\\
&\quad\left.-\left\{\Gamma(\mu-1-\rho)\,\Gamma(\rho+1)/\Gamma(4\nu)\right\}p\ {}_1F_4\left(\rho+1,\ \tfrac{3}{2},\ 2\nu,\ 2\nu+\tfrac{1}{2},\ \rho+2-\mu,\ -\tfrac{1}{16}p^2\right)\right]\\
&+\Gamma(2\rho+1-2\mu)\ p^{2\mu-2\rho}\ {}_1F_4\left(\mu,\ \mu-\rho,\ \mu-\rho+\tfrac{1}{2},\ \mu-\rho+2\nu-\tfrac{3}{4}\pm\tfrac{1}{4},\ -\tfrac{1}{16}p^2\right) \qquad (11)
\end{aligned}
$$

The total conditions under which the foregoing analysis is valid are

$$p>1,\ R(\nu+\mu-\rho)>\tfrac{5}{8},\ R(\rho)>-\tfrac{1}{2},\ R(4\nu-\mu)>1$$

Of these the first two may be waived by A C

Taking $\frac{1}{2}\mu-\frac{1}{4}=2\nu-1=\rho=\frac{1}{2}\lambda$ and using F(2, 12, 13) the relation (6 0) is obtained It is interesting to note that the two hypge functions within the brackets [ ] combine together to become expressible by the single infinite series

$$\sum_{r=0}^{\infty}\Gamma(\rho+\tfrac{1}{2}+\tfrac{1}{2}r)\ \Gamma(\mu-\rho-\tfrac{1}{2}-\tfrac{1}{2}r)\ (-p)^r/r!\ \Gamma(4\nu-1+r)$$

Result 12 Expanding and interpreting as before, we have

$$f(p)=\frac{p^{\mu+\rho-\nu}}{(1+\sqrt{p})^{1+2\mu}}=\sum_{r=0}^{\infty}\frac{(1+2\mu,\ r)\ (-)^r\ x^{\nu+\frac{1}{2}-\rho+\frac{1}{2}r}}{\Gamma(\nu+\frac{3}{2}-\rho+\frac{1}{2}r)\ r!}=\phi(x)$$

The corollary to Th II then yields the relation

$$x^{\nu-1}\phi\ (1/x)=p^{1-\frac{1}{2}\nu}\int_0^{\infty}t^{\mu+\rho-\frac{1}{2}\nu-1}\ J_\nu(2\sqrt{tp})\ dt/(1+\sqrt{t})^{1+2\mu}$$

Evaluating the integral by a known formula [B F, p 436] and expressing $\phi(1/x)$ as a linear combination of two ${}_2F_2$'s we have

$$
\begin{aligned}
\psi(x)&=\frac{x^{\rho-\frac{3}{2}}}{\Gamma(\nu+\frac{3}{2}-\rho)}\,{}_2F_2\left(\begin{matrix}\mu+1,\ \mu+\frac{1}{2},\\ \frac{1}{2},\ \nu+\frac{3}{2}-\rho,\end{matrix}\ \frac{1}{x}\right)-\frac{(2\mu+1)x^{\rho-2}}{\Gamma(\nu-\rho+2)}\,{}_2F_2\left(\begin{matrix}\mu+1,\ \mu+\frac{1}{2},\\ \frac{1}{2},\ \nu+2-\rho,\end{matrix}\ \frac{1}{x}\right)\\
&=\left[\frac{\Gamma(\rho-\frac{1}{2})p^{\frac{1}{2}-\rho}}{\Gamma(\nu-\rho+\frac{3}{2})}\,{}_2F_3\left(\begin{matrix}\mu+\frac{1}{2},\ \mu+1,\\ \frac{1}{2},\ \frac{3}{2}-\rho,\ \frac{3}{2}+\nu-\rho,\end{matrix}\ -p\right)+\right.\\
&\qquad\left.+\frac{\Gamma(\rho-1)(2\mu+1)}{\Gamma(\nu-\rho+2)p^{\rho-2}}\,{}_2F_3\left(\begin{matrix}1+\mu,\ \frac{3}{2}+\mu,\\ \frac{3}{2},\ \nu+2-\rho,\ 2-\rho,\end{matrix}\ -p\right)\right]+\\
&\quad+\frac{2\Gamma(2\rho+2\mu)\Gamma(1-2\rho)}{\Gamma(1+\nu)\Gamma(1+2\mu)}\,p\ {}_2F_3\left(\begin{matrix}\rho+\mu,\ \rho+\mu+\frac{1}{2},\\ \nu+1,\ \rho,\ \rho+\frac{1}{2},\end{matrix}\ -p\right)=g(p),\text{ say} \qquad (12)
\end{aligned}
$$

The various processes involved in the above investigation require that $p>1$, $R(\frac{1}{2}\nu+\frac{3}{4})>R(\rho)>R(-\mu)$ But since $\psi(x)=O(x^{\rho-1})$ as $x\to\infty$ and $O(x^{\rho+\mu-1})$ as $x\to 0$ by B(6) and $g(p)$ is analytic for $p>0$ the final result is valid by A C if only $R(\rho+\mu)>0$

The two terms in [ ] are expressible by the single series

$$\{1/\Gamma(2\mu+1)\}\ \sum_{r=0}^{\infty}\{\Gamma(2\mu+1+r)\Gamma(\rho-\tfrac{1}{2}-\tfrac{1}{2}r)p^{\frac{1}{2}-\rho+\frac{1}{2}r}/\Gamma(\nu-\rho+\tfrac{3}{2}+\tfrac{1}{2}r)r!\}$$

PARTICULAR CASES (i) When $\mu = 0$ or $\nu-\rho-\mu = -\frac{1}{2}$ or -1, $\psi(x)$ is susceptible to F(5)

(ii) Taking $\mu+\rho = \nu+\frac{1}{2}$ or $\nu+1$ and using F(7) we get essentially the same relation, viz,

$$\frac{2^{\mu+\frac{1}{2}}}{\sqrt{\pi}} x^{\rho-\frac{3}{4}} e^{1/2x} D_{-2\mu-1}\left(\sqrt{\frac{2}{x}}\right) = \frac{\Gamma(\rho+\mu)\Gamma(1-2\rho)4^{\rho+\mu}}{\sqrt{\pi}\Gamma(1+2\mu)} p \;\; {}_1F_2\left(\begin{matrix}\rho+\mu, \\ \rho, \rho+\frac{1}{2},\end{matrix} -p\right)$$
$$+\frac{\Gamma(\rho-\frac{1}{2})}{\Gamma(\mu+1)} p^{\frac{3}{2}-\rho} {}_1F_2\left(\begin{matrix}\mu+\frac{1}{2}, \\ \frac{1}{2}, \frac{3}{2}-\rho,\end{matrix} -p\right) - \frac{2\Gamma(\rho-1)}{\Gamma(\frac{1}{2}+\mu)} p^{2-\rho} {}_1F_2\left(\begin{matrix}1+\mu, \\ \frac{3}{2}, 2-\rho,\end{matrix} -p\right),$$
$$R(\rho+\mu)>0 \qquad (12\ 1)$$

(iii) Taking $\mu = 0$ and using F(2, 3) we are led to the relation

$$x^{\rho-\frac{3}{4}} e^{1/x}\mathrm{Erfc}(1/\sqrt{x}) = \tfrac{1}{2}\pi^{\frac{1}{2}} p^{\frac{5}{4}-\frac{1}{2}\rho} \operatorname{cosec} \rho\pi\, [H_{\frac{1}{2}-\rho}(2\sqrt{p})] - Y_{\frac{1}{2}-\rho}(2\sqrt{p})],\ R(\rho)>0$$

Result 13. Using F(3, 1) to expand $f(p)$ and interpreting term by term we have

$$f(p) = 2^{\nu} p^{\frac{1}{2}\nu+\frac{1}{2}}\{I_\nu(1/\sqrt{p}) - L_\nu(1/\sqrt{p})\}$$
$$= \sum_{r=0}^{\infty} \frac{(-\frac{1}{2})^r p^{\frac{1}{2}-\frac{1}{2}r}}{\Gamma(\nu+1+\frac{1}{2}r)\Gamma(1+\frac{1}{2}r)} = \frac{1}{\sqrt{\pi}} \sum_{r=0}^{\infty} \frac{(-)^r x^{\frac{1}{2}r-\frac{1}{2}}}{r!\,\Gamma(\nu+1+\frac{1}{2}r)} = \phi(x)$$

Hence by the corollary to Th II, we find that

$$x^{\nu-\frac{1}{2}} \phi(1/x) = (1/\sqrt{\pi}) \sum_{r=0}^{\infty} (-)^r x^{\nu-1-\frac{1}{2}r}/r!\,\Gamma(\nu+1+\tfrac{1}{2}r)$$
$$= 2^{\nu+1} p^{\frac{5}{4}-\frac{1}{2}\nu} \int_0^\infty t^{2\nu-\frac{1}{2}} J_{\nu-\frac{1}{2}}(2t\sqrt{p})\{I_\nu(1/t) - L_\nu(1/t)\}dt$$
$$= 4p^{1-\nu}K_{2\nu}(2p^{\frac{1}{2}})\, J_\nu(2p^{\frac{1}{2}}), \qquad (13)$$

on using a known integral due to Meijer [10]

Proceeding alternatively with

$$f(p) = p^{-\nu} \exp(-2/\sqrt{p}) \text{ or } f(p) = p^{\frac{1}{2}+\frac{1}{2}\nu}\{I_{\nu+\frac{1}{2}}(1/\sqrt{p}) - L_{\nu+\frac{1}{2}}(1/\sqrt{p})\}$$

and using respectively the integrals [11]

$$\int_0^\infty e^{-a/x} J_\nu(x) dx/x = 2J_\nu(\sqrt{2a})K_\nu(\sqrt{2a})$$

and

$$\int_0^\infty J_{\nu-\frac{1}{2}}(2u)\{I_\nu(z^2/4u) - L_\nu(z^2/4u)\} u^{2\nu-\frac{1}{2}} du = 2^{2\nu} J_{2\nu}(z) K_{2\nu}(z)/2^{3\nu-1}\sqrt{\pi},\ -\tfrac{1}{2}<R(\nu)<1,$$

we arrive at the same result

Result 14 Applying Th I to the relation (9), we obtain by the usual method of procedure the integral

$$\int_0^\infty \frac{J_\beta^\alpha(sw)\, J_\nu(2s^{1/\mu})}{s^{1-(\lambda+1)/\mu}} ds = \tfrac{1}{2}\mu \sum_{r=0}^{\infty} \frac{(-)^r \Gamma\{\frac{1}{2}(\nu+1+\lambda+\mu r)\} w^r}{r!\,\Gamma\{\frac{1}{2}(\nu+1-\lambda-\mu r)\}\Gamma(1+\beta+\alpha r)}, \qquad (14)$$

valid when $0<\mu<1$, $0<\alpha<1$, $R(\lambda+\nu)>-1$, with an additional restriction

$R\{\beta+(1-2\lambda)/\mu\}>-\frac{1}{2}$ in case $\alpha=1$, and then it becomes

$$\int_0^\infty \frac{J_\beta(2\sqrt{sw})J_\nu(2s^{1/\mu})}{s^{1+\frac{1}{2}\beta-(\lambda+1)/\mu}}\,ds=\frac{1}{2}\mu\sum_{r=0}^{\infty}\frac{(-)^r\Gamma\{\frac{1}{2}(\nu+1+\lambda+\mu r)\}w^{r+\frac{1}{2}\beta}}{r!\,\Gamma\{\frac{1}{2}(\nu+1-\lambda-\mu r)\}\Gamma(1+\beta+r)}$$

In the case $\lambda=\nu=\frac{1}{2}\beta$ and $\mu=1$, this gives after some simplification,

$$\int_0^\infty J_{2\nu}(2\sqrt{sw})J_\nu(2s)\,ds=\frac{1}{2}J_\nu(\frac{1}{2}w),\ R(\nu)>-\frac{1}{2}$$

The relation (14) also holds, by A C, when $\alpha=1$, $\mu=2$, provided that (i) when $0<w<1$ or when $w=1$ and $\beta-\nu=$ an odd integer, then $R(\lambda+\nu)>-1<R(\beta-\lambda)$ and (ii) when $w=1$ and $\beta-\nu\neq$ an odd integer, then $R(\lambda+\nu)>-1$, $R(\beta-\lambda)>0$

These cases give the values of the Weber Schafheitlin integrals [B F, §§ 13 4, 13 41] when $a<b$ or $a>b$

Result 15 Termwise interpretation gives the original of

$$f(p)=\sum_{r=0}^{\infty}\Gamma_2(\lambda+\mu r+k+\frac{1}{2}\pm m)(-)^r/\Gamma(1+\lambda+\mu r)r!\ p^{(r+1+\beta)/\alpha-1}$$

Since $p^{\alpha-\beta}f(p^{-\alpha})$ is the image function of the relation (8), Th I leads us to the formula

$$\int_0^\infty J_\beta^\alpha(sw^{1/\mu})\sum_{r=0}^{\infty}\frac{\Gamma_2(\lambda+\mu r+k+\frac{1}{2}\pm m)(-)^r}{\Gamma(1+\lambda+\mu r)\,\Gamma\{(1+\beta+r)/\alpha\}r!}s^{1+(1+\beta+r)/\alpha}\,ds=\frac{e^{\frac{1}{2}w}W_{k,m}(w)}{\mu w^{(\alpha+1)/\mu-\lambda-k}},\ (15)$$

valid when $0<-1+1/\alpha<\mu<1$, $R(2\alpha+\beta)>-1$, $\mu>0$ and in addition

$$R\{\mu(\frac{1}{2}\beta+\frac{1}{4})-(\lambda+k)+|m|)\}<\frac{1}{2},\ \text{if}\ \alpha=1$$

Result 16 Starting with the relation (5 0) and using the expansion of F(8), the corollary to Th II furnishes the integral

$$\int_0^\infty y^{\lambda+1}I_\nu(\surd y)K_\nu(\surd y)\,J_\lambda(2y\surd p)\,dy=\frac{\Gamma(1+\nu+\lambda)4^{-(\nu+1)}}{\Gamma(1+\nu)p^{\nu+1+\frac{1}{2}\lambda}}\,{}_2F_1\left(\begin{matrix}1+\nu+\lambda,\ \nu+\frac{1}{2},\\ 2\nu+1,\end{matrix}\ -\frac{1}{p}\right),$$

$$\frac{1}{2}>R(\lambda)>-1,\ R(\lambda+\nu)>-1.$$

Result 17 Starting with the relation

$$2pK_\nu(\sqrt{2p}\,e^{\frac{1}{4}\pi i})K_\nu(\sqrt{2p}\,e^{-\frac{1}{4}\pi i})=x^{-1}K_\nu(2/x),$$

which can be easily deduced from Macdonald's integral [B F, §13 71] and applying the corollary, we obtain the integral

$$\int_0^\infty y^{1+\lambda}J_\lambda(y\sqrt{2p})K_\nu(ye^{\frac{1}{4}\pi i})K_\nu(ye^{-\frac{1}{4}\pi i})\,dy=2^{\frac{1}{2}\lambda-3}\Gamma(-\nu)\Gamma(1+\nu+\lambda)p^{-(1+\nu+\frac{1}{2}\lambda)}\times$$

$\times\,{}_2F_1(\frac{1}{2}\lambda+\frac{1}{2}\nu+\frac{3}{4}\pm\frac{1}{4},1+\nu,4/p^2)+$a similar exp with $-\nu$ written for ν, $R(\lambda\pm\nu)>-1$

Theorem III *If* $f(p) = h(x)$, $p^{\beta-\alpha}\, h(p^{\beta}) = \phi(x)$ *and* $p^{\beta-\alpha} f(p^{-\beta}) = \psi(x^{\beta})$, *then* $^{-1+\alpha/\beta}\phi(x^{-1/\beta}) = p^{1-\alpha/\beta}\psi(p)$, *provided that the integral* $\int_0^{\infty} h(s)J_{\alpha}^{\beta}(sw)\, ds$ *converges*

PROOF Theorem I applied to the first relation shows that

$$p^{\beta-\alpha} f(p^{-\beta}) = x^{\alpha}\int_0^{\infty} h(s)J_{\alpha}^{\beta}(sx^{\beta})ds = I, \text{ say} \tag{18 0}$$

On comparing this with the third given relation and using Lerch's theorem we conclude that $I = \psi(x^{\beta})$

Now substitute p for x^{β} and t^{β} for s in (18 0), thus

$$p^{1-\alpha/\beta}\, \psi(p) = \beta p\int_0^{\infty} t^{\beta-1}\, h(t^{\beta})\, J_{\alpha}^{\beta}(pt^{\beta})\, dt$$

An application of Th II to the second of the given relations shows that the original of the right side of this equation is $x^{-1+\alpha/\beta}\phi(x^{-1/\beta})$

Hence the theorem It may also be stated in the alternative form

If $\qquad h(p) = \phi(x)$, $x^{\alpha/\beta}\phi(x^{\ 1/\beta}) = \psi(p)$ and $x^{\alpha}\psi(x^{\beta}) = f(p)$,

then provided that the integrals involved converge

$$p^{\ \alpha/\beta} f(p^{-1/\beta}) = x^{\alpha/\beta}h(x^{1/\beta})$$

Result 18 We firstly apply the theorem to the relation (10) in which the image and original are respectively

$$f(p) = \frac{\Gamma(\mu+\nu)\Gamma(\rho)\Gamma(\mu)}{\Gamma(\rho+\mu)\Gamma(\nu)}\, p\, {}_1F_2\left(\begin{matrix}\rho, \\ 1-\mu, \nu,\end{matrix}\ p\right) + p^{\mu+1}\,\Gamma(-\mu)\ {}_1F_2\left(\begin{matrix}\rho+\mu, \\ \mu+1, \mu+1,\end{matrix}\ p\right),$$

and $$h(x) = x^{-\mu-1}\ {}_1F_1(\,\rho+\mu\,;\ \mu+\nu,\ -1/x)$$

Hence by termwise interpretation

$$p^{\beta-\alpha}\, h(p^{\beta}) = \sum_{r=0}^{\infty} (\rho+\mu, r)(-)^{r}\, x^{\alpha+\beta(\mu+r)}/(\mu+\nu, r)r!\ \Gamma\{1+\alpha+\beta(\mu+r)\} = \phi(x),$$

and $$p^{\beta-\alpha} f(p^{-\beta}) = \frac{\Gamma(\mu+\nu)\ \Gamma(\mu)}{\Gamma(\rho+\mu)}\sum_{r=0}^{\infty} \frac{\Gamma(\rho+r)\, x^{\alpha+\beta r}}{r!\,(1-\mu, r)\ \Gamma(\nu+r)\ \Gamma(1+\alpha+\beta r)}$$

$$+\sum_{r=0}^{\infty}\frac{\Gamma(-\mu)\ (\mu+\rho, r)\ x^{(r+\mu)\beta+\alpha}}{(1+\mu, r)\, r!\ \Gamma\{1+\alpha+\beta(\mu+r)\}(\mu+\nu, r)} = \psi\ (x^{\beta})$$

Consequently Th III leads us to the relation

$$\sum_{r=0}^{\infty}\frac{(\rho+\mu, r)(-)^{r}\ x^{-\mu-1-r}}{(\mu+\nu, r)r!\ \Gamma\{1+\alpha+\beta(\mu+r)} = \sum_{r=0}^{\infty}\frac{\Gamma(-\mu)\ (\mu+\rho\ \ r)\ p^{\mu+1+r}}{r!\ (\mu+\nu, r)\ (1+\mu, r)\ \Gamma\{1+\alpha+\beta(\mu+r)\}}$$

$$+\Gamma(\mu)\,\Gamma(\mu+\nu)\ \sum_{r=0}^{\infty}\Gamma(\rho+r)\, p^{r+1}/\Gamma(\nu+r)\, r!\ (1-\mu, r)\ \Gamma(1+\alpha+\beta r)\ \Gamma(\rho+\mu), \tag{18 01}$$

which is valid by A C when $0<\beta\leq 1$ and $R(\rho)>0$ In case $\beta = 1$, there is the additional restriction $R(\mu-\nu+\rho-\alpha)<\frac{1}{2}$

It is obvious that we may now start with the relation (18 01) and again apply Th III, and the process may be repeated indefinitely We would thus obtain the relation

$$\sum_{r=0}^{\infty}\frac{\Gamma(\rho+\mu+r)(-)^r x^{-\mu-1-r}}{\Gamma(\nu+\mu+r)\,r!}G(r) \doteq \Gamma(-\mu)\sum_{r=0}^{\infty}\frac{\Gamma(\rho+\mu+r)G(r)p^{\mu+r+1}}{(\mu+1,\,r)\Gamma(\mu+\nu+r)\,r!}$$

$$+\Gamma(\mu)\sum_{r=0}^{\infty}\frac{\Gamma(\rho+r)G(r-\mu)\,p^{r+1}}{\Gamma(\nu+r)\,(1-\mu,\,r)\,r!}, \qquad (18.02)$$

where $G(r) = \prod_{q=1}^{m}\{\Gamma(a_q+\alpha_q r)\}\Big/\prod_{q=1}^{n}\{\Gamma(b_q+\beta_q r)\}$

Evidently the factor $\Gamma(\rho+\mu+r)$ in the numerator and $\Gamma(\nu+\mu+r)$ in the denominator of the original function of (18.02) may be absorbed* in $G(r)$ without any loss of generality, giving the final simplified form

$$\sum_{r=0}^{\infty}(-)^r x^{-\mu-1-r}\,G(r)/r! = \Gamma(-\mu)\sum_{r=0}^{\infty}p^{1+\mu+r}\,G(r)/(1+\mu,\,r)\,r!+$$

$$+\Gamma(\mu)\sum_{r=0}^{\infty}p^{1+r}\,G(r-\mu)/r!\,(1-\mu,\,r) \qquad (18)$$

Using the formulae of **Table B**, we find that this is valid when

$$-1<\Sigma\beta_q-\Sigma\alpha_q=\sigma\leq 1,\ R(a_q-\mu\alpha_q)>0,\ \beta_q>0<\alpha_q$$

There is the additional restriction $R\{2\mu+\tfrac{1}{2}(n-m)+\Sigma a_q-\Sigma b_q\}<0$ when $\sigma=1$

PARTICULAR CASES If all the α's and β's are unity, the relation (18) becomes

$$x^{-\mu-1}{}_mF_n\begin{pmatrix}a_1, a_2, \ldots, a_m; & -\frac{1}{x}\\ b_1, b_2, \ldots, b_n, & \end{pmatrix} = \Gamma(-\mu)p^{1+\mu}{}_mF_{n+1}\begin{pmatrix}a_1, a_2, \ldots\ldots, a_m, & p\\ b_1, b_2, \ldots, b_n, 1+\mu, & \end{pmatrix}$$

$$+\Gamma(\mu)\{G(-\mu)/G(0)\}\dagger\,p\,{}_mF_{n+1}(a_1-\mu, \ldots, a_m-\mu,\ b_1-\mu, \ldots, b_n-\mu,\ 1-\mu,\ p), \qquad (18.1)$$

where $m-n=0$ or -1 and $R(a_q-\mu)>0$ In case $m-n=-1$, there is the additional restriction $R(2\mu+\tfrac{1}{2}+\Sigma a_q-\Sigma b_q)<0$

(i) Taking $m=2$, $n=3$, $a_2=a_1+\tfrac{1}{2}$, $b_1=2a_1=1+\mu+\nu$, $b_2=1+\mu$ and $b_3=1+\nu$ and using **F(11)** we get the image of $x^{\rho}J_{\mu}(1/\sqrt{x})J_{\nu}(1/\sqrt{x})$

(ii) Taking $m=1=a_1$ and $n=2$, and using F(2, 3) the original may be expressed as a Lomell's or as a Struve's function

(iii) Taking $m=0$, $n=1$ and $b_1=\lambda+1$ and using $F(1)$ we have

$$x^{\frac{1}{2}\lambda-\mu-1}J_{\lambda}(2/\sqrt{x}) \doteq \{\Gamma(-\mu)/\Gamma(1+\lambda)\}p^{1+\mu}{}_0F_2(1+\mu,\,1+\lambda,\,p)$$

$$+\{\Gamma(\mu)/\Gamma(\lambda-\mu+1)\}p\,{}_0F_2(1-\mu,\,1-\mu+\lambda,\,p)$$

It will be noticed that by applying Tricomi's theorem to this relation and interpreting term by term we are led to Hanumant Rao's integral (B F, p 437)

(iv) The special cases of (18.1) when $m=1=n$ have already been worked out in Result 10

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* This purpose is served by supposing $\rho=\nu$ so that the two gammas cancel out The final relation (18) could also be obtained by proceeding with the original $x^{\lambda}e^{-1/x}$ and its image instead of (10)

† See (18.02)

(v) Taking $m=2$, $n=1$, the function ${}_2F_1$ of the original may be made susceptible to the formulae F(17) to F(22) For instance, on taking $a_2=a_1-\frac{1}{2}=\rho+\mu=\frac{1}{2}b_1$ and using F(17) we have

$$2\frac{(4x)^{\rho-1}}{\sqrt{(1+x)}}(\sqrt{1+x}-\sqrt{x})^{2\rho+2\mu-1}=\frac{\Gamma(2\rho)\Gamma(\mu)}{\Gamma(2\rho+\mu)}p\,{}_2F_2\left(\begin{matrix}\rho,\ \rho+\frac{1}{2},\\ 2\rho+\mu,\ 1-\mu,\end{matrix}\ p\right)$$
$$+4\Gamma(-\mu)(\tfrac{1}{4}p)^{\mu+1}{}_2F_2(\rho+\mu,\ \rho+\mu+\tfrac{1}{2},\ 2\rho+2\mu,\ 1+\mu,\ p)$$

(vi) Lastly on taking $n=0$, $m=1$ we obtain the relation (1·4) (i)

Result 19. Adopting exactly the same procedure with the relation (11) as with (10) we have

$$\sum_{r=0}^{\infty}(-)^r G(r)/r!\,x^{1+2\mu+2r}=\sum_{r=0}^{\infty}\{(-)^r/r!\}\,\{\Gamma(-2\mu-2r)G(r)p^{1+2\mu+2r}$$
$$+\tfrac{1}{2}\Gamma(\mu-\tfrac{1}{2}r)G(\tfrac{1}{2}r-\mu)p^{r+1}\},\qquad(19)$$

where $G(r)$ is as defined in (18·02), the conditions of validity being the same as those of (18)

Particular Cases (i) When all the α's and β's are unity, this gives the image of the function

$$x^{-1-2\mu}\,{}_mF_n(a_1,\quad,a_m,b_1,\quad,b_n,-1/x^2)$$

from which the image of

$$x^{\nu}J_{\alpha}(1/x)J_{\beta}(1/x)$$

may be deduced by a use of F(11) and that of

$$x^{\nu}e^{-1/x^2}M_{k,\lambda}(1/x^2)$$

by a use of F(5) In the latter case if we further suppose that $k=\lambda+\frac{1}{2}$ [or simply that $m=0=n$ in the main relation (19)] and use F(10), we get

$$x^{-1-2\mu}\exp(-1/x^2)=2\pi^{3/2}(\tfrac{1}{3}P)^{2+\mu}\operatorname{cosec}2\mu\pi\{J_{-\mu-\frac{1}{3}}(P)\cos\mu\pi$$
$$+J_{\frac{1}{3}-\mu}(P)\sin\mu\pi-J_{\mu-\frac{1}{3}}(P)\},$$

where
$$P=3(\tfrac{1}{2}p)^{2/3}$$

(ii) Taking $m=1$, $n=0$ and using F(16), the relation (19) gives

$$\frac{x^{2\rho}}{(1+x^2)^{\mu+\rho+\frac{1}{2}}}=\frac{\Gamma(\rho+\frac{1}{2})\Gamma(\mu)}{2\Gamma(\mu+\rho+\frac{1}{2})}p\,{}_1F_2\left(\begin{matrix}\rho+\frac{1}{2},\\ \frac{1}{2},\ 1-\mu,\end{matrix}\ -\frac{p^2}{4}\right)$$
$$-\frac{\Gamma(\rho+1)\Gamma(\mu-\frac{1}{2})}{2\Gamma(\rho+\mu+\frac{1}{2})}p^2\,{}_1F_2\left(\begin{matrix}\rho+1,\\ \frac{3}{2},\ \frac{3}{2}-\mu,\end{matrix}\ -\frac{p^2}{4}\right)+$$
$$\Gamma(-2\mu)p^{1+2\mu}{}_1F_2(\mu+\rho+\tfrac{1}{2},\ \mu+\tfrac{1}{2},\ \mu+1,\ -\tfrac{1}{4}p^2)\qquad(19·1)$$

When $\rho=0$, this reduces by F(3, 4) to the relation

$$(1+x^2)^{-\mu-\frac{1}{2}}\doteq\sqrt{\pi}\,\Gamma(\tfrac{1}{2}-\mu)\,(\tfrac{1}{2}p)^{1+\mu}\{H_{-\mu}(p)-Y_{-\mu}(p)\},$$

which was given in the special case $\mu=0$ by Machlachlan[9]

Result 20. The relation (12) gives like (10) or (11), the relation

$$\sum_{r=0}^{\infty}\frac{(-)^r G(r)}{r!\,x^{1+\frac{1}{2}(\mu+r)}}=\sum_{r=0}^{\infty}\frac{(-)^r}{r!}\left\{\Gamma(-\tfrac{1}{2}r-\tfrac{1}{2}\mu)\,G(r)\,p^{1+\frac{1}{2}(\mu+r)}+2\Gamma(\mu-2r)\,G(2r-\mu)\,p^{r+1}\right\}$$

Particular Cases (i) Taking $m=0=n$, we get

$$x^{-1-\frac{1}{2}\mu}\exp(-1/\sqrt{x})=2\operatorname{cosec}\mu\pi(p\pi)^{3/2}(2p)^{-\frac{1}{2}+\frac{1}{2}\mu}\{J_{-\frac{1}{2}\mu,\,-\frac{1}{2}\mu-\frac{1}{2}}(P)$$

$$-\cos(\tfrac{1}{2}\mu\pi)\,J_{\frac{1}{2}\mu,\,-\frac{1}{2}}(P)+\sin\tfrac{1}{2}\mu\pi\,J_{\frac{1}{2}+\frac{1}{2}\mu,\,\frac{1}{2}}(P)\},\ P=3(\tfrac{1}{4}p)^{\frac{1}{3}}$$

(ii) Taking $m=1$, $n=0$ and using **F**(16) we get

$$\frac{x^{\rho+\mu-1}}{(1+\sqrt{x})^{1+2\mu}}=\frac{2\Gamma(1-2\rho)}{\Gamma(1+2\mu)}\Gamma(2\rho+2\mu)p\,{}_2F_2\left(\begin{matrix}\rho+\mu,\ \rho+\mu+\frac{1}{2}\\ \rho,\ \rho+\frac{1}{2},\end{matrix}\ -p\right)+\Gamma(\rho-\tfrac{1}{2})p^{\frac{3}{2}-\rho}\times$$

$${}_2F_2(\mu+\tfrac{1}{2},\ \mu+1,\ \tfrac{1}{2},\ \tfrac{3}{2}-\rho,\ -p)+\Gamma(\rho-1)(1+2\mu)p^{2-\rho}{}_2F_2(\mu+1,\ \mu+\tfrac{3}{2},\ \tfrac{3}{2},\ 2-\rho,\ -p)$$

Other Cases of Relations (18)–(20) There are some interesting cases when the original can be expressed in terms of functions which are not expressible by a single hypergeometric function For instance, by taking $n=0$, $m=1$ and $\alpha_1=\frac{1}{2}$, the originals may be expressed by **F**(7) in terms of the parabolic cylinder function, or again, by taking $m=1=n$ and $a_q=\alpha_q=\beta_q=\frac{1}{2}$, the originals may be expressed by means of F(1, 3) in terms of the function $I_\nu(z)-L_\nu(z)$ Furthermore by making suitable combinations of the original we may obtain the operational images of a variety of other functions like the $W_{k,m}$ and product functions $J_\nu K_\nu$ and $I_\nu K_\nu$.

Theorem IV. *The functions $f(x)$, $h(x)$ and $\phi(x)$ being continuous in $x \geqslant 0$, if $f(p)=h(x)$ and $p^{1-\lambda+\mu}h(p^\mu)=\phi(x)$, then*

$$p^{-1}f(p)=\int_0^\infty x^{-\lambda}\,G_\lambda^\mu(p/x^\mu)\,\phi(x)\,dx, \qquad (21\cdot0)$$

where $G_\lambda^\mu(x)$ is represented by the series

$$\mu\sum_{r=0}^{\infty}\frac{(-x)^r}{r!}\Gamma(\mu r+\lambda)\quad or\quad \sum_{r=0}^{\infty}\frac{(-)^r}{r!}\frac{\Gamma\{(r+\lambda)/\mu\}}{x^{(r+\lambda)/\mu}}\quad or\ the\ function\ \frac{\Gamma(\lambda)}{(1+x)^\lambda}$$

according as $\mu<1$ or >1 or $=1$, provided that $R(\lambda)>0$ and the integral (21·0) converges In case $\phi(x)$ changes sign in $x>0$, the integral

$$\int_0^\infty \phi(x)\,dx$$

also must converge

Proof From the two given operational relations, we have

$$f(p) = p\int_0^\infty e^{-px}h(x)\,dx, \qquad \text{(i)} \qquad \text{and} \qquad p^{1-\lambda+\mu}\,h(p^\mu) = p\int_0^\infty e^{-ps}\phi(s)\,ds \qquad \text{(ii)}$$

Hence

$$f(p) = p\int_0^\infty e^{-px}x^{-1+\lambda/\mu}\,dx\int_0^\infty e^{-sx^{1/\mu}}\phi(s)\,ds$$

$$= p\int_0^\infty \phi(s)\,ds\int_0^\infty x^{-1+\lambda/\mu}\exp\left(-px-sx^{1/\mu}\right)dx$$

on changing the order of integration which is justified by de La Vallée Poussin's theorem[2] For if $\phi(t)$ is of the same sign in $t \geq 0$ then the s-integral converges by virtue of the integral (ii) and the x integral converges if $R(\lambda)>0$ and the repeated integral exists by the last condition However, if $\phi(t)$ changes sign in $t \geq 0$ then the change in the order of integration is justified since then

$$\int_0^t \phi(t)\,dt$$

converges and consequently the s integral and the x integral converge uniformly being multiplied by $\exp(-sx^{1/\mu})$ and e^{-px} [Copson p 115]

The theorem now follows at once on expanding in ascending powers of x the factor e^{-px} when $\mu<1$ and the factor $\exp(-sx^{1/\mu})$ when $\mu>1$ and integrating term by term by means of the formula

$$\int_0^\infty e^{-ax}x^{\mu-1}\,dx = \Gamma(\mu)/a^\mu \quad R(\mu)>0,\ a>0 \qquad (21\ 01)$$

The term-by term integration effected above is easily justified

To settle the convergence problem we observe that as

$$x\to 0,\ G^\mu_\lambda(x) = O(1) \text{ and as } x\to\infty,\ G^\mu_\lambda(x) = O(x^{-\lambda/\mu})$$

It may be of interest to note that the contour integral of Barnes's type for this function $G^\mu_\lambda(z)$ is

$$-\frac{\mu}{2\pi i}\int_C \Gamma(-s)\Gamma(\mu s+\lambda)z^s\,ds,$$

where λ/μ is not a negative integer and $|\arg z| \leq \frac{1}{2}(\mu+1)\pi-\epsilon,\ \epsilon>0$ This shows the interconnection between the two series representing the function In fact the asymptotic expansion for large values of the argument of one series is given by the other

The function $G^\mu_\lambda(x)$ reduces to a parabolic cylinder function when $\mu = \frac{1}{2}$ or 2, we actually have

$$G^{\frac{1}{2}}_\lambda(x) = 2^{-\lambda}\Gamma(2\lambda)D_{-2\lambda}(x/\sqrt{2})e^{\frac{1}{4}x^2},\quad G^2_\lambda(x) = 2\Gamma(\lambda)\,(2x)^{-\frac{1}{2}\lambda}\,e^{1/8x}D_{-\lambda}\,(1/\sqrt{2x})$$

Result 21. Take the relation (9) as $f(p) \doteqdot h(x)$ and obtain the original of $p^{1-\lambda+\mu} h(p^{\mu})$ by termwise interpretation, Th IV and Lerch's theorem then lead us to the integral

$$\int_0^{\infty} \frac{G^{\beta}(px^{-\beta}) J_{\nu}(2x^{1/\mu})}{x^{1+\alpha-(\lambda+1)/\mu}}\, dx = \tfrac{1}{2}\mu \sum_{r=0}^{\infty} \frac{(-)^r \Gamma\{\frac{1}{2}(\nu+1+\lambda+\mu r)\} \Gamma\{(r+\alpha)/\beta\}}{r!\, \Gamma\{\frac{1}{2}(\nu+1-\lambda-\mu r)\} p^{(r+\alpha)/\beta}},$$

valid when $\mu > 0,\ \mu \leq 1-1/\beta,\ R(\lambda+\nu) > -1,\ R(\lambda-\mu\alpha) < \frac{1}{2}$

Result 22 Starting with (1) and using F(5, 6) to expand $f(p)$ for termwise interpretation, the theorem gives the integral

$$\int_0^{\infty} G_{\alpha}^{\beta}(px^{-\beta}) \sum_{r=0}^{\infty} \frac{(-)^r \Gamma_2(r+k+\frac{1}{2}\pm m)}{r!\, \Gamma(1+\lambda+\mu r)}\, x^{\lambda+\mu r-\alpha}\, dx = \sum_{r=0}^{\infty} \frac{(-)^r}{r!} \frac{\Gamma(R_1)}{pR_1} \Gamma(-2m-r) \times$$

$$\Gamma(\tfrac{1}{2}+m+k+r) + \text{a similar expression with } -m \text{ written for } m,$$

where $\beta R_1 \equiv \alpha-\lambda-1+\mu(\frac{1}{2}+m+k+r)$ The formula is valid if $\mu \leq \beta,\ 1 \leq \mu \leq 3$, $R(\lambda) > -1$ and $R\{\mu(k+\frac{1}{2}-|m|)-\lambda+\alpha\} > 1$ There is the additional condition $R(3k-\frac{1}{2}\lambda-\alpha) < \frac{1}{4}$ in case $\mu = 3$

We may similarly use the relation (2) instead of (1)

Corollary Taking $\mu = 1$ in Th IV, we have *If*

$$f(p) = h(x) \textit{ and } p^{2-\lambda} h(p) = \phi(x),$$

then provided that the integral converges

$$f(p) = \Gamma(\lambda) p \int_0^{\infty} (p+x)^{-\lambda} \phi(x)\, dx$$

Result 23 Ramanujan[12] has shown that if

$$J(y) = \int_0^{\infty} y^x\, dx / \Gamma(1+x),$$

then
$$\int_0^{\infty} e^{-sy} J(y)\, dy = 1/s \log s,\ s > 1$$

and
$$J(y) = e^y - \int_0^{\infty} e^{-xy}\, dx / x\{\pi^2+(\log x)^2\}$$

The former integral shows on using the relation $e^x = p/(p-1),\ p > 1$, that

$$h(x) \equiv e^x - J(x) = \frac{p}{p-1} - \frac{1}{\log p} \equiv f(p),\ p > 1,$$

and the latter shows that

$$ph(p) \equiv p\{e^p - J(p)\} = 1/x\{\pi^2+(\log x)^2\} \equiv \phi(x)$$

Hence by the corollary to Th IV and A C, we have

$$\int_0^{\infty} \frac{dx}{x(p+x)\{\pi^2+(\log x)^2\}} = \frac{1}{p-1} - \frac{1}{p \log p},\ R(p) > 0,$$

wherein the right side is, by the theory of limits, to be replaced by $\frac{1}{2}$ when $p = 1$

TABLE A OF FORMULAE

F(1) $$J_\nu(z) = \frac{(\frac12 z)^\nu}{\Gamma(\nu+1)}\, {}_0F_1(\nu+1, -\tfrac14 z^2)$$

$$I_\nu(z) = \frac{(\frac12 z)^\nu}{\Gamma(\nu+1)}\, {}_0F_1(\nu+1, \tfrac14 z^2)$$

F(2) $$K_\nu(z) = \tfrac12\pi \operatorname{cosec} \nu\pi\{I_{-\nu}(z) - I_\nu(z)\}$$

$$= \tfrac12\Gamma(-\nu)(\tfrac12 z)^\nu {}_0F_1(1+\nu, \tfrac14 z^2) + \tfrac12\Gamma(\nu)(\tfrac12 z)^{-\nu}\, {}_0F_1(1-\nu, \tfrac14 z^2)$$

F(3) $$H_\nu(z) = \frac{2(\frac12 z)^{\nu+1}}{\Gamma(\nu+\frac32)\sqrt{\pi}}\, {}_1F_2(1, \tfrac32, \nu+\tfrac32, -\tfrac14 z^2),\ L_\nu(z) = i^{-\nu}\ {}^{1}H_\nu(iz)$$

F(4) $$Y_\nu(z) = J_\nu(z)\cot\nu\pi - J_{-\nu}(z)\operatorname{cosec}\nu\pi$$

F(4a) $$S_{\mu,\nu}(z) = \{(1+\mu)^2-\nu^2\}^{1} z^{\mu+1}\, {}_1F_2(1, \tfrac32+\tfrac12(\mu\pm\nu), -\tfrac14 z^2)$$

F(5) $$M_{k,m}(z) = z^{m+\frac12}e^{-\frac12 z}\, {}_1F_1\left(\begin{matrix}\frac12+m-k, \\ 1+2m,\end{matrix}\ z\right) = z^{m+\frac12}e^{\frac12 z}\, {}_1F_1\left(\begin{matrix}\frac12+m+k, \\ 1+2m,\end{matrix}\ -z\right)$$

F(6) $$W_{k,m}(z) = \frac{\Gamma(-2m)}{\Gamma(\frac12-m-k)}M_{k,m}(z) + \frac{\Gamma(2m)}{\Gamma(\frac12+m-k)}\ M_{k,-m}(z)$$

F(7) $$W_{\frac12\nu+\frac14,\pm\frac14}(z^2) = 2^{-\frac12\nu}\sqrt{z}\, D_\nu(z\sqrt{2}) = \sum_{r=0}^{\infty}\frac{\sqrt{(\pi z)}\ e^{\frac12 z^2}(-2z)^r}{r!\Gamma\{\frac12(1-\nu-r)\}}$$

$$= \sum_{r=0}^{\infty}\frac{(-2z)^r\Gamma(\frac12 r-\frac12\nu)\sqrt{z}}{2^{1+\nu}\Gamma(-\nu)e^{\frac12 z^2}\, r!}$$

F(8) $$e^{\pm z} I_\nu(z) = \frac{(\frac12 z)^\nu}{\Gamma(\nu+1)}\, {}_1F_1(\nu+\tfrac12, 2\nu+1, \pm 2z)$$

F(9) $$(-)^n\, T^n_\nu(z) = \frac{L^\nu_n(z)}{\Gamma(1+n+\nu)} = \sum_{r=0}^{n}\frac{(-z)^r}{(n-r)!\ r!\ \Gamma(1+\nu+r)}$$

$$= \frac{1}{n!\Gamma(\nu+1)}\ {}_1F_1\left(\begin{matrix}-n, \\ 1+\nu,\end{matrix}\ z\right)$$

F(10) $$J_{m,n}(z) = (\tfrac12 z)^{m+n}\frac{1}{\Gamma(m+1)\Gamma(n+1)}\, {}_0F_2\left(m+1\quad n+1, -\frac{1}{27}z^3\right)$$

F(11) $${}_2F_3\left(\begin{matrix}\frac12(\mu+\lambda-1), \frac12(\mu+\lambda), \\ \mu, \lambda, \mu+\lambda-1,\end{matrix}\ -z^2\right) = {}_0F_1(\mu, -\tfrac14 z^2)\times{}_0F_1(\lambda, -\tfrac14 z^2)$$

$$= \Gamma(\mu)\,\Gamma(\lambda)\,(\tfrac12 z)^{-\lambda-\mu+2}\, J_{\mu-1}(z)\, J_{\lambda-1}(z)$$

F(11a) $$2\nu\, I_\nu(z)\, K_\nu(z) = {}_1F_2(\tfrac12, 1+\nu, 1-\nu, z^2)$$

$$- \frac{\Gamma(1-\nu)\,(\frac12 z)^{2\nu}}{\Gamma(1+\nu)}\, {}_1F_2\left(\begin{matrix}\frac12+\nu, \\ 1+\nu, 1+2\nu,\end{matrix}\ z^2\right)$$

F(12) $${}_0F_3\,(\tfrac{1}{2}+\nu,\ 1+\nu,\ 1+2\nu\ ,\ -\tfrac{1}{4}z^4) = {}_0F_1\,(1+2\nu\ ,\ -z^2)\times{}_0F_1\,(1+2\nu\ ,\ z^2)$$
$$= \{\Gamma(1+2\nu)\}^2\, z^{-4\nu}\, I_{2\nu}(2z)\, J_{2\nu}(2z)$$

F(13) $$2\Gamma(\nu)\Gamma(1-\nu)I_{-\nu}(z)J_\nu(z) = \sum_{r=0}^{\infty}\Gamma(\tfrac{1}{2}\nu-\tfrac{1}{2}r)(-\tfrac{1}{4}z^2)^r/r!\ \Gamma(\tfrac{1}{2}\nu+\tfrac{1}{2}r+1)$$
$$= \frac{2}{\nu}\,{}_0F_3\left(\tfrac{1}{2},\ 1+\tfrac{1}{2}\nu\ \ 1-\tfrac{1}{2}\nu\ \ -\frac{1}{64}z^4\right)+\frac{z^2}{1-\nu^2}\,{}_0F_3\left(\tfrac{3}{2},\ \tfrac{3}{2}\pm\tfrac{1}{2}\nu,\ -\frac{1}{64}z^4\right)$$

F(13a) $$I_\nu(z)-L_\nu(z) = \sum_{r=0}^{\infty}(-)^r(\tfrac{1}{2}z)^{r+\nu}/\Gamma(\tfrac{1}{2}r+1+\nu)\Gamma(\tfrac{1}{2}r+1)$$

F(14) $${}_1F_1(\alpha\ \ \rho\ -x)\times{}_1F_1(\alpha,\ \rho\ \ x) = {}_2F_3(\alpha\ \ \rho-\alpha\ ,\ \rho,\ \tfrac{1}{2}\rho,\ \tfrac{1}{2}\rho+\tfrac{1}{2},\ \tfrac{1}{4}x^2)$$

F(15) $${}_1F_1(\alpha,\ \rho\ \ x)\times{}_1F_1(\alpha-\rho+1,\ 2-\rho\ -x)$$
$$= {}_2F_3\left(\alpha-\tfrac{1}{2}\rho+\tfrac{1}{2},\ \tfrac{1}{2}\rho-\alpha+\tfrac{1}{2},\ \tfrac{1}{2},\ \tfrac{1}{2}\rho+\tfrac{1}{2}\ \ \tfrac{3}{2}-\tfrac{1}{2}\rho\ ,\frac{x^2}{4}\right)$$
$$-\{(2\alpha-\rho)(1-\rho)/\rho(2-\rho)\}x\ {}_2F_3(\alpha-\tfrac{1}{2}\rho+1\ \ \tfrac{1}{2}\rho-\alpha+1,\ \tfrac{3}{2}\ \ \tfrac{1}{2}\rho+1,\ 2-\tfrac{1}{2}\rho\ \ \tfrac{1}{4}x^2)$$
$$= \tfrac{1}{2}(\rho-1)\sum_{r=0}^{\infty}\Gamma(\alpha-\tfrac{1}{2}\rho+\tfrac{1}{2}+\tfrac{1}{2}r)\Gamma(\tfrac{1}{2}\rho-\tfrac{1}{2}-\tfrac{1}{2}r)(-x)^r/\Gamma(\alpha-\tfrac{1}{2}\rho+\tfrac{1}{2}-\tfrac{1}{2}r)\Gamma(\tfrac{1}{2}\rho+\tfrac{1}{2}+\tfrac{1}{2}r)r!$$

F(16) $${}_1F_0(a,\ -x) = (1+x)^{-a}$$

F(17) $${}_2F_1(\tfrac{1}{2}+\mu,\ 1+\mu\ ,\ 1+2\mu\ ,\ -x) = (1+x)^{-\frac{1}{2}}\{2x^{-1}(\sqrt{1+x}-1)\}^{2\mu}$$

F(18) $${}_2F_1(\mu\ \ \tfrac{1}{2}+\mu,\ 1+2\mu\ ,\ -x) = \{2x^{-1}(\sqrt{1+x}-1)\}^{2\mu}$$

F(19) $${}_2F_1(\tfrac{1}{2}-\alpha,-\alpha\ ,\ \tfrac{1}{2}\ ,\ -x^2) = (1+x^2)^\alpha\cos\,(2\alpha\tan^{-1}x)$$

F(20) $${}_2F_1(1-\alpha,\ \tfrac{1}{2}-\alpha\ ,\ \tfrac{3}{2},\ -x^2) = \{(1+x^2)^\alpha/2\alpha x\}\sin\,(2\alpha\tan^{-1}x)$$

F(21) $${}_2F_1(\tfrac{1}{2}+\alpha,\ \tfrac{1}{2}-\alpha\ ,\ \tfrac{1}{2},\ -x^2) = (1+x^2)^{-\frac{1}{2}}\cosh\,\{2\alpha\sinh^{-1}x\}$$

F(22) $${}_2F_1(1+\alpha,\ 1-\alpha\ ,\ \tfrac{3}{2},\ -x^2) = (2\alpha x\sqrt{1+x^2})^{-1}\sinh\,\{2\alpha\sinh^{-1}x\}$$

F(23) $${}_2F_1(\alpha,\beta\ ,\ \alpha-\beta+1\ ,\ -1) = \sqrt{\pi}\,\Gamma(\alpha-\beta+1)/2^\alpha\Gamma(\tfrac{1}{2}+\tfrac{1}{2}\alpha)\Gamma(\tfrac{1}{2}\alpha-\beta+1)$$

F(24) $$(-n\ \ r) = (-)^r\Gamma(n+1)/\Gamma(n+1-r)$$

F(25) $$\Gamma(a+nr) = n\ \ \Gamma(a)\left(\frac{a}{n},r\right)\left(\frac{a+1}{n},r\right)\qquad\left(\frac{a+n-1}{n},r\right)$$

TABLE B

ASYMPTOTIC BEHAVIOURS [18] of

$$ {}_pS_q(z) = \sum_{r=0}^{\infty} \frac{\prod_{n=1}^{p} \Gamma(\beta_n+\alpha_n r)(-z)^r}{\prod_{n=1}^{q} \Gamma(\lambda_n+\mu_n r)\, r!} \quad \text{and} \quad J^{\mu}_{\lambda-1}(z) = \sum_{r=0}^{\infty} \frac{(-z)^r}{r!\,\Gamma(\lambda+\mu r)}, $$

where α's and μ's are real and positive and λ's, β's are unrestricted

Notation $\zeta = \arg z$, $\eta = \arg(-z)$, so that $-\pi < \zeta \leq \pi$, $-\pi < \eta \leq \pi$,

$$ h = \prod_{n=1}^{p} (\alpha_n)^{\alpha_n} \Big/ \prod_{n=1}^{q} \mu_n^{\mu_n}, \quad k = 1+\Sigma\mu_n - \Sigma\alpha_n > 0, $$

$$ \theta = \sum_{n=1}^{p} \beta_n - \sum_{n=1}^{q} \lambda_n + \tfrac{1}{2}(q-p), $$ δ, ϵ are small positive numbers,

$$ Z = k(h|z|)^{1/k} e^{i\eta/k}, \quad Z_1 = |Z| e^{i(\zeta+\pi)/k}, \quad Z_2 = |Z| e^{i(\zeta-\pi)/k} $$

$$ I(x) \equiv x^{\theta} e^{x} \left\{ \sum_{m=0}^{M-1} A_m x^{-m} + O(x^{-M}) \right\} $$ M being a +ve integer

$$ J(y) \equiv \sum_{n=1}^{p} \sum_{l=0}^{l_n} P_{n,l}\, y^{-(l+\beta_n)/\alpha_n} + O(y^{-M+\delta}) $$ M, $P_{n,l}$ being some constants

Formulae

B(1) $0 < k-1 = \mu \leq 1$, $|\zeta| \leq \pi-\epsilon$, $\epsilon > 0$ $J^{\mu}_{\lambda-1}(z) \sim I(Z_1) + I(Z_2)$ which is exponentially small if $|\zeta| < \frac{1}{2}\pi(2-k)$

B(2) $|\eta| \leq \pi-\epsilon$, $\epsilon > 0$, $J^{\mu}_{\lambda-1}(z) \sim I(Z)$, which is exponentially large if $|\eta| < \min(\pi, \frac{1}{2}\pi k)$

B(3) $k > 0$, $|\eta| \leq \frac{1}{2}\pi \min(k, 2) - \epsilon$ ${}_pS_q(z) \sim I(Z)$

B(4) $k > 2$, $|\zeta| \leq \pi$ ${}_pS_q(z) \sim I(Z_1) + I(Z_2)$

B(5) $k = 2$, $|\zeta| \leq \pi$ ${}_pS_q(z) \sim I(Z_1) + I(Z_2) + J(z)$

B(6) $0 < k < 2$, $|\zeta| \leq \frac{1}{2}\pi(2-k) - \epsilon$ ${}_pS_q(z) \sim J(z)$

B(7) $0 < k < 2$ $|\eta| \leq \min(\pi, \frac{1}{2}\pi k - \epsilon)$ and $|\zeta| \leq \pi$ ${}_pS_q(z) \sim I(Z) + J(z)$

In particular,

B(8) $$ {}_pF_{p+m-1}\left[\begin{matrix} \beta_1, \beta_2, & \ldots & \beta_p, \\ \lambda_1, \lambda_2, & \ldots & \lambda_{p+m-1}; \end{matrix} \mp \left(\frac{x}{m}\right)^m \right] $$

$$ \sim x^{\theta} \exp\left(x e^{(\frac{1}{2} \pm \frac{1}{2})\pi i/m}\right) + \sum_{n=1}^{p} a_n x^{-m\beta_n} $$

B(9) $$ {}_{p+1}F_p\left(\begin{matrix} \beta_1, \beta_2, & \ldots & \beta_{p+1}, \\ \alpha_1, \alpha_2, & \ldots & , \alpha_p, \end{matrix} -x\right) \sim \sum_{n=1}^{p+1} a_n x^{-\beta_n} $$

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A TIME-HYSTERESIS IN THE CONDUCTIVITY OF BROMINE VAPOUR UNDER SILENT ELECTRIC DISCHARGE

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(*Received July 17, read August 1, 1947.*)

The non-identity of the characteristic curves for the rising and falling applied potential V indicative of hysteresis, is attributed primarily to changes in the gas phase produced by a self-maintained discharge, by the pre-existing electrical fields. A like phenomenon in respect of the time of discharge was observed incidentally during work on the *Joshi*-effect, viz. a photo-variation of the conductivity i in halogens and other gases under electrical discharge (Joshi, 1943). In bromine vapour, this has revealed a marked dependence on V of the 'aging' of i under the discharge and on its discontinuation its progressive recovery. The results have suggested an additional factor, viz. a reversible conditioning of the surface excited under the applied fields.

EXPERIMENTAL

The experimental arrangement and the electrical circuit used were essentially the same as described elsewhere (Deshmukh, 1947). The discharge was produced in the annular space of a Siemens type (glass) ozoniser filled with bromine vapour purified by fractionation over liquid air. The ozoniser was surrounded by a large size glass jacket with a water circulating arrangement (fig. 1) to reduce any temperature fluctuations during a given series of observations.

Single phase alternating current of 50 cycles frequency was obtained from a rotary converter worked off 220 volt D.C. mains. The potential was stepped up by a H.T. transformer. A moderately concentrated salt solution filled in the inner tube and the bath surrounding the outer tube represented the two terminals of the ozoniser (fig. 1). The high tension terminal, i.e. the inner electrode was connected to one of the secondaries of the transformer through a 20,000 ohm Dubilier type stabilising resistance (fig. 1). The potential applied to the ozoniser V, expressed in kilo-volts (r.m.s.) kV, was calculated from a knowledge of the primary potential and the transformer ratio. The discharge current i was measured by a double wave, oxide rectifier type, Cambridge A.C. microammeter (μA in fig. 1) introduced in the low tension part of the ozoniser circuit.

Bromine vapour excited in the range 5–9 kV (50 cycles) at 30°C. showed the occurrence of 'aging', i.e. a time variation of i at a constant applied V. The secondary potential was switched off when i reached a constant minimum stage due to 'aging' and remained unaltered for an appreciably long time. The system was then allowed to stand over for different intervals of *rest period*. The discharge was again switched on at the end of a given *rest-period* and the time-variation of i at the (previous) constant V was observed. These results for one typical series of the 'aging', and also for the influence of *rest period* in restoring i, at the original V, to its pre-'aging' value are shown in fig. 2.

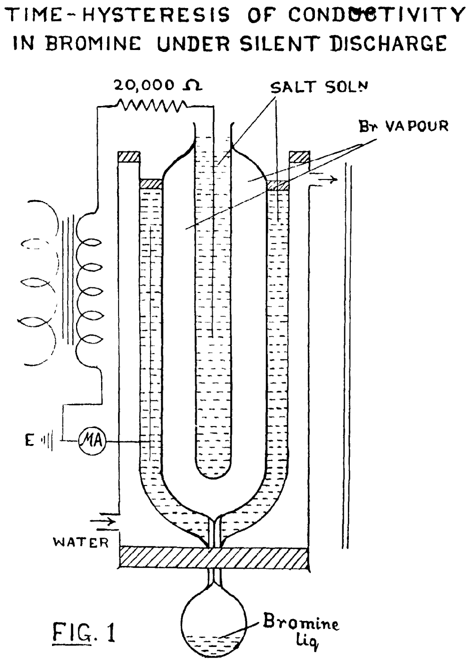
TIME-HYSTERESIS OF CONDUCTIVITY
IN BROMINE UNDER SILENT DISCHARGE
20,000 Ω
SALT SOLN
Br VAPOUR
E
MA
WATER
Bromine
Liq

FIG. 1

DISCUSSION OF RESULTS

The conductivity i in an ozoniser type vessel, excited at a potential V, is given by the following general equation due to Joshi (1947)

$$i = V \Bigg/ jL\Sigma_f + \frac{1}{jC_w\Sigma_f} + \frac{1}{\frac{1}{R_g} + jC_g\Sigma_f} \tag{1}$$

where Σ_f represents the frequencies developed under the discharge, including those of the supply and its harmonics, C_w and C_g are the capacities associated with the combined annular walls and the excited gas respectively, the circuital inductance L is constant. It follows from (1) that the decrease of i, such as shown by curves in fig. 2, may be attributed (*a*) to a rise of R_g, a fictitious resistance in parallel with C_g, $1/R_g$ represents the ionisation current produced in the gas under fields due to V. It depends primarily on the average velocity and the number per unit volume of the ions. Changes in either or/and both these quantities should be sensibly instantaneous, if the reaction leading to the observed 'aging' effect in bromine is restricted to the vapour phase. Alternatively, the decrease of i can originate from a decrease of C_w. This might come about due to adsorption of the gas excited by the discharge. As this is not an instantaneous process the gradual decrease of i as observed is explicable, if the assumption is made that Σ_f does not alter due to 'aging'. Joshi (1946) has postulated the formation of an 'activated adsorption layer (s)' formed, in part, from a wall adsorption of the ions and molecules from the gas phase, and that this layer might be a chief seat of the periodic variation of i in N_2O—H_2 reaction (Joshi & Deshmukh, 1945) and the newly discovered *Joshi* effect Δi, an instantaneous and reversible photo variation of i (*vide infra*). It is now suggested that the time variation of i at constant V, constituting the 'aging' effect in bromine vapour (fig. 2), and like systems, may also be associated with the behaviour of this boundary layer.

The disappearance of a gas under the discharge due to adsorption by the walls has been observed by numerous workers. Plücker, Vegard (1916), and Hippel (1926) established it during exhaustive experiments on cathode sputtering. There is also evidence to show that the adsorption of the gas is pronounced with metals that sputter most copiously. That adsorption, however, is not restricted to conditions favouring metallic sputtering is shown by Hill (1912) from the results obtained under the ring electrodeless discharge through air in glass bulbs. He attributed the adsorption produced under the above conditions to the formation of the oxide of the metal in the glass. Mey found that all gases other than the inert gases combine readily with the cathode coated with an alkali metal. Willows (1901) advanced evidence to show that the metal in the glass was chiefly responsible for the disappearance of the gas under discharge, and that the rate of diminution of pressure due to absorption depended on the kind of glass, being, e.g. greater for soda than for lead or Jena glass. The 'clean-up' of iodine vapour after long continued sparking between platinum electrodes has been attributed by Luedeking (1890), in part, to the formation of alkali iodides or/and iodates as a result of the interaction between the vapour and the glass wall of the discharge tube. Kellner (1902) observed the formation of a sulphur yellow deposit on the container (glass) walls, when bromine vapour was exposed to an electric discharge in ozone tubes. No formation of such a deposit was, however, observed by the authors during these experiments.

It is suggested by Joshi (1946) that the formation under fields intense enough to sustain ionisation by collision, of the 'activated ionic+molecular adsorption layer' leads (or is tantamount) to a dielectric strain in the system, it tends to revert to the normal state on the discontinuation of the discharge, due presumably, to a desorption of the bromine vapour. Like its formation, the break-up (on the discontinuation of the discharge) may well be a time-reaction. This is illustrated by the general result (fig. 2), that if the discharge is discontinued after i has reached the constant minimum in the stationary stage due to 'aging', and is restarted at the original applied potential V after different durations for the *rest period*, the discharge current is restored partially or fully, depending on its magnitude Thus e g, at 8 kV applied to the ozoniser, i decreases due to 'aging' from the initial 83 to 51 μA, i e by about 40% of the original current, the *rest period* necessary for its complete restoration is about 180 minutes (fig 2) At lower applied potentials, e g 7 5 and 6 9 kV, the

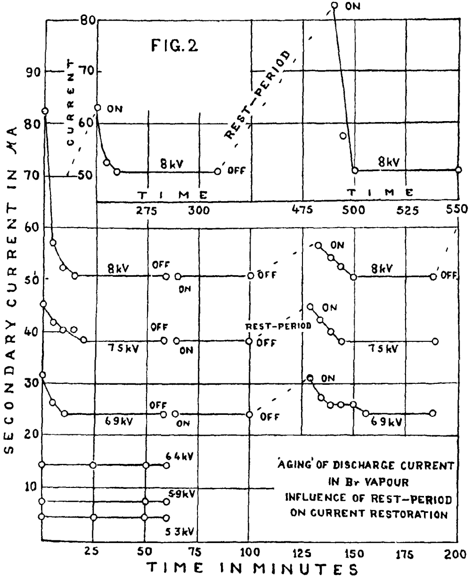
FIG. 2

decrease in the current i observed after 'aging' is about 22% of its initial value From the standpoint of the adsorption layer mechanism, it is significant that excitation at lower potentials entails a smaller *rest-period*, necessary for the complete restoration of the original conductivity, thus e g , at 7 5 and 6 9 kV this *rest period* is about 30 minutes During this, the restoration of i is, however, only 20 per cent, when the original exciting potential is higher, viz 8 kV Over the entire range of the applied potentials mentioned above, the constant minimum value of i produced after 'aging' remained unaltered when the *rest period* was reduced to 5 minutes (fig 2)

Detailed experiments and long exposures showed that the 'aging' effect was not observed at/below 6 4 kV (fig 2), this suggests that Joshi s general finding that a discharge reaction occurs only above its characteristic threshold potential' V_m, is applicable to the wall, gas type reaction, significant for the aging' mechanism Furthermore, the results in fig 2 show that the current decrease due to aging' is much faster at 8 kV than at lower applied potentials, this is also explicable on Joshi's general finding that the velocity of a discharge reaction at a given applied potential V depends on $V—V_m$ (Joshi, 1929, 1939, 1945, 1946)

Earlier results in these Laboratories in the case of the chlorine gas have shown that with a freshly prepared ozoniser, the magnitude of the net *Joshi* effect Δi increases after 'aging' (Deo, 1945) It is, however, interesting that the variation in Δi and the relative effect $\%\Delta i$ with the time of exposure to discharge *decreases* with the increase in the duration of 'aging' Thus e g , Δi and $\%\Delta i$ in chlorine after 10 hours of exposure to discharge were about 8 and 10 respectively, both these quantities decreased further by about 90% when the 'aging' was prolonged to 160 hours It may also be mentioned that in the present investigation, prior to aging', Δi in bromine vapour was markedly high, it became negligibly small after aging for about 60 minutes despite intense irradiation and varied applied potentials Thus e g , at 6 9 kV, before aging', the discharge current in dark and under light, i_D and i_L, was 93 and 85 μA respectively, the corresponding Δi and $\%\Delta i$ were 8 and 9 respectively After 60 minutes exposure to discharge, i_D decreased from the initial 93 to 81 5 μA , and on irradiating the system at this stage i_L was 79 μA, the corresponding Δi and $\%\Delta i$ being 2 and 3 respectively The diminution to a constant minimum in the final stage (fig 2) of i and therefore, of C'_w in equation (1) denotes the attainment of a stationary equilibrium condition A prolonged 'aging beyond this stage, enhances the stability of the boundary layer this should reduce the corresponding Δi as observed, since it is determined by the activated character of the boundary layer, associated with instability Using Geissler, Crookes and like type discharge tubes, Joshi (1945) found that the effect Δi is either not detected or (compared with glass tubes) short lived, if both the electrodes are metallic It is, therefore, suggested that the behaviour of the above adsorption layer of bromine (responsible for both Δi and the 'aging' effect) should be more metallic than that of chlorine The comparatively much greater diminution of Δi with bromine vapour due to long 'aging' is, therefore, to be anticipated This deduction is in accord with like results under silent discharge excitation in the case of iodine, and mercury vapour now being studied in these Laboratories

Our grateful thanks are due to Professor S S Joshi, D Sc (London), F R I C , F N I , for suggesting the problem, advice and kind guidance during the work

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ON A TREATMENT OF IMPERFECT GAS AFTER FERMI'S MODEL (II).

By M Dutta

(Communicated by Prof N R Sen, D Sc , Ph D , F N I)

(*Read August 1, 1947*)

Abstract

In this paper, a statistical theory of imperfect gas has been developed by application of a principle analogous to Pauli's Exclusion Principle and of the method of Fermi The equation of state, giving Van der Waal's equation up to first approximation, has been obtained by taking into account the cohesion and the finite size of molecules simultaneously and by dividing the physical space into different potential layers and cells of volume b equal to the finite extension of molecule, as is done in the discussion of Fermi Dirac Statistics

Introduction

In a previous paper (Dutta, 1947), the equation of state for an imperfect gas has been very simply deduced in two steps In the first step, the effect of the finite dimension of molecules has been calculated by a method similar to that of Fermi in Fermi-Dirac Statistics, by the introduction of a principle quite analogous to Pauli's Principle of Exclusion In the second step, the correction for cohesive forces has been introduced by an increase in the effective volume of the assembly

In considering the effect for finite dimension of molecules, in that paper, the physical space, instead of the phase space, has been divided into cells of dimensions b, the volume of exclusion of each of the molecules, and then, the effect of this exclusion in physical volume due to finite dimensions of rigid molecules has been taken as a new *principle of exclusion*, quite similar to that of Pauli-Fermi in the discussion of Fermi-Dirac Statistics The principle has been formulated thus No cell of the physical space can be occupied by more than one molecule at the same instant'

In the present paper, it is found that, if the method of Fermi is followed more closely, then the effect of finite dimension of molecules and that of cohesion amongst the molecules can be considered in a very simple way in a single step For this, over and above the division of the physical space into cells of small volumes b, the physical space is to be divided into potential energy layers, just as the phase space is divided into energy layers in the deduction of the Fermi-Dirac Statistics Then, as in the previous paper, the distributions in the physical space and in the momenta space have been considered separately The thermodynamic probability for the distribution in the physical space has been calculated after Fermi, and that for the momenta space in the usual classical way, the product of these two gives the total thermodynamic probability

Description of the Assembly

The assembly under consideration consists of N non dissociating and non-associating molecules, each commanding an equal rigid volume of exclusion of magnitude b The assembly is enclosed within an enclosure of volume V

The effect of cohesion between molecules manifests itself as the formation of a very thin surface layer of potential energy slightly greater than that of the interior So, in considering the distribution in the physical space, the physical space will be at first divided into two layers, the interior is of volume V_1 and of potential energy w_1, the surface layer is of volume V_2 and of potential energy w_2, where $w_2 < w_1$ and according to usual assumptions $V_2 << V_1$

After this, the layers will be divided into cells of volume b as usual It is also assumed that $b << V_1, V_2$

Let N_1, N_2 be numbers of molecules in the interior and in the surface layer respectively at an instant Let a_i represent number of molecules with kinetic energy ϵ_i

CALCULATIONS

Then, the thermodynamic probability, calculated in the usual way, becomes

$$W = \left[\left(\frac{V_1}{b}\right)! \Big/ \left\{N_1! \left(\frac{V_1}{b} - N_1\right)!\right\}\right] \times \left[\left(\frac{V_2}{b}\right)! \Big/ \left\{N_2! \left(\frac{V_2}{b} - N_2\right)!\right\}\right] \times [N! / \prod_i a_i!] \qquad (1)$$

To get entropy, this is to be maximised subject to the following restrictions

$$\sum_i a_i \epsilon_i + N_1 W_1 + N_2 W_2 = E$$

$$\sum a_i = N$$

$$N_1 + N_2 = N$$

where E, N, V_1, V_2 are constants

Now,

$$\log W = \left(\frac{V_1}{b}\right) \log \left(\frac{V_1}{b}\right) - N_1 \log N_1 - \left(\frac{V_1}{b} - N_1\right) \log \left(\frac{V_1}{b} - N_1\right)$$

$$+ \left(\frac{V_2}{b}\right) \log \left(\frac{V_2}{b}\right) - N_2 \log N_2 - \left(\frac{V_2}{b} - N_2\right) \log \left(\frac{V_2}{b} - N_2\right)$$

$$+ N \log N - \sum a_i \log a_i$$

The variation of this after use of undetermined multipliers gives,

$$a_i = e^{-\lambda - \mu \epsilon_i}, \qquad (2)$$

$$\left.\begin{aligned} \frac{V_1}{N_1 b} - 1 &= e^{\nu + \mu w_1}, \\ \frac{V_2}{N_2 b} - 1 &= e^{\nu + \mu w_2}, \end{aligned}\right\} \qquad (2a)$$

or

$$N_1 = \frac{V_1}{b} \; \frac{1}{e^{\nu + \mu w_1} + 1}, \qquad (3)$$

$$N_2 = \frac{V_2}{b} \; \frac{1}{e^{\nu + \mu w_2} + 1} \qquad . \qquad . \qquad (4)$$

Now, in gases,

$$1 < < \frac{V_1}{N_1 b}, \frac{V_2}{N_2 b},$$

$$\frac{V_1}{N_1 b} = e^{\nu+\mu w_1}, \frac{V_2}{N_2 b} = e^{\nu+\mu w_2},$$

or

$$N_1 = \frac{V_1}{b} e^{-\nu-\mu w_1}, \; N_2 = \frac{V_2}{b} e^{-\nu-\mu w}$$

This is the well known Boltzmann Law for gases
Now,

$$N = N_1 + N_2 = \frac{V_1}{b} e^{-\nu-\mu w_1} + \frac{V_2}{b} e^{-\nu-\mu w_2},$$

$$e^{-\nu} = \frac{Nb}{V_1 e^{-\mu w_1} + V_2 e^{-\mu w_2}},$$

$$N_1 = N \frac{V_1 e^{-\mu w_1}}{V_1 e^{-\mu w_1} + V_2 e^{-\mu w}},$$

$$N_2 = N \frac{V_2 e^{-\mu w_2}}{V_1 e^{-\mu w_1} + V_2 e^{-\mu w_2}}$$

Since $\frac{V_2}{V_1} < < 1$, so, up to 1st approximation

$$N_1 = N \frac{1}{1 + \frac{V_2}{V_1} e^{-\mu(w_2 - w_1)}} = N\left(1 - \frac{V_2}{V_1} e^{-\mu w}\right), \tag{5}$$

$$N_2 = N \frac{1}{1 + \frac{V_2}{V_1} e^{-\mu(w_2 - w_1)}} = N \frac{V_2}{V_1} e^{-\mu w} \tag{6}$$

where $w = w_2 - w_1 > 0$
Now, as shown in the (Dutta, 1947),

$$N = \Sigma a_i = e^{-\lambda} \iiint e^{-\mu\epsilon_i} \frac{dp_x dp_y dp_z}{h^3/b}$$

or

$$\lambda = \log \left\{ \frac{1}{N} \frac{b}{h^3} \left(\frac{2\pi m}{\mu} \right)^{3/2} \right\}$$

$$\begin{aligned} S = k\Bigg[& \left(\frac{V_1}{b}\right) \log \left(\frac{V_1}{b}\right) - N\left(1 - \frac{V_2}{V_1} e^{-\mu w}\right) \log \left\{ N\left(1 - \frac{V_2}{V_1} e^{-\mu w}\right) \right\} \\ & - \left\{ \frac{V_1}{b} - N\left(1 - \frac{V_2}{V_1} e^{-\mu w}\right) \right\} \log \left\{ \frac{V_1}{b} - N\left(1 - \frac{V_2}{V_1} e^{-\mu w}\right) \right\} \\ & + \left(\frac{V_2}{b}\right) \log \left(\frac{V_2}{b}\right) - \left(N \frac{V_2}{V_1} e^{-\mu w}\right) \log \left(N \frac{V_2}{V_1} e^{-\mu w}\right) \\ & - \left\{ \frac{V_1}{b} - N \frac{V_2}{V_1} e^{-\mu w} \right\} \log \left\{ \frac{V_1}{b} - N \frac{V_2}{V_1} e^{-\mu w} \right\} \\ & + N \log N + N\lambda + \mu E - N\mu w_1 - N_2 \mu w \Bigg] \end{aligned}$$

Now, by well-known thermodynamic relation,

$$\frac{1}{T}=\left(\frac{\partial S}{\partial E}\right)_{\Gamma, V}=\mu k, \quad \text{or } \mu=\frac{1}{kT}$$

On the substitution and the simplification, this gives

$$\begin{aligned} S = Nk\Bigg[& \log\left(\frac{V_1}{b}\right)-\left(1-\frac{V_2}{V_1}e^{-\frac{w}{kT}}\right)\log\left(1-\frac{V_2}{V_1}e^{-\frac{w}{kT}}\right) \\ & -\frac{V_1}{Nb}\left\{1-\frac{Nb}{V_1}\left(1-\frac{V_2}{V_1}e^{-\frac{w}{kT}}\right)\right\}\log\left\{1-\frac{Nb}{V_1}\left(1-\frac{V_2}{V_1}e^{-\frac{w}{kT}}\right)\right\} \\ & -\frac{V_2}{b}\left\{1-\frac{Nb}{V_1}e^{-\frac{w}{kT}}\right\}\log\left\{1-\frac{Nb}{V_1}e^{-\frac{w}{kT}}\right\} \\ & +\log\left\{\left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}}b\right\}-\frac{w_1}{kT}+\frac{E}{kT}\Bigg] \end{aligned} \tag{7}$$

Then,

$$\begin{aligned} \Psi = Nk\Bigg[& \log V_1-\left(1-\frac{V_2}{V_1}e^{-\frac{w}{kT}}\right)\log\left(1-\frac{V_2}{V_1}e^{-\frac{w}{kT}}\right) \\ & -\frac{V_1}{Nb}\left\{1-\frac{Nb}{V_1}\left(1-\frac{V_2}{V_1}e^{-\frac{w}{kT}}\right)\right\}\log\left\{1-\frac{Nb}{V}\left(1-\frac{V_2}{V_1}e^{-\frac{w}{kT}}\right)\right\} \\ & -\frac{V_2}{Nb}\left\{1-\frac{Nb}{V_1}e^{-\frac{w}{kT}}\right\}\log\left\{1-\frac{Nb}{V_1}e^{-\frac{w_1}{kT}}\right\} \\ & +\log\left\{\left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}}\right\}-\frac{w_1}{kT}\Bigg] \end{aligned} \tag{8}$$

Now, if V_2/V, Nb/V, etc., are treated as small quantities of 1st order, and the small quantities of higher order are neglected, then, this becomes

$$\Psi=Nk\left[\log V-\frac{1}{2}\frac{Nb}{V}-\frac{V_2}{V}\left(1-e^{-\frac{w}{kT}}\right)+1+\log\left\{\frac{1}{N}\left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}}\right\}-\frac{w_1}{kT}\right] \tag{9}$$

As

$$P=T\left(\frac{\partial\Psi}{\partial V}\right)_T,$$

so,

$$\begin{aligned} P=NkT\Bigg[& \frac{1}{V}+\frac{1}{2}\frac{Nb}{V^2}+\left\{\frac{V_2}{V^2}-\frac{1}{V}\left(\frac{\partial V_2}{\partial V}\right)_T\right\} \\ & \times\left(1-e^{-\frac{w}{kT}}\right)-\frac{V_2}{V}e^{\frac{w}{kT}}\frac{1}{kT}\left(\frac{\partial w}{\partial V}\right)_T-\frac{1}{kT}\left(\frac{\partial w_1}{\partial V}\right)_T\Bigg], \end{aligned}$$

or,

$$P+\frac{\alpha}{V^2}=\frac{NkT}{V-\frac{1}{2}Nb}=\frac{NkT}{V-\beta}, \qquad . \tag{10}$$

where $\beta = \frac{1}{2}Nb$, and so of usual interpretation, and

$$\alpha = NkT\Bigg[\left(1-e^{-\frac{w}{kT}}\right)\left\{V\left(\frac{\partial V_2}{\partial V}\right)_T - V_2\right\}$$
$$-\frac{V_2}{kT}e^{-\frac{w}{kT}}\;V\;\left(\frac{\partial w}{\partial V}\right)_T - \frac{V^2}{kT}\left(\frac{\partial w_1}{\partial V}\right)_T\Bigg]$$
$$= f(T, V, N) \qquad (11)$$

as it is the case for imperfect gas in general

To fit it with Van der Waal's Equation of State

Now, to show the agreement of the above equation with Van der Waal's Equation, only α is to be shown to be independent of T, V and is proportional to N^2

As usual, it will be assumed that,

$$w_1 = \frac{N_1}{V_1}c_1 = c_1\frac{N}{V}\left\{1+\frac{V_2}{V}\left(1-e^{-\frac{w}{kT}}\right)\right\}$$

$$w = w_2 - w_1 = \frac{N}{V}\left[c_2e^{-\frac{w}{kT}} - c_1 + \frac{V_2}{V}\left\{c_2e^{-\frac{w}{kT}} - c_1\left(1-e^{-\frac{w}{kT}}\right)\right\}\right]$$

where c_1, c_2 are independent of T, V, N and depend only on the nature of molecular attraction and mass of molecules

$$\left(\frac{\delta w}{\delta V}\right)_T = -\frac{N}{V^2}\frac{\left[c_2e^{-\frac{w}{kT}} - c_1 + \left\{\frac{2V_2}{V} - \left(\frac{\delta V_2}{\delta V}\right)_T\right\}\left\{c_2e^{-\frac{w}{kT}} - c_1\left(1-e^{-\frac{w}{kT}}\right)\right\}\right]}{1+\frac{N}{V}\left[c_2\left(1+\frac{V_2}{V}\right) - \frac{V_2}{V}c_1\right]\frac{1}{kT}e^{-\frac{w}{kT}}}$$

$$= -\frac{N}{V_2}g(T, V, N)$$

where $g(T, V, N)$ is a function, finite and with non-zero constant term in Taylor's expansion

$$\alpha = NkT\Bigg[\left(\frac{w}{kT}+\qquad\right)V\left\{\left(\frac{\delta V_2}{\delta V}\right)_T - \frac{V_2}{V}\right\} + \frac{1}{kT}\frac{V_2}{V}\;N\;g(T\;V, N)$$
$$+\frac{Nc_1}{kT}\left\{1+\left(1-e^{-\frac{w}{kT}}\right)\left(\frac{2V_2}{V}+\left(\frac{\delta V_2}{\delta V}\right)_T\right)+\frac{N}{kT}\frac{V_2}{V}g\;(T, V\;N)\,e^{-\frac{w}{kT}}\right\}\Bigg]$$
$$= N^2\Bigg[\left(c_2e^{-\frac{w}{kT}}+c_1\right)\left(1-\frac{w}{2!\,kT}+\qquad\right)\left(\frac{\delta V_2}{\delta V}\right)_T - c_1\left\{1+\left(\frac{w}{kT}\qquad\right)\right.$$
$$\left.\times\left(\frac{\delta V_2}{\delta V}\right)_T\Bigg] \qquad \left(\text{neglecting } \frac{V_2}{V}\right)$$
$$= N^2\left[\{(c_2-c_1)\,a+c_1\}+\frac{w}{kT}h\,(T, V, N)\right]$$
$$= N^2\left[c_1+a\,(c_2-c_1)\right]$$

when the 2nd term is neglected as small quantity of higher order, and V_2 be assumed to be equal to aV

Then, it is found that with these simplifying conditions, the equation of state, obtained here, reduces to that of Van der Waal, if a is independent of T, V, N up to 1st approximation

The equation of state obtained here also reduces to that of Van der Waal, even if V_2 is taken to be $a_1 V^{\frac{2}{3}}$ from consideration of dimension Then,

$$\left(\frac{\delta V_2}{\delta V}\right)_T = \tfrac{2}{3} a_1 V^{-\frac{1}{3}} = \tfrac{2}{3}\frac{V_2}{V},$$

therefore neglecting quantities of order $\frac{V_2}{V}$, $\alpha = N^2 c_1$

CONCLUSION

Here, it is found that the method of Fermi (as used in deduction of Fermi Statistics) is not only suitable for considering effect of volume, but also, can be very simply extended to the cases where there is certain field of force as that of cohesion It is also expected that this method can also be conveniently used in the case of a field of which the gradient is small

It is also to be noted that, up to the approximation retained here, the identical results are obtained for the thermodynamic functions, and, for the distributions in physical or momenta space even if the total thermodynamic probability is calculated as the product of the thermodynamic probabilities for the interior, and for the surface layer considered separately According to this idea the thermodynamic probability is

$$W = \left\{ \frac{\left(\frac{V_1}{b}\right)!}{\left(\frac{V_1}{b} - N_1\right)!} \, \frac{1}{\prod_i b_i!} \right\} \left\{ \frac{\left(\frac{V_2}{b}\right)!}{\left(\frac{V_2}{b} - N_2\right)!} \, \frac{1}{\prod_m c_m!} \right\} \tag{1a}$$

where b_i is number of molecules with kinetic energy ϵ_i and in the interior, and, c_m is that with kinetic energy η_m and in the surface layer, and the restricting equations are

$$\Sigma b_i = N_1, \ \Sigma c_m = N_2 \ \ \Sigma b_i + \Sigma c_m = N$$

$$\Sigma b_i(\epsilon_i + w_1) + \Sigma c_m(\eta_m + w_2) = E$$

where N, E, V_1, V_2 are constants

All expressions for N_1, N_2, b_i, c_m, Ψ, P are same as obtained here

This is not quite unexpected The difference of the present idea with that put forward in this paper may be looked upon as introduction of the hypothetical partition wall in the homogeneous thermodynamic system dividing volumes V_1 and V_2 and so there can be no deviation

The author takes this opportunity to express his gratitude and thanks to Dr S C Kar and Prof N R Sen for helpful discussions and keen interest taken in this paper

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SPECTROGRAPHIC DETERMINATION OF GALLIUM IN INDIAN BAUXITE BY CARBON ARC CATHODE LAYER METHOD

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(Communicated by Prof P B Sarkar, Dr ès Sc , F N I)

(*Received November 3, 1946, read September 5, 1947*)

ABSTRACT

The carbon arc cathode layer method has been applied to the determination of gallium in nine different samples of Indian bauxite by means of the Hilger El Quartz Spectrograph Details of a suitable arc stand designed for very minute arc regulation and efficient operation of the cathode layer method, of Twyman Simeon lens arrangement and of other necessary equipments are described A consistent operating technique is adopted in the preparation of standard comparison spectra of gallium in the range of 05% to 0001% and of the sample spectra The blackness of the Ga line 2943 7Å with proper background correction for each spectrum plate is obtained with a photo electric non recording microphotometer A calibration curve is drawn relating the corrected blackening values and concentrations of gallium obtained from the standard plates Percentages of gallium in the bauxite samples are estimated by interpolating the corrected blackening values of the Ga line in the sample spectra on the calibration curve That the method gives reproducible data is proved by the fact that duplicate spectra of the same standard mixture and also of the same sample are almost identical

INTRODUCTION

Lecoq de Boisbandran discovered gallium spectrographically Ultimate lines of gallium, according to Gramont were situated in the violet, 4033 01 Å and 4172 05Å Bertrand (1941) found that because of the presence of a continuous background in the violet, these ultimate lines were not visible So these lines could not be utilised in the case of pure metallic gallium According to Bardet (1926) the ultraviolet lines 2874 2Å (coincident with an iron line) and 2943 7Å permitted to recognise one by one hundred thousand of gallium in aluminium

Goldschmidt and Peters (1931) determined semi-quantitatively the gallium content in different aluminium containing rocks and minerals with the carbon arc cathode layer method of Mannkopff and Peters (1931) The comparison substance was made of a mixture of quartz with different quantities of Ga_2O_3 and also of a mixture of Ga-free aluminium oxide with known quantities of gallium oxide The contents were given as percentage Ga_2O_3 and each result had a wide range of value, as 0 001—0 01%, nearer to the first figure and was designated as 10^{-3}—(10^{-2}) per cent They also found that the sensitivity of the Ga line was not influenced by the presence of alkali metals (in amounts usually present in rocks and minerals) for which several artificial alkali aluminium silicates were prepared by adding decreasing amounts of Ga_2O_3

Detailed developments and applications of the cathode layer spectrum analysis method developed by Mannkopff and Peters were described by Strock (1936) Lines of the elements which showed the 'Glimmschicht' effect were wedge shaped with maximum intensity near the base and the intensity ratio of two lines in the cathode layer was more constant than in the central arc gas column

In the present investigation as the internal standard line of suitable excitation characteristic was not available near the Ga-line 2943 7Å, the author also followed the 'Comparison Standard Method'

EXPERIMENTAL

(a) *Equipment necessary for the carbon arc cathode layer method*

The dispersion of the Hilger quartz spectrograph (E1) was such that the characteristic groups of Fe lines 3099 97Å, 3100 30Å, 3100 67Å and 3021 07Å, 3020 64Å were quite distinctly resolved

For the efficient operation of the cathode layer method, Gramont's universal arc stand was found inconvenient Quill and Selwood (1929) designed an electrode holder for ordinary arc spectrum analysis, which was more efficient than the Gramont's arc stand An improved arc stand was designed which permitted instantaneous control of the arc position, easy and very minute arc regulations and rapid change of electrodes from the holders Details of the construction are shown in Figure 1

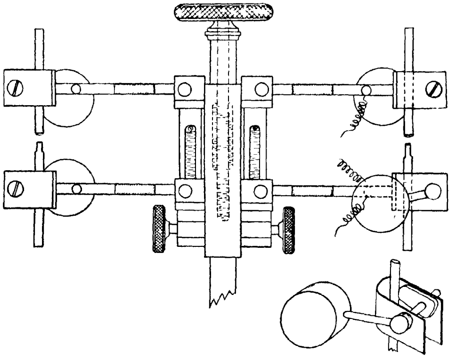
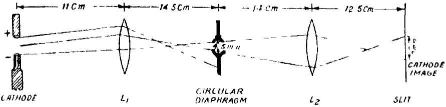


Fig 1

The two systems of holders could be used conveniently one after the other simply by rotating the arrangement through a half cycle without actually changing the hot electrodes The arc stand permitted separate regulation of the cathode, total arc height and rapid horizontal adjustment during the exposure Vertical adjustments of the two electrodes at a time were made by the circular movement of the screw-head at the top and of the lower electrode (cathode) separately by the screw-head at the side attached to rack and pinion arrangement Horizontal adjustment of the two electrodes was made simultaneously by a slight movement of the above screw-head sideways The construction of the special type of clamps suitable for holding the short carbon electrodes and for making easy and rapid change of electrodes is also shown separately in the figure All the necessary insulations were made from ebonite and bakelite blocks

An electric supply of up to 15 amp. at 220 v.d.c. with suitable slide-wire resistance in the circuit was arranged which permitted the arc to be ignited at 2 amp. and gradually increased in steps of 0.5 to 1 amp. to as high as 15 amp.

The lens arrangement recommended by Twyman and Simeon (Strock, 1936) with necessary modification was found very satisfactory for the cathode layer effect. This arrangement is diagrammatically shown in Figure 2.



LENS ARRANGEMENT

FIG. 2

A sharp image of the cathode layer, the region adjoining the cathode was focussed enlarged onto the spectrograph slit. This was achieved by focussing the arc, placed at a distance of 52 cm. from the slit, sharply with a spherical quartz condensing lens on a screen which had an adjustable circular diaphragm. An opening of 4.5 mm. of the diaphragm allowed to pass through it only the edge of the cathode and the adjoining arc column. The light from both the incandescent carbon electrodes was intercepted by the screen which would otherwise make the background heavy. The arc was adjusted so that the sharp cathode image coincided with a mark just over the edge of the diaphragm and the incandescent anode with another mark on the screen 1 cm. below the cathode image. This illuminated opening was focussed enlarged by another spherical quartz condensing lens onto the spectrograph slit. The light passing the circular diaphragm formed nearly an elliptical image over the slit with its major axis 8 mm. in length in the vertical direction. A constant electrode separation was best controlled by means of an enlarged image thrown over a graduated screen by a spherical condensing lens. This graduated screen was placed very near to the spectrograph slit so that the cathode and the anode image could easily be seen over both the screen and the diaphragm simultaneously. Near about one half of the total arc length overlying the cathode layer was used for slit illumination. As the illuminated image was 8 mm. in height over the spectrograph slit instead of Hartmann diaphragm V shaped diaphragm was used. In this lens arrangement any slight variation such as wandering of the arc as usually occurred and slight lateral lack of alignment could not affect spectrum intensity nearly so much as when a single lens was used for slit illumination.

For qualitative analysis, a separate arrangement was made. A second arc stand was placed at right angles to the former one in nearly the same line with the slit. The light of the arc was focussed onto the slit by passing through a spherical quartz lens and a right angle (90°) quartz prism, placed properly near the slit.

Carbon rods of very great purity were produced in this laboratory by the author (Mukherjee, 1947). The starting materials, 'Kino-Homogen' carbon rods, contained above all traces of gallium as impurities. The purified final products were completely free even from the minute traces of gallium.

A cathode with 5.5 mm. deep bore and 3 mm. inner diameter with a wall thickness of 0.7 mm. was found convenient for the analysis. This cathode originally 6.0 mm. in diameter likewise of purified carbon rod, was reduced to 3.7 mm. in diameter over the upper 12 mm. length in which the bore was made in order to make the wall thickness to 0.7 mm. The anode was 5 mm. in diameter with a flat plain surface at the end The thin wall of the cathode was suitable for obtaining a steady arc, with little wandering and the arc was concentrated like a narrow cone of light within the electrodes In each carbon rod the depth of the crater and the diameter of the thin wall was regulated by proper adjustments in a lathe machine in order to obtain the same type of cathode always

(b) *Preparation of comparison standards*

As bauxite is an aluminium containing mineral aluminium oxide was taken as the base substance By qualitative arc spectrographic analysis at 10 amp current it was found that the aluminium hydroxide to be used as base contained traces of gallium Grahame and Seaborg (1938) with the help of artificially produced radio active gallium as an indicator showed that Ga could be extracted successfully by repeated extraction with ether from a 6*N* hydrochloric acid solution with as low a concentration of gallium as 10^{-12} parts per 1 part So gallium chloride was readily extracted from 1 1 hydrochloric acid by ether extraction Nearly 10 gm of aluminium hydroxide was dissolved in 1 1 hydrochloric acid and the process of ether extraction was followed which provided a separation from gallium and many other metals Sufficient quantity of liquor ammonia was added to the solution until it was just alkaline, then thoroughly washed, filtered and ignited The process of ether extraction was repeated several times till the test spectrum showed the complete absence of gallium line

In preparing the standard mixtures, standard solutions of 0 01% and 0 001% gallium prepared as gallium chloride from pure ammonium gallium alum were used For each mixture 100 mg of the gallium free aluminium oxide was taken and measured quantity of the standard solution was added from a microburette In order to obtain an accurate homogeneous mixture, this was diluted with little distilled water, evaporated over an electric heater and then ignited at 800°C over a mecker burner Intimate mixing was achieved by grinding thoroughly in a clean, smooth agate mortar By this means, a series of standard mixtures of the following compositions was prepared 0 05%, 0 02%, 0 01%, 0 007%, 0 005%, 0 003%, 0 001%, 0 0005% and 0 0001%

(c) *Production of the spectra*

3 mg of a standard mixture was accurately weighed in a micro-balance in a 5 c c crucible and thoroughly mixed with 3 mg pure carbon powder obtained during drilling of the carbon rods with a silver rod of 2 5 mm in diameter, whose one end was flat while the other end was like a spoon The mixture was introduced into the boring of the carbon rod with the silver spoon and then slightly compressed with its flat end, proper precaution being taken not to lose even very small amount of this mixture The boring was then loosely packed in the upper portion with some carbon powder The advantage of this packing was that when the arc was first struck due to spurting only a small amount of the carbon powder was lost from the upper portion and when the arc was separated, after about 10 minutes the mixture began to burn along with the carbon steadily

Before an exposure, the position of the anode was accurately adjusted by a preliminary arcing with two pure carbon rods The lower electrode was then replaced by the carbon rod packed with the mixture, the anode remaining in position The arc was ignited at 3 amp when the cathode was allowed to strike steadily at the tip of the anode and then separated slightly The current was raised to 6 amp for few seconds, and the cathode was slowly racked down so that the cathode image coincided with the mark on the screen The current was then raised to 10 amp and the small shutter covering the spectrograph slit was opened, the shutter of the plate holder was kept open long before During the exposure the images of the cathode and anode were observed on the screen and accurately adjusted over the marks. The time of exposure was 150 seconds. The colour of the arc was found to change after about 15 seconds, indicating that the sample began to vaporise in the arc. At about 120 to 130 secs the colour of the arc sharply changes to its original hue, indicating that all the sample was consumed. About 7 mm of the cathode was burnt away during this time. Maintaining a consistent operating technique such as amount of standard mixture, packing of the cathode bore, time of exposure and developing procedure, different standard plates corresponding to the above standard mixtures were prepared.

For the production of the spectra of the different samples of bauxite, the same operating technique was followed. Ilford (H & D 100) plates and metol hydroquinone developer prepared according to Ilford's formula were used. Fixed time of development (120 seconds) and fixation (10 minutes) at 18°C with fresh solution for each plate and uniform rocking procedure during development were followed in order to obtain plates of almost identical character. The effects of the slight spitting in the arc during vaporisation due to the carbons and of other disturbing factors involved such as the rate of vaporisation of the different samples which could not be eliminated were found to be small when the duplicate spectra of the same standard mixture were examined.

(*d*) *Analysis of spectra by photometric measurement*

A comparator of suitable magnification was used for visual inspection of the spectra.

TABLE I

λ in Å	Per cent Ga in standard mixtures				
	.05%	.01%	.005%	.001%	.0001%
2874.24	Coincident with Fe line 2874.17Å				
2943.7	++	++	+	+	→0
2944.18	++	+	→0	0	
3020.5	Coincident with Fe line 3020.5Å				

+ = Strong, →0 = Faint, 0 = Undetectable

The sensitivity of the gallium line 2943.7Å was high and accepted for the analysis especially for lower concentrations. The Ga line 2943.7Å was less sensitive in the region 0.0005% to 0.0001%. The Ga-line 2874.2Å was very sensitive over the ranges of concentration up to 0.0005% in the standard spectra but as this line was coincident with an iron line, it could not be accepted as the comparison line.

The blackness was defined as the difference between the peaks of the galvanometer deflections of the microphotometer for the Ga line 2943.7Å and for the background, taken adjacent to the line. The blackness of the Ga-line was obtained with a photo electric non recording microphotometer and measured in arbitrary units from the scale of the galvanometer. The background correction was applied, subtracting from the galvanometer reading of the background for each spectrum taken at a height of 1 mm from the base adjacent to the Ga-line the reading of the Ga-line. As the spectrum lines in the cathode layer were wedge shaped and more intense towards the base, the photometric measurement of the line 2943.7Å was carried out at a height of 1 mm from the base. In order to make an accurate measurement of the position and height of the Ga line, there were two sliding arrangements at right angles to one another in the plate holder of the microphotometer fitted with millimeter scales.

A calibration curve was then drawn (Fig. 3) with the corrected blackening values of the Ga-line in the different standard spectra plotted as ordinates against percentage of gallium in the corresponding standard mixture Percentage of gallium present in the different samples of bauxite were determined by correlating the corrected blackening values of the Ga-line in sample spectra, in the standard calibration curve

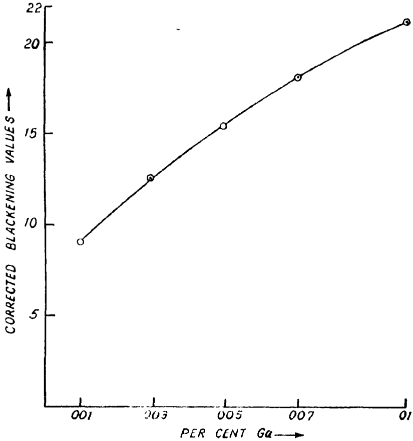


Fig 3

TABLE II

Bauxite No	Locality	Percentage of gallium
291	Salgipat, N Lohardaga, Bihar	0 002
737	N E of Amarkantak, S Rewah	0 0035
788A	N W of Radhanagri	0 0043
797	Yelurgarh Fort, Bombay	0 003
1128	Surganj State	0 007
1142	”	0 008
4696	Katni, Jubbulpore C P	0 0055
5038	1 mile N W of Dhagarvadi, Kolhapur, Bombay	0 0026
5100	Salal, Riasi Tehsil, Kashmir	0 007

DISCUSSION

In the carbon arc cathode layer method, when the cathode was adjusted to keep its image fixed just on the edge of the rectangular diaphragm, the spectrum revealed the presence of a black streak at the bottom which was due to the continuous light from the cathode, (Strock, 1936) As the continuous light masked the lines at the bottom of the spectrum, no accurate photometric measurement was possible at this most sensitive region of the cathode layer spectrum specially at low concentrations of the element In the present investigation, in order to avoid this continuous background at the bottom cathode was constantly regulated to keep its image fixed just over the edge of the diaphragm, so that the incandescent cathode was cut off by the screen while the high sensitivity of the cathode layer method was by no means affected

In Twyman-Simeon lens arrangement rectangular diaphragm was used in Gottingen (Strock, 1936) In the present investigation the author used a circular diaphragm, for which the elliptical light image formed over the spectrograph slit has the central layer of maximum intensity along the vertical direction with the vertical distribution of the light in the cathode layer arc column proportionately enlarged

During preparation of standard mixtures and production of spectra that a consistent operating technique was always maintained was proved by the fact that duplicate spectra of the same standard mixture and also of the same sample were almost identical

ACKNOWLEDGMENT

The author wishes to express his grateful thanks to Prof P B Sarkar for valuable discussions on the subject and for providing laboratory facilities, to the Director, C S I R , for having granted him a research scholarship, and to Prof K Banerjee and Mr R Sen of the Indian Association for the Cultivation of Science for their kind permission to use the microphotometer constructed by them

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A NOTE ON THE PHYSICAL CHARACTERISTICS OF THE PARTIALLY DEGENERATE MODEL STARS OF SMALL MASSES

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(Communicated by Dr. D. S. Kothari, Ph.D., F.N.I.)

(*Received September 24, read 7th November, 1947.*)

ABSTRACT

Certain physical properties of the partially degenerate model stars of masses 5⊙, 4⊙, 2⊙, 1⊙ and 05⊙ are calculated for the central degeneracy ψ_0 0, 2 and 5, using Ware's numerical integration of the differential equation for the partially degenerate standard model. In the calculations account has been taken of both the radiative and the conductive opacities at the centre of the star. The results show that the central stars of the planetar nebulae are in partially degenerate state.

§ INTRODUCTION

It has been suggested by Chandrasekhar (1939) that the supernova outburst may result from the inability of a star of mass greater than M_3 ($M_3 = 5\ 6\ \mu_e^{-2}\odot$) to settle down to the final state of complete degeneracy without getting rid of excess mass. After the excess mass is blown off the star will, for the first time, be in a position to develop a degenerate core. Minkowski's (1942) spectroscopic study of the Crab nebula seems to support Chandrasekhar's suggestion. His analysis of the central star of the Crab nebula—the stellar ramanant of a supernova shows that it is an object of extremely high surface temperature $T_e \sim 5 \times 10^5$°C, high density $\rho \sim 1\ 8 \times 10^6$ gm/c.c., and high luminosity $L/L_\odot \sim 3 \times 10^4$, but it has a small radius $R = 02R_\odot$. It is a white dwarf in which degeneracy is incomplete.

The planetary nebulae, as for example the famous Ring nebula in Lyra, are composed of a shell of tenuous gases at the centre of which is a small but exceedingly hot star. The question whether these planetary nebulae result from supernova explosion is as yet unanswered. If we accept Chandrasekhar's suggestion, then the central stars of the planetary nebulae are on their way to complete degeneracy. It would, therefore, be interesting to investigate certain physical properties of partially degenerate model stars for varying central degeneracy. Our calculations seem to support the suggestion that the planetary nuclei are in partially degenerate state.

§ 2 MATHEMATICAL FORMULAE

Using Ware's (1944) results, the central temperature T_c and the central density ρ_c for a given central degeneracy ψ_0 are respectively given by

$$T_c = 4\ 47 \times 10^9\, \mathcal{F}^{2/3}_{3/2}(\psi_0)\left(\frac{1-\beta}{\beta}\right)^{\frac{1}{2}}, \tag{1}$$

and

$$\rho_c = 2\ 72 \times 10^6 \mu_e \frac{1-\beta}{\beta}\ \mathcal{F}_{3/2}(\psi_0) F_{\frac{1}{2}}(\psi_0), \tag{2}$$

where $F_{\frac{1}{2}}(\psi)$ and $\mathfrak{F}_{3/2}(\psi) = \frac{2}{3}F_{3/2}(\psi)$ are the well-known Fermi-Dirac functions, μ_e the molecular weight of the stellar material and β the ratio of the gas pressure to the total pressure

The general quartic equation is

$$M = -9\,67 \times 10^{33}\mu_e^{-2}\left(\frac{1-\beta}{\beta^4}\right)^{\frac{1}{2}}\left[\mathfrak{F}_{3/2}^{2/3}(\psi_0)\xi_1^2\psi'(\xi_1)\right], \tag{3}$$

where M denotes the mass of the star The value of the quantity in the square bracket on the right hand side of (3) is tabulated in Ware's paper (1944) for $\psi_0 = 0$, 2 and 5 ($\psi_0 \rightarrow \infty$ for complete degeneracy, and $\psi_0 \rightarrow -\infty$ for complete non-degeneracy)

The radius of the configuration is

$$R = \frac{0\,00944}{\mu_e\,[\beta^2(1-\beta)]^{\frac{1}{2}}}\ \xi_1, \tag{4}$$

where the boundary value constant ξ_1 depends on central degeneracy ψ_0 and is tabulated in Ware's paper

§ 3 THE OPACITY, THE MOLECULAR WEIGHT AND THE LUMINOSITY

In the transition region between non degeneracy and degeneracy the radiative opacity k_r is of the same order of magnitude as the conductive opacity k_c The effective opacity k is taken to be

$$\frac{1}{k} = \frac{1}{k_r} + \frac{1}{k_c} \tag{5}$$

The expressions for the radiative and the conductive opacities were taken from Marshak's paper (1940) The concentration X_r of the Russell mixture was assumed to be unity, and the guillotine factor τ was calculated, following Stromgren, for different temperatures For temperature $T > 10^{8}$°C and density $\rho > 10^{8}$ gm /c c , the scattering opacity k_s predominates over the radiative opacity The expression for k_s was taken from Morse's paper (1940)

In order to know the conductive opacity accurately in the transition region, it is necessary to know k_c both on the non-degenerate and the degenerate side For a given temperature T, Log k_c is plotted against ψ_0 and the value is then read from the graph Curves of this type were plotted for different temperatures (as has been done by Marshak, 1940)

In all our calculations we have used the value of μ at the centre of the model star Assuming $\mu = 2$, as a first approximation, ρ_c and T_c are known from equations (1) and (2) Knowing ρ_c the value of μ at the centre is calculated following Marshak (1940)

In calculating the luminosity of the model stars we use Eddington's mass luminosity relation, in which for $\bar{k}$ we substitute the effective opacity at the centre and $\bar{\eta}$ is taken to be equal to unity (since the energy generation is due to gravitation)

§ 4 The results of our calculations for the model stars of masses 5⊙, 4⊙, 2⊙, 1⊙ and 05⊙ are given in the tables below for three different values of $\psi_0 = 0$, 2 and 5 It may be pointed out here that our calculated values for the surface temperature T_e and the luminosity $L/L_{\odot}$ will be higher than the actual values which the model stars would have for a given central degeneracy, since we have taken the values of μ and $\bar{k}$ at the centre of the star

TABLE 1

$\psi_0 = 0$

Mass M/⊙	Log $(1-\beta_c)$	Log ρ_c	Log T_c	Log k_r *	Log k_c	Log $L/L_{\odot}$	Log $R/R_{\odot}$	T_e
5	$\bar{2}$·2126	4·73	8·38	$\bar{1}$·46		2·57	0·079	90,030
4	$\bar{2}$·1484	4·70	8·34	$\bar{1}$·47		2·40	0·075	83,660
2	$\bar{3}$·6802	4·26	8·03	$\bar{1}$·52		1·58	0·083	49,580
1	$\bar{3}$·3294	3·98	7·79	0·43		0·02	0·082	20,300

* Wherever k_r is unimportant compared to k_c, the latter is used

TABLE 2

$\psi_0 = 2$

Mass M/⊙	Log $(1-\beta_c)$	Log ρ_c	Log T_c	Log k_r	Log k_c	Log $L/L_{\odot}$	Log $R/R_{\odot}$	T_e
5	$\bar{3}$·6128	5·31	8·44	$\bar{1}$·45	0·52	2·02	0·041	91,110
4	$\bar{3}$·4190	5·12	8·31	$\bar{1}$·48	0·68	1·69	0·044	72,610
2	$\bar{4}$·9982	4·75	8·03	$\bar{1}$·52	0·88	0·92	0·047	45,300
1	$\bar{4}$·6570	4·47	7·80	0·44	1·08	$\bar{1}$·43	0·046	19,400
0·5	$\bar{4}$·2034	4·05	7·50	1·03	1·44	$\bar{2}$·14	0·050	8,831

TABLE 3

$\psi_0 = 5$

Mass M/⊙	Log $(1-\beta_c)$	Log ρ_c	Log T_c	Log k_r	Log k_c	Log $L/L_{\odot}$	Log $R/R_{\odot}$	T_e
5	$\bar{4}$·8902	5·79	8·42	$\bar{1}$·45	$\bar{1}$·64	1·48	0·023	89,540
4	$\bar{4}$·6964	5·59	8·29	$\bar{1}$·47	$\bar{1}$·96	1·06	0·025	67,660
·2	$\bar{4}$·0942	4·99	7·89	$\bar{1}$·58	0·24	$\bar{1}$·50	0·031	24,550
1	$\bar{5}$·7500	4·71	7·66	0·87	0·48	$\bar{2}$·54	0·03[illegible]	14,240
0·5	$\bar{5}$·3660	4·38	7·41	1·39	0·64	$\bar{3}$·62	0·031	8,318

§ 5. CONCLUSION

The results tabulated in § 4 show that our model stars are extreme objects of high density, high surface and central temperatures and high luminosity but have small radii. They may be classified as blue dwarfs. From his spectroscopic study of a large number of planetary nebulae, Page (1942) estimates the surface temperature of the central stars to lie between 20,000 and 100,000. Though the masses of the planetary nuclei are not known accurately it is reasonable to assume them to lie between 5⊙ and 1○ (certainly much less than the mass of the central star of the Crab nebula, which has been estimated by Minkowski to [illegible] ⊙) The planetary nuclei are no doubt extreme objects, but are milder than the central star of the Crab nebula It would not be unreasonable to say that the preceding tables are a fair representation of the physical characteristics of the planetary nuclei under varying central degeneracy

It is a pleasure to express my thanks to Prof D S Kothari for his very kind interest in this work

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APRIL, 1948

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23 NOV 1948

STUDIES IN THE EMBRYOLOGY OF *ANISOMELES INDICA* O KZE AND *LEONURUS SIBIRICUS* LINN

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(Communicated by Dr I Banerji, D Sc)

(*Received September 1, 1946, read August 4, 1947*)

Since the beginning of this century a considerable amount of work has been done on various genera of Labiatae The earlier literature on the morphology and embryology have been reviewed by Schnarf (1931) During the last few years a number of important papers have been published Laws (1930) described the embryology of *Lavandula* Ruttle (1931, 1932) investigated the embryo sac development of five species of *Mentha* and the embryo-sac and seed development of *Lycopus europaeus* Junell (1934) has given an account of the structure and morphology of the gynoecium in some genera of Verbenaceae and Labiatae He also studied the development of seed in some 'verbenoid Labiatae' Recently Bushnell (1936*a*) has studied the development of ovule and macrogametophyte of *Monarda fistulosa, M didyma, M punctata* and *Nepeta cataria* She found all of them similar except for minor differences Carlson and Stuart (1936) have investigated the development of spores and gametophytes of six New World species of *Salvia* The most important paper in recent years is that of Junell (1937) whose extensive study comprises nineteen genera and twenty-four species belonging to the three sub-families Lavanduloideae, Stachyoideae and Ocimoideae Though he has attempted to generalise the stages of endosperm formation, his account lacks in detailed description of all the individual species

In India, Narasimha Murthy (1940) has described the embryology of three species of *Ocimum*. The same author has subsequently published two short notes on the endosperm formation in *Leucas aspera* (1941) and in two species of *Anisomeles* viz , *A malabarica* and *A indica* (1942)

The present investigation was undertaken before the publication of Narasimha Murthy's note (1942) on *Anisomeles* and a detailed and critical study has revealed fundamental differences with the latter As regards *Leonurus*, Billings (1909) and Junell (1937) have recorded a few mature embryological stages in *Leonurus cardiaca*

MATERIALS AND METHODS

Flower buds of varying stages of development and developing seeds of *Anisomeles* and *Leonurus* both of which were found growing in waste places and fields of Ballygunge, Calcutta, were fixed in field between 10 a m to 3 p m on bright days Allen's modified Bouin's and Nawaschin's fluids were employed as fixatives To facilitate proper fixation an exhaust pump was used at the time of fixing Nawaschin's fluid gave better results for all stages of *Anisomeles* while post fertilisation stages in *Leonurus* came out better in Allen's modified Bouin's fluid Materials were dehydrated, cleared and embedded in paraffin in the customary ways In case of ovaries containing post-fertilisation stages, clearing was done in ceder-wood oil Sections were cut 8μ–30μ thick depending on the stage required for study Heidenhain's iron-alum haematoxylin and Newton's Iodine Gentian Violet were used as stains

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OBSERVATIONS

The Development of the Floral Parts —The development of the floral parts has been studied in detail in *Anisomeles indica* As the stages of development in *Leonurus* were found to conform to that of *Anisomeles*, the description of latter species is alone given here

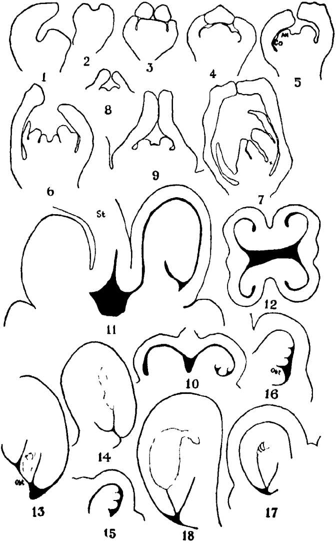
The primordium of the flower is at first visible in the axil of a bracteole as a domeshaped protuberance (Fig 1) The sepals are the first members to appear as prominences from the sides of the dome (Fig 2) As the calyx grows, the central mass of cells (the primordium) again becomes broad and completely covered up by the sepals (Fig 3) The staminal primordia then arise from the base of the sepals which are pushed out (Fig 4) Soon after this the primordia of the petals emerge from the dorsal surface of the stamens (Fig 5) The central primordium still remains undifferentiated, it becomes wavy in appearance when the petals have just overgrown the anthers From this central portion the carpellary primordia arise (Fig 6)

It appears from above that the order of development of the floral parts is sepals, stamens, petals and carpels Thus there is alternately acropetal and basipetal successions of the different cycles, and this is a deviation from the general order of development of cyclic flowers in which the cycles appear mostly in acropetal succession Such deviations are reported in other families like Rosaceae, Compositae, Dipsacaceae, Valerianaceae, Rubiaceae and Cruciferae The order of appearance of cycles noted in *Anisomeles* has also been observed in various members of Scrophulariaceae by Schertz (1919), Srinath (1940), Srinivasan (1940) and by Krishna Iyengar (1937-40) Webb (1902) has observed in *Astilbe* that the order of succession of the floral parts is sepals, inner stamens, carpels, outer stamens and petals The appearance of petals on dorsal surface of stamens is also reported in Primulaceae by Pfeffer (1872) That the petals and stamens have a common origin one appearing a little later than the other is evident when a later stage is examined (Fig 7)

The primordia of carpels come out at the base of the stamens, after the latter have been overgrown by the petals In longitudinal section they appear as two protuberances leaving a depressed area at the centre (Fig 6) The origin of the carpels is thus lateral Examination of later stages shows that these really form the wall of the ovary By further growth of the wall and a simultaneous broadening of the central portion, the cavity of the ovary is formed When the wall meets at the tip the ovules are seen to arise on the broadened central portion at the base of the ovarian wall (Fig 8) A transverse section of the ovary shows that the ovules arise on the placenta formed at the margin of the two united carpels (Fig 12) The placental cushion thus takes its origin as a united structure, the two ovules growing on the two sides of it in diametrically opposite directions The ovarian wall, in the meantime, join at the centre producing the ovarian cavity The united carpels continue their growth upwards to form the style and stigma (Fig 9)

Formation of the Gynobasic style and lobing of the Ovary —After the style has grown to a certain length and the ovules enlarged in size (before the inception of the archesporial cell), the basal portion of the former begins to grow downwards in between the ovules (Fig 10) This process continues *pari passu* with the growth of the ovules, and ultimately at the four-nucleate stage of the gametophyte it reaches the basal portion, dividing the ovary into four lobes from inside each lobe containing one ovule (Fig 11) Along with this process the external surface of the ovary opposite to the placental tissue inside becomes involuted gradually towards the centre of the ovary (Fig 12) and thus completes the ovarian lobing into four bits At this stage the inner mass of tissue at the mid-rib of the carpels (the 'sterile carpels' of Saunders, 1939) also grows towards each other and form a secondary septum. The style does not fuse at the base but an empty space is left there It follows from the foregoing facts that the placentation is not wholly axile in nature but it is a combined form of axile and parietal placentation.

The general idea of the axile nature of the placentation (cf Rendle (1935)) in this family is thus not supported by the evidence obtained by a study of the organogeny in the two species



FIGS 1–14 *Anisomeles indica* Figs 1–7 —Stages in the development of floral organs × 38 CO petals, AN—stamens, X—point of common origin of petals and stamen. Figs 8–11.—Stages in the ontogeny of the gynoecium, growth and curvature of the ovules and the formation of the gynobasic style (*st*) × 38 Fig 12 —Transverse section of young ovary × 38 Figs 13 14 —Development of the obturator (*obt*) and integuments. Fig 13 —× 115 Fig 14 —× 38 Figs 15–18 *Leonurus sibiricus* Stages showing the curvature of the ovule, development of integuments and obturator (*obt*) × 38 See text for details

The fleshy annular disc which is seen in the mature stages of the flower commences to grow out round the base of the ovary at the time when the style begins to push in between the ovules as described above In *Anisomeles* (Fig 11), this is rounded in outline and of greater thickness, whereas in *Leonurus* it is thinner and forms a cup-shaped structure at the base

Development of the Ovule and the Integuments —The ovule begins to curve away from its orthotropous condition by unilateral growth before the inception of the archesporial cell When the curvature is less than ninety degrees, the archesporial cell is differentiated in the hypodermal layer even before the appearance of the integumental primordium In the Labiatae, such an early inception of the archesporium has been observed and figured by Bushnell (1936*a*), Junell (1937) and Narasimha Murthy (1940) It is characteristic of many Scrophulariaceae, Solanaceae, etc but is also reported in distant families like Juglandaceae (Langdon, 1935) and in *Blyxa*, a monocotyledon (Rangaswamy, 1941)

The integument appears when the megaspore mother cell becomes differentiated, and the primordium is first noted on the side close to the funicle (Figs 19-20) In *Anisomeles* the ovule attains the anatropous condition when the integument appears on both sides of the nucellus (Fig 13) The latter becomes massive at the mature stage of the embryo sac, and at the micropylar portion, it is 12–14 layers on the side away from the axis and 7-8 layers on the other side It is, however, thinner at the lower region bordering the embryo-sac When the integument lies just below the nucellar tip, the *obturator* appears at the funicular region overlying the nucellus and the inner part of the integument (Fig 13) It extends above the nucellar portion before the micropyle is formed by the integuments At the mature stage it takes a massive form fitting over the micropyle like a lid (Fig 14)

In *Leonurus* the attainment of the anatropous condition is delayed by the fact that the obturator takes its origin earlier than it does in *Anisomeles*, i e , when the integuments are halfway around the nucellus (Fig 16) It grows vigorously and assumes a massive appearance and the curvature of the ovule is somewhat arrested at this period When the growth of the ovule is followed up to the attainment of its final form, it is noted that in *Leonurus* it curves along the funicle as well as along its lower portion and the curvature is continued up to the tetrad stage of the megaspores resulting in a transverse orientation of the latter Consequently, this brings about a somewhat campylotropous condition which is more apparent in the mature stage of the gametophyte (Figs 17 18) Though Junell (1937) has figured such forms in *Physostegia*, *Sideritis* and *Notochaete* and referred to the micropyles as beak-like, he has not made any statement regarding the ovular orientation in these cases In *Anisomeles*, however, the ovule curves only along the funicle retaining the anatropous condition (Fig 14)

Formation of the Megaspores —In a number of cases a multiple archesporium has been observed in *Leonurus* In one instance, as many as six archesporial cells have been observed, and among these the upper two are slightly larger (Fig 19) The existence of two megaspore mother cells has also been detected, either superimposed or lying side by side, in *Leonurus* (Figs 21 22) The megaspore mother cell enters into the meiotic prophase long before the integument is fully grown, and soon attains its maximum size and form The fully developed megaspore mother cell is much longer in *Anisomeles* in which it shows a long process lodged into the nucellar tissue below (Fig 38) Both the divisions of the megaspore mother cell are unequal so that the lower dyad cell and the lowermost megaspore are longer (Figs 23, 24, 38, 39) Junell (1937) has recorded such a long megaspore in *Molucella*, but in *Lavandula* the megaspores are of equal size (Laws (1930), Junell (1937)) The rate of division generally slows down in the upper dyad in *Anisomeles* (Fig 39) as figured by Sharp (1911) in *Physostegia*

An abnormal case of a dyad and a tetrad, one partly overlying the other, has been observed in *Anisomeles* (Fig 41) The tetrad is fully formed while the dyad is just organised This appears to have developed from the differential growth of two megaspore mother cells lying side by side In two other instances in *Leonurus*, a triad and a tetrad one overlying the other has also been noted (Fig 24A) A still more peculiar and complex organisation has been noted in *Leonurus*, in which there are ten megaspores arranged in an irregular fashion (Fig 25) Of these, one of the lowermost megaspores has developed into a four nucleate embryo-sac without much enlargement of the latter, three are much larger than those in a normal tetrad while three are small and in different stages of degeneration and the remaining three are of intermediate sizes but not very healthy Thus from amongst the whole set one can trace the concurrent development and degeneration of a number of gametophytes and megaspores respectively

Normally, the lowermost cell of the linear tetrad which is already enlarged functions as the embryo-sac mother cell *It may be suggested that the precocious* enlargement of the lower dyad cell as also of the chalazal megaspore indicates the potential fertility of the basal portion of the megaspore mother cell at earlier stages

The disintegration of the upper three megaspores of the linear tetrad takes place from below upwards in both the species studied Fig 40 shows the irregular outline of the degenerating megaspore nucleus in *Anisomeles* as the first indication of the process and this is also evident in *Leonurus* The degeneration of the non-functional megaspores is not completed until after the late two nucleate stage and is completed in *Leonurus* at the initiation of the four nucleate stage (Figs 26, 27)

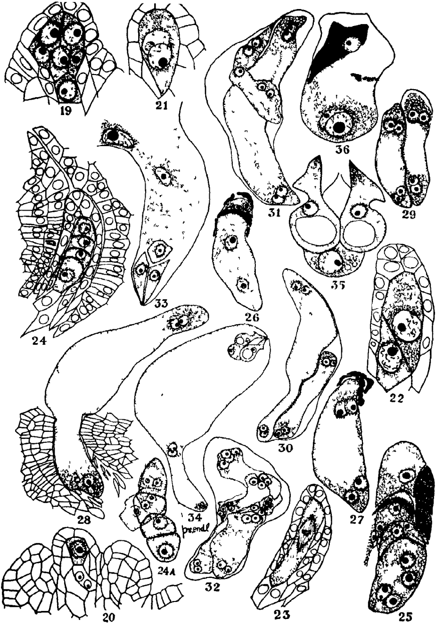
Nucellus —The primary differentiation of the nucellus takes place when the integuments are initiated and the megaspore mother cell enters into the meiotic prophase (Fig 20) It appears as a single layer above, and two layers on the sides of the megaspore mother cell When the latter increases in size, the epidermal layer of the nucellus keeps pace with it by intercalary growth with the result that the lateral cells of the nucellus (below the epidermal layer) are confined to the basal regions of the dyad megaspore mother cell (Fig 37) Ultimately these cells become loosely arranged and degenerate with the organisation of the tetrad The epidermal layer also begins to disintegrate with the degeneration of the upper three megaspores In *Leonurus*, degeneration of this layer is completed before the upper three megaspores degenerate, while in *Anisomeles*, the reverse is the case Such a simple and reduced type of nucellus which has been termed 'nucelle' by Balicka Iwanowska (1899) seems to be characteristic of the sympetalous families Junell (1937) has found it in the various species of Labiatae

In connection with the nucellus the most interesting feature observed is the presence in *Anisomeles* of two or three nuclei in the lateral and lower cells of the nucellus (Figs 37, 38) Among these one of the cells in the basal region is particularly larger, and all these cells characteristically persist through the developmental stages In two instances in *Leonurus* (Fig 20), a basal nucellar cell has been found to be binucleate The nuclei of these cells disintegrate in the mature stages of the embryo-sac It is interesting to note that such binucleate nucellar cells were also found to occur at the tip of the abnormal dyad and tetrad stage of *Anisomeles* The occurrence of such multinucleate nucellar cells has not been recorded by previous workers in this family

Another structural peculiarity is the periclinal division of the two epidermal nucellar cells at the tip In *Anisomeles* (Fig 38) it is noticed after the dyad stage, whereas in *Leonurus* (Fig 22), it is seen to take place earlier This feature has also not been recorded in this family

The Development of the Embryo sac —The nucleus of the enlarging megaspore divides at the centre In *Leonurus* the two nuclei migrate to the extreme poles but in *Anisomeles* the nucleus proceeding towards the chalazal end stops at a certain distance before the pole and divides there (Fig 42) so that in the resulting four-nucleate gametophyte, the big vacuoles lie in between the two pairs of nuclei in the former (Fig 27) and below the two pairs in the latter (Fig 43)

Rapid enlargement of the embryo sac takes place at the four-nucleate stage (Figs 27, 28, 43), this feature has previously been noted by Schnarf (1917), Ruttle



FIGS 19–36 *Leonurus sibiricus* Fig 19 —Multiple archesporium × 400 Fig 20 —Initiation of integuments (This figure is a magnified drawing of the ovule in Fig 15) × 250 Fig 21 22 —Double megaspore mother cells × 400 Fig 23 —First division of the megaspore mother cell × 250 Fig 24 —Linear tetrad of megaspores enclosed by the integumentary tapetal jacket × 250 Fig 24A —Triad overlying tetrad of megaspores × 250 Fig 25 —Complex arrangement of a number of megaspores in two layers. × 250 Figs 26 27 —Two-nucleate and early four nucleate gametophyte and degeneration of megaspores × 250 Fig 28 —Late four nucleate embryo sac × 155 Figs 29–31 —Double embryo sacs Fig 29 — × 250 Figs 30 31 — × 155 Fig 32 —Multiple embryo sacs—three in upper and two in lower part × 155 Fig 33 —Chalazal portion of nearly mature embryo-sac × 400 Fig 34 —Mature embryo sac ***Pr end l*** =primitive endosperm lobe × 115 Fig 35 —Egg apparatus × 400 Fig 36 —Fertilisation × 400. See text for details

(1931, 1932) and Junell (1937) in their investigations on the various species of this family The enlarging embryo-sac pushes through the just degenerated macrospore- and nucellar-cap and grows along the micropyle with a simultaneous dissolution of the integumental issue This leads to the formation of an upper broadened part of the embryo-sac in contrast to the lower narrower part which is more or less cylindrical and invested by a tapetal jacket of the integument (*vide infra*) The enlargement is more pronounced in *Leonurus* where the micropylar portion is not only much longer than the chalazal portion but also much greater in breadth thus offering a sharp contrast which is lacking in *Anisomeles* The latter species also differs from *Leonurus* in having a greater development of the embryo-sac before the micropylar extension which in its turn does not dominate in size and shape over the lower portion

Since the growth of the embryo-sac proceeds along the micropylar canal, the resulting shape of the embryo-sac is also accordingly influenced It is more or less straight in *Anisomeles* and L shaped in *Leonurus* This dependency of the shape of the embryo-sac on the earlier orientation of the ovular parts has already been pointed out in connection with the development of the megasporangium

The four-nucleate gametophyte passes to the eight-nucleate stage During the formation of mature gametophyte from this eight-nucleate stage, which has been studied in detail in *Anisomeles*, one nucleus from each pole migrates towards the centre and the two nuclei lie side by side just above the narrow chalazal part Simultaneously, the egg apparatus and the antipodal cells begin to organise at the two poles At the micropylar end, two nuclei become differentiated as the synergids, which in this primary stage appear somewhat triangular in form as figured by Bushnell (1936*a*) The other nucleus organises the egg cell which is attached to the micropylar end of the embryo-sac The egg is at this time much larger than the developing synergids and a small vacuole is already developed at the upper end At the antipodal end, one of the nuclei becomes invested by a cytoplasmic membrane to form a small triangular cell, while the other two are enclosed within a common membrane below the triangular cell and become longer and bigger in size as well (Fig 44) Later, the binucleate cell divides longitudinally into two long antipodal cells with two divergent winglike processes in between which the third smaller antipodal cell occurs At this stage the antipodals are quite rich in cytoplasm and have prominent nuclei indicating a physiological activity (Fig 45) It is to be noted here that the antipodals may be organised before the differentiation of the egg apparatus

The egg apparatus in the later stages in *Anisomeles* becomes further differentiated The synergids become very long each having one large vacuole at the basal portion which is perfectly round in outline In the upper portion prominent hooks develop and the nucleus lies just above the vacuole The tips of the synergids become extended into the micropyle as long pointed structures The egg cell hangs below the synergids and possesses the usual structure The cytoplasm seems to have reserve food matters (Fig 46)

In *Leonurus*, as in *Anisomeles*, the two polar nuclei migrate at that region of separation of the upper and lower part of the embryo sac, i e , just at the mouth of the narrow chalazal portion It is interesting to note that the upper polar nucleus is more than twice as large as the lower one, this smaller size of the chalazal polar nucleus is significant in that the antipodals degenerate early (Fig 33)

The shape of the antipodal cells has been found to be identical with that observed in *Anisomeles*, there being two long lower cells with a smaller one fitting into the upper portion of the long cells (Fig 33) In the egg apparatus, each of the synergids has a characteristic hook and a large spherical basal portion with a vacuole, and the nucleus lies above it The egg has a distinct stalk and the bulging lower portion containing the nucleus hangs below the synergids (Fig 35).

In both the cases the antipodal cells degenerate very early and it is difficult to trace them in a mature embryo-sac (Figs 34, 46) The two polar nuclei remain side by side for a long time before fertilisation In *Leonurus*, they fuse before fertilisation, but in *Anisomeles*, they have been found to fuse at the time of double fertilisation as will be described later on

Double and Multiple Embryo-sacs —Several interesting cases of double and multiple embryo-sacs have been observed in *Leonurus* In one case (Fig 29) two embryo-sacs, each with a four nucleate gametophyte, have been found in which both the embryo-sacs are almost of equal size, one of them has just started to grow above the other In two other instances (Fig 30), one of these four-nucleate embryo-sacs has grown considerably above the other which lags behind being pressed at the side by the former This stage must have been derived from a condition similar to the double embryo sac just described In another case the smaller embryo-sac lies at the micropylar part at the side of the larger sac and contains five nuclei—one pair at each of the two poles and one at the centre The lower portion of this smaller embryo sac is, however, deflected on the larger one (Fig 31)

A multiple embryo sac is illustrated in Fig 32 There are three embryo-sacs in the upper broadened part and two in the lower Of the three in the upper, one is an eight-nucleate gametophyte with four nuclei at the two poles, and the other two are four-nucleate, each having two nuclei at the two ends, of the two nuclei at the lower end in each of the said four nucleate embryo sacs, one has developed a cytoplasmic membrane forming an antipodal-like cell and the other remains free The two embryo-sacs in the chalazal portion are typically four-nucleate

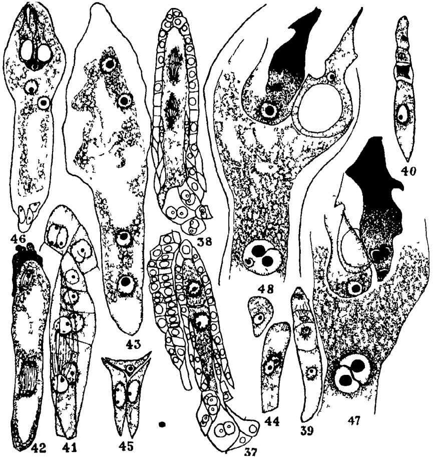
Integumentary tapetum —The most specialised part of the integument is the formation of a jacket of cells around the lower portion of the embryo-sac The earliest indication of these tapetal cells, is the differentiation (of 1-2 layers in *Anisomeles* and 2 3 layers in *Leonurus*) of the innermost cells of the integument, when the megaspore mother cell is in the meiotic prophase These cells become richer in plasma which take up more stain than the surrounding cells In *Anisomeles*, the cells of this innermost layer become long and narrow at the dyad stage of the megaspore mother cell due to periclinal division and radial elongation of these cells (Fig 37) The final form of the tapetum is attained at the four-nucleate stage when the cells of the innermost layer become longer, the nuclei having a flattened and lobed appearance, the cells of the next one or two layers do not differ in shape from the other cells of integument, but they remain deeply stained and also seem to function like the innermost layer At this stage the nucellus is destroyed and the integument lies in direct contact with the wall of the embryo-sac

In *Leonurus*, the cells of both the layers become flattened at the tetrad stage and divide anticlinally to form three to four layers which extend beyond the nucellar tip (Fig 24) In the mature four-nucleate stage of the embryo sac the tapetal layers become fully differentiated investing the originally jacketted narrow chalazal portion (Fig 28)

In both the cases a large mass of the integument tissue is disorganised due to the growth of the embryo-sac as mentioned earlier Naturally, the upper part of the integument does not get the opportunity to organise a tapetal covering, though it has got the potentiality, as indicated by the tapetal nature of the inner cells of the integument above the nucellar tip before the extension of the micropylar part of the embryo-sac It will be pointed out later that this part of the integument becomes the host, not only of the embryo-sac, but also of the micropylar haustorium which is organised by the endosperm

Fertilisation —The stages of fertilisation have been observed in both the species examined Since pollen tubes have been seen to travel along the obturator the function of the latter seems to direct the pollen tube towards the micropyle as suggested by Narasimha Murthy (1940) who also detected it in species of *Ocimum*

In both the species the pollen tube has been found to pass through one of the synergids and discharge its contents into it Consequently, the synergids in question become dense black in colour as evident from their hooked structure, shown in the figures 47, 48 and 36 Fig 47 also illustrates (in *Anisomeles*) the two male nuclei which are somewhat elongated and the tube nucleus at the base of the affected synergid At the same time the polar nuclei are seen to be adpressed to each other as a preliminary to fusion It has been observed in *Anisomeles* that before polar fusion both the nuclei migrate a little below the mouth of the chalazal portion of the embryo-sac, and it is in a deeper region of the latter that triple fusion occurs, the polar nuclei fusing along with the second male nucleus (Fig 48) It has also been found in this species that double fertilisation precedes syngamy as illustrated in the above figure It will be seen that the first male nucleus has not yet reached the egg, though the second one is in a state of fusion This also suggests the possibility that the male nucleus which is seen at the lowermost portion of the



FIGS 37–48 *Anisomeles indica* Fig 37 —Dyad megaspore mother cell ×250 Fig 38 —Same undergoing division ×250 Fig 39 —Almost completed tetrad of megaspores ×250 Fig 40 —Same showing mode of degeneration from below ×250 Fig 41 —Linear tetrad overlying dyad megaspore mother cell ×400 Fig 42 Two nucleate gametophyte undergoing division ×400 Fig 43 —Late four nucleate embryo sac ×250 Figs 44 45 —Formation of antipodal cells ×400 Fig 46 —Mature embryo-sac ×155 Figs 47 48 —Stages of fertilisation and triple fusion latter preceding syngamy ×400 See text for details

black synergid (Fig 47) travels down quickly and unites with the fusing polar nuclei before the other male nucleus could reach the egg Such a state of occurrence was claimed by Guignard (1901) in *Zea* and Thomas (1900) in *Caltha* Fig 36 shows in *Leonurus* that the vermiform male nucleus is adpressed to the egg The tube nucleus is seen at the cut end of a synergid which has been affected by the pollen tube

Development of the Endosperm and Endosperm-Haustoria —The endosperm formation is of the cellular type in both the species studied and the first division results in a partition of the embryo sac into two chambers The directions of the wall formation in the few primary divisions are characteristic and of systematic value The later developmental stages of endosperm have been found to be characteristic of the two species Since these primary and later stages of development vary in the two species, the accounts are given separately A comparative study is deferred for the present

Anisomeles indica —The first division of the secondary endosperm nucleus is accompanied by a transverse wall by which the embryo-sac is divided into two compartments—a smaller chalazal chamber, and a large micropylar chamber which includes the upper half of the narrow chalazal portion and the whole of the wide micropylar portion (Fig 49) According to the nature of the second division the development may be classified into two types viz , *Type I* Both the cells divide by a longitudinal wall (Fig 51) This appears to be the dominating type *Type II* Only the upper cell divides by a longitudinal wall, the lower becoming a binucleate structure without any wall formation (Fig 50 shows the binucleate lower cell, the upper being still undivided) The wall separating the two upper cells is in either type extended upwards and completely separates them The uninucleate two-celled or binucleate unicellular structure resulting from a division in the lower cell does not undergo any further division but directly becomes metamorphosed into a haustorium which is recognisable by the gradual intensity of stain taken up during the succeeding stages, as also by the slight increase in length

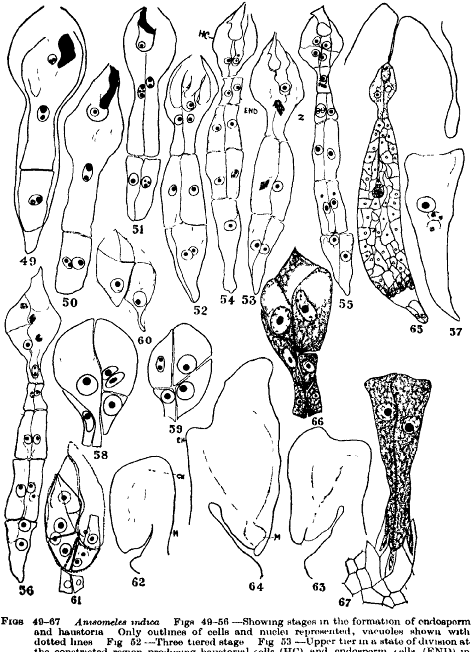
In the two upper cells which result from the second division, the nuclei again divide, the cell walls being laid down transversely Three rows of cells are thus formed, each row being composed of two longitudinal cells The upper cells consist of the whole micropylar part plus a portion of the chalazal part of the embryo-sac (Fig 52)

In these three tiers of cells the nuclei in the upper two tiers divide almost simultaneously with their spindles oriented more or less longitudinally the walls being laid down transversely The spindles in the upper tier are oriented at the mouth of the chalazal portion so that the walls are laid down here, the result being the complete separation of the upper and lower portions of the embryo-sac by cell-partitions (Figs 53, 54) This separation has got a physiological significance because, generally, as a result of a transverse division of the two cells contained in the micropylar part, a four-celled micropylar haustorium is formed (Fig 55) Thus the whole micropylar portion is devoted to the formation of a cellular haustorium

The majority of the endosperm tissue is derived from the middle tier of the three rows of cells represented in Fig 52 It should be noted in this connection that due to the nature of division of the uppermost tier described before, a part of the endosperm is contributed by the latter This point bears some significance which will be discussed elsewhere (*vide* p 208)

Deviations from the usual developmental stages as described above have been detected in a fairly large number of preparations Fig 65 illustrates a fairly advanced stage of the seed in which there are two long haustorial cells at the micropylar portion. This appears to have resulted by direct transformation of the two upper cells in Fig 54 into the haustorial structure

Again it has been found that of the four cells, which generally result in a four-celled haustorium, three are in a dividing state (Fig 56) An eight-celled haustorium



FIGS 49–67 *Anisomeles indica* Figs 49–56 —Showing stages in the formation of endosperm and haustoria Only outlines of cells and nuclei represented, vacuoles shown with dotted lines Fig 52 —Three tiered stage Fig 53 —Upper tier in a state of division at the constricted region producing haustorial cells (HC) and endosperm cells (END) in Fig 54 Z—Oospore Figs 49 50 —×155 Figs 51–56 ×115 Fig 57 —Abnormal 3 nucleate 2 celled chalazal haustorium ×250 Figs 58 61 Various types of micropylar haustoria (8 celled in Fig 61) ×155 Only outlines of cells and nuclei represented in Figs 57–61 Figs 62–64 —Developmental stages of seed to show the gradual unilateral extension of endosperm tissue (represented by dotted lines) and the responsive growth of the integuments, M & CH representing micropylar and chalazal haustorial regions respectively ×25 Fig 65 —Advanced stage of seed Fig 66 —Mature 4 celled micropylar haustorium, the lowermost are endosperm cells Topmost is cut end of synergid. ×155 Fig 67.—Mature chalazal haustorium ×250 See text for details.

represented in Fig 61 seems to have been derived from such a configuration provided all the four completed divisions Even in a four-celled haustorium, there may be considerable variations in structure Fig 60 shows four cells arranged peculiarly in three layers In the uppermost layer there is one cell with a cup-like upper portion and a narrow base, in the next layer there are two long cells lying side by side the right one having a long beak-like process below, a pear-shaped cell lies in the lowermost layer Fig 66 illustrates a four celled haustorium, in which the upper two cells have grown vigorously with much larger nuclei and apparently they are the most active, the other two cells having been suppressed, as it were, by the two larger ones An intermediate stage between a normal haustorium in which all the cells are equally active, and the stage described above may be seen in Fig 59, where the cells are relatively unequal in size In Fig 58 one finds a binucleate upper haustorial cell which must have resulted from free nuclear division of one cell

In one instance, a ten-celled haustorium has been detected in which there are intergrading forms of cells in size and contents It is, however, difficult to demarcate in this case as well as in many others the 'haustorial' and the endosperm cells proper, because the haustorial cells also gradually merge into the endosperm cells in size and staining capacity A consideration of this feature is postponed for a later discussion (*vide* para 5, p 206)

The active micropylar haustorial cells in general have got hypertrophied nuclei with conspicuous nucleoli, and the cytoplasm is also very highly staining In these nuclei, dark staining bodies are present (Fig 66)

The chalazal haustorium functions in the early stages of endosperm formation and it attains its maximum development when the endosperm is only a few cell layers at its maximum thickness It is broader in the upper portion and gradually tapers downwards It elongates to a certain extent towards the vascular trace and sends in sucker-like haustorial branches into the cells below which appear empty in contrast to the surrounding rich cells of the vascular trace (Fig 67)

As deviations from the normal two-celled, or the less occurring binucleate chalazal haustorium, a three nucleate two-celled chalazal haustorium (one of the cells being binucleate) and an absolutely uninucleate chalazal haustorium were noticed as rare cases (Figs 54, 57)

The chalazal haustorium begins to degenerate when the micropylar haustorium has just started its activity Gradually the whole haustorium disintegrates leaving a large cavity bordered by the endosperm cells At this time the latter become very rich in plasma and have larger nuclei They assume the rôle of an absorptive organ deriving the food materials absorbed already by the chalazal haustorial cells

As has been mentioned before, the real endosperm tissue develops from the middle tier and partly from the upper tier of the three-tiered stage The bulk of the endosperm is, however, produced from the middle tier by repeated divisions which are longitudinal in earlier stages, but transverse in the advanced stages The growing tissue thus becomes massive, and it increases in surface chiefly at the lower region, so that in a fairly advanced stage the endosperm becomes somewhat pear-shaped in appearance With the growth of the endosperm, the innermost tapetal layers which were most prominent in the form of a nutritive jacket of the developing gametophyte, persist for some time though they do not appear to be conspicuous The cells gradually become more and more isodiametric due to the stretching of the growing endosperm and they are not differentially stained as seen in the earlier stages When the globular form of the embryo is reached, the tapetal layers begin to be crushed by the endosperm tissue Along with this the destruction of the integumentary tissue also begins The nuclei in these cells degenerate, the cells become loose, and the walls also disorganise gradually When the cotyledonary lobes of the embryo are just differentiated, the integument becomes greatly disorganised, large cavities appear in the tissue, and only disintegrating loose cell walls are present.

With the growth of the endosperm tissue as well as that of the micropylar haustorium, a constriction is produced at the junction of the latter and the endosperm This is evidently due to the longitudinal stretching growth of the endosperm and the haustorium *pari passu* with the growth of the ovule as a whole

It is interesting to note that when the micropylar haustorium has reached the maximum development, the endosperm cells bordering the latter at the constricted region become very large with rich contents and larger nuclei They evidently resemble the micropylar haustorial cells of the earlier stages It also appears that these have an absorbing and conducting function resembling those that have been observed at the chalazal end bordering the disintegrated haustorium

The growth of the endosperm tissue is not uniform It extends unilaterally on one side of the chalazal end of the ovule (Fig 62) As a response to this the ovule also begins to grow on that side as a knob like protuberance (Fig 63) Consequently, the rounded shape of the ovule becomes angular giving a different appearance altogether The growth of the ovule at the chalazal end is thus limited and it is pushed at one side at a later stage when the endosperm has extended considerably (Fig 64)

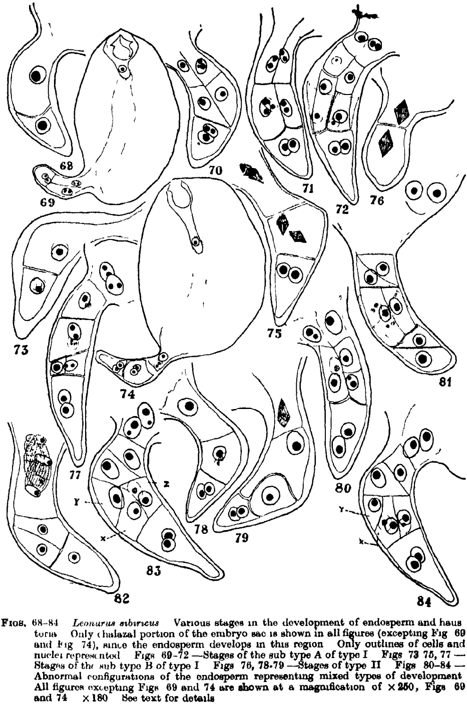
It will be noted from what has been said above that the form of the mature seed is not a consequence of an uniform enlargement of the ovule but it is the result of an unilateral growth initiated at the response of the growing endosperm in a particular direction This feature is of importance in comparing the developmental stages of the seed in this species with those met in other members of this family, so far studied

Leonurus sibiricus —Considerable difficulty was experienced to follow the developmental stages due to the peculiar orientation of the chalazal portion of the embryo-sac (wherein the endosperm tissue develops) The deeply staining nature of the cells in the primary stages and the further curving of the chalazal region which gradually approximates the micropylar cavity in later stages makes the study difficult In fact Junell (1937) has admitted in studying *Sideritis* and *Physostegia*, the genera resembling *Leonurus* in structural complexities that he could not study the developmental stages due to some of these impediments and he has described only a few mature stages of endosperm of *Leonurus cardiaca* without any illustration He does not throw any light on the early developmental stages of endosperm in *Sideritis, Physostegia* and *Notochaete* which can provide any basis for comparison with *Leonurus* In the present investigation, however, all the details have been worked out which present on critical examination an immense amount of variation in structural and developmental sequences

Considering as a whole the development corresponds to two major types and the first type can again be conveniently divided into two sub-types Besides, other variations have also been described below, indicating the probable stages in the major types and sub-types from which they might have been derived

In all the types the first division of secondary endosperm nucleus is always followed by a transverse wall dividing the embryo-sac into a smaller chamber towards the chalazal end and a larger chamber consisting of the rest of the upper chalazal portion as well as the micropylar sub-division of the embryo-sac (Fig 68) The main criterion for distinguishing the two types is the nature of the second division in the upper chamber as to whether it is followed by longitudinal or transverse walls The descriptions of the different types and sub types are, however, extended up to the stages in which the micropylar haustorial apparatus is initiated

Type I.—The upper chamber divides by a longitudinal wall at right angles to the transverse wall already laid down at the first division This longitudinal wall does not divide the upper chamber completely as in *Anisomeles* but it extends only to the mouth of the chalazal portion (Figs 69, 73) The criterion for separating the first two sub-types under the relevant type (Type I) is the orientation of this longitudinal wall, i e , whether the plane of the latter is at right angles to the plane



FIGS. 68–84 *Leonurus sibiricus* Various stages in the development of endosperm and haustoria Only chalazal portion of the embryo sac is shown in all figures (excepting Fig 69 and Fig 74), since the endosperm develops in this region Only outlines of cells and nuclei represented Figs 69–72 —Stages of the sub type A of type I Figs 73 75, 77 —Stages of the sub type B of type I Figs 76, 78-79 —Stages of type II Figs 80–84 —Abnormal configurations of the endosperm representing mixed types of development All figures excepting Figs 69 and 74 are shown at a magnification of ×250, Figs 69 and 74 ×180 See text for details

of the section or not There can be no ambiguity in utilising this criterion because it is in a single plane of the ovule that we can see the structures of the developing endosperm in its proper perspective

Sub-type A —In this sub type, the plane of the longitudinal wall is *at right angles to the plane* of the section and therefore this wall is visible (Fig 69) The nucleus of the lower chamber divides once without the formation of a cell wall As will be pointed out later, the two resulting nuclei do not divide any more but the whole chamber is directly transformed into a binucleate chalazal haustorium which gradually grows in size

The longitudinal cells produced at the second division in the upper chamber divide again, almost simultaneously, followed by transverse partitions (Fig 70) Thus three tiers of cells are produced in which the two upper tiers consist of two cells each and a lowermost tier composed of a binucleate cell The cells of the middle tier divide longitudinally (Fig 71), but those of the uppermost result in the formation of four nuclei the two upper nuclei being separated by transverse walls laid down at the mouth of the chalazal opening (Fig 72) (cf Fig 54 in *Anisomeles*) These two upper nuclei are the primary haustorial nuclei which are thrown into the micropylar cavity to form a multinucleate haustorium by rapid divisions

Sub-type B —The longitudinal wall of the second division is oriented *in the plane* of the section so that one can see two nuclei in different foci representing the two cells (Fig 73) The chalazal haustorium is organised from the lower chamber as in the first sub-type The cells of the upper chamber divide transversely giving rise to four cells and the whole thing results in a three tiered endosperm corresponding to Fig 70, in the sub-type 'A' (Fig 74) All the four nuclei divide simultaneously (Fig 75), but the result is somewhat different from the first sub type Here the two cells of the middle tier divide transversely instead of longitudinally as in sub-type 'A' Fig 77 shows a little later stage where the lower cell resulting from the transverse division of one of the cells in the middle tier has again divided longitudinally The two cells of the upper tier also divide transversely followed by walls so that two free nuclei are separated out in the micropylar portion of the embryo-sac

It is now clear that in both the sub-types, the cells of the upper tier of the three tiered stage of the endosperm do not become directly converted into the micropylar haustorium but they divide again, and a part of the endosperm which lies at the constricted portion of the embryo-sac is derived from this tier, the other part being converted into a free nucleate haustorium

Type II —In the type under consideration, after the first transverse division the spindle in the upper chamber is oriented longitudinally so that it necessarily results in a transverse division (Fig 76) A three tiered stage homologous to that of the first type is formed the difference being that in this type the two upper tiers are composed of one cell each The lowermost cell is, however, binucleate as in the previous type Fig 78 shows a little advanced stage than this, in which the middle cell (tier) has divided longitudinally In Fig 79 again, one observes that the middle cell has divided by a dome shaped oblique wall, whereas the nucleus of the uppermost cell is in a state of division in the longitudinal direction (i e it will result in a transverse division) Though this type could not be followed to the end the few stages clearly establish a different type of endosperm development It is probable that the nucleus which is in a state of division (Fig 79) sets free a nucleus in the micropylar cavity to develop into the haustorium This is also supported by some of the peculiar configurations described below (Figs 82–84)

In one instance, of the two longitudinal cells of the middle tier in the three-tiered stage of the first sub-type 'A' one cell has divided longitudinally and the other transversely (Fig. 80) This may be regarded as an intermediate form between the first two sub-types, since here we find both longitudinal and transverse divisions of the middle tier. Fig 81 shows another case which is essentially a derivative of the second sub-type and which possesses the characteristics of the variations of the first sub-type just described above Here in the middle tier (cf fig 74) one of the longitudinal superimposing cells has divided longitudinally while the other has divided transversely The divisions of the uppermost tier have taken place as usual (cf Fig 77) the only difference being that the lower two cells are seen to be separated by an oblique wall In the lowermost tier, however, one finds a two-celled chalazal haustorium and this feature is to be noted as an exception A similar two-celled chalazal haustorium has been noted in another instance

Two more abnormal cases have been observed, the essential feature of which is that the sequence of divisions follows the first type in the primary stages, but later shows the second type of development Fig 82 shows that the two cells of the middle tier are separated by an oblique longitudinal wall which is transverse in effect, that is, one of the cells is blocked by the other, this is probably an aberrant form of the stage represented in Fig 69 where the spindle was a little oblique In the uppermost chamber one finds three superimposed cells which have almost completed divisions These three cells must have been produced by the cell derived from the transverse division of the upper cell of the middle tier

Figs 83 and 84 illustrate cases which can be explained as follows After the second division as in Fig 69 the cell away from the 'primitive endosperm lobe' has divided longitudinally 'Z' while the cell on the side of the latter has divided obliquely to produce two cells 'X' and 'Y' of which 'Y' produced a series of transversely oriented cells ultimately setting free a nucleus at the micropylar cavity, which by free nuclear division has organised the two primary haustorial nuclei In Fig 84 the cell 'X' has been shown to divide longitudinally after the oblique transverse division

Examination of later stages of the endosperm development shows that sub-type 'A' of the first type is the predominant type though the other types also occur in sufficiently large number of cases to warrant their generalisation The ultimate result in all the developmental types is the same, viz , the formation of a multinucleate micropylar haustorium from the free nuclei set out by the endosperm cells, a binucleate chalazal haustorium which is also endospermous in origin, and a central endosperm tissue Exceptional cases of two-celled chalazal haustorium have already been described

The micropylar and chalazal haustoria—The large micropylar cavity of the original mature embryo-sac becomes converted in a haustorium with free nuclei embedded in it The one or two nuclei originally produced by the endosperm as the primary haustorial nuclei rapidly multiply by division The number of nuclei actually produced is found to be variable Only six large nuclei have been found in a fairly mature haustorium, whereas twelve nuclei occur in a young haustorium As many as sixteen nuclei are present in a few cases (Fig 88), though other numbers like eight and ten are also common On examining a large number of mature haustoria twelve seems to be the most general number (Figs 89–91) It may be recalled that in *Leonurus cardiaca* Billings (1909) claimed the number to be four to six, while Junell (1937) counted eight and sixteen in two cases of the same species

The micropylar haustorium begins to enlarge laterally destroying the integumental tissue with the simultaneous increase in size of the ovule which also grows vigorously When the endosperm has grown to the size of the micropylar haustorium (Fig 88) a constriction is produced at the junction of the two Junell (1937) refers to this constricted portion in *Leonurus cardiaca* as a long canal But so far as this species is concerned it cannot be described as a long canal as is found in other genera like *Stachys* (Schnarf 1917), *Sideritis*, *Molucella* (Junell, 1937) etc * However, the length of the constriction is not maintained long as the growing endosperm gradually pushes the micropylar chamber which also begins to enlarge

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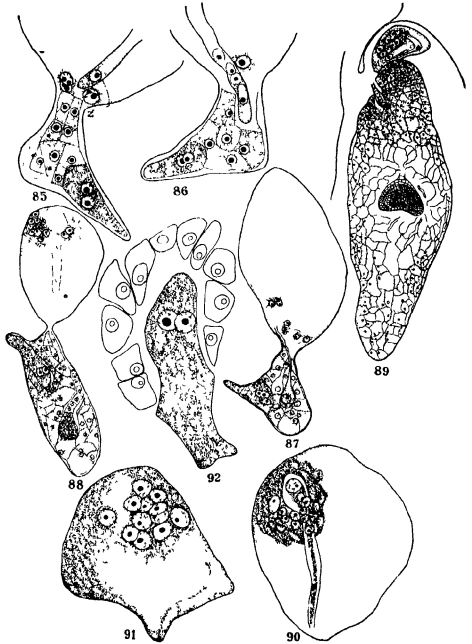
* Unfortunately Junell has not illustrated any figure of *Leonurus cardiaca*.



Figs 85–92 *Leonurus sibiricus* Later stages in the development of endosperm and haustoria Figs 85 86 —Endosperm tissue just extending along 'primitive endosperm lobe' Oospore entering into endosperm Z—Oospore × 180 Fig 87 —Young stage of the developing haustoria, endosperm and oospore × 115 Fig 88 —Later stage with 16 nucleate micropylar haustorium. × 70. Fig 89 —Still later stage showing degenerating chalazal haustorium, massive endosperm and embryo and active micropylar haustorium (12 nucleate) × 38 Fig 90 —Rather immature 12 nucleate micropylar haustorium Note persistent suspensor tube with smaller nuclei × 115 Fig 91.—Mature micropylar haustorium. × 115 Fig 92 —Chalazal haustorium × 180 at the basal portion laterally (Fig 89) Measurements show that this is 209μ long and 104μ broad at the mature embryo-sac condition, whereas at the mature haustorium stage it becomes 182μ broad at the basal region, the length becoming reduced to 143μ (Fig 89) Thus the change in shape is effected by the increase in size laterally and the simultaneous pushing of the basal portion by the growing endosperm

The nuclei in the micropylar haustoria enlarge enormously and become 26μ–$33\ 8\mu$ long and $14\ 4\mu$–$20\ 8\mu$ broad in the mature stages They are elliptical or ovoid in shape as in *Anisomeles* The cytoplasm also becomes dense and takes up so much stain that it becomes often difficult to make out the outline of the nuclei

After setting free the primary haustorial nuclei in micropylar cavity the endosperm begins to grow in bulk mainly along the 'primitive endosperm lobe' bulging out, as it were, in the direction just opposite and almost in the same line with the micropylar cavity It extends both laterally as well as longitudinally, the growth being restricted to a particular direction already determined by the 'primitive endosperm lobe' The chalazal region of the endosperm with its haustorium is pushed to one side approximating gradually the micropylar cavity (Figs 85–89)

The chalazal haustorium begins to function during the early stages of endosperm when the micropylar haustorium is still in the process of differentiation It reaches its maximum development when the endosperm has grown considerably, the embryo has become a globular mass, and the micropylar haustorium has just started functioning The nuclei of the haustorium become hypertrophied being as large as those attained later by the micropylar haustorial nuclei The cytoplasm is also marked by its rich contents (Fig 92) It is notable in this connection that during its development the chalazal haustorium is drawn partly into the endosperm evidently due to the stretching of this part along with the extension of the endosperm Some portion, however, remains outside the endosperm area to connect the vascular trace from which the food matters are transported (Figs 88, 89)

The fate of the integumental tapetum is somewhat different from the corresponding stage in *Anisomeles* The jacket portion surrounding the 'primitive endosperm lobe' is immediately destroyed as the endosperm grows in this particular direction breaking through the tapetum, as it were Their identity is altogether lost except at the constricted region, the most conservative part of the integument, where least growth either of the endosperm or of the integument takes place The individuality of the other side of the jacket is retained for a short time but this also gives way with the progressive pushing of the chalazal region which is brought close to the micropylar cavity at the mature stages The tapetal cells at the constricted portion is disorganised only when the endosperm progresses in this upward direction at a rather advanced stage

The disorganisation of the integument tissue is apparent when the embryo attains a massive globular structure It is first visible in the form of cavities at the lower region bordering the growing endosperm tissue Gradually other cells also become loose with their nuclei showing signs of disintegration It is interesting to note that the same thing happens simultaneously to the endosperm cells bordering the embryo (Fig 89)

An interesting case of the endosperm has been observed where all the cells are multinucleate with very dense cytoplasm and prominent nuclei just like a mature haustorium The embryo appears unusually vacuolated and disfigured and the integument is much more alveolated The whole structure is in striking contrast to the normal cases where the endosperm is active only at the region bordering the integument and disorganised at the portion surrounded by the endosperm

The further growth of the endosperm tissue in the upward direction gradually begins to obliterate the micropylar haustorial region which reaches its final form when the cotyledons have differentiated distinctively The haustorium is almost completely effaced as the embryo assumes nearly its full size Section of a very mature embryo shows only one or two layers of the integument at the two extremities of the embryo and it is probably completely destroyed at a time when the seed falls off. A few layers of endosperm are also seen at this stage filled with deeply stained reserve matters which indicate that some amount of endosperm persist in the mature seed.

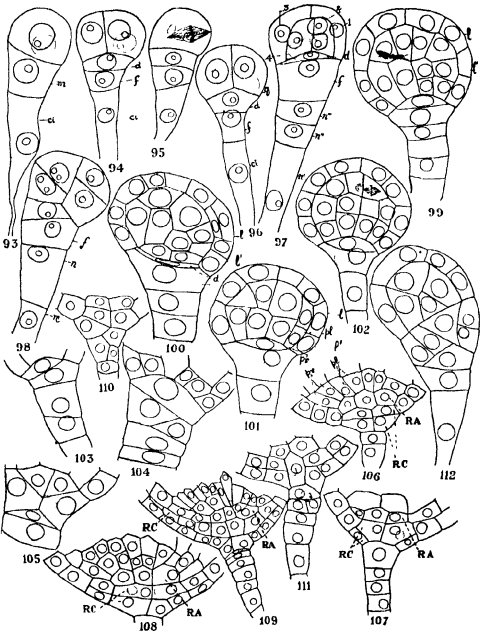
Development of the Embryo.—A considerable time elapses between the first division of the secondary endosperm nucleus and the first division of the oospore. In fact the latter process takes place when a fair amount of endosperm tissue is formed. The oospore elongates extensively as a single cell forming a long tubular structure. It traverses the whole micropylar cavity in this condition and enters the summit of the endosperm tissue at the mouth of the chalazal portion of the embryo-sac (Figs. 69, 74, 85, 86 in *Leonurus*, Figs. 51–55 in *Anisomeles*). The greater length of the micropylar portion in *Leonurus* necessitates an equally long tube to pass through, and consequently this structure is much longer than that of *Anisomeles* where the micropylar portion is relatively shorter. The entrance of the oospore into the endosperm is very interesting in *Leonurus* where it has been found to break through the wall of one of the two endosperm cells lying at the constricted region of the embryo-sac and pass into the endosperm tissue destroying immediately the cell in question (Fig. 86). The process reminds one of the entrance of the pollen tube through a synergid. Even after entering the endosperm tissue it does not divide, but still goes deeper, taking a more or less central position in the endosperm mass (Fig. 87).

The details of the development of the embryo have been followed in *Leonurus sibiricus* and therefore the following account refers to that species only.

The first division of the oospore is followed by a transverse wall forming a two-celled pro-embryo. Both the apical and basal cells now divide, the former longitudinally and the latter transversely, producing a tetrad pro-embryo (Fig. 93). The two juxtaposed cells at the apex then divide almost simultaneously (Fig. 95), or one before the other (Fig. 94) by two vertical walls giving rise to a quadrant embryo at the tip (Fig. 96). Just after this, the four cells of the latter divide periclinally to form four long outer cells and four inner cells, and not transversely as in most other angiosperms (Fig. 97). The dermatogen is thus differentiated with the formation of the octant embryo, where all the cells are arranged in linear files instead of being formed into transverse layers. An abnormal configuration illustrated in Fig. 98 indicates that transverse division of the cells of the quadrant embryo may also be initiated. In fact this stage represents an intermediate form between the two types. An exactly similar aberrant pro-embryo has been found by Soueges (1921) in *Glechoma hederacea*.

The division of the basal cells of the tetrad pro-embryo is initiated simultaneously with those of the apical cells. The formation of cells d and f by the division of the middle cell of the tetrad pro-embryo is completed before the quadrant embryo is developed (Figs. 94, 95) after which the lower cell ci divides transversely to form two cells n and n', the upper cell n again divides to form two transverse cells n'' and n''' (Figs. 97, 98). These divisions also take place before the octant embryo is formed. The basal cell of the pro-embryo therefore gives rise to five cells viz. d, f, n''', n'' and n'. It must be noted in this connection that the products of the cell n (i.e. the cells d and f) are generally unequal, the upper cell d being smaller than the lower cell f. The apparently very small size of the cell d represented in Fig. 97 is perhaps due to a curving of the pro-embryo at that region, which is also indicated in the later stages. Moreover, the partition between these two cells may also be obliquely oriented, being inserted at one end of the horizontal wall of an octant cell (Figs. 98, 103).

The series of divisions (mostly transverse) which follow after the separation of the dermatogen layer both in the latter as well as in the four axial cells results in the differentiation of the different parts destined to be formed in the mature embryo. Figs. 99–102 illustrate a little later stages where further tangential and transverse divisions of the inner cells have taken place. These figures show the differentiation



FIGS 93 112 *Leonurus sibiricus* Stages of the development of the embryo Figs 93–96 —Pro-embryo Fig. 97 —Periclinal division of all the quadrant cells (*q*) in Fig 96 has resulted in four outer dermatogen cells (1, 2, 3, 4) and four inner axial cells Fig 98 —Abnormal octant stage showing both transverse and tangential divisions Figs 99 100 —Showing differentiations of the histogens of the embryo *l*—stem tip and cotyledons; *l′*—hypocotyl Figs. 101, 102 —Differentiation of periblem (*pr*) and plerome (*pl*) Figs 103–105 —Diverse mode of wall formation at the hypocotyl region Fig 106.—Showing the products of hypophysis cell, *pr* —periblem, *pl* —plerome Fig 109 —Completion of root-cap by tangential divisions of the dermatogen cells of the hypocotyl Figs. 110–112.—Abnormalities Figs 93–105 and 112 —× 400 Figs 106–108 and 110 111.—× 250 Fig 109.—× 185 RA = Root-apex. RC = Root-cap. For details see text of the three histogens in different developing stages Fig 101 shows that the inner octant cells which have produced two transverse layers of cells have again divided tangentially to give rise to a layer of periblem (*pr*) on each side below the dermatogen and two inner layers of plerome (*pl*) The demarcations of the different parts of the pro-embryo are also clear The apical row of cells (*l*) in Fig 101 constitutes the cotyledon and stem apex, while the lower layer (*l′*) represents the hypocotylar region

The differentiation of the hypophysis which also takes place simultaneously with the distinct separation of the layers *l* and *l′* does not follow any strict rule with regard to the corresponding stages of advancement of the hypocotylar tissue All the Figs 99–103 show that the cell *d* has divided into two apparently unequal cells, either by a straight horizontal wall or by a curved wall thus clearly establishing its hypophysial nature by inserting the wall on the dermatogen cells of the hypocotyl Fig 103 illustrates a case where an oblique wall has been laid after the division of the cell *d*, one end of the wall being inserted on the peripheral membrane of the pro-embryo, as has been reported by Souèges (1921) in *Mentha viridis* The separating wall may even lie entirely on the proembryonal membrane (Fig 104, where both the cells have again undergone vertical and oblique divisions respectively), the lower cell from *d* actually forming a part of the suspensor thus extending the differentiation of the hypophysis to a later stage Souèges (1921) expected such a sequence in *Mentha* and *Glechoma* but could not actually detect any such case The upper cell of the products of *d* appears narrow (Figs 99–102) possibly because of the curving of the embryo at this region as already suggested

The first horizontal walls appear in the layer *l′* after the differentiation of the two cells of periblem on two sides and two cells of plerome at the centre (Fig 99) When the cotyledonary lobes have just differentiated there appear two layers of periblem and two layers of plerome on each side of the hypocotylar axis just below the hypophysial derivatives (Fig 106) The outermost layer of the plerome cells is the pericycle

An interesting case of a periclinal division of a dermatogen cell to add to the periblem of the layer *l* has been observed (Fig 100) This is significant in that it may contribute to a precocious differentiation of the root cap

The contributions of the hypophysis to the embryo is the next important consideration Both the cells derived from *d* usually divide vertically producing a tetrad hypophysis (Figs 101, 102, 105) but the lower may exceptionally segment transversely adding to the suspensor filament (Fig 99) The two lower cells of the tetrad hypophysis divide tangentially to produce four cells which give rise to the median portion of the root-cap by further vertical divisions (Figs 106-7, RC) The two upper cells derived from *d* contribute to the root apex by continuing vertical divisions (Figs 106–8, RA) or both by vertical and transverse divisions (Fig 109, RA)

In the upper portion of the root on the two sides the two root cap layers are joined by the tangential divisions of the dermatogen pertaining to the hypocotyl where it extends to a certain distance (Figs 108-9)

Several irregular configurations of cells of the hypophysial region have also been noticed as illustrated in Figs 110–112 It is interesting to note that Souèges (1921) has also observed similar abnormalities As stated by him it is impossible to trace the sequence of divisions which lead to these configurations

DISCUSSION

A Development of the ovule and integuments—Working on Labiatae Schnarf (1917) and Junell (1937) have generalised their opinion regarding the nature of the ovule which they consider as anatropous But the course of curvature of the ovule in *Leonurus sibiricus* shows that though in the primary stages (up to the formation of the tetrad of megaspores) the ovule curves along the funicle, which is the feature of anatropous forms, it begins to bend along the body of the ovule in the later stage Due to this the micropyle forms a long curve and consequently the mature embryo-sac presents a shape in which the longitudinal axis of the latter is curved along a definite angle It is therefore obvious that the ovule in *Leonurus* cannot be strictly described as anatropous, and a semi-campylotropous form would be the proper designation

An interesting feature of the ovule is the development of a massive *obturator* The presence of an obturator has been reported in this family by Narasimha Murthy (1940) in *Ocimum* species, Ruttle (1931-32) in *Mentha* and *Lycopus* and Bushnell (1936*a*) in *Monarda* species According to Bushnell it is absent in *Nepeta cataria* and the sketches of Sharp (1911) and Junell (1937) indicate that it is absent in most of the genera studied by them except in *Hormium pyrenaicum* (Junell, 1937) Thus it appears that this structure is not universally present in this family The presence of pollen tube along the obturator suggests that the latter acts as a guide to the pollen tube The occurrence of the obturator therefore indicates a special adaptive feature taken up by genera of diverse affinities, and cannot be attributed to a progressive evolutionary line in this family

B Development of the megaspores and the female gametophyte —The degeneration of the upper three megaspores of a linear tetrad from below upwards observed in both the species studied is a unique feature in contradistinction to the most general condition of degeneration from above downwards as observed by Junell (1937) in several species of Labiates, Narasimha Murthy (1940) in *Ocimum* species and Bushnell (1936*a*) in *Monarda*

The degeneration of the non-functional megaspores and the nucellus which starts more or less at the two-nucleate stage of the embryo-sac in *Leonurus* and *Anisomeles* (*vide* text) has also been observed by Junell (1937) in *Pogostemon patchouli*, *Hormium pyrenaicum* and *Lavandula spica* In *Leonurus* the megaspores and in *Anisomeles* the nucellus completes degeneration only at the initiation of the four-nucleate stage Such a late degeneration of the megaspores and nucellus seems to be characteristic of this family

The shape of the mature embryo-sac in different members of the Labiatae is worthy of consideration Schnarf (1917) has already pointed out that in the majority of Labiatae with a few exceptions, the embryo-sac is differentiated into two parts,—'a lower narrow part surrounded by epithelium and an upper broadened part whose surrounding is not epithelial' Carlson and Stuart (1936), however, found out two distinct forms of female gametophytes in the same genus *Salvia*—a short form without a bounding epithelium in *S splendens*, *S leucantha*, *S greggii*, distinguished as the *S splendens* type, and a long form with a bounding epithelium extending about two-thirds the length of the gametophyte in *S mellifera*, *S apiana*, and *S columbariae*—designated as the *S mellifera* type Junell (1937) added to some of the exceptions and stated that in *Lallemantia iberica* and *Salvia tiliifolia*, the distinction between the two parts is not possible It seems that the latter species would come under the *S splendens* type of Carlson and Stuart (1936)

From a comparative study of the form of embryo-sac in *Anisomeles*, *Leonurus* and those of other species described previously, two distinct types of gametophytes can be distinguished amongst those characterised by the presence of an epithelial jacket *Anisomeles* and *Leonurus* represent each of the types respectively *Leonurus* agrees with the type of gametophyte where the upper micropylar part is very broad and long and the lower chalazal portion surrounded by the tapetal jacket is comparatively much narrower and shorter This type is also illustrated by *Sideritis*, *Physostegia*, *Notochaete*, *Plectranthes*, etc *Anisomeles* on the other hand can be conveniently grouped with the proposed second type of gametophyte where the micropylar part is comparatively shorter in length and the chalazal portion is not much narrow, and this form is exemplified by *Hormium*, *Pogostemon*, *Molucella*, *Lavandula*, *Ziziphora*, *Perowskia*, *Salvia splendens*, etc Other genera like *Ocimum*, *Hyptis*, etc would then be intermediate in character It is to be noted further that in the two types thus distinguished the post-fertilisation developments agree in their general features

The structure of the gametophyte as observed by Narasimha Murthy (1940) in *Anisomeles* does not agree with my observations The present study reveals a perfectly rounded base and a long pointed upper portion of the synergids and this is inconsistent with Narasimha Murthy's figure which shows a flattened base and a short upper portion The size and arrangement of the antipods is also at variance with the observations of the said author It will be of interest to note in this connection that Carlson and Stuart (1936) have found the antipodals to consist of nuclei in *Salvia* species

The disintegration of the antipodal cells before fertilisation has also been noted by Schnarf (1917) and Bushnell (1936*a*) Junell (1937) has referred to the antipodals in Labiatae he studied as 'inconspicuous' but that is probably because he found them in later stages when they generally degenerate Such ephemeral antipodals have been reported in diverse families, e g , Potamogetonaceae, Rubiaceae, Solanaceae, Verbenaceae, etc

Schnarf (1917) stated that fusion of polar nuclei before fertilisation is a characteristic of Labiatae, but as observed in *Anisomeles*, Junell (1937) also mentions the occurrence of polar fusion at the time of fertilisation in *Lycopus europaeus* and in certain species of *Mentha*, and he has also found indications of this condition in *Pogostemon patchouli* So the general statement made by Schnarf (1917) seems to be invalidated

C Abnormalities in the development of megaspores and embryo-sac and related structures—The occurrence of a number of cases of a multiple archesporium has already been described Schnarf (1917) and Strasburger (quoted by Schnarf) have noted the presence of two archesporial cells in *Galaeopsis pubescens*, and *Lamium* respectively Junell (1937) has reported the presence of potential archesporial cells in *Molucella* and *Pogostemon* around the base of the developing megaspore mother cells as also in earlier stages Multiple archesporium has been recorded in numerous other families, but its origin is a debatable question It may be derived from one archesporial cell as indicated in Figs 21, 22 showing two superposed megaspore mother cells Alternatively, the multiple archesporium observed at such an early differentiation of the ovule as also the position of the archesporial cells (hypodermal and two sub-hypodermal layers) point to the possibility of their individual differentiation

The formation of two megaspore mother cells, double and multiple tetrads some of which show all megaspores having a tendency to function have been recorded in the text The similar development of two megaspore mother cells has been observed by Junell (1937) in *Pogostemon patchouli* where he found in one instance two tetrads developing side by side Furthermore, in the later stages in *Leonurus* two to five embryo-sacs have been seen to develop side by side, one of them reaching up to the eight-nucleate stage

It is therefore not very difficult to explain the occurrence of double and multiple embryo-sacs It is very likely that from the double and multiple archesporium the double megaspores are formed, the latter again resulting in the double tetrads (Fig 24A) out of which only the two chalazal megaspores developed into the characteristic double embryo-sacs The multiple embryo sacs can easily be contemplated to arise from the functioning of five of the megaspores of the tetrad complex in Fig 25, which possibly resulted from a double or multiple archesporium and megaspore mother cells Unfortunately no preparation has been obtained which shows a degeneration or differentiation into typical mature embryo-sacs of one or more gametophytes of the double and multiple embryo-sacs respectively However, it seems that ultimately only one of these functions by the suppression of others, as such a tendency is indicated by the several instances in the double embryo-sacs

The occurrence of double and multiple embryo-sacs has been reported for a long time in a number of families Coulter and Chamberlain (1903) summarised comprehensively the occurrence of such abnormal gametophytes in angiosperms Hurst (1931) found two archesporial cells developing up to tetrad in *Rosa mollis*, and more than one embryo-sac in diploid and polyploid species of *Rosa* More recently Joshi and Venkateswarlu (1935) in *Lawsonia*, and Bhaduri (1935) in *Withamnia*, *Physalis* and *Nicotiana* have recorded the occurrence of more than one embryo-sac (see also Puri (1914) in *Moringa oleifera*)

Compton (1912) in *Lychnis alba* × *L floscoculi* and Woodworth (1930) in *Alnus rugosa* have attributed this occurrence of double and multiple embryo-sacs to the hybrid origin of the plants concerned The occurrence of the multiple embryo-sacs in families which are absolutely unrelated does not permit to make out phylogenetic significance Only those cases where all the megaspores of the tetrad seem to function can be said to have analogy with the microspores

The abnormal occurrence in *Anisomeles* of binucleate and trinucleate cells in the chalazal and lateral region of the nucellus below the megaspore mother cell and subsequent stages needs consideration Bhaduri (1935) found a chalazal nucellar cell to be binucleate in *Brunfelsia* Since he found previously in *Solanum melongana* a chalazal nucellar cell functioning as a megaspore mother cell up to the linear tetrad stage, he concluded that in *Brunfelsia* the binucleate chalazal cell, which has been followed up to the four-nucleate stage might represent a stage in 'Lilium type' of embryo sac development, 'since no partition wall has been observed during the binucleate condition' But such a conclusion seems to be untenable in the light of the present investigation in *Anisomeles*, where a number of binucleate and trinucleate nucellar cells persist characteristically through the different stages of development and ultimately the nuclei degenerate, and further in the abnormal dyad and tetrad stage the two epidermal nucellar cells at the tip are binucleate, and in one of them, again, the nuclei are found to be in a fusing state (Fig 40)

Another feature of structural importance in the embryo-sac of *Leonurus* is the rounded chalazal portion ('*pr end l*' in Fig 34) which I have called the '*primitive endosperm lobe*' The terminology is based on the designation of a similar structure in *Physostegia* by Sharp (1911) who found a small protrusion of the embryo-sac at the same position which he termed 'endosperm lobe' on the ground that 'it is soon to contain all the endosperm formed' Sharp's figure shows that the whole of the extremely narrow region including the 'endosperm lobe' excepting the antipodal end contains the first formed endosperm cells * In *Leonurus* also this portion of the chalazal region of embryo-sac contains all the endosperm cells formed primarily, and the chalazal haustorium lies at the extreme end of the embryo-sac In fact the structural difference in the two genera lies in a precocious enlargement of this portion in *Physostegia* In *Leonurus* the endosperm cells grow in the same direction, and it is difficult to distinguish the structure in the two genera at a little later stage It, therefore, seems justified to call the particular region of the embryo-sac of *Leonurus* as 'primitive endosperm lobe' which appears as a prominent protrusion in the mature embryo-sacs of genera like *Physostegia* and *Notochaete*

D Integumentary tapetum—The occurrence of tapetal jacket of the integument surrounding the lower part of embryo-sac is almost universal in this family except in a few species of *Salvia* (Carlson and Stuart, 1936) and *Lallemantia iberica* (Junell 1937) It is present in the majority of the sympetalous families as a characteristic structure The restriction of the tapetal jacket to the lower portion of the embryo-sac is also a characteristic of the Labiates, but this is also found in several members of the Scrophulariaceae, and Lentibulariaceae (Kausik, 1938)

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* The narrow antipodal region of *Physostegia virginiana* contains the chalazal haustorium, as shown later by Schnarf (1917) and not a hypertrophied binucleate antipodal cell as Sharp (1911) claimed earlier.

The significance of the tapetum from the functional point of view is a long debated question Diverse interpretations have appeared in accordance with the development and structure of the tapetum observed by investigators in different families In the late nineties of the last century, Balicka Iwanowska (1899) ascribed a nutritive function to the tapetum which was supported later by Goebel (1923) Palm (1915) expressed the view that the tapetum serves as an embryonal tissue in the earlier stages and has the chief function of providing the transport of nutritive materials to the endosperm tissue and embryo The idea of the digestive function of the tapetum suggested by Lavialle (1922) got support from Junell (1937) in *Hyptis pectinata*, where the author found the glandular development of the tapetal cells According to Junell, the tapetal cells in *Hyptis* helped in dissolving the integumental tissue, and the absorption of the food matter from the dissolved integument takes place chiefly through the micropylar haustorium, he rejects the nutritional hypothesis in view of the cutinized nature of the tapetal cell walls

The loss of identity of the tapetum in the post-fertilisation stages in *Anisomeles* and *Leonurus* and particularly at the very early stages of endosperm development in the latter genus would not lend any support to the theory of nutritive and digestive functions in the later stages of seed development On the other hand the conspicuous form of the tapetal jacket in the earlier stages of embryo-sac development as well as the differential staining capacity from the outer part towards the innermost layers of the integument comprising the tapetum indicate a function of nutrition to the embryo-sac in the pre-fertilisation stages The greater development of the tapetal jacket on the side of the 'primitive endosperm lobe' in *Leonurus* and the rapid extension of the endosperm on this part only would favour the idea that the meristematic character of the tapetum helps in intercalary growth of the integument for the increasing size of the endosperm which grows chiefly along the tapetal jacket in the primary stages This explanation of the function of tapetum was also given by Schnarf (1921) and Svensson (1925)

The argument of a protective function of the tapetum to the endosperm and embryo advocated by investigators of other families like Compositae, Campanulaceae, Scrophulariaceae, Podostemaceae, etc is out of question here as the tapetal jacket is non existent during these later stages

E Endosperm and endosperm-haustoria —Before embarking on any discussion of the development of endosperm and the coincident formation of the haustorial apparatus, it is desirable to discuss Narasimha Murthy's (1942) observations in this respect

Narasimha Murthy has stated that 'three primary tiers of two cells each are formed in the embryo-sac The uppermost cells divide once more by transverse walls into four cells which enlarge and organise micropylar haustorial apparatus' But the present study reveals that the two uppermost cells divide in such a manner as to produce two cells which lie below the tapetal opening (Fig 54), and two upper cells which divide again and form the micropylar haustorial cells The derivation of the micropylar haustorial cells thus takes place entirely differently Further, he has not been able to find out the other type, viz , the free nuclear division of the chalazal chamber ultimately converting into a haustorium The frequent variations in the number of micropylar haustorial cells cannot be expected from such a superficial study (see pp 207 208 for further details)

It is now proposed to consider the position of the two genera in relation to the types of endosperm as determined perviously by various workers The entirely different nature of endosperm development leading to the formation of haustorium will also be pointed out

As early as 1917 Schnarf working on a large number of Labiatae distinguished four types of endosperm development in this family on the nature of the second division in the two cells resulting from the transverse division of the secondary endosperm nucleus *Scutellaria type* (also in *Prostanthera*).—In upper as well as lower chamber longitudinal walls are formed *Brunella type* (also in *Satureja, Thymus, Salvia*) In the upper chamber a longitudinal wall is formed The lower chamber is transformed into a binucleate haustorium *Galaeopsis type* (also in *Physostegia*) As in 'Brunella type,' the difference being that the chalazal chamber is small and degenerate soon *Stachys type* —The upper chamber is divided by a transverse wall The chalazal chamber is transformed into a binucleate haustorium

He further stated that in *Scutellaria* a cellular micropylar haustorium is formed, and in the rest the micropylar haustorium is composed of free nuclei, the number of nuclei being larger in 'Stachys type' and fewer in 'Galaeopsis type'

Junell (1937) was the first to take exception to Schnarf's classification when he stated 'It is certainly best to consider the 'Galaeopsis type' as a special case of the Brunella type' In *Salvia mellifera* Carlson and Stuart (1936) has described an unusual type of development, but their account is *misleading* They have stated that a binucleate micropylar haustorium is formed primarily as a result of a division of the daughter nucleus derived from a division of the endosperm nucleus in which the spindle was longitudinally oriented Curiously enough, they have neither figured nor described any transverse wall resulting from the latter division and in the legend of the figure the micropylar haustorium is described as two-celled They further state that a longitudinal division takes place in the lower daughter nucleus and one of the nuclei migrates at chalazal region and organises a binucleate chalazal haustorium by one division, but again, they have figured a two-celled haustorium (according to the explanation of the figure) In fact they have made a captious blending of the conception of cellular and free-nuclear structure which is of foremost importance in endosperm studies Their description of the *S splendens* type shows the development to be of the 'Stachys type' It is difficult to take the account of *S mellifera* as substantiated and probably a re-investigation will lead to the same conclusion for the latter species

From the standpoint of Schnarf's classification, *Anisomeles* would evidently fall under the 'Scutellaria type', when we consider the dominant type of development in the genus, i e , in cases, where a longitudinal wall is formed in the chalazal chamber Again according to the second type, i e , when the chalazal chamber is a binucleate structure, it comes under the 'Brunella type' and particularly resembles *Pogostemon patchouli* and *Elsholtzia cristata*, where Junell (1937) found a binucleate chalazal haustorium associated with a cellular micropylar haustorium * *Anisomeles indica*, thus, evidently represents the intermediate form between the 'Scutellaria type', which has been considered as the most primitive type on account of the undifferentiated cellular haustorium, and the 'Brunella type' The transitional nature of the genus is further evidenced by the occasional binucleate micropylar haustorial cell (Fig 58) and one of the chalazal haustorial cells (Fig 57)—a tendency towards a free nuclear haustorial apparatus

The primitive nature of the cellular haustoria which is characteristic of the 'Scutellaria type' is obvious in *Anisomeles*, where frequently the micropylar haustorial cells gradually merge into those of endosperm without a distinct demarcation, and this is very clearly represented by the occurrence of ten haustorial cells and a number of intergrading endosperm cells below It evidently indicates that the haustorial system is gradually evolved by the differentiation of the terminal cells of the endosperm developed for a better supply of nutrition to the embryo via endosperm The occasional presence of uninucleate haustorial cells (Fig 54) would suggest the same feature, where one terminal endosperm cell is devoted to the absorption of nutritive materials

The varied type of endosperm development in *Leonurus* presents a puzzling situation at the outset On the one hand, the first type (Type I) essentially follows the 'Brunella type' in so far as the nature of the second division is concerned (From Schnarf's point of view no distinction can be made between the two sub-types of the first type) The second type, on the contrary, unmistakably represents the 'Stachys type'. Fortunately, however, all the intermediate stages between the different types and sub-types have been observed The abnormal configurations in Figs 80-81 represent the forms intermediate between the first two sub-types Again, Figs 82–84 clearly represent the stages by which the 'Brunella type' can give rise to 'Stachys type' It is particularly remarkable that in these abnormal cases, the development of the endosperm primarily follows the more primitive 'Brunella type' and then gradually leads to the 'Stachys type' by the adoption of transverse divisions On the other hand, in the abnormalities representing the first type (Type I) which essentially follows the 'Brunella type', the occasional presence of a two celled chalazal haustorium illustrates the primitive cellular condition of the chalazal haustorium characteristic of the 'Scutellaria type', which thus reappears in the unusual cases This is a further evidence of a derivation of the 'Brunella type' from the 'Scutellaria type' (*vide supra*)

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* Actually, however, the 2 micropylar cells do not develop into a haustorial structure but degenerate early (Junell, 1937)

It is now clear that the observations of the varied types of endosperm development in *Leonurus* enables one to draw up the stages through which the 'Brunella type' gradually passed into the 'Stachys type' The 'Stachys type' is undoubtedly the highest type of endosperm development in Labiatae with a highly differentiated multinucleate micropylar haustorium and a free-nuclear chalazal haustorium

It is a matter of coincidence to investigate these species of *Anisomeles* and *Leonurus* concurrently, the former showing the evolution of 'Scutellaria to Brunella type' and the latter indicating the stages of derivation of the 'Stachys type' from the 'Brunella type' Even in the same species of *Leonurus* there are indications of the most primitive to the highest type of endosperm development from the standpoint of Schnarf's unmistakable classification

Junell (1934, 1937) added *Amethystea coerulea* and *Prostanthera lasianthos* to the 'Scutellaria type' and 'probably *Ajuga*' to the 'Stachys type' and stated that (apart from *Stachys*) in the S F s Lavanduloideae, Stachyoideae and Ocimoideae, i e in the Labiatae with gynobasic styles, the endosperm development takes place after the 'Brunella type', the contributions of Schnarf (1917), Laws (1930) and Ruttle (1931, 1932) confirming his statement But the present study offers the strongest criticism to such a generalisation, both being under the S F Stachyoideae, *Anisomeles* and *Leonurus* together show all the types of development Narasimha Murthy's (1940, 1941) accounts of *Ocimum* species (S F Ocimoideae) and *Leucas aspera* (S F Stachyoideae) show the developments to be of the 'Stachys type' and these further do not support Junell's conclusion

It is now desirable to consider the nature of the further development of endosperm leading to the formation of micropylar haustoria as it differs from the processes described by the previous investigators

Though Schnarf (1917) did not make any rigid statement in regard to the further development of endosperm leading to the formation of haustorial apparatus, Junell (1937) generalised a developmental sequence in the 'Brunella type' as follows 'In the division of the central nucleus a transverse wall is formed The nucleus in the basal cell formed thereby divides itself once without the accompaniment of a wall formation during this division In the upper primary cell a longitudinal wall is formed in connection with their division, and in each of the two long cells formed thereby a transverse wall is laid down at the level of the tapetal opening or a little deeper In this stage one can distinguish between the different parts in the endosperm Below lie the basal, usually two-nucleate cell Next to this and above, I have found two parallel cells These are usually surrounded by a tapetum layer and forms the beginning of a real endosperm tissue. The third part which includes in general an upper broadened part of the embryo-sac usually becomes formed into a haustorium like the basal cell ' *

It has already been pointed out how the present study in *Anisomeles* reveals the exact nature of the endosperm and haustorial development differing from Narasimha Murthy's observations Still more interesting is the occurrence of a similar feature in the developmental stages of *Leonurus* The descriptive portion has shown beforehand that the micropylar haustoria in both the sub-types of Type I is derived as a part of the uppermost cells of the three tiered stage of the endosperm Unfortunately the last stage of the second type (Type II) could not be obtained, but as stated previously, there seems to be no doubt that the developmental position is the same as in the previous type, when we look into the abnormal cases where the later stages are nothing but representing the second type This is further suggested by the deepseated position (in the tapetal jacket) of the spindle in the stage in Fig 79

It is of profound interest to notice that the so long generalised sequence of the stages of endosperm which gives rise to the micropylar haustorial cells or nuclei is entirely altered in the two species studied As already generalised by Junell for the 'Brunella type' and also evident from the other types worked out by the various investigators, the micropylar haustorium is derived in those species (according to the said authors) wholly from the upper part of a primary stage of endosperm represented by three superposed regions In fact this is also the feature existing in the other families like Scrophulariaceae, Lentibulariaceae, etc It is but quite natural for Narasimha Murthy (1942), who probably biased with the results of previous investigations concluded the same general mode of development for *Anisomeles* The segregation of the said uppermost part into a portion of endosperm and the micropylar haustorium in *Anisomeles* and *Leonurus* is thus the principal feature which demarcates these two species from the other members of the family in a comparative study

The sudden deviation (in *Anisomeles* and *Leonurus*) from the general process might appear as very striking but a closer analysis of Junell's paper (1937) would reveal that he has been able to trace the details of the ontogeny of the micropylar haustorium in only six out of the nineteen genera he has attempted, and in the rest either he could not find any stages or his interpretations are purely inferential His illustrations of *Molucella*, *Ziziphora* and *Pogostemon* (Figs 3*b*, 4*d* and 6*e* of Junell, 1937) are easily comparable to Fig 52 of *Anisomeles* and in these cases he himself mentions that the uppermost tier lies within the tapetal jacket It is very probable that more critical study will bring out the crucial stage by which the segregation of this tier is effected

Two other features of the micropylar haustoria remain to be discussed The first is the persistence of one synergid in a few instances of *Anisomeles*, which has taken up a definitely haustorial function, apparent in comparison with the other haustorial cells (Figs 65, 66) Persistent synergids have been reported to occur in several members of Compositae, Scrophulariaceae, etc Its significance seems to lie in the development of an accessory haustorial organ which is to be regarded as an improved micropylar absorbing system The other structure of importance consists of the persistence and enlargement of the tubular portion of the uppermost suspensor cell lying in the micropylar chamber and connected with the filamentous suspensor and embryo in *Leonurus* (Figs 88-90) The presence of nuclei in this structure as well as the considerable enlargement (in breadth) is a strong evidence which leads one to suppose that this has got a definite physiological function It seems probable that the nutritive substances absorbed by the micropylar haustorium are conveniently passed through this suspensor cell to the embryo by the cellular filament The wide funnel shaped portion of the tip of this cell is a further evidence in support of this explanation Junell (1937) mentions the occurrence of persistent suspensor tubes in *Hyptis* and *Molucella* and Carlson and Stuart (1936) describes 'active' suspensor tubes in the micropylar haustoria of *Salvia splendens*

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* Translated from Junell's paper.

It will not be out of place to discuss a significant point in regard to the extension of the endosperm tissue in the two species which as I have previously said represent two distinct types of embryo-sacs The unilateral growth of the endosperm in a direction opposite to the micropylar haustorium occurs in both the species But in *Leonurus* it takes place from the very beginning, whereas in *Anisomeles* it is delayed to a later stage Thus the tendency for this unilateral growth which showed signs in the more primitive *Anisomeles* at a later stage in seed development is initiated in *Leonurus* at the very earlier stages, being already determined in the structure of the mature embryo sac, indicating an advancement in the latter species which is also evident in the structures of endosperm and haustoria

F Embryo—With the limited literature available to the author, the only important work on the embryogeny of this family seems to be that of Souèges (1921) who has investigated three species of Labiatae, viz , *Glechoma hederacea*, *Mentha viridis* and *Lamium purpureum* Besides this, Sharp (1911) has described only a few primary segmentations in *Physostegia virginiana* but the account is too restricted to favour a comparison

According to Souèges (1921), the embryogenic behaviour of *Mentha viridis* and *Glechoma hederacea* are more or less similar, while *Lamium purpureum* presents a very irregular mode of development The two former species have been distinguished by Souèges from that of *Capsella bursa pastoris* in the following developmental features (*a*) Early differentiation of the hyphophysis from the middle cell of the tetrad pro-embryo, (*b*) The suspensor is represented by a simple thinned filament, (*c*) The difference in the speed of segmentations of the two apical and basal cells of the embryo

The embryogeny of *Leonurus sibiricus* differs from those of *Mentha viridis* and *Glechoma hederacea* in several important points (1) In the two species mentioned the longitudinal cells of the quadrant embryo divide by transverse walls to produce an octant, but in *Leonurus* the four cells divide periclinally differentiating the four dermatogen cells in the octant stage itself (2) As a consequence the differentiation of the embryo into the layers *l* and *l'* is delayed to the sixteen celled stage of the embryo in *Leonurus*, in contradistinction to *Mentha* and *Glechoma* where the layers *l* and *l'* can be made out in the octant stage itself (3) The early division of the cell *m* to produce *d*, the progenitor of the hypophysis tissue, which occurs simultaneously with the division of the two apical cells It thus takes place still earlier than that of *Mentha* and *Glechoma* when the cell *m* divides after the formation of the quadrant and the octant embryo respectively (4) Souèges has mentioned in *Glechoma* that the cell *ci* rarely divides to form two cells *n* and *n'* and may even lose its power of division, the filamentous suspensor being derived mostly from the cell *m* But the early division of the cell *n* even at the octant stage in *Leonurus* indicates that the derivation of the suspensor is equally favourable from elements *n* and *m* (5) The occasional derivation of the periblem by a periclinal division of the dermatogen cells (6) The frequent divisions of a few upper suspensor cells to form two layers

Since *Leonurus* differs from *Mentha* and *Glechoma* in several important points, the resemblance of the former to *Capsella* becomes still more far-fetched The early differentiation of the dermatogen and the consequent delay of the formation of the layers *l* and *l'* in *Leonurus* represent a fundamental deviation from the 'Capsella' type

The hypophysial region shown in Fig 112 is strikingly similar to some of Souèges' figures for *Lamium purpureum* The oblique nature of the walls in the abnormal cases (Figs 98, 112) indicates the possibilities by which the comparatively more 'Capsella' like developments of *Mentha*, *Glechoma* and *Leonurus* could have been derived from that of *Lamium purpureum* or *vice versa* Again the presence of a tangential wall in the aberrant octant stage in *Leonurus* (Fig 98) a similar occurrence of which was found by Souèges in *Glechoma*, represents a more or less intermediate form which correlates the early dermatogen differentiation in *Leonurus* with the 'Capsella' like formation of octant in *Mentha* and *Glechoma*

The heterogeneous nature of the embryogenic behaviour in the different members of Labiatae which has already been pointed out by Souèges is further emphasised by the still more deviating mode of development in *Leonurus* It now appears from a study of the embryogeny of *Leonurus* that it would not be safe to make any general statement regarding the sequence of segmentations which would characterise the family as a distinct type

SUMMARY

1 The paper deals with the ontogeny of the flower, development of carpels, ovules, integuments, female gametophyte, fertilisation and formation of endosperm and haustoria in *Anisomeles indica* O Ktze and *Leonurus sibiricus* Linn It also embodies the embryogeny of the latter species

2 The carpellary primodia arise from the base of the stamens and grow up to form the wall of the ovary The latter grows inwards through the four ovules and forms the gynobasic style which fuses with the funicle at a later stage

3 The curvature of the ovule is of different nature in the two species and produces two types of ovules, ultimate form of embryo sac also depends on this mode of curvature, two distinct types of gametophyte have been noted

4 The degeneration of the upper three megaspores takes place from below upwards Development of the gametophyte is normal

5 The synergids have got distinct hooks The antipodals are ephemeral

6 Polar fusion takes place just before fertilisation in *Leonurus* In *Anisomeles*, the fusion occurs along with double fertilisation

7 Rapid development of the embryo sac takes place at the four nucleate stage The mature embryo sac consists of two distinct regions

8 The nucellus is of a simple type A division of the two upper nucellar cells occurs in both the species The occurrence of bi or tri nucleate nucellar cells throughout the earlier stages in *Anisomeles* and occasionally in *Leonurus* is remarkable

9 An integumentry tapetal jacket is organised which encloses the chalazal portion of the embryo sac, and lies against the latter after the earlier degeneration of the nucellus

10 Various abnormalities have been noted in connection with megasporogenesis and female gametophyte development in *Leonurus* Of these the occurrence of multiple archesporium, double and multiple tetrads, double and multiple embryo sacs, are notable

11 In *Anisomeles* double fertilisation precedes syngamy

12 After fertilisation the secondary endosperm nucleus divides immediately to form two transverse chambers The upper chamber by a series of characteristic divisions organises a micropylar haustorium which is generally a four celled structure in *Anisomeles*, and twelve nucleate in *Leonurus* The chalazal chamber forms a binucleate haustorium (in *Leonurus* and occasionally in *Anisomeles*), or a two celled haustorium (in *Anisomeles* and rarely in *Leonurus*)

13 The sequence of divisions leading to the formation of the micropylar haustoria are described in detail in *Anisomeles* and *Leonurus*, and they have been shown to be different from those observed by previous investigators The important point in this connection is that the uppermost tier of a three tiered stage, which results from the divisions of the secondary endosperm nucleus, is not directly transformed into the micropylar haustorium but divides again, the products being a portion of endosperm and haustorial cells or nuclei

14 The methods of endosperm development and subsequent formation of haustoria in *Leonurus* have been classified into two broad types of which the first type is, again, subdivided into two sub types The probable stages of evolution of the primitive types of endosperm development to the more advanced types have been shown with the aid of abnormal configurations in *Leonurus*

15 After the degeneration of the respective haustoria, the endosperm cells bordering the latter become very rich in cytoplasm, especially in *Anisomeles*

16 A general similarity exists in the extension of the massive endosperm tissue in both the species though they are structurally different before the growth

17 Due to the activity of the micropylar haustoria and endosperm cells the integument is largely destroyed and totally disappears during the later stages The endosperm cells surrounding the embryo also become destroyed by the activity of the latter

18 Three nutritive portions are thus organised in the seeds for the embryo, viz , the micropylar and chalazal haustoria and the endosperm tissue

19 The persistence and increase in size of the upper portion of the suspensor and the occasional persistence of the synergids in the micropylar haustoria of *Leonurus* and *Anisomeles* respectively are noteworthy

20 Detailed embryogeny has been studied in *Leonurus* The main features of distinction with the 'Capsella type' have been discussed

21 The embryo is filled with starch in the mature stages and is surrounded by a thin starchy sheath of endosperm

In conclusion, the writer wishes to express his deep sense of gratitude and indebtedness to Dr Ilabonto Banerji, D Sc, F N I, under whose able guidance and constant encouragement the work was carried out His thanks are also due to Dr J C Sen Gupta, Head of the Department of Botany, Presidency College, and to Mr P C Maulik, Lecturer in Languages, Calcutta University, for their kind help in translating certain German and French literature

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PROCEEDINGS
OF THE
NATIONAL INSTITUTE OF SCIENCES OF INDIA

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19 APR 1950

NATIONAL INSTITUTE OF SCIENCES OF INDIA

Thirteenth Annual General Meeting

The Thirteenth Annual General Meeting of the National Institute of Sciences of India was held on 1st January, 1948, at 4 p m , in the Science College, at Patna

Present

Dr Baini Prashad, O B E , D Sc , F R S E , F L S , F Z S , F R A S B , a past President, in the Chair

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Dr A C Ukil

Dr D S Kothari } *Secretaries*
Dr H S Pruthi }

Besides, a large number of visitors were present

In the absence of the President and the Vice Presidents, Dr Baini Prashad was voted to the Chair (according to Rule No 58(b))

1 The minutes of the Ordinary General Meeting of the Institute held at Delhi on the 5th December, 1947, were read and confirmed

2 The Chairman announced that the American Philosophical Society and the National Academy of Sciences, Washington, had presented a Medallion to the Institute The Medallion was exhibited to the Fellows present

> Resolved that the present be accepted and a letter of thanks sent to the Societies concerned

3 The Chairman announced that as a result of the scrutiny of voting papers received from the Fellows, the following fifteen persons were elected Ordinary Fellows of the Institute —

r, B S , D Sc (Cal), Fisheries Officer with the Government of Mysore, Bangalore, distinguished for his work on fisheries

Bose, Pratap Chandra, B Sc (Eng'g) (Hons) (Glasgow), Chief Engineer, Corporation of Calcutta, distinguished for his work on drainage problem and water supply of Calcutta

Chatterjee, Satya Charan, M Sc , D Sc (Cal), Head of the Department of Geography, Patna College, Bankipur, Patna, distinguished for his work on the basic and ultra basic rocks of Bihar, Orissa and Bengal

Dastur, Jehangir Fardunji, M Sc , D I C , Head of the Division of Mycology, Indian Agricultural Research Institute, New Delhi, distinguished for his work on various crop diseases caused by fungi.

Dutta, Arun Kumar, D Sc (Dacca), Reader in Physics, Dacca University, Dacca, distinguished for his work on absorption spectra, supersonics and viscosity of gases and liquids

Dutta Roy, Rukmini Kishore D Sc (Dacca), Dr Ing (Hanover), Chemist, Geological Survey of India Calcutta distinguished for his work on the chemical study of coals

Heilig, Robert E, M D (Vienna) Chief Physician, Jaipur, distinguished for his work on various pathological problems connected with hookworm and malaria

Krishnaswami Kolar Ramakrishnaiyer, D Sc (Lond), F R I C , Director of Industries, Bihar, Patna , distinguished for his work in Industrial Chemistry

MacGregor, Robert Anderson Formerly Chief Metallurgist to the Government of India, Calcutta, distinguished for his work on fatigue in metals

Mitter, Ganes Chandra, O B E , M Sc (Cal) F R I C , M Inst Met , Chief Assayer, His Majesty's Mint, Bombay, and Honorary Professor of Industrial Chemistry, Royal Institute of Science, Bombay distinguished for his work on chemical and metallurgical problems of coinage metals

Moghe, Mahadeo Atmaram, M A M Sc , Ph D (Lond) F Z S Professor of Zoology, College of Science Nagpur and Head of the Department of Zoology and Dean of the Faculty of Science, Nagpur University Nagpur distinguished for his work on Helminthology and Embryology

Raju, S P, B A B L Dr Ing (Munich) M I E (India), Member of the International Association for Hydraulic Structures Research Stockholm Director, Engineering Research Department, H E H the Nizam s Government Hyderabad, Deccan distinguished for his work on Engineering subjects particularly hydraulics

Ramanujam, Srinivasa M A (Madras) Ph D (London) Director, Central Potato Research Institute, New Delhi distinguished for his work in the fields of cytology, genetics and plant breeding

Rao, Subbarao Ramchandra, M A (Madras), Ph D (Lond), D Sc (Lond), Professor of Physics, Central College Bangalore distinguished for his work on soft X rays, secondary electron emission and molecular magnetism

Ray, Jyotis Chandra, M D (Berlin) Director, Indian Institute for Medical Research, Calcutta, distinguished for his work on physiological subjects

4 The Chairman appointed Dr K N Bahl and Dr B B Mundkur as scrutineers for the voting papers in connection with the election of Council Members and Office bearers of the Institute for 1948

After scrutiny the following were declared as duly elected

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5 At the request of the Secretary the item with regard to the scrutiny of voting papers regarding modification of Rules 9 and 10 was deleted from the Agenda

6 The Secretary (Dr H S Pruthi) read out the Annual Report of the Council of the Institute The report was unanimously adopted (*Vide page 225*)

7 In the absence of the authors, the following papers were taken as read --

(i) *The Geometry of Extra Ordinary Refraction*— by Prof J B Seth, Government College, Lahore (*Communicated* by Dr K P Kichlu)

(ii) *Some Non Ramanujam Congruence Properties of the Partition Function*— by Dr D B Lahiri (*Communicated* by Mr S N Roy)

8 Due to the unfortunate absence of the President (Sir S S Bhatnagar) his Presidential Address was read by the Chairman, Dr Baini Prashad (*Vide page 227*)

ANNUAL ADDRESS TO THE NATIONAL INSTITUTE OF SCIENCES OF INDIA

By SIR S S BHATNAGAR, F R S, F N I

Patna, 1st January, 1948

I am deeply grateful to the Fellows of the National Institute of Sciences of India who elected me their President This is the highest honour which they can bestow upon their fellow-colleagues and I am thankful to them for this recognition of my humble services to science

The year under review has been a momentous one and the most outstanding event being the establishment of Indian independence and the re birth of a really National Government in India The scientists have not lagged behind and the last year's session of the Indian Science Congress under the presidentship of our national hero, Pandit Jawaharlal Nehru, was a great success, the memorable feature of that session being the presence in India of the following delegations, *U K* Sir Charles Darwin, Sir Harold Spencer Jones, Prof P M S Blackett, Sir D'Arcy Thompson, Prof L J Mordell, Prof W Brown, Mr Phillip Bruce White, Sir Angus Gillan, Prof Munro Fox, Sir Arthur Fleming, Prof Dudley Stamp, *Canada* Prof W F Hanna, Prof R B Thomson, Dr T L Tanton, *U S A* Mr Albert F Blakeslee, Dr Harlow Shapeley, Dr E Newton Harvey, Dr Ascar Riddle, Dr W E Deming, *France* Prof J S Hadamard and Madame Hadamard, *China* Prof S S Chern, *U S S R* Prof V P Volgin, Prof E N Pavlovsky, Prof Bolshaikov, Prof S Umarov.

To those who attended the opening session, the event will always remain fresh and will be remembered by them in their old age as something which will bring cherished memories of the past In introducing Sir Harold Spencer Jones, the Astronomer Royal, to the President, I mentioned his great influence with the heavenly bodies and his prowess with the god of rain, which had prevented our meeting from becoming a catastrophe The words were almost prophetic because as soon as the inaugural ceremony was over, came a thunderous down pour of rain the like of which had not been witnessed before in Delhi at that time of the year. Heavens having helped us at the opening day, the session was a tremendous success, particularly well attended were the lectures by foreign delegates which roused a great deal of enthusiasm amongst the peoples of India Even the Russians and the Americans remarked that they had never seen such enthusiastic crowds at lectures of a scientific character

Two more notable events have taken place First of all, the Prime Minister whose interest in science is well known, has agreed to be the President of the Council of Scientific and Industrial Research and the Scientific Consultative Committee Secondly, he has become one of us as we elected him a Fellow of the National Institute of Sciences of India, under Rule 7

With the great change in the outlook of political India, events in the world of science are bound to march forward It is no doubt true that the political developments and rioting which followed the partition of India brought to a standstill the programme of development which was in hand particularly in the northern regions. Our construction programmes of the National Physical Laboratory, National Chemical Laboratory and the Central Glass and Ceramic Research Institute, were completely stopped With great difficulty we are restarting, though slowly. Nevertheless progress has been spontaneous in many directions In the Ministry of

Works, Mines and Power, the projects for the development of power by daming rivers have made vast strides

The following projects are well in hand

Mahanadi Project

Work has already started This scheme is expected to cost Rs 47½ crores and will be able to produce 350,000 K W of electric energy Total land expected to be irrigated when the project is completed is 1 1 million acres

Bhakra Dam

Work has already started This scheme is expected to cost about Rs 65–75 crores and the total land expected to be irrigated will be about 2 million acres 300,000 K W of electric energy are likely to be produced

Damodar Valley Project

Work is expected to be started in a few months' time The cost of the scheme is expected to be about Rs 35 crores 300,000 K W of electric energy is expected to be produced when the scheme is completed 8 dams are likely to be constructed

Kosi Project

This dam is expected to be 800 ft high, the highest in the world and this will be able to produce 1 8 million K W of power Work is expected to be started sometime in 1948-49

Besides these four projects, several other projects are being actively planned and work is expected to be started in about a year's time Some of these are Narbada and Tapti projects, Indrawati project and Godavari project

Possibility of power development in Assam are also under active consideration

The Central Government maintains a number of institutes for training and research in agricultural sciences, animal husbandry, veterinary sciences, dairy, forestry and fisheries The Government have sanctioned schemes for the expansion of facilities for research and training in these institutions The Scientific and Technical Manpower Committee, about whose recommendations I propose to deal later, have recommended that the time has come when a co ordination of agricultural education and training might be accepted through the creation of an All-India Agricultural Council The proposal is now under the consideration of the Government The Indian Agricultural Research Institute is expanding facilities for training and research in all the major branches of agricultural sciences The greatest necessity is felt that facilities for training and research in the basic agricultural sciences are provided for our young men at the highest level so that it becomes less necessary to send young men abroad excepting for specialised training in specific subjects and gaining uptodate experience of technique and ideas abroad This was the main purpose for which the Indian Agricultural Research Institute was founded I am glad to say that it is expanding now on these lines During the last year the Agricultural Department gave serious consideration to the importance of measures of control of wheat against rust The serious damage to the wheat crop last year which meant a loss of more than a million tons of wheat drew the attention of the Government to the subject A scheme of research based on the note prepared by the staff of the Indian Agricultural Research Institute—this note has been published in *Science and Culture*—was sanctioned by the Indian Council of Agricultural Research with a view to effectively controlling rust problem Very important work has already been done and the new scheme aims at strengthening the existing work and expanding further on all India basis the research work on the subject

Another matter which should receive the attention of all of us is the question of increased production from land Two reports concerning soil productivity and soil conservation have been prepared and are under consideration of the Indian Council of Agricultural Research

For some years, the Council of Scientific and Industrial Research has been making efforts to bring into being a number of national laboratories The National Chemical Laboratory at Poona, the National Physical Laboratory at Delhi and the National Metallurgical Laboratory at Jamshedpur, have already been planned and the work of construction has already started The Central Glass and Ceramic Research Institute at Calcutta and the Fuel Research Station at Dhanbad are also busy in completing their building operations The technological block of the Central Glass and Ceramic Research Institute is already working When these laboratories are ready—they have been unfortunately very much delayed for reasons over which we had no control—they will constitute some of the finest laboratories in the world Meanwhile the research work in these various fields has also been started at these centres in hired or improvised buildings

Amongst the new schemes of expansion sanctioned by the Council of Scientific and Industrial Research may be mentioned the following —

(1) A nucleus of Building Research Unit which has started functioning at Roorkee with a skeleton staff and has published results of some interest
(2) A Road Research Institute at Delhi is being actively planned and attempts are being made to secure a plot of land for the Institute on the Muttra Road
(3) The Council has agreed to the establishment of a Central Drug Research Institute
(4) The Council has also agreed to the establishment of a Food Technological Laboratory

The Finance Ministry has agreed to provide funds for the effective planning of the Drug Research Institute and the Food Technological Laboratory

The Council welcomes the establishment of the Indian Standards Institution We are particularly happy as this is a child of the Council which sponsored its early establishment

In the field of scientific and technical manpower, the Scientific Manpower Committee appointed by the Ministry of Education submitted their Interim Report some time ago The Final Report is expected to be submitted by the end of February next The Interim Report has been considered by the Cabinet and the recommendations made by the Committee have been generally accepted

The Committee have recommended measures which the Government should initiate without loss of time in order to meet, to some extent, the existing shortage of scientific and technical personnel These measures relate to—

(i) Expansion of facilities for higher scientific and technical education and training,
(ii) Medical education and training,
(iii) Scientific and industrial research and training,
(iv) Industrial training,
(v) Technical training for Defence Services

The Committee are of the opinion that the present facilities for scientific and technical education are utterly inadequate and in order that the immediate needs of the country for scientific and technical personnel may be met in as short a time as possible, it is necessary for the Government to initiate measures which may start bearing fruit within a year or two The Committee have expressed the opinion that there should be a fourfold increase in the output of technical personnel so that industrial progress may be possible

The Committee have recommended that for an immediate improvement in the out-turn of scientific manpower the Government should utilise the existing sources, viz, universities, special institutions and the industrial concerns, by helping to create in these places adequate facilities for higher education, research and practical training Such help as is to be given should be largely in the form of grants for (*a*) the creation of scholarships on a generous scale, (*b*) the purchase of equipment, and (*c*) the opening of post-graduate research departments in the universities which do not have any at the moment

Some of the general recommendations of the Committee may be summarised below —

(*a*) Top priority should be given to imports of scientific equipment and apparatus for educational institutions and the Government should allow rebate on the import duty on such equipment

(*b*) Surplus war material of scientific value should be made available to educational and research institutions, free of cost as far as possible

(*c*) To facilitate the training of additional workers in institutions and industrial concerns, the Government should evolve suitable machinery for expediting building construction at various training centres and also provide necessary funds for the purpose

(*d*) To overcome the great difficulties experienced by the Government in implementing most of their schemes, the Government should—

- (i) permit the administrative departments concerned to make direct recruitment to scientific and technical posts in special cases instead of depending on the Federal Public Service Commission,
- (ii) create a Scientific Service at par in status and emoluments with Administrative Service similar to that created in Great Britain,
- (iii) improve the salary scales of teachers especially those engaged in technical education,
- (iv) provide funds to enable institutions to send their experienced staff overseas to visit important centres of scientific and technical education and research,
- (v) institute a large number of scholarships for post-graduate and research training in institutions in India,
- (vi) permit the construction of buildings for educational institutions through private agencies,
- (vii) delegate a greater amount of power than at present to heads of educational and research institutions,
- (viii) place at the disposal of institutions block grants for implementing the programme approved by the Government, instead of grants from year to year,
- (ix) consult scientific and technical men at all stages when taking decisions on technical matters,
- (x) take steps towards maintenance of a National Register of Scientific and Technical Personnel by the Council of Scientific and Industrial Research through the agency of the National Institute of Sciences of India

The keen interest taken by the Prime Minister and the Cabinet of the Indian Government inspires us with a new hope and it looks as if Science will get a chance of service New opportunities and new responsibilities offer themselves to us and the National Institute of Sciences has to play an important part in building the New India

Incipit Vita Nova—here begins a new life There are moments in the lives of a nation as well which forcibly recall Dante's words, for one glimpse of freedom may spell for a whole people what a glimpse of Beatrice did for Dante With the ushering of Indian freedom on August 15, Indian science too cried out in ecstasy—***Incipit vita nova.***

The tasks of scientific education in this country henceforward have to be vastly different from those assigned to it hitherto In the past, scientific education has aimed mostly at equipping a number of people for the profession of teaching, or of routine testing, whether of a medical variety or of an engineering or industrial variety A certain number of such scientific workers were in any case needed to keep the imperial machine going—to build and maintain its roads and railways, to run its communications on modern lines, to serve its armed forces and its administrative personnel, to man the various surveys for collecting the data needed by the foreign exploiter To some extent the people, no doubt, benefited, but undeniably an imperialist bias underlay the entire activity The Geological, Botanical and other Surveys collected scientific data but they did so inspired by an imperialist purpose The linguistic and ethnographic data, collected no doubt often with much skill and industry, could be used for the imperialist political purpose of indicating the moral that no nationhood could be claimed for a mere congeries The scientific Surveys of India's resources were primarily inspired by the imperialist purpose of the exploitation of these resources by her alien rulers Even scientific research carried out in India was often not freely available in this country During the late war formulae and processes evolved in Indian laboratories were often handed over to other countries and since they were industrially more developed, they could utilise them for greater advantage In a free India science is no longer to be the tool of a foreign imperialism, and its two great tasks now are to develop Indian scientific talent to its utmost capacity so that it can make a worthy contribution to humanity's pool of scientific thought and knowledge, and to develop India's resources so that the lot of the common man in this country may be improved To raise the economic standards of the common man, it is necessary that the speed of industrialisation be a good deal accelerated This would require expansion in the available scientific personnel and it is among the immediate tasks of scientific education in this country to meet this demand for enhanced scientific personnel Indian industrialisation demands an adequate exploitation of her vast power resources No one will contend that today she has the technical personnel she needs for this task Again, India will have to develop her science and industry for defence purposes, if she is to maintain her freedom and to pursue an independent foreign policy, or to pull her weight in defence arrangements in a commonwealth or a system of alliances Under foreign tutelage her scientific workers were not permitted to peep into the secrets of the War Office But henceforward it is going to be the responsibility of Indian science to see that the Indian defence organisations do not suffer for want of scientific knowledge and scientific personnel

What is to be the language of science in India? Hitherto science has been taught and studied in this country, from the secondary school onwards through English—though there have been laudable efforts to produce scientific terminology and some scientific literature in Indian languages The new context would not permit this state of things to continue The teaching of science will henceforward have to be done through Indian languages and our universities and learned bodies and scientists and teachers must now be called upon to take effective steps to make this possible without any impairment of efficiency—in fact the efficiency must be a good deal raised if we are to prove equal to our new big tasks in free India In a very few years, we may take it, the teaching in the topmost university classes will be through Indian languages, though of course those going up for higher studies will find a working knowledge of one or two European languages essential Those engaged in research are sure to find a knowledge of English and other European languages almost as necessary as they do at present But there seems no reason why students in our universities who will be listening to lectures in the science theatres in their own languages should not have a supply of scientific text books in these languages, though for sometime these will necessarily have to be supplemented by those in European languages particularly in English Our universities and learned bodies and education departments have to give immediate attention to the task of producing a supply of scientific text books and scientific journals in Indian languages The task, I should think, is one which would require the offices of a central co-ordinating agency The chief hitch at present would be the want of ready-made scientific terminology This is a problem that requires very careful thought and much expert and erudite labour and it must not be decided in a hurry The Ministry of Education of the Government of India sometime ago set up a committee to go into this question, and so also did the National Institute of Sciences The Committee set up by the Institute seems to favour, when it informally met, the retention of English for the time being, as a vehicle of thought for advanced scientific knowledge, and the retention of the English technical and scientific terms in scientific writing in Indian languages Scientific writing in India may have to be done in a number of languages, but it will be stupendous waste of labour if each of these sets about coining its own scientific terms—such an enterprise might cost as much labour as did the Tower of Babel and for science in India its results might be no more propitious than those commonly associated with that monument

To have a uniform scientific and technical terminology for all Indian languages is a desideratum that all interesting themselves in this problem will do well to bear in mind In the West the scientific terms generally do not differ much as you pass from one European language to another The common ancestry of European scientific thought in that of Greece, and the common acceptance of Latin as the language of learning during some centuries, have been of great help in creating this uniformity The mention of Latin will to many be an inevitable reminder that the potentialities of Sanskrit with its richness of vocabulary, its facility in new formations, its having been the mother of Indian languages, and with its religious and cultural position in India not less important than that of Latin in Christendom—are no less significant But that is a question I would much rather not go into here For the immediate future I think we have to be content with the English terminology, though it is plain this is just 'making do' and not a satisfactory or final solution of our problem We shall soon start looking for such a solution, perhaps a fresh stocktaking may be necessary after the Constituent Assembly has made its decision with regard to India's national language At the moment all I can say is that we must not lose sight of our objectives and they are

(*a*) Indian scientific workers must be able to draw upon the world pool of scientific knowledge, and in repayment of this debt their own scientific work must be available as a contribution to this pool in a language which is not too difficult to learn and may have to be English

(*b*) For the free exchange of scientific knowledge among the various centres in India a uniformity of scientific terminology is essential It is hoped that the Fellows of the Institute would give the country a real lead in these matters

The country has witnessed a catastrophe the like of which has not been seen in the world in what has followed the partitioning of India A large number of people have been displaced and they are all over the place as refugees Science has not been safe from this tragedy and a great many scientists of note as well as professors and students have been rendered homeless The Government of India in spite of the vast magnitude of this tragedy have done a great deal to relieve the situation The Delhi University has come to the aid of the former Punjab University and have taken up the Honours Schools in Chemistry and Physics under their auspices Attempts are also being made to start camp and double shift colleges so that a large number of displaced teachers and students may be usefully occupied A deep sense of patriotism pervades the country and the rather difficult problem of rehabilitating medical and engineering students has been satisfactorily solved and all the medical colleges in India have agreed to divide the Punjab students amongst themselves Similarly the Thomason College of Engineering at Roorkee has absorbed all the Punjab students who were working for engineering subjects It is the prime duty of the National Institute of Sciences to see that these brethren of ours who have been so displaced are suitably employed. It is much to be regretted that this premier body of Indian scientists has not yet addressed itself to the solution of this important problem. I appeal to the fellow scientists who have gathered here to remember these unfortunate brother scientists and see that any help that is possible is given to them in their domains.

I cannot conclude this address without saying a word about the practical utilisation of results of research in various fields in the new India. Although scientific research is a search for truth for its own sake it will be considered an expensive luxury and no exchequer will vote funds for it unless results of practical utility or for reducing suffering or poverty were the outcome of our investigations and while I should not like to minimise the importance of scientific research for its own sake I must draw the attention of your scientists to the great need of applying their knowledge to the good and betterment of India.

In this connection, it is perhaps not out of place to point out that the utilisation and development of research results is a difficult problem in itself to which considerable thought and time has been and is being devoted in other countries particularly in the United Kingdom, U.S.A., and Canada, etc. It would perhaps be desirable to describe some of the methods employed in these countries to attract the attention of the industrialists and potential individual who is likely to use and derive the ultimate benefit from the research activities. A review of these methods has recently appeared in the British Commonwealth Scientific Office Memo. No. 525 (United Kingdom Scientific Mission Memorandum No. 66/47 dated the 25th August, 1947). The British Commonwealth Scientific Official Conference set up in 1946 a Standing Committee Working Party to determine ways and means of promoting the utilisation of non-patentable scientific and technical data. The Committee contacted various American and Canadian agencies to determine methods in operation in these countries in connection with both patentable and non patentable discoveries. Of these the methods found to be most satisfactory and now in general use are briefly outlined below.

The methods employed by the Federal and State Agricultural Colleges (U.S.A.) in interesting farmers and others likely to benefit by the implementation of their results fall in three categories, viz.

(a) Methods that reach the masses. News stories, circular letters, radio, cinema, exhibits, bulletins and posters.

(b) Methods that reach groups. General meetings, demonstration meetings, leader training meetings, extension schools and study courses.

(c) Methods that reach individuals. Demonstrations, farm and house visits, office calls, telephone calls and correspondence.

The Bureau of Agricultural and Industrial Chemistry (U.S.A.) has given much thought to determining the best methods of keeping industry informed and to getting commercial firms to develop the laboratory scale results obtained in its Regional Laboratories. After several years of experimentation, the Bureau has found that the best results are obtained by one or more of the following five methods:—

(a) Demonstration by pilot plants which show to industrialists how the process can be operated on full production basis. It is the Bureau's experience that if this course is not adopted then more often than not the potentially interested industrialist will not take the trouble to give a new process a tryout.

(b) Frequently the Bureau enters into a co-operative agreement with industry, i.e. a firm's plant is used to develop a process worked out on the laboratory scale by the Bureau's staff. To avoid possible jealousy or recriminations from other potential users, the Bureau wherever possible asks the appropriate industrial trade organisation to nominate the firm with which it enters into a formal or informal collaborative agreement.

(*c*) Each Regional Laboratory has one scientist on its staff who devotes his whole time to liaison work between the Bureau and industry The liaison officers keep industry informed of the Bureau's activities by personal visits and they also endeavour to determine nature of industry's problems and utilise the information so gathered in planning future research programmes of the Bureau Many large firms reciprocate by appointing liaison officers who work in the reverse direction

(*d*) The Bureau tries to get itself represented on as many trade organisations as possible This has been found to be one of the best means of winning industrial confidence and ensuring interest in the Bureau's research activities

(*e*) The Bureau makes the fullest possible use of publications, encouraging its scientists to publish their research findings in a wide variety of recognised scientific journals

All these methods and more will have to be adopted by us in India to make science really effective and understandable to masses Realising the importance of this aspect the Government of India set up an Industrial Research Utilisation Committee in 1941, a year after the creation of the Board of Scientific and Industrial Research Since April last this Committee has been replaced by an Industrial Liaison Committee whose functions remain the same as those of the Utilisation Committee The success of similar utilisation methods will show even to the lay Indian public the great advantages of scientific research

To take one concrete case only, it is estimated that vegetable oil lubricants valued at Rs 5 crores were produced by the oil companies from the processes worked out by the Council of Scientific and Industrial Research The Council did not derive any direct monetary benefit from this process but if the oil companies made a nett 10 per cent profit on these, they paid taxes on Rs 50 lakhs profit and if this figure is worked out it will come to quite a substantial amount In addition to this some 50,000 tons of shipping space with its sea freight and handling charges was saved and if this is calculated, the savings will amount to a handsome figure Similarly, antigas cloth valued at over 1 crore of Rupees was manufactured from the Council's formula Here again the Council did not derive any direct monetary benefit but the indirect benefits to the country's economy will be found to be considerable

There are, however, processes in which both direct and indirect advantages are obtainable and enhance the value of research in the eyes of financially minded individuals and corporations in the country

Amongst the indirect benefits may be mentioned the increased earnings in income-tax, super-tax, excess profits tax and corporation tax, besides provincial and local taxes, which accrue to the Central Government as a result of the profits derived by industry from the Council's processes To these might be added the benefits derived by the country's economy from savings in exchange resulting from diminution of imports of products developed from several of the processes In addition to these the industries and processes developed have provided employment to a considerably large number of men and the provision of this additional employment of labour brings in its train innumerable benefits which it is difficult to detail

Before I close, it is my pleasant duty to offer a hearty welcome to the foreign delegates who are attending the current session of the Indian Science Congress It is very gratifying to learn that an Australian Scientific Delegation which is visiting this country on the invitation of the Government of India for the first time is expected to be present in Patna in time for the Science Congress These visits have amply shown the growing signs of internationalism of science in India and have proved that science is really international in character and transcends politics To these foreign delegates India's young scientists give the assurance that in the new India they will prove their altruistic interest in science by constant collaboration with their colleagues all the world over and by working disinterestedly in the cause of scientific research and development It is only through the efforts of the scientists of all countries that a real United National Organisation can emerge

ANNUAL REPORT, 1947

The Council of the National Institute of Sciences of India have pleasure in submitting the following Report on the general concerns of the Institute for the year 1947 as required by the provisions of Rule 48(*f*)

Membership

The number of Fellows on the roll of the Institute at the beginning of the year was 253—228 Ordinary and 25 Honorary The Hon ble Pandit Jawaharlal Nehru was elected an Ordinary Fellow of the Institute under the provisions of Rule 7(*e*) In accordance with the new Calendar for meetings adopted by the Council of the Institute the election of Ordinary Fellows for the year 1947 will take place in January 1948 at the Annual General Meeting of the Institute One Ordinary Fellow who had resigned his Fellowship previously applied for withdrawal of his resignation and was readmitted in terms of Rule 32 as modified in the Ordinary General Meeting of the Institute held on 18th October 1946 Four Honorary Fellows were elected Four Ordinary Fellows and two Honorary Fellows died during the year under report The total number of Fellows on the roll at the end of the year was therefore 253—226 Ordinary and 27 Honorary

Meetings

The Twelfth Annual General Meeting of the Institute was held in the physics lecture theatre of the University of Delhi on the 1st January 1947

Prof D N Wadia the retiring President delivered his Annual Address on India in Transition—the Role of Science in the Building of New India

During the year under report 10 Ordinary General Meetings were held At these meetings papers were read and discussed At the Ordinary General Meeting held on the 4th April 1947 at Bangalore Dr S L Hora gave a résumé of General Impressions and Specific Contributions of the Empire Scientific Conference held in the United Kingdom in 1946 The following public lectures were also delivered at the Ordinary General Meetings —

August 1 1947 Prof H J Bhabha on *Fundamental Particles* (At Bombay)
November 7 1947 Sir K S Krishnan on *Liquid Metals* (At Delhi)

The Council

The Officers and Members of the Council for the year 1947 were elected at the Twelfth Annual General Meeting of the Institute held on the 1st January 1947 The Council including the representatives of the co operating Academies the Indian Science Congress Association and the Government of India was constituted as follows —

President	Dr Sir S S Bhatnagar *Delhi*
Vice Presidents	Prof H J Bhabha *Bombay*
	Prof S N Bose *Calcutta*
Additional Vice Presidents	Prof A C Banerji *Allahabad*
	Khan Bahadur M Afzal Husain *Lahore*
	Lt Col C L Pasricha *Calcutta*
	Representative of the Indian Academy of Sciences (nomination not received)
Treasurer	Dr Bashir Ahmad, *Delhi*
Foreign Secretary	Dr J N Mukherjee, *Delhi*
Secretaries	Prof D S Kothari, *Delhi*
	Dr H S Pruthi, *Delhi*

Editor of Publications	Dr S L Hora, *Benares*
Members of the Council	Dr S P Agharkar, *Poona*
	Dr K N Bagchi, *Calcutta*
	Dr K N Bahl, *Lucknow*
	Dr S K Banerji, *Delhi*
	Dr B B Dey, *Madras*
	Dr Verrier Elwin, *Benares*
	Prof B C Guha, *Calcutta*
	Prof Sir K S Krishnan, *Delhi*
	Prof S K Mitra, *Calcutta*
	Dr P Parija, *Cuttack*
	Prof M Qureshi, *Hyderabad Dn*
	Dr L A Ramdas, *Poona*
	Mr M S Randhawa *Delhi*
	Dr M R Siddiqi, *Hyderabad Dn*
	Sir S S Sokhey *Bombay*
	Dr A C Ukil, *Calcutta*
	Dr W D West, *Calcutta*
Ex Officio Members of the Council (Past Presidents)	Sir R N Chopra (1939 1940), *Jammu*
	Sir J C Ghosh (1943 44), *Bangalore*
	Dr Baini Prashad (1941 1942) *Delhi*
	Prof M N Saha (1937 1938), *Calcutta*
	Prof D N Wadia (1945 46), *Delhi*
Additional Members of the Council	Prof P C Mahalanobis, *Calcutta*
	Dr H R Mehra, *Allahabad*
	Dr B Mukerji, *Calcutta*
	Representative of the Indian Academy of Sciences (nomination not received)

Prof D N Wadia represented the Government of India also on the Council of the National Institute

In the absence of Dr Bashir Ahmad from station, first Dr B C Guha, and then Prof D N Wadia, was appointed Treasurer

The Council held nine meetings during the year Abstracts of the proceedings of the Council relating to questions which are likely to be of interest to Fellows are given in Appendix III

Publications

Six numbers of the *Proceedings* and one number of the *Transactions* were published during the year 1947

Exchange List

The following additional institutions were placed on the distribution list of the publications of the Institute bringing the total number on the list to one hundred and nine

1. Royal Society, London
2. Academy of Sciences, U S S R , Leningrad
3. Hungarian Academy of Natural Sciences, Budapest
4. National Research Council of Canada, Ottawa
5. Jefe del Servicio Meteorologico Mexicano, Tacubaya, Mexico.
6. Oceanographic Institute of Taiwan, China

Presents and Donations

The Council thank the Academy of Sciences, U S S R for presentation of nearly 200 copies of their publications to the library of the Institute

The following donations were also received—to be utilised, as far as possible, for popularisation of Science, this being the wish of the donors —

		Rs
1	Mr Panna Lal, Delhi	500
2	Mr S B Gupta, Delhi	500
3	Mr Kali Charan, Delhi	100
4	Mr T R Jawahar, Delhi	150
5	Mr Devi Charan Gupta, Delhi	100
6	Mr Chetan Swarup, Delhi Shahdra	200
7	Mr Padam Chand, Delhi	351
8	Mr Khazanchi Mal Jain, Delhi	501
9	Mr C B Gupta, Delhi	500
10	Mr Hardayal, Delhi	500
11	Sardar Sohan Singh Anand, Delhi	500
12	M/s Tek Chand Atma Ram, Delhi	500
		4,402

Grants-in-aid of Publications

The following grants were received during the year for distribution to scientific institutions and academies in aid of their publications —

1	From the Rockefeller Foundation	Rs 15,000
2	From the Government of India	Rs 15,000

The above grants were distributed as follows —

		Rockefeller Grant	Government Grant
		Rs	Rs
1	Royal Asiatic Society of Bengal	500	
2	National Academy of Sciences, India	1,300	
3	Indian Academy of Sciences		6,000
4	Indian Science Congress Association	1,300	
5	Indian Science News Association	1,000	750
6	Current Science Association	1,000	750
7	Indian Statistical Institute	650	
8	Indian Mathematical Society	650	
9	Calcutta Mathematical Society	650	
10	Benares Mathematical Society	300	
11	Indian Physical Society	1,000	250
12	Indian Chemical Society	1,300	500
13	Society of Biological Chemists, India	150	
14	Institution of Chemists, India	650	
15	Geological, Mining & Metallurgical Society of India	750	
16	Calcutta Geographical Society	350	
17	Indian Ecological Society		500
18	Indian Botanical Society	650	
19	Indian Society of Genetics and Plant Breeding	350	
20	Bombay Natural History Society	750	
21	Entomological Society of India	700	400
22	Indian Institute for Medical Research	350	
23	Indian Anthropological Institute	250	
24	Man in India	350	
25	Lahore Philosophical Society		250
26.	Physics Quarterly (Allahabad)		250
	Reserve	150	350

National Institute of Sciences of India Research Fellowships

The following National Institute of Sciences of India Research Fellows were carrying out research work at places mentioned against their names during the year under report

Senior Research Fellowship

Dr F C Auluck (Physics), Delhi University, Delhi
Dr P N Bhaduri (Botany), Calcutta University, Calcutta
Dr M B Pithawalla (Geography), Karachi
Dr M A H Qadri (Zoology), Muslim University, Aligarh (Up to July 31, 1947)
Dr K Subba Rao (Chemistry), Central College, Bangalore

Junior Research Fellowship

Dr C Datta (Botany), Calcutta University, Calcutta
Mr P A R Iyer (Zoology), Central College, Bangalore
Dr N L Phalnikar (Chemistry), Sir Parashurambhau College, Poona
Dr A K Saha (Physics), Calcutta University, Calcutta (Up to June 30, 1947)
Mr K S Singwi (Physics), Delhi University, Delhi
Dr V R Thiruvenkata Char (Mathematics), Central College, Bangalore
Dr N K Sarkar (Chemistry), Calcutta University, Calcutta

For want of formal sanction of the additional Government grant necessary for the purpose, no Research Fellowships were awarded by the Institute this year Applications have, however, been invited and appointments will be made as soon as funds are sanctioned

Imperial Chemical Industries (India) Research Fellowships

The following Research Fellows continued to carry out research work at places noted against their names —

Mr P C Bhattacharya (Physics), Calcutta University, Calcutta
Dr P C Datta (Chemistry), Calcutta University, Calcutta (Up to October 15, 1947)
Dr R N Singh (Botany), Benares Hindu University, Benares
Dr R V Sitholey (Palaeobotany), Lucknow University, Lucknow
Dr M K Subramaniam (Biology), Indian Institute of Science, Bangalore
* Dr Ram Parshad (Physics), National Physical Laboratory, Delhi (Up to June 30, 1947)
* Mr S P Basu (Zoology), Fisheries Department, Government of Bengal, Calcutta
* Mr H N Bose (Physics), Calcutta University, Calcutta

The Imperial Chemical Industries (India), Ltd , sanctioned a grant of Rs 67,200 for the award of I C I Research Fellowships in Physics, Chemistry and Biology

National Register of Scientists

At the instance of the Scientific Man-Power Committee of the Government of India, the National Institute of Sciences of India has issued a questionnaire to assess the 'drift' or 'leakage' of scientific talents in the country The list will include names of persons who possess high scientific and technical qualifications and are either unemployed or engaged in non-technical or unproductive occupations

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* **Appointed during 1947.**

The institute has also started work on the preparation of a National Register of Scientific and Technical Personnel available in India

Grants from the Government of India

The Government of India sanctioned for the year 1947 48 the following grants —

(1) *Recurring* —Rs 1,17,000 for expenses on staff, research fellowships, publications and other general expenses

(2) *Non-recurring* —Rs 2,20,000 for the new building (not likely to be utilised this year)

Other Grants

The following grant in-aid was also received by the Institute during the year —

Rs 500 from the Calcutta University

Site for the new Building.

The question of allotment of a suitable site in New Delhi is nearing decision Sanction to the allotment of a site near the All-India Radio Station or near Kotla Ferozshah on the Muttra Road is expected at an early date

Delegations to Foreign Conferences

The following Fellows of the Institute were appointed delegates to Foreign Conferences by the Government of India

Dr H S Pruthi, Sc D , Ph D , Plant Protection Adviser to the Government of India, led the Indian Delegation to the International Food Infestation Conference held in London in August 1947

Dr M N Saha, D Sc , F R S , F R A S B , Palit Professor of Physics, Calcutta University and Dr H J Bhabha, Ph D , D Sc (Hon), F R S , Director, Tata Institute of Fundamental Research, Bombay, were deputed Government of India representatives to attend the International Conference for Control of Atomic Energy, held in Paris in November 1947

Report by Foreign Secretary

In accordance with No 51 of the Rules and Regulations of the Institute, the Foreign Secretary sent in his report detailing action taken by him regarding the following matters

(1) Conveying thanks of the Institute to institutions concerned for presents of books and publications and for hospitality shown to delegates of the Institute to Conferences abroad

(2) Acknowledging receipt of 74 declassified reports on Atomic Energy from the Ministry of Supply, Directorate of Atomic Energy, London

(3) Information of persons elected by the Institute as Honorary Fellows

The Council at its meeting on November 7, 1947

Resolved—

(i) that it would be desirable to widen the scope of the work of the Foreign Secretary,

(ii) that it would be desirable to establish more international contacts,

(iii) that quarterly report of the work and activities of the Institute should be sent to the Royal Society, London, for publication,

(iv) that the President be authorised to move in the matter of representation of the Institute on various Committees established by the Government of India for dealing with the question of International Scientific Unions.

APPENDIX I

LIST OF FELLOWS

ORDINARY FELLOWS, 1947

1 Abraham, W E V , Lt Col , A R C S (I), F G S , M Inst P T , Senior Geologist, Burmah Oil Co , Ltd , Burma (1936)

2 Agharkar, S P , M A , Ph D , F L S , Maharashtra Association for the Cultivation of Science, Law College, Poona

3 Ahmad, Bashir, M Sc , Ph D , Director, University Institute of Chemistry, Punjab University, Lahore (1944)

4 Ahmad, Nazir, O B E , M Sc , Ph D , Contractor Building, 3rd Floor, Nicol Road, Ballard Estate, Bombay 1

5 Aiyar, R Gopala, M A , L T , M Sc , Professor of Zoology, Andhra University, Waltair (1938)

6 Ajrekar, S L , B A , I E S (Retd), Bhandarkar Institute Road, Poona 4

7 Ananda Rao, K , Rao Bhadadur, M A , I E S , Professor of Mathematics, Presidency College, Madras

8 Ash, W C , B Sc , M Inst C E , A M I Mech E , c/o Lloyds Bank Ltd , 6 Pall Mall, London

9 Auden, J B , M A (Cantab), Geologist, Geological Survey of India, 27 Chowringhee, Calcutta (1938)

10 Awati, P R , B A , D I C , I E S (Retd), 759/20 Deccan Gymkhana, Poona 4

11 Bagchee, K D , D Sc , D I C , Forest Botanist, Forest Research Institute, New Forest, Dehra Dun

12 Bagchi, K N , Rai Bahadur, B Sc , M B , D T M , F R I C , 5 Ballygunge Place, Ballygunge, Calcutta 19 (1940)

13 Bahl, K N , D Sc , D Phil , Professor of Zoology, Lucknow University, Lucknow

14 Banerjee, K , D Sc , Mahendralal Sircar Professor of Physics, Indian Association for the Cultivation of Science, 210 Bowbazar Street, Calcutta 12 (1939)

15 Banerji, A C , M Sc , M A , F R A S , I E S , Gyan Kutir, Beli Road, Allahabad

16 Banerji, I , D Sc , Lecturer in Botany, Calcutta University, 35 Ballygunge Circular Road, Calcutta 19 (1945)

17 Banerji, S K , O B E , D Sc , Director General of Observatories, Lodi Road, New Delhi.

18 Bardhan, J C , D Sc (Cal & Lond), Khaira Professor of Chemistry, Calcutta University, 92 Upper Circular Road, Calcutta 9 (1942)

19 Basu, J K , M Sc , Ph D (Lond), Soil Physicist to the Government of Bombay, Sholapur (1941)

20 Basu, N M , M A , 63 Hindusthan Park, Ballygunge, Calcutta (1944)

21 Basu, S , M Sc , Dy Director General of Observatories, Meteorological Office, Ganesh Khind Road, Poona 5 (1946)

22 Basu, U P , M Sc , Chief Chemist, Bengal Immunity Co , Ltd , 153 Dharamtala Street, Calcutta 13 (1946)

23 Beeson, C F C , C I E , D Sc , Thames House, near Eynsham, Oxford

24 Behari, Ram, M.A , Ph D , Professor of Mathematics, Delhi University, Delhi (1941)

25 Bhabha, H J , Ph D , D Sc (Hon), F R S , Director, Tata Institute of Fundamental Research, 53 Pedder Road, Bombay 26 (1941)

26 Bhaduri, P N , Ph D , F R M S , F R H S , F L S , Lecturer in Botany, Calcutta University, 35 Ballygunge Circular Road, Calcutta 19 (1944)

27. Bharadwaja, Y , M Sc , Ph D (Lond), F L S , University Professor and Head of the Department of Botany, Benares Hindu University, Benares (1937)

28 Bharucha, F R , B A , B Sc , M Sc , D Sc , Professor of Botany and Head of the Department, Royal Institute of Science, Mayo Road, Bombay 1 (1939)

29 Bhaskara Shastri, T P , Rao Saheb, M A , F R.A.S., Director Nizamiah Observatory (Retired), 'Manorama', Begumpet, Hyderabad (Deccan).

30 **Bhatnagar**, Sir S S, Kt, O B E, D Sc, F R S, F R I C, F Inst P, Director of Scientific and Industrial Research, Imperial Secretariat, North Block, New Delhi

31 **Bhattacharya**, D R, Rai Bahadur, D Sc, Ph D, F Z S, Professor of Zoology, Allahabad University, 7 Malaviya Road, Allahabad

32 **Bomford**, G, Lt-Col, R E, Westways, Bothenhampton, near Bridport, Dorset, England (1935)

33 **Bor**, N L, C I E, M A, D Sc, F L S, c/o Messrs Lloyds Bank (Cox and Kings Branch), 6 Pall Mall, London (1941)

34 **Bose**, D M, M A, B Sc, Ph D, Director, Bose Institute, 93 Upper Circular Road, Calcutta 9

35 **Bose**, G S, D Sc, M B, Head of the Department of Experimental Psychology, Calcutta University, 92 Upper Circular Road, Calcutta 9

36 **Bose**, N K, M Sc, Ph D, Director, River Research Institute, Anderson House, Alipur, Calcutta (1938)

37 **Bose**, P K, D Sc, Director, Indian Lac Research Institute, Namkum, Ranchi (1944).

38 **Bose**, R C, M A, Lecturer in Statistics, Calcutta University, Presidency College, Calcutta (1942)

39 **Bose**, S N, M Sc, Khaira Professor of Physics, Calcutta University, 92 Upper Circular Road, Calcutta 9

40 **Bose**, S R, M A, Ph D, F R S E, Professor of Botany, Carmichael Medical College, Calcutta (1935)

41 **Burridge**, W, D M, M A (Oxon) (Address not known)

42 **Calder**, C C, B Sc (Agr), F L S, 18 Gladstone Place, Aberdeen, Scotland

43 **Chatterjee**, N C, Rai Bahadur, D Sc, F R E S, A I I Sc, 18 Rajpur Road, Dehra Dun (1942)

44 **Chopra**, B N, D Sc, F L S, Zoological Survey of India, Kaiser Castle, Benares Cantt (1935)

45 **Chopra**, Sir R N, Kt, C I E, M.D, Sc D, F R A S B, F R C P, Brevet Col, I M S (Retired), Director, Drug Research Laboratory, Jammu Tawi, Jammu and Kashmir State

46 **Chowdhury**, J K, M Sc, Dr Phil (Berlin), Department of Chemistry, Bose Institute, 93 Upper Circular Road, Calcutta 9 (1938)

47 **Chowdhury**, K **Ahmad**, M B E, B A, B Sc, M S, D Sc, Wood Technologist, Forest Research Institute, New Forest, Dehra Dun (1940)

48 **Chowla**, S, M A, Ph D, Professor of Mathematics, Government College, Ludhiana

49 **Coates**, J, A R S M, F G S, Senior Geologist, India, The Burma Oil Co (India Concessions), Ltd, Digboi (1946)

50. **Crookshank**, H, B.A, D Sc, B A I, Geological Survey of India, 27 Chowringhee, Calcutta (1938)

51. **Das**, A K, D Sc, Director, Solar Physics Observatory, Kodaikanal, S India (1943)

52. **Dastur**, R H, M Sc, Cotton Physiologist, Institute of Plant Industry, Indore

53. **Datta**, S, M Sc, D Sc, D I C, 39 Hindusthan Park, R B Avenue P O, Calcutta (1935)

54. **Datta**, S, D Sc, F R S E, M R C V S, Q A D V S, Major, Indian Veterinary Research Institute, Izatnagar, Bareilly, U P (1938)

55. **De**, M. N, M B (Cal), M R C P (Lond), Professor of Medicine, Medical College and First Physician, Medical College Hospital, Calcutta (1942)

56 **De**, P, M B (Cal.), F R C P E, Professor of Physiology, Medical College, Calcutta (1943)

57. **Desai**, R D, M Sc (Lond), D I C, Department of Chemical Technology, Bombay University, Matunga Road, Bombay (1942)

58. **Dey**, B. B, D Sc., F R I C, I E S, 68 High Road, San Thome, Madras

59 **Dhar**, N R, D Sc, F R I C, I E S, Professor of Chemistry, Allahabad University, Allahabad.

60 **Dikshit**, B B, Ph D (Edin), M R C P (Edin), D P H (Calcutta), M B B S (Bombay), Principal, B J Medical College, Poona (1941)

61. **Dunn**, J A, D Sc, D I C, F G S, Superintending Geologist, Geological Survey of India, 27 Chowringhee, Calcutta (1935)

62. **Dutt**, S B, D Sc, D I C, Professor of Chemistry, Delhi University, Delhi (193[illegible])

63. **Elwin**, **Verrier**, M.A, D Sc (Oxon), F R A I, Fellow of Morton College, Oxford, Deputy Director, Anthropological Survey of India, 64 Cantonment, Benares Cantt (1943)

64 **Evans**, P , B A , F G S , c/o Messrs Burmah Oil Co , Ltd , Britannic House, Finsbury Circus, London, E C 2

65 **Fermor, Sir Lewis** L , Kt , O B E , D Sc , A R S M , M Inst M M , F G S , F R A S B , F R S , 24 Durdham Park, Bristol 6

66 **Fowler, Gilbert** J , D Sc , F R I C , Consulting Chemist, Central Hotel, Bangalore

67 **Fox, Sir Cyril** S , Kt , D Sc , M I Min E , F G S , F R A S B , 5 Loudon Court, Moira Street, Calcutta 16

68 **Ganapathi**, K , D Sc , Assistant Director, Chemotherapy Department, Haffkine Institute, Parel, Bombay (1946)

69 **Ganguly**, P B , D Sc (Lond), Principal, Science College, Patna (1945)

70 **Gee**, E R , M A , F G S , Deputy Director, Geological Survey of India, 27 Chowringhee, Calcutta (1935)

71 **Ghosh**, B N , D Sc (Lond), Department of Chemistry, Calcutta University, 92 Upper Circular Road, Calcutta 9 (1942)

72 **Ghosh**, J , M A , Ph D , Principal, Hooghly Mohsin College, Chinsurah, Bengal (1935)

73 **Ghosh, Sir** J C , Kt , D Sc , Director General of Industries and Supply, Imperial Secretariat, New Delhi

74 **Ghosh**, P K , M Sc , D I C , D Sc (Lond), Geologist, Geological Survey of India, 27 Chowringhee, Calcutta (1941)

75 **Ghosh**, R N , D Sc , Reader in Physics, Allahabad University, 152 South Malaka, Allahabad (1939)

76 **Ghosh, Sudhamoy**, D Sc (Edin), F R I C , 15 Justice Chundermadhab Road, P O Elgin Road, Calcutta (1945)

77 **Ghurye**, G S , M A , Ph D , Professor of Sociology, University of Bombay, Fuller Road, Bombay (1941)

78 **Gill**, P S , M Sc , Ph D (Chicago), Professor of Experimental Physics, Tata Institute of Fundamental Research, 53 Pedder Road, Bombay 26 (1945)

79 **Glennie**, E A , D S O , Brigadier, R E , Steepways, Cross Oak Road, Berkhamsted, Herts, England

80 **Gravely**, F H , D Sc , F R A S B , 52 London Road, Reading, England

81 **Guha**, B C , D Sc , Professor of Applied Chemistry, Calcutta University, 92 Upper Circular Road, Calcutta 9 (1941)

82 **Guha**, B S , M A , Ph D , F R A S B , Director, Anthropological Survey of India, 64 Cantonment, Benares Cantt

83 **Guha**, P C , D Sc , Professor of Organic Chemistry, Indian Institute of Science, Malleswaram, Bangalore (1935)

84 **Gupta**, J C , M B (Cal), Professor of Pharmacology and Officer in charge, Indigenous Drugs Inquiry (I R F A), School of Tropical Medicine and Medical College, Calcutta (1945)

85 **Haddow**, J R , O B E , M R C V S , D V S M , I V S (Address not known)

86 **Heron**, A M , D Sc , F G S , F R G S , F R S E , F R A S B , Mines and Geology Office, Hyderabad (Deccan)

87 **Hora**, S L , Rai Bahadur, D Sc , F R S E , F Z S , F R A S B , Director, Zoological Survey of India, Kaiser Castle, Benares Cantt

88 **Husain**, M **Afzal**, Khan Bahadur, M A , M Sc , I A S , 56 Jail Road, Lahore

89 **Ishaq, Mohammed**, M Sc , Ph D , D I C , Head of the Department of Physics, Muslim University, Aligarh (1940)

90 **Iyengar**, M O P , M A , Ph D , F L S , University Professor of Botany, Madras University, 71 Venkatarangam Pillai Street, Triplicane, Madras

91 **Joshi**, A C , D Sc , c/o Botany Department, Indian Agricultural Research Institute, New Delhi (1938)

92. **Joshi**, S S , D Sc (Lond), Principal and Head of the Department of Chemistry, Benares Hindu University, Benares (1945)

93. **Kadam**, B S , Ph D , Director, Tobacco Research Station, Rajahmundry, M & S M. Rly. (1946)

94. **Kapur**, S N , Ph D , Officer in charge, Wood Working Section, Forest Research Institute, 16 New Forest, Dehra Dun

95. **Khan, Hamid**, M Sc , LL B , Ph.D , Warden of Fisheries, Punjab, 2 Sanda Road, Lahore (1944)

96 **Khanolkar**, V. R, M D, Director of Laboratories, Tata Memorial Hospital, Hospital Avenue, Parel, Bombay (1946)

97 **Khastgir**, S R, Ph D, D Sc, F R S E, Head of the Department of Physics, Dacca University, Ramna, Dacca (1944)

98 **Kichlu**, P K, D Sc, Reader in Physics, Delhi University, Delhi (1936)

99 **Kini**, M G, M B, M Ch (Orth), F R S E, Captain, M C, Surgeon and Superintendent, Stanley Hospital, Royapuram, Madras (1944)

100 **Kosambi**, D D, S B (Harvard), Professor of Mathematics, Tata Institute of Fundamental Research, 53 Pedder Road, Bombay 26 (1946)

101 **Kothari**, D S, M Sc, Ph D, Professor and Head of the Department of Physics, Delhi University, Delhi (1936)

102 **Krishna**, S, C I E, Ph D, D Sc, F R I C, Chemist, Forest Research Institute, New Forest, Dehra Dun

103 **Krishnan**, **Sir** K S, Kt, D Sc, F R S, Director, National Physical Laboratory, University Buildings, Delhi

104 **Krishnan**, K V, M B B S, L R C P, D B, D Sc, Professor of Microbiology, All India Institute of Hygiene and Public Health, 110 Chittaranjan Avenue, Calcutta

105 **Krishnan**, M S, A R C S, Ph D, D I C, Geologist, Geological Survey of India, 27 Chowringhee, Calcutta (1935)

106 **Kundu**, B C, Ph D (Leeds), F L S, Director, Jute Agricultural Research, Tejgaon, Dacca (1945)

107 **Lal**, R B, M B B S, D P H, D T M & H, D B, Professor of Vital Statistics and Epidemiology, All India Institute of Hygiene and Public Health, 110 Chittaranjan Avenue, Calcutta (1935)

108 **Law**, S C, M A, B L, Ph D, F Z S, M B O U, 50 Kailas Bose Street, Calcutta (1936)

109 **MacMahon**, P S, M Sc, B Sc (Oxon), F I C, I E S (Address not known)

110 **Madhava**, K B, M A, A I A (Lond), Economist and Statistician, Ministry of Transport, Govt of India, Shahjahan Road, New Delhi (1940)

111 **Mahadevan**, C, M A, D Sc, Professor of Geology, Erskine College of Natural Sciences, Andhra University, Waltair (1945)

112 **Mahajani**, G S, M A, Ph D, Principal and Professor of Mathematics, Fergusson College, Poona 4

113 **Mahalanobis**, P C, O B E, M A, B Sc, F R S, I E S, Statistical Laboratories, Presidency College, Calcutta

114 **Mahanti**, P C, D Sc, F Inst P, Assoc A I E E, Ghose Professor of Applied Physics, Calcutta University, 92 Upper Circular Road, Calcutta (1945)

115 **Maheshwari**, P, D Sc, Head of the Department of Biology, Dacca University, Ramna, Dacca (1935)

116 **Majumdar**, D N, M A, Ph D, F R A I, Lecturer in Anthropology, Lucknow University, Lucknow (1940)

117 **Majumdar**, G P., Ph D, Professor of Botany, Presidency College, Calcutta, 19 Ekdalia Place, Ballygunge, Calcutta (1943)

118 **Majumdar**, R C, Dr Phil Nat (Jena), Professor of Physics, Delhi University, Delhi (1941)

119 **Malhotra**, D R, S B (Harvard), Sc D (Brussels), A M I Ch E, M I E, Metallurgist, B B & C I Ry, Golf Course Road, Ajmer (1946)

120 **Mehra**, H R, M Sc, Ph D, Reader in Zoology, Allahabad University, Allahabad

121 **Mehta**, K C, Rai Bahadur, M Sc, Ph D, Sc D, Principal and Professor of Botany, Agra College, Agra

122 **Mills**, J P, M A, I C S, F R A S B, Lum Sohphoh, Shillong, Assam (1936)

123 **Mitra**, **Subodh**, M D, F R C S, F R C O G, 3 Chowringhee Terrace, Calcutta 9 (1944)

124 **Mitra**, S C, M A, D Phil (Leip), Lecturer in Psychology, Calcutta University, 92 Upper Circular Road, Calcutta 9 (1941)

125 **Mitra**, S K, M B E, D Sc, Ghose Professor of Physics, Calcutta University, 92 Upper Circular Road, Calcutta 9

126 **Mitter**, P C, M A, Ph D, Palit Professor of Chemistry Calcutta University (Retired), 22 Garpar Road, Calcutta

127 **Mookerjee**, H. K, D Sc, D I C, Sir Nilratan Sirkar Professor of Zoology and Head of the Department, Calcutta University, 35 Ballygunge Circular Road, Calcutta 19 (1939).

128 MOWDAWALLA, F N, M A, M I E E, Mem A I E E, M I E, 301 Frere Road, Fort, Bombay

129 MUKERJI, B, M D, D Sc, Director, Central Drugs Laboratory, Government of India, 110 Chittaranjan Avenue, Calcutta 12 (1943)

130 MUKHERJEE, J N, C B E, D Sc, F C S, F R A S B, Director, Indian Agricultural Research Institute, New Delhi

131 MUNDKUR, B B, Ph D, Deputy Director (Plant Diseases), Plant Protection and Quarantines Organisation, Ministry of Agriculture, Pusa Buildings, New Delhi 3 (1946)

132 NAIK, K G, D Sc, F R I C, Retired Principal, Baroda College, Baroda, Camp Pratapganj, B B & C I Ry

133 NAIR, U S, M A, Ph D (Lond), Head of the Department of Statistics, Travancore University, Trivandrum (1945)

134 NARAYANA, BASUDEV, M Sc, M B, Ph D, F R S E, Principal and Professor of Physiology, Prince of Wales Medical College, Patna (1946)

135 NARAYANAMURTI, D, B Sc, Dr Ing, Officer in charge, Wood Preservation Section, Forest Research Institute, New Forest, Dehra Dun (1940)

136 NARLIKAR, V V, B Sc (Bom), B A (Cantab), F R A S, Professor of Mathematics and Head of the Department, Benares Hindu University, Benares (1939)

137 NATH, RAJ, Ph D, D I C, Professor of Geology, Benares Hindu University, Benares (1943)

138. NATH, VISHWA, M Sc, Ph D, F R M S, Professor of Zoology, Government College, Lahore (1940)

139 NEHRU, PANDIT JAWAHARLAL, Prime Minister, Government of India, New Delhi (1947)

140 NEOGI, P, M A, Ph D, I E S (Retired), 44A New Shambazar Street, Calcutta (1936)

141 NORMAND, SIR CHARLES W B, Kt, C I E, M A, D Sc, c/o Midland Bank, Gloucester Road, London, S W 7

142 OLVER, COL SIR ARTHUR, C B, C M G, F R C V S (Address not known)

143 PAI, B P, Ph D, F L S, Economic Botanist and Joint Director, Indian Agricultural Research Institute, New Delhi (1946)

144 PANDIT, C G, O B E, M B B S, Ph D, D P H, D T M, Director, King Institute of Preventive Medicine, Guindy, Madras (1939)

145 PANJA, G, M B, D Bact, Professor of Bacteriology and Pathology, School of Tropical Medicine, Chittaranjan Avenue, Calcutta (1944)

146 PANSE, V G, Ph D, Deputy Director (Research), Institute of Plant Industry, Indore (1946)

147 PARANJPE, G R, O B E, M Sc, A I I Sc, I E S (Retired), 'Sudarshan', 202/1 Sadashiv, Poona 2 (1937)

148 PARANJPYE, SIR R P, Kt, D Sc, Purushottam Ashram, Poona 4

149 PARIJA, P, O B E, M A, D Sc, I E S, Vice Chancellor, Utkal University, Cuttack

150 PASRICHA, C L, M A, M B, B Chir, M R C S, L R C P, Lt Col, I M S, School of Tropical Medicine, Chittaranjan Avenue, Calcutta (1939)

151 PERCIVAL, F G, O B E, Ph D, F G S, Agent, Tata Iron & Steel Co, Ltd, Jamshedpur. (1936)

152 PICHAMUTHU, C S, B Sc, Ph D (Glasgow), F R S E, Principal, Intermediate College, Bangalore (1942)

153 PINFOLD, E S, M A, F G S, c/o Messrs Steel Bros & Co, Ltd, 24/28 Lombard Street, London, E C 2

154 PRASAD, B N, M Sc, D Sc, Ph D, Reader in Mathematics, Allahabad University, Lakshmi Niwas, George Town, Allahabad (1936)

155 PRASAD, MATA, D Sc, F R I C, Principal, Royal Institute of Science, Mayo Road, Bombay (1935)

156. PRASHAD, BAINI, O B E, D Sc, F R S E, F L S, F Z S, F R A S B, Fisheries Development Adviser, Government of India, Ministry of Agriculture, New Delhi

157 PRUTHI, H S, M Sc, Ph D, Sc D, Plant Protection Adviser to the Government of India, Ministry of Agriculture, 12 Akbar Road, New Delhi

158 QURESHI, MUZAFARUDDIN, Ph D, Director of Scientific and Industrial Research, H E H. Nizam's Government, Hyderabad (Deccan)

159 RAJ, B SUNDARA, Dewan Bahadur, M.A, Ph D, Fisheries Development Officer, U P, Civil Secretariat, Lucknow (1935).

160 **Rakshit**, H , D Sc , F Inst P , Lecturer in Applied Physics, Calcutta University, 92 Upper Circular Road, Calcutta 9 (1943)

161. **Ramanathan**, K R , Dewan Bahadur, M A , D Sc , Deputy Director General of Observatories, Meteorological Office, Lodi Road, New Delhi

162 **Ramdas**, L A , M A , Ph D , Agricultural Meteorologist, Meteorological Office, Poona 5 (1935)

163 **Ramiah**, K , M B E , L Ag , M Sc , Dip Agr (Cantab), Director, Central Rice Research Institute, P O Chauliaganj, Cuttack (1942)

164 **Randhawa**, M S , M Sc , I C S , Deputy Commissioner, Delhi (1943)

165 **Rangaswami Ayyangar**, G N , Rao Bahadur, B A , I A S (Retired), Rama Mandiram, 4 Ramaswami Street, Thyagarayanagar, Madras

166 **Rao**, B **Rama**, M A , D I C , F G S , Director, Geological Survey Department, Mysore State, Bangalore

167 **Rao**, B **Sanjiva**, M A , Ph D , D Sc (Lond), Principal and Professor of Chemistry, Central College, Bangalore (1944)

168 **Rao**, C V **Hanumanta**, M A , Professor of Mathematics, Punjab University

169 **Rao**, H **Srinivasa**, M A , D Sc , Central Marine Fisheries Research Station, Triplicane P O , Madras (1937)

170 **Rao**, K **Rangadhama**, D Sc (Madras and London), Professor of Physics, Andhra University, Waltair (1937)

171 **Rao**, L **Rama**, M A , F G S , Professor of Geology, Central College, Bangalore (1939)

172 **Rao**, M V **Radhakrishna**, M B B S , Ph D (Andhra), Assistant Director, Haffkine Institute, Parel, Bombay (1945)

173 **Ray**, J N , D Sc , Ph D , F R I C , Deputy Director General (Development), Directorate General of Industries and Supplies, Shahjahan Road, New Delhi (1935)

174 **Ray**, P R , M A , Palit Professor of Chemistry, Calcutta University, 92 Upper Circular Road, Calcutta 9

175 **Ray**, R C , D Sc (Lond), F R I C , Professor of Chemistry, Science College, B M Das Road, Bankipore, Patna (1943)

176 **Roonwal**, M L , M Sc , Ph D (Cantab), Assistant Superintendent, Zoological Survey of India, Kaiser Castle, Benares Cantt (1945)

177 **Row**, R , M D , D Sc , Lt Col , I M S (Hon), 27 New Marine Lines, Bombay 1

178 **Roy**, S C , M Sc , F R Met Soc (Lond), Deputy Director General of Observatories (Instruments and Supplies), India Meteorological Department, Lodi Road, New Delhi (1940)

179 **Roy**, S K , Ph D , In charge of Development of Coal Mines, Daton Colliery, P O Samdih, Dist Burdwan (1940)

180 **Roy**, S N , M Sc , Lecturer in Statistics, Calcutta University, Presidency College, Calcutta (1944)

181 **Saha**, M N , D Sc , F R S , F R A S B , Palit Professor of Physics, Calcutta University, 92 Upper Circular Road, Calcutta 9

182 **Sahni**, B , D Sc (Lond), Sc D , F R S , Professor of Botany, Lucknow University, Lucknow

183 **Sarkar**, P B , Dr és Sc , A I C , Ghose Professor of Chemistry, Calcutta University, 92 Upper Circular Road, Calcutta 9 (1935)

184 **Sarkar**, P B , D Sc (Dacca), Offg Director of Technological Research Laboratories, Indian Central Jute Committee, 33 Aswini Dutt Road, Calcutta 29 (1946)

185 **Savur**, S R , M.A , L T., Ph D , Regional Director, The Observatory, 15 Mowbrays Road, Teynampet P O , Madras (1941)

186 **Sen**, B M , M A , M Sc , I E S (Retired), 12 Ballygunge Circular Road, Ballygunge, Calcutta 19

187 **Sen**, J M , Rai Bahadur, B Sc , M Ed (Leeds), Dip Ed (Oxford), T D Calcutta (London), F R G S , 28 New Road, Alipur, Calcutta (1935)

188 **Sen**, N R , D Sc , Ph D , Ghose Professor of Applied Mathematics, Calcutta University, 92 Upper Circular Road, Calcutta 9

189 **Seshadri**, T R , M A , Ph D (Manchester), Vikrama Deo Professor and Head of the Department of Chemistry and Chemical Technology, Andhra University, Waltair (1942)

190 **Setna**, S B , M Sc , Ph D (Cantab), F R M S , Director of Fisheries Bombay, Old Customs House Yard, Bombay (1943).

191 **Seymour Sewell**, Lt -Col R B, C I E, M A, Sc D, F R S, M R C S, L R C P, F Z S, F L S, 18 Barrow Road, Cambridge, England (1936)

192 **Shah**, R C, M Sc, Ph D (Lond), Professor of Organic Chemistry, Royal Institute of Science, Mayo Road, Bombay (1941)

193 **Sharif**, M, D Sc, Ph D, Asst Director, Entomology Department, Haffkine Institute, Parel, Bombay (1939)

194 **Siddiqi**, M R, M A, Ph.D, Director, Research Institute, Wooden Bridge, Saifabad, Hyderabad (Deccan) (1937)

195 **Singh**, B K, M A, Sc D, F R I C, I E S, c/o Imperial Bank of India, Chandni Chowk, Delhi

196 **Singh**, B N, D Sc, 3 P C Banerji Road, Allenganj, Allahabad (1941)

197 **Sinton**, J A, V C, O B E, M.D, D Sc, D P H, D T.M, Lt Col, I M S (Retired), Malaria Laboratory, Horton Hospital, Epsom, Surrey, England

198 **Sirkar**, S C, D Sc, Lecturer in Physics, Calcutta University, 92 Upper Circular Road, Calcutta 9 (1942)

199 **Sohoni**, V V, B A (Hons), M Sc, Superintending Meteorologist, India Meteorological Department, Lodi Road, New Delhi (1942)

200 **Sokhey**, Sir S S, Kt, M A, M D, D T M & H, Major General, I M S, Director, Haffkine Institute, Parel, Bombay

201 **Sondhi**, V P, M B E, M Sc, F G S, Geologist, Geological Survey of India, 27 Chowringhee, Calcutta (1941)

202 **Soparkar**, M B, M D, B Hy, 117 Khar, Bombay 21 (1937)

203 **Spencer**, E, D Sc, Ph D, F R I C, A R S M, M I M M, F G S, Consulting Chemist, Bird & Co, Chartered Bank Buildings, Clive Street, Calcutta

204 **Srivastava**, B N, D Sc, Lecturer in Physics, Allahabad University, Allahabad (1946)

205 **Srivastava**, P L, Rai Sahib, M A, D Phil, Reader in Mathematics, Allahabad University, Allahabad (1935)

206 **Subrahmanyan**, V, D Sc, F R I C, Professor of Biochemistry, Indian Institute of Science, Malleswaram, Bangalore

207 **Sukhatme**, P V, D Sc (Lond), Ph D, Statistical Adviser, Indian Council of Agricultural Research, New Delhi (1943)

208 **Sur**, N K, D Sc, 74 Tagore Town, Allahabad (1938)

209 **Tawde**, N R, M Sc, Ph D (Lond), F Inst P, Professor of Physics, Royal Institute of Science, Mayo Road, Bombay (1942)

210 **Taylor**, Sir John, Kt, C I E, D S O, M D, D P H, Major General, I.M S (Retired), 58 Cornwall Gardens, Kensington, London, S W 7

211 **Temple**, F C, (Hony Col), A F (I), C I E, V D, A D C, 28 Victoria Street, London, S W 1 (1937)

212 **Toshniwal**, G R, D Sc, Meteorologist, India Meteorological Department, Poona 5 (1943)

213 **Tribedi**, B P, M B (Cal), D B (Lond), Professor of Pathology, Medical College and Bacteriologist to the Government of Bengal, Calcutta (1943)

214 **Ukil**, A C, M B, M S P E, F S M F (Hon Causa), Principal, Medical College, Calcutta. (1935)

215 **Uppal**, B N, M B E, Ph.D, Director of Agriculture (Research and Education), Bombay Province, Poona 5 (1944)

216 **Vachell**, E T, M.A (Cantab), F G S, F Inst P, Coley, Lydwell Road, Torquay, Devon, England (1942)

217 **Venkataraman**, K, M S Tech, Ph D, D Sc, Director, Department of Chemical Technology, Bombay University, Matunga Road, Bombay (1939)

218 **Venkatesachar**, B, Rao Bahadur, M A, F Inst P, Ambika Vilas, Bull Temple Road, Basavangudi, Bangalore

219 **Verman**, L C, B S Eng, M S, Ph D (Cornwall), F Inst P, Director, Indian Standard Institution, Jaisalmer House, Mansingh Road, New Delhi (1946)

220 **Vijayaraghavan**, T, Ph D (Oxon), Staff Quarters, Andhra University, Waltair

221 **Viswanath**, B, D Sc, F R I C, Principal, College of Agriculture, Benares Hindu University, Benares

222 **Wadia**, D N, M A, B Sc, F G S, F R A S B, Mineral Adviser, Ministry of Works, Mines and Power, Government of India, 10 King George Avenue, New Delhi.

223 **West, W D**, M A, Sc D (Cantab), F R A S B, Director, Geological Survey of India, 27 Chowringhee, Calcutta

224 **Wheeler, R E M**, D Litt (London), Hon D Litt (Bristol), Director General of Archaeology in India, Curzon Road Barracks, New Delhi (1945)

225 **Wheeler, T S**, D Sc, Ph D, F R C Sc I, F R I C, F Inst P, M I Chem E, Professor of Chemistry, University College of Dublin (National University of Ireland), Upper Merrion Street, Dublin

226 **Yajnik, N A**, M A, D Sc, A I C, 17 Manchubhai Road, Malad (Bombay Suburb) (1940)

HONORARY FELLOWS, 1947

1 **Appleton, E V**, M A, D Sc, F R S, N L, Secretary, Department of Scientific and Industrial Research of Great Britain, London

2 **Bailey, E B**, F R S, Director General, Geological Survey of Great Britain, Exhibition Road, South Kensington, London, S W 7

3 **Blakeslee, A F**, Smith College, Northampton, U S A

4. **Bohr, Niels**, N L, Institute for Advanced Studies, Princeton, U S A

5. **Bower, F O**, Sc D (Cantab), LL D, F R S, Emeritus Professor of Botany, Glasgow University, 2 The Crescent, Ripon, Yorks, England

6. **Burn, J H**, M D, F R S, Professor of Pharmacology, Oxford University, Oxford, England

7 **Christophers, Sir Samuel Rickard**, Kt, C I E, O B E, M B, Brevet Colonel, I M S (Retired), 186 Huntingdon Road, Cambridge, England

8 **Dale, Sir Henry Hallett**, Kt, O M, G B E, M A, M D, F R C P, Hon D Sc, Hon M D, Hon LL D, Director of the Davy Faraday Research Laboratory and Fullerian Professor of Chemistry in the Royal Institution, London

9. **Diels, Ludwig**, Director General of the Botanical Garden and Museum, 7 Konigin Luise Strasse, Berlin Dahlem, Germany

10 **Dirac, P A M**, F R S, N L, Lucasian Professor of Mathematics, Cambridge University, Cambridge, England

11. **Donnan, F G**, F R S, Formerly Director, Sir William Ramsay Laboratory, University College, 23 Woburn Square, London, W C 1

12 **Edmunds, Charles W**, A B, M D, Professor of Pharmacology and Therapeutics, University of Michigan Medical School, Ann Arbor, Michigan, U S A

13. **Einstein, Albert**, N L, Princeton University, New Jersey, U S A

14 **Fisher, R A**, Sc D, F R S, Galton Professor in the University of London, England

15 **Goodrich, E S**, M A, D Sc, F R S, Linacre Professor of Zoology and Comparative Anatomy, University Museum, Oxford, England

16 **Greenwood, Major M**, D Sc, F R C P, F R S Professor of Epidemiology and Vital Statistics, London School of Tropical Medicine and Hygiene, London England

17 **Hill, A V**, O B E, Sc D, F R S, M P, Foulerton Research Professor, University College, London, England

18 **Lawrence, E O**, Radiation Laboratory, California University, Berkeley, U S A

19 **Marshall, Sir Guy A K**, C M G F R S, Director, Imperial Institute of Entomology, London, England

20 **Millikan, R A**, President of the California Institute of Technology, U S A

21 **Niggli, P**, Professor of Mineralogy and Petrology, Federal Polytechnical University and University of Zurich

22 **Robinson, Sir Robert**, D Sc, F R S, N L, Waynflete Professor of Organic Chemistry in the Dyson Perrins Laboratory, Oxford University

23 **Russell, Sir E John**, D Sc, F R S, Director, Rothamsted Agricultural Experimental Station, Harpenden, Herts, England

24 **Sherrington, Sir Charles S**, O M, G B E, N L, F R S, Formerly Waynflete Professor of Physiology in the University of Oxford, Broomside, Valley Road, Ipswich, England

25 **Szent Gyorgyi, A**, N L, Professor of Biochemistry, University of Budapest, Budapest, Hungary

26 **Urey, Harold C**, N L, Professor of Chemistry, Institute of Nuclear Studies, University of Chicago, Chicago 37, Illinois

27 **Wenyon, C M**, C M G, C B E, F R S, Director in chief, Welcome Bureau of Scientific Research, 183 Euston Road, London, N W 1

APPENDIX II

COMMITTEES

SECTIONAL COMMITTEES, 1948

(1) 'Mathematics' Committee for Mathematics, Astronomy and Geodesy —

	To serve unt Dec 31
Mr R C Bose	1948
Dr T Vijayaraghavan	1948
Prof A C Banerji (Secretary and Convener)	1949
Dr B N Prasad	1949
Prof Ram Behari	1950
Prof V V Narlikar	1950

(2) 'Physics' Committee for Physics and Meteorology —

Prof Sir K S Krishnan	1948
Prof D S Kothari	1948
Dr P K Kichlu	1949
Prof S N Bose (Secretary and Convener)	1949
Prof H J Bhabha	1950
Dr R N Ghosh	1950

(3) 'Chemistry' Committee for Pure and Applied Chemistry —

Sir S S Bhatnagar	1948
Sir J C Ghosh	1948
Dr K Venkataraman	1949
Dr J N Mukherjee (Secretary and Convener)	1949
Dr J N Ray	1950
Dr B C Guha	1950

(4) 'Engineering Sciences' Committee for Engineering, Metallurgy, Electrotechnics and kindred subjects —

Mr W C Ash	1948
Dr Gilbert J Fowler (Secretary and Convener)	1948
Dr D R Malhotra	1949
Prof G R Paranjpe	1949
Mr P C Bose	1950
Dr S P Raju	1950

(5) 'Geology' Committee for Geology, Palaeontology, Mineralogy and Geography —

Dr M S Krishnan	1948
Prof L Rama Rao	1948.
Mr J Coates	1949
Mr V P Sondhi	1949
Prof C S Pichamuthu	1950
Dr W D West (Secretary and Convener)	1950

(6) 'Botany' Committee for Pure and Applied Botany, Forestry and Agronomy —

Prof S P Agharkar (Secretary and Convener)	1948
Mr K Ramiah	1948
Dr A C Joshi	1949
Prof S R Bose	1949
Dr B S Kadam	1950
Mr M S Randhawa	1950.

(7) 'Zoology Committee for Pure and Applied Zoology and Anthropology including Ethnology —

	To serve until Dec 31
Dr Verrier Elwin	1948
Prof H K Mookerjee	1948
Dr B S Guha	1949
Khan Bahadur M Afzal Husain	1949
Dr S L Hora (Secretary and Convener)	1950
Dr Baini Prashad	1950

(8) Physiology Committee for Animal Physiology Pathology Bacteriology, Psychology and other Medical and Veterinary subjects —

Rai Bahadur K N Bagchi (Secretary and Convener)	1948
Lt Col C L Pasricha	1948
Sir S S Sokhey	1948
Prof N M Basu	1949
Capt M G Kini	1949
Dr A C Ukil	1949
Dr G Bose	1950
Dr B Mukerji	1950
Dr C G Pandit	1950

FINANCE COMMITTEE, 1947

The President
The Treasurer
The two Secretaries
Dr B C Guha

EDITORIAL BOARD 1947

Editor of Publications	Dr S L Hora
Members: Mathematics	Prof D D Kosambi
Physics	Prof H J Bhabha
Chemistry	Prof B B Dey
Geology	Dr W D West
Botany	Prof S P Agharkar
Zoology	Dr S L Hora
Anthropology	Dr B S Guha
Physiology	Dr K N Bagchi

RESEARCH GRANTS COMMITTEE

(Appointed January 1 2, 1947)

Office bearers and Conveners of All Sectional Committees

SCIENTIFIC PUBLICATIONS GRANTS DISTRIBUTION BOARD

(Appointed on June 3 1946)

The President
One of the Secretaries
Sir S S Bhatnagar
Prof M N Saha
Dr A C Ukil
Prof K B Madhava

NATIONAL INSTITUTE OF SCIENCES OF INDIA RESEARCH FELLOWSHIPS COMMITTEE

(To function up to November, 1948)

The President
The Treasurer } *ex officio*
The two Secretaries

Prof M R Siddiqi	Mathematics
Dr S K Banerji	Physics
Sir J C Ghosh	Chemistry
Mr D N Wadia	Geology
Dr P Parija	Botany
Prof K N Bahl	Zoology
K B M Afzal Husain	Zoology
Dr B Mukerji	Physiology
Dr S L Hora	
Sir S S Sokhey	

IMPERIAL CHEMICAL INDUSTRIES (INDIA) RESEARCH FELLOWSHIPS COMMITTEE

(To function up to April 1948)

The President
The Treasurer } *ex officio*
The two Secretaries

Physics	Prof M N Saha Dr Sir K S Krishnan Prof S N Bose
Chemistry	Prof J N Mukherjee Dr K Venkataraman Sir J C Ghosh
Biology	Dr P Parija Mr K Ramiah Dr S L Hora

LIBRARY COMMITTEE

(*Appointed on November 22 23 1946*)

The President
The Treasurer } *ex officio*
The two Secretaries
Prof A C Banerji
Dr B N Prasad
Dr Baini Prashad
Mr M S Randhawa

CHANDRAKALA HORA MEMORIAL MEDAL ADVISORY BOARD, 1947

Prof K N Bahl
Dr H R Mehra
Dr A B Misra
Dr H K Mookerjee
Dr Baini Prashad
Dr S L Hora } *Donors (ex officio)*
Mrs Vidya Hora

COMMITTEE APPOINTED TO CONSIDER THE PROPOSAL FOR A UNITED ACADEMY OF SCIENCES OF INDIA

(Appointed on March 7, 1947)

The President
The Treasurer } *ex officio*
The two Secretaries
Dr S P Agharkar
Prof H J Bhabha
Sir J C Ghosh
Dr J N Mukherjee
Prof M N Saha
Sir S S Sokhey

UNIVERSITIES COMMITTEE

(Appointed on November 22 23, 1946)

The President
The Treasurer } *ex officio*
The two Secretaries
Sir J C Ghosh
Sir Maurice Gwyer (Delhi University)
Dr J N Mukherjee
Dr P Parija (Utkal University)
Prof M N Saha
Sir S S Sokhey
Prof Tara Chand (Allahabad University)
Mr D N Wadia
Dr Ziauddin Ahmed (Muslim University, Aligarh)

COMMITTEE FOR CONSIDERING WAYS AND MEANS FOR THE PROMOTION AND POPULARISATION OF SCIENCE IN INDIA

(Appointed on January 12, 1947)

The President
The Treasurer } *ex officio*
The two Secretaries
Prof K N Bahl
K B M Afzal Husain
Dr J N Mukherjee
Prof M Qureshi
Mr M S Randhawa
Dr A C Ukil
Mr D N Wadia

COMMITTEE TO EXPLORE POSSIBILITIES OF CARRYING ON ADVANCED SCIENTIFIC TEACHING AND RESEARCH IN AN INDIAN LANGUAGE AND THE QUESTION OF THE SCRIPT IN WHICH THIS COULD BE DONE

(Appointed on August 1, 1947)

The President
Prof S P Agharkar
Dr B B Dey
Dr D S Kothari
Sir K S Krishnan
Dr J N Mukherjee
Prof H J Bhabha (Convener)

The Committee was requested to consult, among others, the following —

Dr Tara Chand
Dr Suniti Kumar Chatterji
Maulvi Abdul Haq
Dr Zakir Hussain
Prof D D Kosambi
Prof S N Bose
Prof N R Sen
Prof Humayun Kabir
Prof K Swaminathan
Dr Raghu Vira

COMMITTEE TO CONSIDER MODIFICATION OF REGULATIONS REGARDING ELECTION OF ORDINARY AND HONORARY FELLOWS AND FOR SECTIONAL COMMITTEES

(Appointed on November 22 23 1946)

Prof S P Agharkar
Dr A C Ukil
Dr K N Bagchi (Convenor)

COMMITTEE TO TAKE STEPS TO ENSURE THAT NAMES OF SUITABLE PERSONS ARE NOT LEFT OUT FROM THE LIST OF PROPOSALS FOR ELECTION AS ORDINARY FELLOWS

(Appointed on April 4 5 1947)

Dr K N Bagchi (Convener)
and Conveners of all Sectional Committees

COMMITTEE APPOINTED TO CONSIDER ESTABLISHMENT OF INTERNATIONAL LABORATORIES PROPOSED BY U N E S C O

(Appointed on April 4 5, 1947)

The President
The Treasurer } *ex officio*
The two Secretaries
Prof S P Agharkar
Dr S K Banerji
Prof H J Bhabha
Dr B C Guha
Dr S L Hora
Dr A C Ukil (Convener)

APPENDIX III

IMPORTANT RESOLUTIONS OF THE COUNCIL

1st January, 1947 —It was resolved that 2,000 copies (instead of 400 as heretofore) of the *Proceedings* and *Transactions* of the Institute be published for distribution to Scientific bodies and University Departments in India and abroad in exchange for their publications for building up of the Library of the Institute

7th March, 1947 —The Council appointed a Committee to consider the proposal for the formation of a United Academy of Sciences by combining with the National Institute of Sciences of India, the three co operating academies [The question was considered in a number of later meetings and has not yet been finally decided]

4th April, 1947 —(1) The Council resolved to recommend to the Government of India the establishment of a National Union of Geodesy and Geophysics for India

(2) The Council resolved to recommend to the Department of Social Affairs, Studies and research of the United Nations the establishment of the following international laboratories for Research activities on international scale —

(*a*) A laboratory for high altitude atmospheric and cosmic ray research high up in the Himalayas

(*b*) A food and nutritional research laboratory with special reference to food and food resources of South East Asia

(*c*) An institute for fisheries and oceanography

1st August, 1947 —The Council decided to prepare a National Register of Scientists, for which work the Council of Scientific and Industrial Research had sanctioned a grant of Rs 10,000 to the National Institute of Sciences of India The work is in progress

1st August 1947 —The Council appointed a Committee (with eminent linguists co opted for the purpose) for considering the possibilities of carrying out advanced research work in an Indian language and of the best script for the purpose

7th November, 1947 —The Council revised the Regulations for election of Ordinary Fellows with a view to ensure a more thorough scrutiny of the special research work on which the claims of a candidate to election are based as also to improve the procedure for expression of opinion by the Sectional Committees concerned and the Council

APPENDIX IV

ROCKEFELLER FOUNDATION GRANT FUND

	Rs	A	P		Rs	A	P
To Balance	15,000	0	0	By Distribution during the year	14 850	0	0
Receipt during the year	14,850	0	0	Balance	15,000	0	0
	29,850	0	0		29,850	0	0

APPENDIX V

CHANDRAKALA HORA MEMORIAL MEDAL FUND

Founded in 1945 from a donation of Rs 3 000 by Dr S L Hora and Mrs Hora in memory of their daughter to be bestowed triennially on the person who has made conspicuously important contributions to the development of fisheries in India during the five year preceding the year of award

	Rs	A	P		Rs	A	P
To Balance	3,099	13	0	By Investment	3 000	0	0
Interest realised *less* Bank charges	12	9	0*	Balance	112	6	0
	3,112	6	0		3 112	6	0

* This amount was realised as interest on 3½% G P Notes 1842/43 These G P Notes have now been converted into 3% Conversion Loan of 1946 The interest accrued on the 3% Conversion Loan during the year has not yet been realised by the Institute s Bankers awaiting certificate of exemption of income tax, for which application has already been made

APPENDIX VI

IMPERIAL CHEMICAL INDUSTRIES (INDIA) RESEARCH FELLOWSHIPS FUND

	Rs	A	P		Rs	A	P
To Balance	11,512	9	6	By Fellowships and contingencies	24,077	0	0
Receipt during the year	21,885	8	0	Administration	1,315	6	0
					25,392	6	0
				Balance	8,005	11	6
	33,398	1	6		33,398	1	6

APPENDIX VII

INDIAN SCIENCE ABSTRACTS RESERVE FUND

	Rs	A	P		Rs	A	P
To Balance	11,532	6	6	By Balance	16,532	0	0
Set apart for completing 'Indian Science Abstracts' in the budget estimates for 1946 47	5,000	0	0				
	16,532	6	6		16,532	0	0

APPENDIX VIII

ACTUALS FOR THE YEAR DEC 1, 1946—NOV 30, 1947 AND BUDGET ESTIMATES FOR THE YEAR DEC 1, 1947—NOV 30, 1948

	Actuals 1946 47	Budget Estimates 1947 48
Ordinary Receipts		
	Rs	Rs
Members' subscriptions	6,082	8,000
Sale of Authors copies and publications	49	100
Interest on Investments	1,154	1,300
Grant from Universities	500	500
Donation	4,402	2,000
Contribution from General Fund	5,453	2,640
	17,640	14,540
Extraordinary Receipts		
Admission Fee	480	480
Compounding Fee	548	450
	1,028	930
Ordinary Payments		
Printing of Publications	5,100*	5 100*
Printing and Stationery	3,207	2 000
Contribution to other Science Academies under Rule 19	84	720
Postage and Telegrams	1,000	2,000†
Office Equipment	4,837‡	1 000
Advertisements	727	500
Servants' Liveries	330	200
Miscellaneous Expenses	882	500
Rents and Taxes	308	1,300§
Subscription to Indian Standards Institution	250	250
Freights, Cartage and Conveyance	800	800
Bank Charges	65	70
Audit Fee	50	100
	17,640	14,540
Extraordinary Payments		
Funding of Admission Fee and Compounding Fee	1,028	930

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* In addition to Government of India grant

† Excess due to publications to be despatched to foreign countries (for the past years)

‡ Excess over Government of India grant for Office equipment and cost of transfer of office from Calcutta

§ Includes rent of Calcutta Office for the last year

APPENDIX IX

THE NATIONAL INSTITUTE OF SCIENCES OF INDIA, DELHI

Receipts and Payments Account for the year ended 30th November, 1947

RECEIPTS	Rs A P	Rs A P
To Cash and other Balances		85,626 12 10
„ Members' Admission Fees		480 0 0
„ Members' Subscription		6,070 2 6
„ Members' Subscription in Advance		12 0 0
„ Members' Compounding Fee		548 0 0
„ Sale of Authors' Extra Copies and Publications		48 11 0
„ Interest on Investments		1,265 11 4
„ Grant from Government of India		1,02,200 0 0
„ Grant from Rockefeller Foundation		14,850 0 0
„ Grant from Universities		500 0 0
„ Donations		4,402 0 0
„ Imperial Chemical Industries (India) Research Fellowships		21,885 8 0
„ Refund of Advance		47 0 0
„ Miscellaneous Receipts		0 10 0
„ Employees' Contribution to Provident Fund		793 7 0
„ Grant in-aid for Compilation of Register of Scientific Personnels in India		5,000 0 0
„ Refund of Delhi Advance	1,608 0 0	
Less—Amount credited to Subscription	108 0 0	
		1,500 0 0
TOTAL		2,45,229 14 8

PAYMENTS	Rs	Rs A P	Rs A P
By Salaries and Allowances			17,909 1 0
„ Printing and Stationery			3,207 1 0
„ Printing of Publications, etc			10,123 11 6
„ Contribution to other Science Academies under Rule 19			84 0 0
„ Postage and Telegrams			895 0 3
„ Travelling of Members of Council			17,822 12 0
Travelling of Staff			1,168 13 6
Distribution of Rockefeller Foundation Grant			14,850 0 0
Imperial Chemical Industries (India) Research Fellowship Expenses			24,077 0 0
I C I Miscellaneous Expenses			1,315 6 0
„ National Institute of Sciences Research Fellowship			39,463 6 0
„ Grant-in aid to Scientific Publication			9,650 0 0
„ Subscription to Indian Standards Institution, Delhi			250 0 0
„ Expenses for Compilation of Register of Scientific Research Personnels			2,538 8 0
„ Office Equipment			7,837 2 6
„ Advertisements			727 2 0
„ Servants' Liveries			329 8 0
„ Miscellaneous Expenses			882 8 6
„ Rents and Taxes			308 0 0
„ Freight, Cartage and Conveyance			801 2 6
„ Office Transfer Expenses			1,999 13 0
„ Contribution to Provident Fund			37 8 0
„ Advance to Calcutta Office			3,057 6 3
„ Advance for Buildings			1,000 0 0
„ Bank Charges			64 12 9
„ Refund of Miscellaneous Receipts			35 0 6
„ Suspense			36 0 0
„ Cash and other Balances—			
Investments—			
4% Loan 1960—70	17,000		
3% Conversion Loan 1946	29,600		
	46,600	47,189 3 10	
With Imperial Bank of India, Delhi—			
On Savings Bank Account		2,491 6 6	
On Current Account		34,900 1 4	
Cash in hand		178 7 9	
			84,759 3 5
TOTAL			2,45,229 14 8

Checked and found correct subject to our report of even date

S VAIDYANATH AIYAR & CO
Registered Accountants } *Auditors*

DELHI, KASHMERE GATE,
28th February, 1948

APPENDIX X

NATIONAL INSTITUTE OF SCIENCES OF INDIA

Revised Budget Statement for the year 1947-48

No	Name of Sub-head	Original Budget estimates Rs	Revised figures Rs	REMARKS
1	Office and Publication staff	27,100	Permanent staff Rs 20,198 Temporary staff for Library Rs 1,314	Rs 21,512
2	Travelling expenses	15,000	20,000	The grant of Rs 15,000 for T A is hardly sufficient for the whole of the year The minimum expenditure under this sub head is expected to be the neighbourhood of Rs 20,000
3	N.I S Research Fellowship	61,800	55,400	The Government of India have under consideration the question of providing an additional grant of Rs 60,000 for N I S Research Fellowship to be granted during 1947 48 If this is sanctioned, revised figure under this sub head will be Rs 1,15,400
4	Grants in aid for publication of Scientific Journals	15,000	15,000	
5	Purchase of Books and Journals for Library	5,100	5,088	
	TOTAL	1,24,000	1,17,000	

N.B —The Council at their meeting of November 22-23, 1946, requested for a total grant of Rs 1,24,000 excluding the money (Rs 60,000) required for award of new N.I S Research Fellowships for the year 1947 48 The Government of India, however, sanctioned a total sum of Rs 1,17,000

DISTRIBUTION OF WATER VAPOUR IN THE ATMOSPHERE OVER AGRA

By N K Saha, *D Sc*, *University of Delhi*

(Communicated by Dr R C Majumdar, F N I)

(*Received March 18, read August 4, 1947*)

§1 Statement of the Problem

It is well known that the presence and distribution of water vapour in the atmosphere over any place is of supreme importance in determining meteorological conditions of the place The air containing water vapour being subjected to convectional or advectional processes gives rise to a rich variety of meteorological phenomena resulting from the condensation of the water vapour Further the distribution of temperature at different levels of the atmosphere is closely related to the distribution of water vapour The works of Gold (1909), Humphrey (1909), Emden (1913) and Simpson (1928) have definitely established that the gases of the atmosphere allow almost free passage to the solar radiation to the surface of the Earth, but the long wave radiation emitted by the heated earth is absorbed strongly by the water vapour and other gases of the atmosphere, which in turn, by radiating out their characteristic wavelengths form a condition of radiation equilibrium and maintain the observed distribution of temperature of the atmosphere

For all quantitative approaches to the problems of Meteorology arising out of condensation of water vapour and those depending upon the mechanism of temperature distribution in the Earth's atmosphere, an exact knowledge of the distribution of water vapour in the different layers of the atmosphere is therefore essential To cite one example, we may mention the classical work of Simpson (1928) on the calculation of terrestrial radiation Simpson has assumed that at the base of the stratosphere the air is completely saturated with water vapour and within the stratosphere the water vapour forms a self consistent atmosphere of its own, according to Dalton's law of pressure distribution, supported by the saturation pressure at the base • From what follows in this paper, it appears, however, that the relative humidity at the base of the stratosphere (over Agra) does not exceed 11% Further the complete absence of convection and turbulence which is a necessary condition for the formation of a self-consistent water vapour atmosphere within the stratosphere is also probably doubtful, as has been shown by the works of Maris (1929) according to which the diffusion equilibrium of the atmospheric gases does not completely set in below 150 km height

Experimental measurement of the water vapour content of air cannot unfortunately be successfully made up to a great height of the atmosphere The dry and wet bulb thermometers which are ordinarily employed for such measurements at the surface become ineffective at higher levels due to condensation of water vapour and deposition of ice particles on the wet bulb The hair hygrograph in a rapidly ascending (Dine's) meteorograph is therefore commonly used in upper air soundings But the hair hygrograph has got inherent sources of inaccuracy due to its very uncertain time-lag in responding to the relative humidity (R H) of the surrounding air This error can be partially eliminated by taking the hygrograph record both in ascent and descent of the instrument But the greatest uncertainty of the hair as R H -measurer arises out of the fact that its properties in this respect have not been studied at such low temperatures as occur at the high levels of the atmosphere

An approach to the theoretical calculation of water vapour content of the atmosphere at different layers presents no less difficulty The theoretical calculation might appear principally easy Assuming that a thorough mixing of the atmospheric gases takes place at all heights beginning from ground at least up to the base of the stratosphere, we can take the same percentage proportion of water vapour to exist at all heights as at the ground (for Agra the average proportion of water vapour at ground is about 2% by volume) The rate of fall of water vapour with height will then follow the law of fall of barometric pressure with height We are however led to a paradoxical result if we proceed on this line (vide §4) For example, with the upper air temperature data of Agra (§2) and the observed average vapour pressure at ground at Agra $f_0 = 17\ 8$ mb we get the following results by the above method

TABLE 1

z	0	2	4	6	8	10	gKm
Temperature	31 5	16 0	3 5	−5 5	—17 0	−33 0	°C
E/E_0	1	0 39	0 17	0 087	0 035	0 009	
f/f_0	1	0 79	0 625	0 49	0 37	0 28	

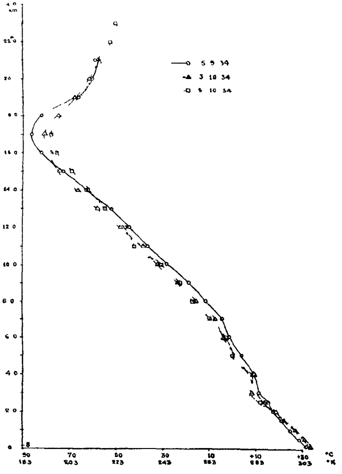

FIG 1 Air Temperature at different heights over Agra from Dine's Meteorograph Ascents

where E and f are the saturation pressure and the calculated water vapour-pressure at the height z, and E_0 and f_0 the corresponding quantities at the surface It is seen that the calculated vapour-pressure at any height far exceeds the saturation pressure, which, as is well known, is solely determined by the temperature prevailing at the height This difficulty has led to the concept that unlike the other permanent gases N_2 and O_2 of the atmosphere, an independent water vapour atmosphere cannot really exist The vertical temperature distribution of the atmosphere sets a limit to the free diffusion of water vapour into higher levels The air temperature decreases so rapidly with height that the total amount of the water vapour at the ground, if allowed to diffuse freely into the atmosphere would begin to condense at a small height above the ground and will partially return to the ground as precipitation

The India Meteorological Department employs the hair hygrograph in their Dine's meteorograph ascents for recording the R H of the upper air The method is obviously subject to the limitations mentioned above An examination of the data obtained by meteorograph ascents over Agra during the year 1932–35 shows that none of the hair hydrograph records goes beyond the height of 10 gKm and 80% of them terminate at 8 0 gKm As this height falls far short of the height of the stratosphere over Agra (about 17 gKm), there is at present no means of direct measurement of the distribution of water vapour near about the stratosphere over Agra Probably a substantial improvement in the matter cannot be effected unless the hair hygrograph is replaced by a radically different instrument * capable of greater precision In the absence of any such instrument, it is considered worth while to review the existing observed data regarding the distribution of R H up to 10 gKm and to try, as a tentative measure, if any general law on the distribution of water vapour with height can be arrived at This is the purpose of the present paper A theoretical deduction of the observed distribution is also attempted and the difficulties of any such deductions are discussed It is shown here that the observed distribution of water vapour pressure can be satisfactorily explained up to 9 gKm on the theoretical basis if the variation of lapse rate of temperature with height is rigorously taken into account

§2 THE EXPERIMENTAL DATA

For details of the form and specification of Dine s Meteorograph adopted by the I M D , reference may be made to the departmental publications including the 'Upper Air Data', vol VI to vol X, Part 14 Eight complete sets of data out of the entire observations taken during the period 1932–36 were selected The basis of selection was that (i) the day of the meteorograph ascent was, as far possible, a cloudless and undisturbed day, and (ii) the data selected referred to a stratosphere of type I, that is, where the stratosphere began with a well-marked temperature inversion The height of the stratosphere was taken as the height of the beginning of the zero lapse-rate The time of all ascents was between 1100 and 1300 G M T The mean observed temperature and relative humidity as obtained from the above ascents are shown in the columns 1, 2 and 4 of table 2 and Figs 1, 2 and 3 Fig 1 represents the observed distribution of temperature with height from three typical flights Fig 2 shows the mean height-temperature curve (mean of the eight sets of data selected) Fig 3 shows the distribution of observed R H with height As the mean error of observed temperature of Fig 2 does not generally exceed 1 %, the smooth temperature distribution curve of Fig 2 is perhaps fairly accurate From this curve the height of the stratosphere over Agra comes out to be 17 0±0 3 gKm (as the year's mean) The average lapse-rate of the atmosphere assumed perfectly polytropic between the ground and the stratosphere would be given by the dotted

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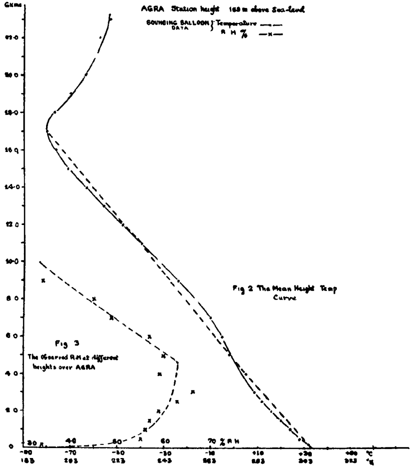
* The method employed by the U S A Meteorological Department in their Radiosonde ascents, I understand, depends on the change of electrical resistance of LiCl solution with absorption of water vapour and appears to be yielding good results.

straight line of Fig 2 and is about 6 6°C /gKm It is seen clearly that the actual height temperature curve is not strictly linear From the ground up to about 4 8 gKm (which is roughly the level corresponding to 0°C) the temperature diminishes more rapidly with height than given by the average lapse-rate, higher up up to about 11 gKm the actual lapse-rate is less than the average and above this height the lapse-rate again exceeds the average till the base of the stratosphere is reached, above which the lapse-rate is zero or negative We shall return to this point later



Figs 2 and 3

Unlike the measurement of temperature, the error of measurement of the R H is very considerable for reasons already explained in §1 Only a very rough height-R H curve can therefore be drawn through the observed points as shown in Fig 3 The main features of the curve, however, seem to be fairly unambiguous There is a rapid increase of R H from the ground up to about 2 0 gKm and then a very slow rise up to about 5 0 gKm (which, it is noteworthy, is very near to the height of 0°C) Afterwards the R H diminishes almost linearly up to 9 0 gKm The R H at any level depends upon the amount of water vapour present at the level as well as on its

TABLE 2

1	2	3	4	5	6	7
Heights gKm	Temperatur t°C	Saturation water vapour pressure E mb	Observed R H %	Observed vapour pressure f mb	f_H calculated from Hergesell's formula mb	f calculated from formula (4) mb
Ground	+31 5	46 9	~40	18 7		
1	24	29 8	56	16 6		
2	16	18 2	60	10 9		
3	9 0	11 5	62	7 1	(8 9)	
4	3 5	7 9	63	5 0	(5 5)	(5 5)
5	−1 2	5 6	61	3 4	3 8	3 46
6	−5 5	4 1	55	2 3	2 6	2 25
7	−10 0	2 87	49	1 4	1 7	1 4
8	−17 0	1 62	45	0 73	0 88	0 66
9	−24 5	0 75	40	0 30	0 4	0 27
10	−33 0	0 28	~35	~0 10	0 16	0 1

temperature The rapid increase of R H from ground up to 2 0 gKm seems largely to be an effect of rapid fall of temperature over this height The variation of saturation pressure of water vapour with height (Fig 4) shows a corresponding large rate of decrease of saturation pressure up to 2 0 gKm The region between 2 0 gKm and 5 0 gKm contains probably the 'Perturbation Zones due to condensation' in the sense in which Süring uses this term Here in spite of a lower lapse-rate of temperature which would tend to decrease the R H with height, it increases at a slow rate due to influence of the 'condensation zones' Between 5 0 gKm and 10 0 gKm the lapse-rate of temperature is again on the increase But the effect of rapid decrease of actual amount of water vapour with height more than compensates the effect of increasing lapse-rate and the result is a steady fall of R H with height observed between these levels Qualitatively we are therefore led to the conclusion that *above the height of 5 0 gKm the amount of water vapour present diminishes rapidly with height but below this height the fall is less rapid on account of the influence of the condensation zones*

The (observed) vapour pressure at different heights obtained by multiplying the saturation pressure by the observed R H at the corresponding levels are shown by the crossed points of Fig 4 and Fig 5 The dotted curve of the figures represents the observed variation of vapour-pressure with height It does not show the irregularities of the R H curve of Fig 3 The rapid increase of R H with height up to 2 0 gKm makes itself apparent in the slow rate of relative decrease of vapour-pressure with height in the very low levels (not shown in Fig 5) The more rapid relative decrease of vapour-pressure in high levels is to be ascribed to almost constant or steadily falling R H with height above 2 gKm

§3 THE DECREASE OF WATER VAPOUR CONTENT OF THE ATMOSPHERE WITH HEIGHT

Early attempts at generalisation of the observed variation of the water vapour-pressure with height have led to the important result that the ratio of vapour-pressures at two given heights is practically constant and independent of the vapour-pressure at the surface Thus Süring has shown that the average fall of vapour-pressure at 2 0 Km height relative to vapour-pressure at the ground, i e f_h/f_0 is about 50% both in Switzerland and in Ceylon, although the surface vapour-pressure in Switzerland is 9 mb and that in Ceylon 29 mb The ratio for Agra with ($f_0 = 18.7$ mb) our present data (table 2) comes out to be 58%, which confirms Sürings generalisation This result has given clue to the empirical formulæ of Hahn and of Süring which express the vapour-pressure as a function of height above sea-level

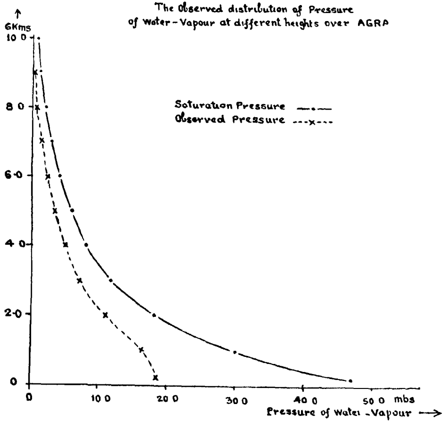


FIG 4

A formula given by Hergesell (1919) based on a careful series of measurements of R H up to 4.0 Km height at Lindenberg is

$$f_H = 3.119 \; e^{23.503 \; t/T} \text{ mm} \tag{1}$$

$$\log_{10} R = 1.833 + 1.603 \; \frac{t}{T} \text{ (per cent)} \tag{2}$$

where f_H = vapour pressure in mm at the level having the air temperature of t°C or T°K The relative humidity (%) at the level is $R = \frac{f_H}{E} \, 100$, E = saturation pressure at the level in mm given by the formula (Scheel)

$$E = 4.581 \, . \, e^{19{\cdot}866 \, t/T} \text{ mm.} \tag{3}$$

The constant 3 119 mm is obviously the vapour-pressure at the level of 0°C temperature Formula (3) makes it clear that the saturation vapour pressure is determined solely by the temperature of the level Similarly (1) and (2) indicate that the vapour-pressure and the R H are apparently reducible ultimately to a function of the temperature alone

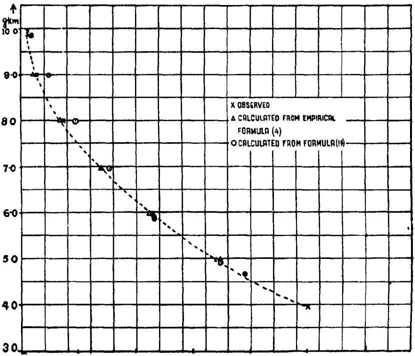


FIG 5 Comparison of Calculated and observed water vapour pressure at Agra

With the constants of the formula (1) and the observed temperature-data over Agra as given in table 1, the vapour-pressure at different heights over Agra is calculated by Hergesell's formula (1) The results are shown in column 6 of table 2 For Agra $t = 0°C$ at the height $z = 4\ 7$ gKm The calculated values of f are therefore given only for the levels 5 0 gKm and above, as Hergesell's formula would be valid only above the condensation level Considering the inaccuracies of the observed values of R H, the agreement between the observed and the calculated values is good Hergesell has asserted that his formula would be roughly valid for all latitudes and up to a very low temperature (very great height) The agreement seen here between the observed and the calculated values of f makes it probable that Hergesell's assertion is correct

At the level of 0°C (about 4 7 gKm) over Agra, the observed R H is ~63%, the saturation pressure $E_0 = 6\ 1$ mb, which together give the pressure of water vapour at this level $\phi_0 = 3\ 9$ mb Hergesell has taken the value of $\phi_0 = 4\ 16$ mb The agreement is again satisfactory

It is, of course, not possible to test the accuracy of Hergesell's formula with our scanty data Nevertheless, a recalculation of the constants of Hergesell's formula in order to fit our observed values of f at Agra at 7 0 gKm level gives the constant

26 90 ($=\beta$) instead of Hergesell's value 23 563, taking $\phi_0 = 3\ 9$ mb as obtained above We thus arrive at the formula

$$f = \phi_0 e^{\beta\ t/T}, \quad \left.\begin{array}{c} \phi_0 = 3\ 9 \text{ mb} \\ \beta = 26\ 90 \end{array}\right\} \qquad . \qquad (4)$$

The values of f calculated from (4) are shown in column 7 of table 2 These show very close agreement with the observed values in column 5

The stratospheric temperature over Agra (at ~ 17 gKm vide table 2) is ~ -79°C Using formula (4) the vapour pressure at the stratosphere comes out to be $\sim 6\ 86\ 10^{-5}$ mb The saturation vapour-pressure at -79°C is $\sim 0\ 61\ 10^{-3}$ mb (over ice) * The R H at the stratosphere therefore comes out to be $\sim 11\%$

§4 INTERPRETATION OF THE EMPIRICAL FORMULA (4)

Assuming a polytropic atmosphere with a linear fall of temperature with height, we can write

$$T = T_0 - \gamma z \qquad (4a)$$

where γ = lapse-rate of temperature in °C /gKm , T_0 = temperature in °K at the surface, T = temperature in °K at the height z gKm The pressure of the atmosphere p at the height z is then given by

$$p = p_0 e^{-\frac{gz}{RT_m}} \qquad (5)$$

where p_0 = pressure of the atmosphere at the surface (sea level), T_m = the integrated mean temperature defined by

$$\frac{z}{T_m} = \int_0^z \frac{dz}{T} = -\frac{1}{\gamma} \ln\left(1 - \frac{\gamma z}{T_0}\right) \qquad (6)$$

$R = 2\ 87\ 10^6$ ergs, and g = acceleration due to gravity (assumed constant) Assuming further that a thorough mixing of the atmospheric gases exists at all levels from ground to the stratosphere, the relative fall of pressure with height of any component i of the atmosphere would be the same as that of the total atmospheric pressure as given by (5) We can therefore write for the vapour pressure f of the component i

$$f_i/f_{i0} = p/p_0 = e^{-\frac{gz}{RT_m}} \qquad (7)$$

Now calling ϕ_{i0} = water vapour pressure at the level with temperature $t = 0$°C , $T = 273$°K $= 1/\alpha$, we have from (5) and (6)

$$\phi_{i0} = f_{i0}\, e^{-\frac{g \ln T_0}{R\gamma}}\ e^{\frac{g}{R\gamma} \ln (273)}, \qquad f_i = \phi_{i0}\ e^{-\frac{g}{R\gamma} \ln \frac{273}{T}} \qquad (8)$$

Further the formula (4) can be written in the form

$$f_{iA} = \phi_{i0}\, e^{-\beta\left(\frac{273}{T} - 1\right)} \qquad (9)$$

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* Washburn, *International Critical Table*, Volume III

where β = const = 26.90. Now the temperatures between the ground and the stratosphere at Agra fall within the range $T = 300°K$ to $173°K$, or $u = 273/T$ lies within the range 0.91 to 1.573. Now since

$$\ln u = (u-1)-\tfrac{1}{2}(u-1)^2+\tfrac{1}{3}(u-1)^3- \qquad (\text{for } 2>u>0),$$

the equation (9) reduces to

$$f_{iA} = \phi_{i0}\ e^{-\beta \ln \frac{273}{T}} \tag{10}$$

in the first approximation. We have therefore

$$\frac{f_{iA}}{f_i} = e^{-\xi \ln \frac{273}{T}} = \left(\frac{273}{T}\right)^{-\xi}, \tag{11}$$

where

$$\xi = \beta - \frac{g}{R\gamma} \approx 26.90 - 5.17 \approx 21.73$$

It is clearly seen that the ratio f_{iA}/f_i would increase gradually with increasing height as T decreases. This means that if we use (8) for calculating f_i in terms of ϕ_{i0}, the calculated values would show more and more disagreement with the observed values (which are reproduced more or less by f_{iA}) as the height increases. That this is a fact is clearly seen from table 3 given below.

TABLE 3

z gKm	5	6	7	8	9	10
f obs	3.4	2.3	1.4	0.73	0.30	~0.10 mb
f calc from (8)	3.82	3.41	3.0	2.57	2.26	1.94 mb
$f_{\text{calc.}}/f_{\text{obs}}$	1.1	1.5	2.1	3.5	7.5	~19
$(273/T)^{\xi}$	1.08	1.6	2.25	4.0	7.4	16.5

The calculated value of vapour-pressure is seen to be 1.5 times the observed value at 6 gKm and more than 10 times that at 10 gKm height. The ratio $f_{\text{calc}}/f_{\text{obs}}$ agrees fairly well with the values of $(273/T)^{\xi}$, which proves the essential correctness of our calculation.

The reason for the discrepancy between the observed and the calculated values cannot be ascribed to the perturbing effect of the condensation zones which possibly comes into play only at lower levels. The reason must be more fundamental.

One source of inaccuracy remains in our calculation of (5), namely, a uniform lapse-rate of temperature γ has been assumed at all heights. This, in reality, does not hold, as can be seen at a glance from Fig. 6. The average lapse rate over ± 0.5 gKm for different heights between the surface and the stratosphere has been calculated from Fig. 2 and plotted in Fig. 6. The curve shows a pronounced minimum ($\gamma \approx 4°C$/gKm) at about 5 gKm which is very near to the 0°C level, and probably another minimum at about 19 gKm. At 17.0 gKm (stratosphere), $\gamma = 0$. Between 0 and 3 gKm and again between 10 and 13 gKm, γ has got a very high value, about 8°C/gKm (vide §2). The effect of variation of γ on (5) cannot be readily seen from our previous formulae, as (4) and the integration of (6) would no longer hold when

γ is a variable But as the nature of the function $\gamma = f(z)$ is not known, only the particular cases that may arise in practice can be treated separately.

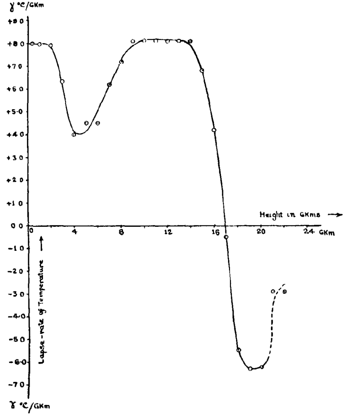

FIG 6 Variation of Mean Lapse rate of Temperature with height

§5 VAPOUR-PRESSURE AS A FUNCTION OF TEMPERATURE AND LAPSE-RATE OF TEMPERATURE

Let f, T = pressure, temperature at the upper level z gKm ,
f_1, T_1 = ,, ,, the lower ,, $z_1 \sim 4.7$ gKm (in this case at level with $T_1 = 273$°K),
γ = lapse-rate of temperature in °C /gKm ,
t, t_1 = temperature in °C corresponding to T°K, T_1°K respectively

Case I

γ = constant, $t = t_1 - \gamma(z - z_1)$,

$$\int_{z_1}^{z}\frac{dz}{1+\alpha t} = \int_{z_1}^{z}\frac{dz}{1+\alpha t_1-\alpha\gamma(z-z_1)} = \int_{0}^{z-z_1}\frac{d(z-z_1)}{1+\alpha t_1-\alpha\gamma(z-z_1)} = -\frac{1}{\alpha\gamma}\ln\frac{1+\alpha t_1-\alpha\gamma(z-z_1)}{1+\alpha t_1}$$

$$= -\frac{1}{\alpha\gamma}\ln\frac{T}{T_1}, \qquad \text{where } \alpha = \frac{1}{273},\ T = 273+t$$

The usual formula for vapour-pressure analogous to (5) taking account of (6) is then

$$\ln\frac{f}{f_1} = -\frac{1}{H}\int_{z_1}^{z}\frac{dz}{1+\alpha t} = -\frac{g}{R\gamma}\ln\frac{T_1}{T}, \text{ where } H = \frac{RT_0}{g} = \frac{R}{g\alpha} \tag{12}$$

The empirical formula (4) can be put into the form (9) comparable to (12)

$$f = f_1 \ e^{-26.90\ln\frac{273}{T}}, \ \ln\frac{f}{f_1} = -26.90\ln\frac{273}{T} \tag{13}$$

Case II $\gamma \neq$ *const*, *but is a linear function of* z The average $\gamma(z)$ between z_1 and z, (where $z > z_1$), is given by

$$\bar{\gamma}(z) = \gamma_1 + \tfrac{1}{2}\int_{z_1}^{z}\frac{d\gamma}{dz}dz = \gamma_1 + \tfrac{1}{2}\gamma(z-z_1), \tag{14}$$

But $\dfrac{d\gamma}{dz} = \gamma =$ const, $t = t_1 - \bar{\gamma}(z-z_1) = t_1 - \gamma_1(z-z_1) - \dfrac{\gamma}{2}(z-z_1)^2$, using (14)

The pressure integral then becomes

$$I = \int_{z_1}^{z}\frac{dz}{1+\alpha t} = \int_{z_1}^{z}\frac{dz}{1+\alpha t_1 - \alpha\gamma_1(z-z_1) - \alpha\frac{\gamma}{2}(z-z_1)^2} \tag{15}$$

Let $x = z - z_1$, then the integral

$$I = \int_0^x \frac{dx}{a+bx+cx^2}, \tag{15a}$$

where $a = 1+\alpha t$, $b = -\alpha\gamma_1$, and $c = -\dfrac{\alpha\gamma}{2}$. In evaluating this integral, two cases may arise

(i) If $b^2 - 4ac > 0$ Then

$$I = \int_0^x \frac{dx}{a+bx+cx^2} = \int\frac{4c\,dx}{(b+2cx)^2-(b^2-4ac)}$$

$$= \int_b^p \frac{2dp}{p^2-q^2}, \text{ where } p = b+2cx, \ q^2 = b^2-4ac, \ dx = \frac{dp}{2c}.$$

With the above meanings of a, b, c, the condition $b^2 > 4ac$ is satisfied automatically when γ is positive For, $b^2 = \alpha^2\gamma_1^2$, $4ac = -4(1+\alpha t_1)\ \dfrac{\alpha\gamma_1}{2} = -2\alpha^2\ T_1\gamma_1$ Now b^2, α^2, T_1 are all positive quantities Hence if γ_1 is positive, $4ac$ must then be a negative quantity, and hence smaller than b^2 The converse is, however, not necessarily true If γ_1 is a negative quantity (lapse-rate of temperature decreasing with height, as towards the stratosphere), it does not follow necessarily that $4ac > b^2$, although then $4ac$ is a positive quantity The question remains to be examined separately Now for $b^2-4ac > 0$,

$$I = \int_b^p \frac{2dp}{p^2-q^2} = \frac{1}{q}\int_b^p \frac{dp}{p-q} - \frac{1}{q}\int_b^p \frac{dp}{p+q}$$

$$= \frac{1}{q}\ln\frac{(p-q)(b+q)}{(p+q)(b-q)} = \frac{1}{q}\ln\left[1+\frac{2q(p-b)}{(pb-q^2)-q(p-b)}\right]$$

Now $p-b = 2cx$, $pb = b^2+2bcx$, $q^2 = b^2-4ac$, which give $pb-q^2 = 2c(bx+2a)$ Then substituting these above

$$I = \frac{1}{q}\ln\left[1+\frac{2q\,2cx}{2c(bx+2a)-q\,2cx}\right] = \frac{1}{q}\ln\frac{x(b+q)+2a}{x(b-q)+2a} \tag{16}$$

To come to the special case ($\gamma =$ const, $\gamma = 0$, $c = 0$), from the general solution (16), put $c = 0$, then $q = b$ Hence from (16),

$$I = \frac{1}{b}\ln\frac{2(a+bx)}{2a} = -\frac{1}{\alpha\gamma_1}\ln\frac{T}{T_1}$$

which at once gives results identical with (12) Now returning to the general case (i), we shall use $q^2 = b^2\left(1-\frac{4ac}{b^2}\right)$, $qx \simeq bx-\frac{2acx}{b}$ in the first approximation

$$I = \frac{1}{q}\ln\frac{(a+bx)+(a+qx)}{2a+x(b-q)} \cong \frac{1}{q}\ln\frac{(a+bx)+bx-\frac{2acx}{b}+a}{2a+\frac{2acx}{b}}$$

$$= \frac{1}{q}\left[\ln\frac{a+bx}{a}+\ln\frac{1-\frac{acx}{b(a+bx)}}{1+\frac{cx}{b}}\right] \tag{17}$$

$$= \frac{1}{q}\left[\ln\frac{T}{T_1}+\ln\frac{1-\frac{cx}{b}\frac{T_1}{T}}{1+\frac{cx}{b}}\right] \tag{18}$$

Hence finally using (18) for the integral in (12a), we get the vapour pressure formula.

$$f_t/\phi_{t0} = e^{-I/H}, \text{ where } H = \frac{R}{g\alpha}$$

$$= \exp\frac{-g\alpha}{R}\;\frac{1}{q}\left[\ln\frac{T}{T_1}+\ln\frac{1-\frac{cx}{b}\frac{T_1}{T}}{1+\frac{cx}{b}}\right] \tag{19}$$

It is easy to see that the quantity $\frac{cx}{b} = \frac{\gamma x}{2\gamma_1}$ is dimensionless The first term in the square bracket is the term corresponding to constant lapse-rate. The second term is the contribution from linear variation of γ with z We are confining, however, ourselves to the case γ = positive, i e to the ascending portion of the curve γ against z Analysis of observed data (Fig 6) shows that this occurs approximately between 4 7 gKm and 9 gKm within the whole range of height from surface to the stratosphere

At 4 7 Km level itself, the curve has got a minimum (corresponding to the condensation point), $\gamma = 0$, $cx/b = 0$ Hence the second term in (18) vanishes Again $T = T_1 = 273°\text{K}$, hence the first term is also zero, or $I = 0$, $f_t = \phi_{t0}$

It is perhaps best to evaluate the second term in (18) numerically by using the experimental values of T, γ, γ for a number of levels between 5 gKm and 10 gKm

From the curve in Fig 6 it follows that between 5 0 and 9 0 gKm , and roughly up to 10 gKm , $\gamma_1 \approx 1°\text{C}/\text{gKm}^2$, $\gamma_1 = 4°\text{C}/\text{gKm}$ (at 4 7 gKm) Then putting the numerical values $\alpha = \frac{1}{273}$, $t_1 = 0°\text{C}$, we have $q = -0\ 0866$, $c/b \approx \frac{1}{8}$, $b = -0\ 0146$ Further with the observed temperature data of the different levels given in table 2 and putting $x = z - z_1$, $z_1 = 4\ 7$ gKm , we obtain from (19) the results shown in the second row of table 4 below The calculated values of f_t (with $\phi_{t0} = 3\ 9$ mb) and the corresponding observed values are also given in the last two rows of the same table and plotted in Fig 5

TABLE 4

z	5 0	6 0	7 0	8 0	9 0	10 0 gKm
(f_t/ϕ_{t0}) calc	0 890	0 60	0 394	0 24	0 129	~0 01
(f_t/ϕ_{t0}) obs	0 87	0 59	0 36	0 19	0 077	0 026
E/E_0	0 92	0 67	0 46	0 27	0 13	0 046
(f_t) calc from eqn (19)	3 47	2 34	1 53	0 93	0 50	0 156 mb
(f_t) obs	3 4	2 3	1 4	0 73	0 30	~0 10 mb

The observed values of f_t/ϕ_{t0}, where $\phi_{t0} = 3\ 9$ mb are given in the third row and the ratio of the saturation pressure at different levels to that at 4 7 gKm ($E_0 = 6\ 1$ mb) in the fourth row The agreement between the calculated and the observed values of f_t/ϕ_{t0} is surprisingly good up to 10 0 gKm except at 9 and 10 gKm where the calculated vapour pressures appear to be somewhat larger than the observed values But in view of the uncertainty of the observed values the agreement can be regarded as satisfactory The calculated values of f are further seen to be smaller than the saturation pressure at all levels up to 10 0 gKm A major difficulty of all previous theoretical calculations of vapour-pressure (vide table 1) based on the assumption of a self consistent water vapour atmosphere from the ground upwards thus appears to be removed

Returning to the solution of the general integral of equation (15), the second case that may arise ($b^2 - 4ac < 0$) cannot be compared with experiment for lack of data and is therefore not attempted here

It is further noted from Fig 6 that above 10 gKm height the temperature lapse-rate γ tends to a constant value up to 13 gKm A calculation of f_t/ϕ_{t0} from (18) using $\gamma = 0$ leads to much higher values at these heights than E/E_0 The observed values at these levels are, however, consistently lower than E/E_0 The disagreement between the observed and the calculated value at these levels is significant It is possible that some new mechanism which arrests further increase of γ with increasing height modifies the basis of our calculation altogether As a possible cause for the new mechanism may be mentioned the rapid increase in the horizontal wind velocity between 10 and 13 gKm over Agra as shown by Sur (1931) from the data obtained from long pilot balloon flights over Agra The rapid increase in wind velocity above 10 gKm may give rise to the mechanical turbulence (Massenaustausch) This is not the turbulence caused by the surface distortion of the stream line flow of wind due to ground resistances or temperature differences, but it is a hydrodynamical property of the fluid flow in which mixing of fluid masses of different layers takes place

In conclusion, I wish to thank Professor D S Kothari, Ph D , and Dr R C Majumdar, Ph D , for many helpful suggestions and the India Meteorological Department whose publications on the 'Upper Air Data' I have made use of

SUMMARY

The mean distribution of temperature between ground and 23 gKm height over Agra and the mean distribution of water vapour pressure up to 10 gKm height are obtained from an analysis of data from selected Dine's meteorograph ascents over the station It is shown that the observed distribution of vapour pressure of water is quite accurately given by an empirical relation of the form $f_h = \phi_0 e^{\beta t/T}$, where ϕ_0 = water vapour pressure at the level with temperature of 0°C, t = temperature in degree centigrade at the height h, $T = 273+t$ and β is a constant An expression giving variation of water vapour pressure with height is derived taking into account the observed variation of lapse rate of temperature with height The results calculated from this expression show good agreement with the observed distribution of water vapour nearly up to 10 gKm height At higher levels there is a discrepancy between the observed and the calculated pressures Probably the basis of our calculation is modified due to the influence of hydrodynamical turbulence caused by rapid increase of wind velocity at very high levels

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STUDIES IN THE PHYSIOLOGY OF RICE.

IV THE EFFECT OF PHOTOPERIODIC INDUCTION ON NITROGEN METABOLISM OF WINTER PADDY

By S M SIRCAR *and* B N DE

(Communicated by Prof P Parija, F N I)

(*Received October, 14 , read November 7, 1947*)

INTRODUCTION

Previous investigations (Sircar, 1942, 1946) on vernalisation and photoperiodism of rice have demonstrated the effectiveness of short days in inducing earliness, increased growth-rate and grain-yield In the preliminary work (Sircar, 1942) the application of 8 hours photoperiods to plants at an advanced stage of the vegetative growth has resulted in a significant increase in tiller number and an earliness of ear emergence In a subsequent work (Sircar, 1944, 1946) different photoperiods have been applied to seedlings in seed beds, as the application of the method in field practices is possible only by the treatment of seedlings before transplantation Of the seed bed treatments for varying periods an exposure to short days of 8 and 10 hours for 6 weeks duration induces maximum earliness and increased grain-yield Tiller number during the vegetative period is maintained at a higher level than in the control

In order to understand fully these effects of photoperiods in accelerating flowering and growth-rate of rice culminating in increased grain yield investigations on the metabolic changes of the plants subjected to different treatments have been undertaken in this laboratory The main physiological processes that have been studied for these effects are carbohydrate and nitrogen metabolism The work on carbohydrate metabolism carried out by Samantaray (1942) has shown that the photoperiodic treatment increases the dry weight of the plant and that at the end of daily photoperiod the earlier leaves show greater accumulation of total sugars than in the control, while there is a fall in the sugar content in the leaves formed later during the life of the plant The accumulation of sugar in the earlier leaves has been found to be associated with more production of tillers at these stages, while a fall in sugar content in the leaves of later stages is related to the translocation of sugars to the developing ears Surprisingly enough it thus appears that the reduction in day length from normal daylight of about 13 hours to 10 or 8 hours has not reduced the photosynthate formed, on the contrary the application of a suitable photoperiod has been found to stimulate the plants to produce increased dry matter

The biochemical changes of plants subjected to photoperiods have been determined by Arthur, Guthrie and Newell (1930), Eckerson (1932), Hard Karrer and Dickson (1934), Murneek (1937), Nightingale (1927), Tincker (1928) and Parker and Borthwick (1939) Some of these workers have attempted to explain the photoperiodic response by the relative amounts of carbohydrate and nitrogen compounds of the plant, generally known as the C/N ratio Suggestions have been made that transition from the vegetative to the reproductive phase is brought about by an accumulation of carbohydrates or by an excess of carbohydrate over nitrogen compound Tincker (1928) from studies on chemical composition of several economic plants which have been submitted to the treatment of different photo periods has come to the conclusion that there is considerable correlation between chemical composition as expressed by this ratio and the behaviour of the plant to varying periods of daily illumination The investigations of Nightingale (1927) and Eckerson (1932) have shown that the length of day influences the production of reductase, the enzyme responsible for the assimilation of nitrate nitrogen The activity of the enzyme is to convert excess carbohydrate and nitrate nitrogen into organic nitrogen and thus a proper balance of C/N ratio necessary for the transition to reproductive growth is maintained Parker and Borthwick (1939) reported that in soybean total nitrogen and soluble non-protein nitrogen were higher in the plant receiving 8 hours photoperiod than in the control Carbohydrates were lower than in the controls with the exception of starch in the leaves which was higher The effects of the short and long day on the chemical composition of the leaves of young wheat plants were studied by Hard Karrer and Dickson (1934) Early flowering in long days was found to be associated with highest carbohydrate and lowest nitrogen per cent in the leaves But this resulted in a very low yield of grain in one variety, *Turkey* On the other hand, the vegetative plants of the short day treatment showed highest nitrogen and lowest carbohydrate per cent and these were associated with good grains in *Hardfederation* and with sterility in *Turkey* The conclusion was drawn that there was no relation between the carbohydrate and nitrogen contents on subsequent grain-yield Murneek (1937) working with Biloxi soybean reported that changes in the relative amounts of carbohydrate and nitrogen compounds do not appear to be of sufficient magnitude to account for the initiation of flower primordia For satisfactory fruiting, however, he suggested that a favourable relationship between carbohydrate and nitrogen compounds may be essential

In some of these investigations plants were grown throughout the season in different photoperiods and changes in the chemical composition of the plants noted In others biochemical changes were determined before and after transfer to various photoperiods Such studies permit a comparison of the influence of different photoperiods on chemical composition of plants, but would not necessarily explain the causal mechanism of photoperiodic response as in most of these cases photoperiods were applied and the chemical composition determined when the plants were already at an advanced stage of development Attempts have been made in this laboratory to approach the problem in a different way Photoperiods necessary for the acceleration of flowering have been applied in the seed bed, and after transplantation the chemical composition of the plant determined at frequent intervals with a view to ascertain the differences in the metabolites produced A difference in the chemical composition of the plants from the early stages before flower initiation takes place is likely to give a clue to the nature of the substances formed during the treatment Recent investigations have indicated that photoperiodic response of the growing points is not a direct result of the light action but is due to the formation of certain substances in the leaves which on transmission to growing points participate in flowering This paper embodies the results of a preliminary work in the nitrogen metabolism in the leaves of rice subjected to a photoperiod in the seed bed

MATERIAL AND METHOD

A pure strain of winter paddy variety *Bhasamanik* was used in this investigation After sterilisation with 0 2% formalin seeds were sown in earthenware seed beds, 15″×5″, on May 28, 1943 Photoperiods of 10 hours for 6 weeks duration were applied to 7 days old seedlings with two leaves They were exposed to the natural day length from 7 a m to 5 p m and for the rest of the day and night they were kept inside a well-ventilated light-proof house The control seedlings were grown in natural day length The seedlings from the treated and control seed beds were transplanted in field plots measuring 12′×12′ on July 18, 1943

ANALYTICAL METHODS

The first sample for chemical analysis was taken at the end of the light period (i e on the day of transplantation) and four more samples at different ages of the plants were analysed

For the analysis of seedlings the whole plant (several taken at random from the seed bed) was uprooted and brought to the laboratory before 8 a m The root system was removed and the shoot cut into small bits, and divided into two halves Care was taken to remove the superficial moisture from the samples with blotting paper One half was dried at 70° to 80°C for 24 hours and finally at 100°C for 30 minutes, cooled in a desiccator and weighed The dried halves were powdered in a mortar, from which total nitrogen was estimated The other half was thoroughly ground with water in a mortar to a paste for estimating soluble nitrogen-fractions The extract was filtered through paper and made up to 50 ml with several washings in distilled water by applying suction with a filter pump Frothing was prevented by a few drops of capryl alcohol In this way 97% to 98% soluble nitrogen can be extracted in this volume of water In the case of leaves they were severed from the plant at 8 in the morning and immediately brought to the laboratory and divided longitudinally in two equal halves, cut into small bits and treated as above for estimating nitrogen content Total nitrogen was estimated by Micro Kjeldahl apparatus of Parnas and Wagner as described by Pregl (1930) 20–30 mg of powdered material were digested by the reduced iron method of Pucher, Leavenworth and Vickery (1930) adapted to a Micro-Kjeldahl scale The digests were distilled for 10 minutes with 15 ml of 30% sodium hydroxide containing 5% sodium thiosulphate and the ammonia absorbed in 10 ml of N/100 hydrochloric acid and the residual acid titrated against N/100 sodium hydroxide using methyl red indicator

For estimating the total soluble nitrogen, 19 ml of the aqueous extract was taken in a 50 ml test tube and 1 ml of 50% trichlor acetic acid was added to precipitate protein, it was then stirred and filtered From this protein free extract aliquot portions are taken and total soluble nitrogen after reduction of nitrates with reduced iron was determined as above by the Micro-Kjeldahl method

Protein nitrogen was calculated from the difference between the total nitrogen and the total soluble nitrogen content Total amide nitrogen and free ammonia were determined by adopting the method of Wolff (1928) The hydrolysis of amides was carried out in a test tube fitted with a long glass tube to serve as a reflux condenser, 2 ml of the protein-free clear extract and 0 5 ml of 0 25 N H_2SO_4 was introduced and heated in a low flame for 4 hours to complete hydrolysis Ammonia was then distilled in vacuo at 40°C after adding 1 ml of 40% sodium hydroxide

Free ammonia was estimated by distilling 2 ml of the aqueous extract with a thick cream of magnesium oxide at a temperature of 40°C Total amino-nitrogen was determined by the Van Slyke micro-amino apparatus (Loomis and Shull, 1937) using a reaction time of 10 min

EXPERIMENTAL RESULTS

The accumulation of dry matter expressed as percentage of fresh wt. in the seedlings and the successive leaves are presented in Table I The results of the analyses of the different nitrogen fractions of the seedlings and the different leaves are presented in Table II as percentage of dry weight The results presented (Figs 1–4) are without statistical evidence as the replicate analysis could not be done Since the data reported here are obtained from a representative number of leaves of the same stage of maturity sampled at random from the field it is worth considering the appreciable differences noticed between the controls and the treated plants The differences are much greater than what could be ascribed to methods of micro-analysis

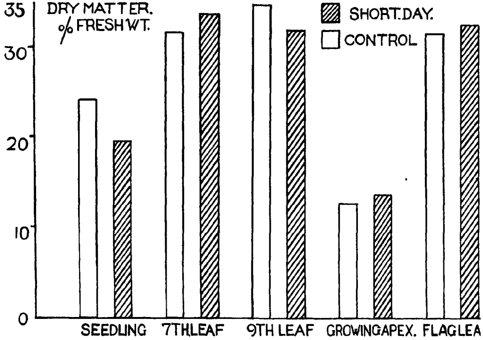

FIG 1

The amount of dry matter in the treated seedlings is less than the controls which received normal day length

TABLE I

Dry matter (% of Fresh weight)

	Seedlings	7th leaf	9th leaf	Growing apex	Flag leaf
Control	24 31	31 67	35 00	12 76	31 60
10 hours for 6 weeks	19 68	33 98	31 98	13 92	32 60

This decreased dry matter in the treated seedlings is possibly due to the effect of short photoperiod which limited the photosynthetic process of the seedlings in the seed bed The total dry matter in other treated samples is greater than the control except in the 9th leaf where a fall is noticed The increased dry weight in the treated plant with an increase in the tiller number (18 8 as against 14 6±1 332) at about the same time would indicate that the photoperiodic treatment acts as a stimulus to plants to greater activity After noting the fall-in the dry weight of

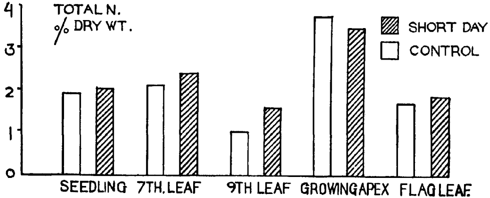

FIG 2

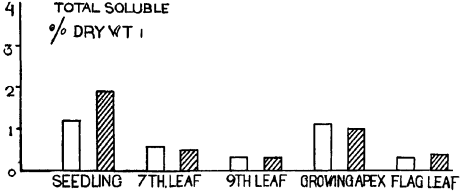

FIG 3.

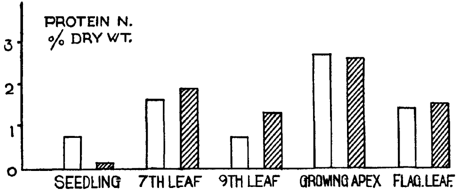

FIG 4.

the 9th leaf it was found necessary to see the conditions of the growing apex On splitting open the sheathing stem it was found that the tip of the axis has just formed the rudiments of inflorescence This would demand a rapid translocation of organic substances prepared in the leaf and would possibly explain the fall in the dry weight of the 9th leaf and a corresponding rise at the growing apex at this stage The growing apex of the control plants was also dissected at the same time but flower initials had not yet been formed

The percentage of total nitrogen (Table II) in the seedlings of the treated and the control remained more or less the same After transplantation the treated plants showed a higher total nitrogen than that of the control and this was noticed in all the leaves analysed In the 7th and 9th leaves the differences in the nitrogen content between the treated and the controls are greater than that of the flag leaf It is interesting to note that at the 7th leaf stage the vegetative growth of the treated plants as indicated by the tiller number is highest and to this corresponds the highest per cent of total nitrogen In the control plants also the tiller number reaches the maximum value at the stage when the maximum nitrogen content is noticed

In the growing apex of the plants receiving short photoperiod the percentage of total nitrogen on dry weight basis is less than that of the control But this difference is not real, as nitrogen content on fresh weight basis is almost the same in both the cases Although percentage of total nitrogen is less it has a higher percentage of dry weight suggesting a greater accumulation of carbohydrates It is known from C/N relation that prior to flowering the tendency of a plant is to accumulate more carbon compounds than nitrogenous material On this hypothesis the growing apex where flower initials are laid down would show a comparatively less nitrogen than the control where the flower primordia are not yet developed

TABLE II

N analyses (% Dry weight)

Stage	Treatment	Total N	Total Sol N	Protein N	Total Amino N	Ammonia N	Amide N
Seedling on date of transplantation	Control	1 9344	1 1929	0 7415		Nil	0 0739
	10 hours 6 weeks	2 0124	1 8740	0 1384		Nil	0 0356
	% changes	+4 0	+57 1	−81 4			−51 8
7th leaf	Control	2 1378	0 5600	1 5778		Nil	0 0955
	10 hours 6 weeks	2 4290	0 5462	1 8828		Nil	0 1820
	% changes	+13 5	−2 5	+19 3			+90 6
9th leaf	Control	1 0211	0 3118	0 7093	0 2010	Nil	Trace
	10 hours 6 weeks	1 5718	0 3060	1 2658	0 2280	Nil	0 0142
	% changes	+54 0	−1 9	+78 4	+13 4		
Growing apex 9th leaf stage	Control	3 7682	1 1022	2 6660	0 2934	Nil	0 1392
	10 hours 6 weeks	3 5326	0 9708	2 5618	0 4353	Nil	0 1425
	% changes	−6 2	−11 9	−3 9	+48 4		+2 4
Flag leaf	Control	1 7117	0 2937	1 4180	0 0559	Nil	Trace
	10 hours 6 weeks	1 9082	0 3714	1 5368	0 1460	Nil	Trace
	% changes	+11 5	+26 4	+8 4	+161 2		

It is of interest to note that in the treated seedlings a considerable amount of protein is hydrolysed, consequently the protein level is low Without further data it is difficult to decide whether this increase in protein hydrolysis is due to increased demand on nitrogen substances by seedlings stimulated by the application of short photoperiod It is also worth considering that in short photoperiods the photosynthetic activity being limited the amount of sugar production would fall which would lead to the hydrolysis of protein In the leaves of the treated plants the protein nitrogen on dry weight basis is always higher than that of the control plants while there is not much difference in the soluble nitrogen content This shows the increased nitrogen content in the leaves of the treated plants is metabolised to protein In the growing apex highest percentage of total nitrogen and protein nitrogen are noticed in both the cases and this is in accordance with the fact that new structures as flowers are to be formed from these regions

The values for total amino-nitrogen of the seedlings and the 7th leaves were not obtained as there was some breakdown in the Van Slyke apparatus at these stages Table II shows that total amino nitrogen is greater in the treated plants than the controls There is a greater content of total amino nitrogen in the growing apex of the treated plant This is largely due to an accumulation of amino acid-nitrogen as is evident by calculating the difference between the total amino figures and the amide figures The value of the difference being 0 2928% which is higher than the value 0 1542% in the control Free ammonia has not been detected in any of the samples analysed in this investigation

DISCUSSION AND CONCLUSION

From the data presented here it appears that the problem is an extremely interesting one Further work is required to elucidate the behaviour of different metabolites after photoperiodic treatment This work gives a clear indication of changes in the nitrogen metabolism of the rice plant subjected to photoperiodic treatment

The results indicate that nitrogen metabolism of rice plants is greatly influenced by the application of a photoperiod of 10 hours in the seed bed and these effects are noticed throughout the life history of the plant Absorption of nitrogen is greatly increased in these plants and the nitrogen absorbed is metabolised to protein The increase in nitrogen content occurs in conjunction with increased tillering It is interesting to note that at the growing apex of short day plant where flower initials have been laid down a large accumulation of amino acids takes place Further work is necessary to throw light on the nature of amino acids accumulated at this stage How far the accumulation of amino acids is related to the initiation of flowering has yet to be investigated further Parker and Borthwick (1939) also noted a general increase in nitrogen fractions of stems and leaves of Biloxi Soybean receiving short days of 8 hours On the basis of the data for tillering (Sircar, 1946), percentage of dry weight, and nitrogen content presented here and also from sugar analysis performed in this laboratory by Samantaray (1942) it is clear that the application of short day treatment of rice seedlings Var *Bhasamanik* in the seed bed for 6 weeks duration stimulated the plants to increased vegetative growth and nitrogen content

SUMMARY

An experiment is described in which seedlings of rice Var *Bhasamanik* were given short days of 10 hours in the seed bed for 6 weeks and subsequently the seedlings were transplanted in field plots The controls in seed beds were exposed all through to normal day length

Analysis of nitrogen fractions of seedlings, leaves and growing apices have revealed that exposure of seedlings to short days in seed beds for 6 weeks modifies the course of nitrogen metabolism of the plants throughout the life history Absorption of nitrogen and synthesis of protein is markedly increased in the short day plants With the initiation of flower primordia an accumulation of amino acids takes place in the growing apex

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ON THE COMPOSITION OF STARS OF SMALL MASSES

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(*Received October 28, read November 7, 1947*)

Abstract

In continuation of a previous paper on the internal constitution of stars of small masses, it is shown how the hydrogen and helium contents of these stars may be calculated from the observed values of their mass, radius, and luminosity, so as to be in complete agreement with Bethe's energy generation law and other mechanical and thermodynamical requirements for equilibrium It is found that in quite a large number of cases hydrogen constitutes about one third of the stellar material while the helium content fluctuates round an average of about 30 per cent

I Introduction

In a previous paper (Sen and Burman, 1945), referred to hereinafter as paper I, we attempted to calculate the hydrogen contents of stars of small masses on the basis of stellar equilibrium under the energy generation law of Bethe and the assumption that the helium content of these stars is negligible Such stars were assumed to be built after the Cowling model, the guillotine factor in the opacity formula being taken account of by a constant average value This value was estimated from a previous numerical calculation of a model of the sun with exact equations and Strömgren's table of the guillotine factor in Kramer's opacity formula One of the two points which came out of our calculations was that for a sunlike star the central temperature should be taken close to 20 million degrees (often lower) and the central density much less than that given by the standard model (higher central temperatures and densities would be inconsistent with Bethe's energy generation law) Secondly, for the assumptions made regarding opacity and helium content (zero), the above necessary effects could be achieved for a sunlike star by assuming hydrogen to constitute about one-third of their composition It was found that for stars of small masses with zero helium content any one of the three observed parameters namely, mass, radius, and luminosity, could be calculated on an assumption of the knowledge of the other two The calculation of the radii of the stars of small masses from their masses and luminosities (on the assumption of zero helium content) gave approximate agreements with the observed radii In the present paper we have dropped the assumption of zero helium content, and using the observed mass, radius, and luminosity of such a star have recalculated its hydrogen and helium contents It is found that the small discrepancies in the observed and calculated radii of the stars in our former calculations can be wiped out by the introduction of suitable amounts of helium, only in those cases, in which the radii calculated on the basis of zero helium content were larger than the observed radii Further in these cases complete adjustment is possible by the introduction of a suitable proportion of helium without much change in the content of hydrogen The numerical results of the present paper may be considered as supplementing our previous results for the alternative assumption of appreciable helium in the composition of stars of small masses In the present state of our knowledge of the composition and internal constitution of stars our conclusions based on the first approximation Cowling model (as necessarily all speculations of this nature) have indeed a tentative character This point has been further elucidated in the remarks at the end of this work

In a recent paper Schwarzschild * (1946) has calculated the hydrogen and helium contents of the sun and has obtained much higher values (47 and 41 per cents respectively) than those given by our present calculations Schwarzschild has taken into consideration the more recent table of the guillotine factor calculated by Morse, and his representation of this factor by an analytical formula has led to a formal modification of Kramer's opacity law which has been written with the ¾th power of the density ρ (instead of simple proportionality with ρ) thus giving smaller opacities Besides, Bethe's energy generation law has been taken in a form different from that used by Bethe and other writers in previous calculations, and also assumed by us in our present calculations The difference in the results obtained in this paper and those by Schwarzschild can be traced to these causes

2 Equations of the Problem

For calculations in the present paper, some of the equations given in paper I have to be modified For completeness, however, we rewrite all the relevent equations here The configurations, assumed to be built after the Cowling model, will be governed by the following equations

$$T_c = 0.9 \frac{\mu H}{k} \frac{GM}{R} \tag{1}$$

$$\rho_c = \frac{\xi_1^3}{4\pi\psi_1} \frac{M}{R^3} \tag{2}$$

$$Q = \frac{\kappa_0 L}{16\pi} \frac{3}{ac} \frac{\rho_c^2}{\alpha T_c^{7.5}} \tag{3}$$

with

$$\alpha^2 = \frac{5k}{8\pi\mu GH} \frac{T_c}{\rho_c} \tag{4}$$

and in addition we have the luminosity equation

$$L(\xi) = A\rho_c^{1/2} I(\xi, T_c) \tag{5}$$

obtained by integrating Bethe's law in the following manner

The luminosity is determined by

$$L(r) = \int_0^r 4\pi r^2 \rho \epsilon \, dr \tag{6}$$

where

$$\epsilon = \epsilon_0' X \rho T^{-\frac{2}{3}} e^{-B/T^{\frac{1}{3}}}, \tag{7}$$

ϵ'_0, B being numerical constants, and X, the hydrogen content of the stellar material

Equation (5) is easily obtained from equation (6) by introducing the usual variables ξ, θ, and σ defined by

$$r = \alpha\,\xi \,,\ T = T_c\,\theta \,,\ \rho = \rho_c\,\sigma$$

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* The calculations of the present paper were completed much earlier than when Schwarzschild's paper was received by us

and using the polytropic relation $\sigma = \theta^{\frac{3}{2}}$, for the convective core, A, and $I(\xi, T_c)$ being given by

$$A = 4\pi \epsilon'_o X \left(\frac{5k}{8\pi\mu G H} \right)^{3/2} T_c^{5/6} \tag{8}$$

and

$$I\,(\xi\,,T_c) = \int_0^{\xi} \theta^{7/3}\, e^{-b/\theta^{1/3}}\; \xi^2\, d\xi \qquad \left(b = \frac{B}{T_c^{1/3}}\right) \tag{9}$$

For the introduction of helium (Y) in the composition of the stellar material, the expressions for the opacity coefficient κ_0 and the average molecular weight μ given in paper I take the modified forms

$$\kappa_o = 3.9 \times 10^{25}(1+X)(1-X-Y)\; \frac{1}{\bar{t}} \tag{10}$$

$$\mu = \frac{2}{1+3X+0.5Y} \tag{11}$$

the guillotine factor being represented by an average value $\bar{t}$, which is taken to have the same value ($\bar{t} = 6$) as before.

As shewn in paper I, the above set of equations can be suitably combined to give

$$\rho_c^{\,3} = \frac{B}{AI\,(\xi,\,T_c)} \tag{12}$$

where

$$B = Q\, \frac{16\pi ac}{3\kappa_o} \left(\frac{5k}{8\pi\mu G H} \right)^{1/2} T_c^{\,8} \tag{13}$$

The four equations (1), (2), (3) and (5) involve 7 quantities L, M, R, ρ_c, T_c, X, and Y, and theoretically if any three quantities, say, L, M and R be known, we can determine the remaining four. In paper I we made the arbitrary assumption $Y = 0$, and from the observed values of L, and M of some stars of small masses we calculated their radii and hydrogen content. In the present paper, however, we calculate X, and Y from the observed values of L, M and R of these stars. For the application of such calculations to actual stars, we build up series of configurations with assigned values of T_c, X and Y, obtaining the corresponding values of L, M, R, and ρ_c for each one of these configurations. For instance, the given values of T_c, X, and Y enable one to calculate the integral $I(\xi, T_c)$, μ, and κ_0, from which further we can calculate A, and B, and finally ρ_c from equation (12). Then equation (5) determines L, and equations (1) and (2) give M, and R.

We have made calculations for four central temperatures $T_c = 19$, 20, 21 and 22 million degrees, and for hydrogen content $X = 0.15$, 0.25, 0.35, 0.45 and helium content $Y = 0.1$, 0.2, 0.3, 0.4. The results of these calculations are shown in Table 1.

3 CALCULATIONS OF X AND Y FOR KNOWN STARS

By plotting the positions of stars of known L, M, R considered in paper I, in diagrams suitably constructed from the following Table 1, it is possible to calculate by following a method given by one of us (Burman, 1947), the values of X, Y, T_c, and ρ_c of these stars. The results of these calculations are presented in Table 2.

TABLE 1

Calculated values of the mass, radius, luminosity and central density for stellar configurations with assigned central temperature and composition

X	Y	$M\,10^{-33}$				$R\,10^{-10}$				$L\,10^{-33}$				ρ_c			
		$T_c\,10^{-6}$				$T_c\,10^{-6}$				$T_c.10^{-6}$				$T_c\,10^{-6}$			
		19	20	21	22	19	20	21	22	19	20	21	22	19	20	21	22
0·15	0·1	0·86	1·03	1·22	1·43	4·40	5·01	5·65	6·32	0·41	1·04	2·47	5·57	89·2	72·4	59·9	50·1
	0·2	0·89	1·06	1·26	1·48	4·40	5·00	5·64	6·31	0·44	1·11	2·64	5·96	92·5	75·2	62·1	52·0
	0·3	0·91	1·09	1·29	1·52	4·37	4·97	5·60	6·27	0·47	1·19	2·83	6·39	96·8	78·6	65·0	54·4
	0·4	0·93	1·11	1·32	1·54	4·31	4·90	5·53	6·19	0·51	1·28	3·05	6·89	10·2	83·2	68·8	57·6
0·25	0·1	1·26	1·51	1·79	2·09	5·36	6·10	6·88	7·70	0·81	2·04	4·86	10·97	72·2	58·7	48·5	40·6
	0·2	1·28	1·54	1·82	2·13	5·31	6·05	6·82	7·62	0·86	2·18	5·18	11·71	75·7	61·5	50·8	42·5
	0·3	1·30	1·55	1·84	2·15	5·23	5·95	6·71	7·50	0·92	2·33	5·55	12·55	80·2	65·1	53·8	45·1
	0·4	1·30	1·56	1·84	2·16	5·10	5·81	6·55	7·32	0·99	2·52	5·99	13·64	86·4	70·2	58·0	48·6
0·35	0·1	1·70	2·08	2·41	2·82	6·19	7·05	7·95	8·89	1·33	3·37	8·02	18·12	63·2	51·3	42·4	35·5
	0·2	1·71	2·05	2·42	2·84	6·08	6·92	7·81	8·73	1·42	3·59	8·55	19·33	67·0	54·4	45·0	37·7
	0·3	1·70	2·04	2·41	2·82	5·92	6·74	7·60	8·50	1·52	3·86	9·20	20·78	72·3	58·8	48·6	40·7
	0·4	1·67	2·00	2·37	2·77	5·68	6·47	7·30	8·16	1·67	4·21	10·03	22·66	80·3	65·2	53·9	45·[illegible]
0·45	0·1	2·16	2·59	3·07	3·59	6·91	7·86	8·86	9·91	2·00	5·07	12·07	27·27	58·0	47·1	39·0	32·6
	0·2	2·15	2·57	3·04	3·57	6·72	7·64	8·62	9·64	2·15	5·43	12·94	29·23	62·7	50·9	42·1	35·2
	0·3	2·10	2·52	2·98	3·49	6·44	7·32	8·26	9·23	2·33	5·90	14·06	31·76	69·6	56·6	46·7	39·2
	0·4	2·00	2·40	2·83	3·31	6·00	6·82	7·69	8·59	2·60	6·60	15·72	35·50	82·1	66·6	55·0	46·1

TABLE 2

Calculated values of the hydrogen and helium contents of some stars and their central temperatures and densities

Star	$L/L_{\odot}$ (obs)	$M/\odot$ (obs)	$R/R_{\odot}$ (obs)	X	Y	$T_c\,10^{-6}$	ρ_c	X (for $Y = 0$)
* Sun	1·00	1·00	1·00	0·32	0·20	20·2	52	0·34
η Cas A	0·81	0·72	0·81	20	38	20·7	71	21
η Cas B	0·07	0·47	0·56	22	58	17·9	140	23
α Cen B	0·37	0·87	0·87	36	24	18·9	69	36
ξ Boo A	0·48	0·87	0·87	34	28	19·3	69	35
ξ Boo B	0·15	0·76	0·80	35	0	18·0	75	35
70 Oph A	0·42	0·89	0·93	34	08	19·3	58	36
70 Oph B	0·14	0·74	0·69	0·45	0·40	17·4	118	0·36

* The case of the Sun does not occur in Table 2 of paper I

The last column which gives the values of X calculated on the assumption of $Y = 0$ is taken from Table 2(I)

4. DISCUSSION OF THE RESULTS AND CONCLUSIONS

There are several interesting points that emerge from a comparison of the above Table 2 with Table 2 of paper I. *Firstly*, in the case of the stars α Cen A, α C Mi A, and ζ Her A, for which the values of the radii calculated on the basis of $Y = 0$, are smaller than their observed values, the discrepancies cannot be wiped out, as no physically significant solutions (subject to $X+Y<1$) of our equations are possible in these cases. This is as is to be expected, as, the introduction of helium in the composition of a star whose mass is not allowed to change, will have the effect of diminishing its volume, i.e., its radius. In the case of all other stars of Table 2 of paper I, the discrepancies between the observed and the calculated values of the radii have been wiped out by suitable proportions of helium. This has been possible because the calculated radius (for $Y = 0$) is greater than the observed radius in each of these cases. *Secondly*, in all cases of agreement in our calculations, except in the case of one star, 70 Oph B, there is no substantial change in the hydrogen content as compared with the calculations on the assumption of negligible helium content. This is effected in these cases as follows. As the luminosity of the star is held fixed, the central temperature to which it is highly sensitive may in a small adjustment change only very slightly. It will then be seen from equation (1) that a change in the composition of the star will be subject to the condition that $\mu M/R$ should remain nearly constant. As the mass is maintained constant, a decrease in R calls for a decrease in μ. Just this can be effected by the introduction of a suitable amount of helium in the composition of the star without much altering its hydrogen content. It might appear that a change in μ would cause a large change in the luminosity, as $L \sim \mu^{7.5}$. How this is prevented can be seen from the mass-luminosity relation (Chandrasekhar, 1939) of the Cowling model, which may be written in the form

$$L \sim \frac{1}{\kappa_0} \frac{\mu^{7.5}}{R^{0.5}} \sim \frac{1}{(1+X)(1-X-Y)} \frac{\mu^{7.5}}{R^{0.5}} \tag{14}$$

retaining only the relevant factors. (The factor $M^{5.5}$ has been dropped as M remains constant.) A slight calculation with the figures of Table 2 shews that the large decrease in L produced by the high power (7.5) of μ (altered by the introduction of helium) is exactly balanced by the combined diminution in the factors $(1-X-Y)$ and $R^{0.5}$. It is thus interesting to note that the Russell mixture part of the stellar material, represented by the factor $(1-X-Y)$ plays a prominent rôle in the present adjustment. Generally, in all the cases of adjustments obtained here, except in the case of the star 70 Oph B, no significant change in X has been necessary. It looks, as if the requisite change in μ is possible in these cases, by giving only a suitable value to Y, without changing X. In the case of the star 70 Oph B, however, a significant change in X as well as an introduction of high percentage of helium is necessary to secure the adjustment. This peculiarity is indeed due to the relative values of L, M and R of this star. It will be noted from Table 2 that this star has a large value of M/R (in fact the largest) compared to other members of the table, while its luminosity is comparatively low. It is probable that this feature is at the root of the above peculiar behaviour.

Thus the new hypothesis of appreciable helium content in stars of small masses, as have been considered here, does not appear to affect our previous conclusion that in a large number of cases at least, hydrogen should constitute about one-third of their composition. Our calculations also lend no support to the view that the helium content of these stars should *in general* be about 40 per cent, though such abundance of helium appears to be quite possible in some cases.

It is, however, necessary to be quite clear about the nature of the conclusions we can derive regarding stellar constitution from the analysis followed here and in paper I. The equations provided by the current theory of stellar structure, viz., those for mechanical equilibrium, conservation of mass, transfer of energy and its generation according to Bethe's law, as well as the observed values of the mass, radius and luminosity of a star, are just sufficient for the calculation of the unknown parameters, viz , the hydrogen and helium contents in the material of the star, and its central temperature and density It is shewn in our papers that in the case of a large number of stars of small masses, it is possible to find physically significant values for their hydrogen and helium contents and also the corresponding values of their central temperature and density, consistent with our equations and data The values of none of these parameters are, however, amenable to direct observational test There is as yet nothing in the current theory of stellar structure providing for any independent internal check on the composition and other data calculated Till such confirmation has been found, the theory cannot be regarded as entirely satisfactory

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19 APR 1950

PROCEEDINGS
OF THE
NATIONAL INSTITUTE OF SCIENCES OF INDIA

JULY, 1948

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CHROMOSOME CONSTITUTION AND CHARACTERISTICS OF GIANT COLONIES IN YEASTS *

By M K SUBRAMANIAM, *M A , D Sc , Imperial Chemical Industries Research Fellow,*
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Communicated by Sir J C Ghosh, Kt , D Sc , F N I

(*Received August 2, 1947, read March 3, 1948*)

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INTRODUCTION.

Correlation between the chromosomal constitution of vegetative yeast cells and giant colony characteristics, if possible, would simplify not only studies on the stability of the various strains, but would also enable those engaged in industry to check from time to time the purity of the particular strains employed by them

Owing to lack of an easily reproducible technique for the demonstration and study of chromosomal behaviour in yeasts—and the resultant confusion—there are no previous records of such attempts The demonstration that a brewery yeast has only two chromosomes (Subramaniam, 1946) and the isolation of a tetraploid (Subramaniam, 1945) and a mutant (Subramaniam and Ranganathan, 1946) by treatment of actively dividing cells of the above strain with acenaphthene has placed us in a better position Now such a correlation could be attempted for the first time

LITERATURE.

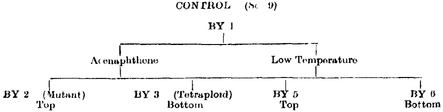
Giant colonies have been used extensively for studies on segregation (Winge and Laustsen, 1937, 1939*b*, Ditlevsen, 1944), hybridization (Winge and Laustsen, 1938, 1939*a*) and mutation (Winge, 1944, Skovsted, 1943) A variety of agencies are capable of inducing polyploidy in higher plants and if correlation between chromosome constitution and giant colony characteristics is possible—something like stomata measurements in higher plants—then, it may even be possible to predict the probable chromosomal constitution of new strains based on the above characteristics There appears to be some justification for the above hope since Skovsted (1943) mentions that it is 'a characteristic of the types shown in the diagram that all of them produce their particular categories of mutants' (p 448) In *Saccharomycodes Ludwigii*, Winge and Laustsen (1939*b*) found that lobed growth was dominant to the smooth type (p 362) However, they mention in a previous paper (Winge and Laustsen, 1939*a*, p 350) that the morphology of the giant colonies may be governed by several factors Lindegren (1945) also suggests that several genes may be responsible for the characteristics of the giant colonies

---

* This paper was read before the Fourth International Congress for Microbiology held at Copenhagen in July 1947.

MATERIAL AND METHODS

Mutations were induced in a bottom fermenting brewery yeast, *Sc 9* (N C T C 3,007) by treatment with acenaphthene and low temperatures (Subramaniam, 1945, Subramaniam and Ranganathan, 1946, 1947)



The chromosome constitution of strains BY 2 and BY 3 isolated from cultures treated for 90 days with acenaphthene are known The cytology of strains BY 5 and BY 6 isolated from cultures of the control kept at low temperatures for 90 days (Subramaniam and Ranganathan, 1947) has not been investigated The strains BY 3 and BY 6 are bottom fermenting types, while BY 2 and BY 5 are top ones The usual method of giant colony production is to cultivate the yeast in a medium composed of 8% wort and 10% gelatin The method of sterilisation appears to be to keep the medium at 100°C for 20 minutes on successive days Since even after this some of our plates got contaminated we have used in our experiments wort-agar Two grams of agar were added to every 100 c c of wort of S G 1 048 and *p*H 6 8 and was sterilised at 10 lbs pressure for half an hour Small petri dishes of 7 cms diameter were used and only a single colony was grown in each plate at room temperature (22–24°C)

OBSERVATIONS

Photographs 1 and 4 show the giant colonies of the two chromosome diploid control in two different samples of wort-agar at intervals of two months While Photo 1 is that of the giant colony made along with those of strains BY 2 and BY 3, Photo 4 is that of the same strain inoculated at the same time as strains BY 5 and BY 6 two months later It was thought desirable to compare the characteristics of the giant colonies grown on the same sample of wort-agar simultaneously and after an identical period of growth. The descriptions and photographs illustrate the appearances observed on the 12th day, as by that time the colonies assume their characteristic shapes and contour The giant colony of the diploid control has a lobed margin and its surface is folded Faint concentric striations near the margin could be seen only in Photo 4 With further growth these striations as well as the indentation of the margin become accentuated The tetraploid colony (Photo 2) is thick with indistinct radial striations and a faintly wavy margin No accentuation of these characteristics was observed on further growth of the colony. The giant colony of the mutant (Photo 3) is thin, glistening and smooth and exhibits a circular furrow near the margin. The appearance of this furrow indicates the limit of growth of the colony. The only change in the colony after the 12th day is a slight broadening of this marginal furrow.

For comparison, the giant colonies of known and unknown chromosomal constitution are shown side by side. Photos 5 and 6 show the giant colonies of BY 6 and BY 5 respectively, and it would be seen how similar these are to the giant colonies illustrated in Photos 2 and 3.

The ranges in size of the cells of the five strains are shown in the Table.

TABLE

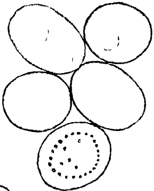
BY 1										
Length in μ	5.94	6.60	6.60	6.60	6.60	6.60	7.26	7.92	7.92	7.92
Width in μ	3.96	5.28	5.28	5.94	5.94	6.60	5.94	6.60	6.60	6.60
BY 3										
Length in μ	5.28	5.94	6.60	6.60	6.60	6.60	7.92	7.92	7.92	7.92
Width in μ	5.28	5.28	6.60	6.60	6.60	6.60	6.60	7.26	7.26	7.92
BY 6										
Length in μ	5.28	6.60	7.26	7.92	7.92	7.92	7.92	9.24		
Width in μ	4.62	6.60	6.60	6.60	7.26	7.92	7.92	7.92		
BY 2										
Length in μ	3.30	3.30	3.96	5.28	6.60	7.26	7.92	10.56	13.2	13.2
Width in μ	1.98	2.64	2.64	2.64	2.64	2.64	1.98	2.64	2.64	2.64
BY 5										
Length in μ	2.64	3.30	4.62	6.60	7.92	9.24	11.28	13.20	14.52	15.84
Width in μ	1.98	2.64	3.96	2.64	2.64	2.64	2.64	2.64	2.64	2.64

Cells from the control, tetraploid and mutant are illustrated in Figs. 1, 2 and 3 respectively, while the cells of the strains BY 6 and BY 5 are shown in Figs. 4 and 5. The cells of BY 6 (Fig. 4) are bigger than that of the tetraploid (BY 3, Fig. 2), while in BY 5 the percentage of long cells are fewer than in BY 2 (Fig. 3). It appears that both acenaphthene and low temperatures produce also changes other than the duplication of chromosomes. Cold shock has been known to produce gene mutations (Kerkis, 1939), and Dermen (1940) mentions that polyploidy may or may not have any effect on the sizes of the cells.

Only a cytological investigation would show whether the chromosome constitution of BY 5 and BY 6 are identical with that of BY 2 and BY 3.

DISCUSSION

Our extended observations indicate that under constant conditions the slight individual variations which occur in giant colonies could be ignored if emphasis is restricted to the major characteristics. The characters of the giant colonies appear to be controlled by a variety of factors. The sample of gelatine or agar, and malt used in making the medium (Skovsted, 1943), the S.G. and *p*H of the wort and even the quantity inoculated (Winge and Laustsen, 1939*a*) all seem to affect the appearance of certain minor characteristics. Naturally, comparisons should be confined to colonies grown for the same length of time. Haploid colonies differ from diploid ones and polyploid ones from both. Winge and Laustsen (1937) had observed changes in the appearance of the colony not only as a result of changes in environmental conditions but also at different periods of growth in some cases. Winge



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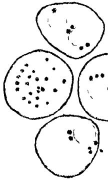

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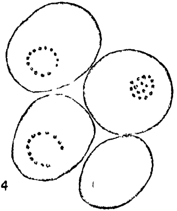

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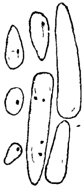

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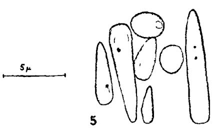

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(1944), however, emphasises 'that under the same conditions the characteristic appearance of the giant colonies of each type remain constant' and presents only photographs of single colonies From observations on 'several hundred different clones of rough colonied yeasts' Lindegren (1945) arrives at the conclusion that no duplicates occur even though each strain gave rise to colonies having distinctive characteristics and capable of identification on transplantation

A perusal of the photographs would leave no doubt that changes in chromosome constitution do produce changes in the characteristics of the giant colonies It is possible to identify particular strains by their characteristic giant colonies The resemblance thus is not confined to their top or bottom fermenting character alone The similarity between colonies of known and unknown chromosomal constitution is striking and hence should be of considerable significance since the same chromosomal mutation could be induced by diverse agencies

SUMMARY

1 An attempt was made to correlate the chromosomal constitution of particular strains with the characteristics of their giant colonies

2 Changes in chromosomal constitution do produce changes in giant colony characteristics

3 The similarity between the colonies of strains of known and unknown chromosomal constitution is striking and hence should be of considerable significance since the same chromosomal mutation could be induced by diverse agencies

ACKNOWLEDGMENTS

We are very grateful to Sir J C Ghosh, Kt , D Sc , F N I , for his active interest and encouragement We would also like to thank the Council of the National Institute of Sciences (India) for the award, to one of us, of an Imperial Chemical Industries Fellowship and the Council of Scientific and Industrial Research for generous financial assistance

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DESCRIPTION OF ILLUSTRATIONS

Photographs are of colonies taken on the twelfth day after inoculation The photos were taken at different magnifications and hence measurements of the longest diameter of the colonies alone are given

Photo 1 Control—Diploid Diameter 2 4 cms
Photo 2 Tetraploid—BY 3 Diameter 2 1 cms
Photo 3 Mutant—BY 2 Diameter 2 2 cms
Photo 4 Control—Duplicate Diameter 2 3 cms
Photo 5 BY 6 Diameter 1 9 cms
Photo 6 BY 5 Diameter 1 7 cms

Cells from Giant Colonies

FIG 1 Control
FIG 2 Tetraploid—BY 3
FIG 3 Mutant—BY 2
FIG 4 BY 6
FIG 6. BY 5

1

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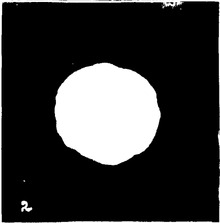

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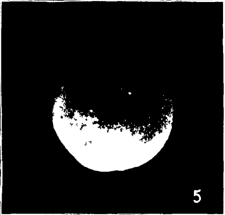

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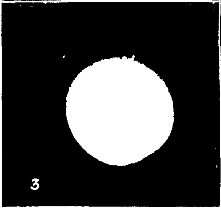

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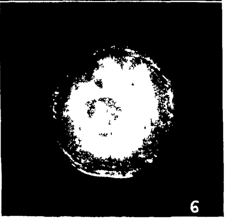

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THE DISTRIBUTION OF CROCODILES AND CHELONIANS IN CEYLON, INDIA, BURMA AND FARTHER EAST

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(*Received April 12, read May 4, 1948*)

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Introduction

In 1901, the completion of the Vertebrata in the *Fauna of British India* afforded an opportunity to Blanford to review and discuss the distribution of vertebrate animals throughout Ceylon, India and Burma His object was 'to determine the divisions into which the Indian Empire can be classed by our present knowledge of the Vertebrata, and especially to ascertain the zoological relations between the Indian Peninsula and the neighbouring countries' To achieve this object, he reviewed the distribution of genera, for 'families and sub families alone being too few in number and ranging in general over wider tracts than genera do, so that it is difficult to determine subregional divisions by means of them, whilst species are too numerous and too unequal in importance' For some years I have been interested in the distribution of freshwater fishes throughout south-eastern Asia and in the revision of genera characteristic of mountainous regions My views and conclusions based on these studies are given in the article 'On the Malayan Affinities of the Freshwater Fish Fauna of Peninsular India, and its bearing on the probable age of the Garo-Rajmahal Gap' (Hora, 1944) In the discussion that followed the reading of this paper, the views of some of the leading Indian geologists on the Garo-Rajmahal Gap are given and there is considerable wealth in their observations regarding some palaeogeographical features of India In the course of these studies it has become abundantly clear to me that the distribution of some of the genera, as given by Blanford, has proved to be erroneous due to incorrect determinations and that this fact has greatly vitiated the value of his work, monumental and outstanding as it is and will always remain, for our present-day studies The revision of the *Fauna* volumes on certain groups of Vertebrata, therefore, affords another opportunity to review the distribution of Indian fauna and to elucidate the zoological relations of the various subregions of India and their relationships with the neighbouring countries The present article is written with this object in view

As the revised editions of the *Fauna* volumes dealing with Batrachia and Fishes have not yet been published, a complete revision of Blanford's article is not possible at the present time, but sufficient information is available regarding the distribution of Mammals, Birds, Reptiles, and certain families of fishes to warrant the publication of some articles. It will at least serve to obtain the views of other zoologists interested in similar studies.

Besides correct and reliable determination of animals, it is equally essential for zoogeographical studies that the ecological factors influencing the lives of different groups of animals should also be known, for knowledge of an animal without some knowledge of its environment is very imperfect indeed. In studying the zoogeography of the mountainous fishes, I had a great advantage in knowing something about the ecology, bionomics and evolution of the torrential fauna (Hora, 1930). The revised edition of the Vertebrata in the *Fauna* are in this respect a great improvement on the earlier volumes, for they contain not only the systematic descriptions of the species but also notes on their habits and habitats.

Though families and sub-families are generally unsuitable for zoogeographical studies, the same may be said of genera to a lesser extent, for species, however unequal in value, are our units of study. Whether we catalogue them or not, it is necessary to keep them in mind always. For the first article of the series, I have, therefore, selected the groups of Crocodiles and Chelonians as the number of species to be dealt with is not large. Besides, the volumes on Reptiles cover a much larger geographical area, as they include the whole of the Indo-Chinese Peninsula. In support of this departure from the usual practice, Smith (1931, p. 13) states that 'The fauna of Siam, French Indo-China and southern China is so closely allied to that of Burma that it would be scientifically incorrect to separate them from one another.' This is true of other groups of animals also. Not only is the fauna of Burma closely allied to that of Siam, French Indo-China and southern China but the same can be said with regard to the fauna of the Eastern Himalayas and the Malabar Tract of the Western Ghats.

For the present series of articles on Reptiles, I have retained the two subregions proposed by Smith and the twelve zoogeographical areas into which he has divided them. The distribution of various species as tabulated below has been checked by Dr. Malcolm A. Smith. I am grateful to him for this.

The geographical limits of the twelve areas, as recognized by Smith, are given below for convenience of reference.

ZOOGEOGRAPHICAL AREAS

The Oriental Region is divided into three Subregions, the Indian, the Indo-Chinese and the Malaysian. The Indian Subregion includes the peninsula of India as far east as Bengal at about longitude 90° and south of the Himalaya Mountains. It has been divided into seven geographical areas as follows —

1. *The Desert Area of North-West India.*—This includes Baluchistan, the North-West Frontier Province, the Punjab, Western Rajputana as far as the Aravalli Range and Sind.

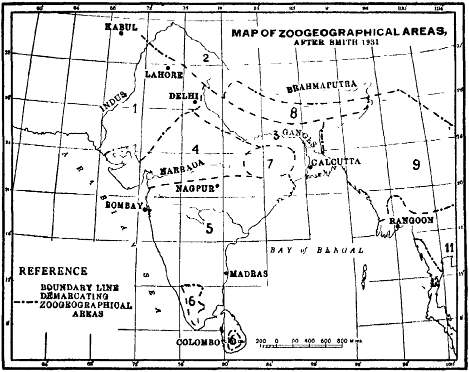
2. *Kashmir and the Western Himalayas up to Nepal.*—Most of the country is mountainous and highly elevated.

3. *The Gangetic Plain.*—The great Gangetic Plain of Northern India extends from the Valley of the Indus in Sind to the right bank of the Brahmaputra in Bengal.

4. *Central India.*—This includes the area between the Gangetic Plain, the Deccan, the Aravalli range and Chota Nagpur.

5. *The Deccan.*—This includes the central tableland of the Deccan between 12° and 21° north latitude. A part of the Western Ghats is included in this area.

6 *The Mountains of the Malabar Tract and Ceylon* The hills of the Peninsula south of lat 12°N together with the mountains of Ceylon constitute a distinct faunal region



TEXT-FIG 1 Map of the Zoogeographical Areas of India, Ceylon and Farther East as recognised by Dr M A Smith in the revised edition of the *Fauna of British India* on Reptilia.

7 *The Chota Nagpur Area* —This includes Bihar south of the Gangetic Plain, the northern part of Orissa, and the eastern part of the Central Provinces

Comparing the above zoogeographical divisions with those proposed by Blanford, it will be seen that there is considerable general agreement between the two systems as is shown below —

Smith's Classification	*Blanford's Classification*
The Desert Area of N W India	Punjab Tract, excluding the plain of the Indus
Kashmir and the Western Himalayas	Western Himalayan Tract
The Gangetic Plain	North Western Provinces (now designated as the United Province and Bihar) Tract including the plain of the Indus *plus* Bengal Tract
Central India	Rajputana or Central Indian Tract
The Deccan	Carnatic or Madras Tract
The Mountains of the Malabar Tract and Ceylon	Malabar Coast Tract and Ceylon Tract.
The Chota Nagpur Area.	Bihar Orissa Tract.

Judging from the distribution of freshwater fishes, Smith's classification would seem to be a better arrangement of the zoogeographical areas into which India west of the Brahmaputra in Bengal can be divided.

Smith has divided his Indo Chinese Subregion into 5 areas, namely,—

1. *The Eastern Himalayas*.—which corresponds with the Assam Tract of Blanford, and extends from the western frontier of Nepal to the termination of the range at the bend of the Brahmaputra.
2. *The Trans Himalayan Mountainous Area*[1].—corresponds to the Upper Burma Tract of Blanford and includes the hills of Assam east of the Brahmaputra, the whole of Burma, except the lowlands in the south, southern Yunnan, the northern part of French Indo-China and the northern part of Siam.
3. *Annam*.
4. *The Great Plain of Indo-China*.—This area includes the lowlands of Burma south of Prome and Toungoo and at the mouth of the Salween, the great plains of Siam, Cambodia, and Cochin China. This corresponds to the Pegu Tract of Blanford.
5. *Tenasserim and Peninsular Siam*.—This area corresponds to the Tenasserim and South Tenasserim Tracts of Blanford and includes the mountain chain which extends down the peninsula.

ECOLOGICAL FACTORS INFLUENCING ANIMAL LIFE IN VARIOUS ZOOGEOGRAPHICAL AREAS

In a recent article, of which I have seen a summary in English, Nikolsky (1947) has discussed biological peculiarities of faunistic complexes and the value of their analysis for zoogeographical studies. He has rightly pointed out that 'A zoogeographical division into definite regions is a stage which belongs chiefly to the past, being now nearly completed with regard to large areas'. In his opinion, with which I agree, 'The criterion on the ground of which a certain species is referable to one or another faunistic complex besides the character of a distribution area is its ecological specificity, *i.e.* its relation to both abiotic and biotic media which are closely connected with each other, being both a sequence of the adaptation to life in a definite geographical zone'. From the above, it will follow as a natural sequence that 'In populating a new basin the members of a faunistic complex occupy its part the conditions of which approach most the zone where the formation of the complex has proceeded'. It will, therefore, be clear that in elucidating the history of the origin of the fauna, its ways of distribution, its interrelations with other zoogeographical areas, etc., etc., *it is absolutely necessary to bear in mind* the biological peculiarities of the fauna and the principal factors in its environment. Evolution is the result of the action and reaction of these important conditions which govern the life of an animal and determine its survival, modification or annihilation under changed conditions of environment, if any.

Before discussing the distribution of Crocodiles and Chelonians, it will be advantageous, therefore, to bear in mind the main ecological conditions which affect the fauna in the twelve zoogeographical areas enumerated above.

The Desert Area of North-West India is desert or semi-desert, except near the rivers. The rainfall averages from nil in certain portions to 20 inches in the year in others. The interior of Baluchistan is from 3,000 to 6,000 feet above sea level.

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[1] It may be pointed out that among geographers the term Trans Himalaya has a definite meaning and generally signifies the high plateau of Central Asia to the north of the Himalayas.

The fauna exhibits some interesting adaptive modifications to withstand extreme heat and drought

In the *Kashmir and the Western Himalayan Areas*, the country is mountainous and highly elevated *The Gangetic Plain* is divisible into a western, more or less arid or semi-arid area and an eastern area with a fertile, alluvial soil which is cleared, cultivated and thickly populated for the most part The rainfall is 20 to 30 inches in the western portion and 50 to 100 inches in the eastern region

The Central India is an undulating and hilly tract cleared and cultivated in parts and covered with brushwood or thin forest of small trees The average rainfall is about 35 inches *The Deccan Area* is similar to the Central India but the rainfall along the western coast is heavy—from 100 to 200 inches annually The Eastern Ghats have not the same unity of structure as the Western Ghats and represent a discontinuous line of mountain country *The mountains of the Malabar Tract and Ceylon* rise to a considerable height and most of the area is well forested The rainfall is heavy, between 90 and 150 inches, and owing to its approximation to the equator, seasonal changes are not so marked *The Chota Nagpur Area* is hilly and, with a few exceptions, is heavily forested The average rainfall is between 50 to 70 inches in the year The highest peak in the Chota Nagpur area is the Parasnath Hills, 4,800 feet above sea level

The Eastern Himalayas are sub tropical and heavily forested The average rainfall is 50 to 150 inches *The Trans Himalayan Mountainous Area* is of hills and dense forests closely resembling the Eastern Himalayas The rainfall is heavy, generally exceeding 100 inches annually over a considerable part of the area *The Tenasserim and the Peninsular Siam Area* is also mountainous and heavily forested, with the yearly average rainfall being 150 to 200 inches *Annam* is entirely mountainous and the conditions are similar to those of Burma *The Great Plain of Indo-China* is for the most part flat, alluvial and more or less cultivated and populated , The annual rainfall in the Pegu area is from 100 to 125 inches, in Bangkok it is 55, in Cochin China 50 to 90 inches

In the case of aquatic forms, continuity of water courses is essential at some period of the earth's history for their dispersal and for the dispersal of land animals, temperature, humidity and vegetation are the chief governing factors Continuity of land-masses is also an essential factor in the distribution of torrential forms Crocodiles and Chelonians could only be distributed over wide areas through natural agencies For the survival of a species in a new geographical area after its dispersal, it is important that similar ecological conditions in its new habitat must persist, otherwise it will have to undergo some sort of modifications to adapt itself to new conditions Such modifications will, of course, depend on the intensity of variation within the species itself under the new environmental conditions and the resulting animal will either become a geographical race, form, subspecies or a new species Even new genera and families may result from long isolation or some kind of habitudinal segregation

Ecologically speaking, it will thus be seen that, with the exception of the Plain of Indo China, the remaining four areas of the Indo-Chinese Subregion are very closely allied to the Malabar Tract and Ceylon and also resemble the Western Ghats and the Chota Nagpur plateau It is no wonder, therefore, that in the migration of the fauna and the development of the present-day features of India's physical geography, pockets are formed where similar or identical species could continue to live undisturbed under environmental conditions more or less similar to those under which they had once lived over a much wider area We shall refer to these points again when dealing with certain aspects of the palaeogeography of India, but now we may consider the distribution of the Crocodiles and Chelonian listed by Smith from the Indian and the Indo Chinese Subregions

THE DISTRIBUTION OF CROCODILES AND CHELONIANS IN CEYLON, INDIA, BURMA AND FARTHER EAST

Zoogeographical Areas of Greater India based on the Distribution of Reptiles (After Smith)

Families, Genera and Species	The Indian Subregion							The Indo Chinese Subregion				*Distribution and general remarks*
	1	2	3	4	5	6	7	8	9	10	11	12	
Family CROCODILIDAE Gray													Southern Asia, the East Indian Archipelago and tropical Australian region, Africa, tropical and subtropical America
Genus *Gavialis* Oppel													Occurs in the Pliocene deposits of the Siwalik Hills and Narbada valley
1 *Gavialis gangeticus* (Gmelin)	x		x						x				The Indus, Ganges, Brahmaputra Rivers and their tributaries, and the Kaladan River, Arakan
Genus *Crocodilus* Gronovius													
2 *Crocodilus porosus* Schneider													The Estuarine Crocodile inhabits the mouths of muddy rivers and canals near the sea
3 *Crocodilus siamensis* Schneider											x	x	Siam, French Indo-China, the Malay Peninsula, Java
4 *Crocodilus palustris*	x	x	x	x	x	x	x	?					India and Ceylon
Family SPHARGIDAE Gray													
Genus *Dermochelys* Blainville													
5. *Dermochelys coriacea* (Linn.)													Marine

1 The Desert Area of North West India 2 Kashmir and the Western Himalayas 3 The Gangetic Plain 4 Central India 5 The Deccan 6 The Mountains of the Malabar Tract and Ceylon 7 The Chota Nagpur Area 8 The Eastern Himalayas 9 The Trans-Himalayan Mountain Area 10 Annam 11 The Great Plain of Indo-China 12 Tenasserim

Families, Genera and Species	The Indian Subregion							The Indo-Chinese Subregion					*Distribution and general remarks*
	1	2	3	4	5	6	7	8	9	10	11	12	
Family CHELONIDAE Gray													
Genus *Eretmochelys* Fitzinger													
6 *Eretmochelys imbricata* (Linn.)													Marine
Genus *Chelonia* Brongniart													
7 *Chelonia mydas* (Linn.)													Marine
Genus *Caretta* Refinesque													
8 *Caretta caretta olivacea* (Eschscholtz)													Marine
Family PLATYSTERNIDAE Gray													
Genus *Platysternum* Gray													
9 *Platysternum megacephalum* Gray									x		x	x	Southern Burma, Siam, French Indo-China, Southern China, Hainan
Family EMYDIDAE Gray													
Genus *Cyclemys* Bell.													
10 *Cyclemys mouhoti* Gray								x	x		x		Assam, French Indo China and Hainan
11 *Cyclemys dentata* Gray								x	x	x	x	x	Assam Hills, Burma, French Indo-China, Annam, the Malay Peninsula and Archipelago and Philippine Islands.

Families, Genera and Species	The Indian Subregion							The Indo-Chinese Subregion					*Distribution and general remarks*
	1	2	3	4	5	6	7	8	9	10	11	12	
Family EMYDIDAE Gray—*contd.*													
12 *Cyclemys annamensis* Siebenrock										x			Annam
Genus *Cuora* Gray													
13 *Cuora amboinensis* (Daudin)											x	x	Tenasserim, Siam, Cambodia, Cochin China, the Malay Archipelago and Peninsula, the Philippine Islands
14 *Cuora flavomarginata* (Gray)									x				Southern China, Formosa, the Lu Chu Islands
15 *Cuora trifasciata* (Bell)									x				Southern China, Hainan
16 *Cuora yunnanensis* (Bigr.)									x	x			Yunnan Fu and Tong Chuan Fu
Genus *Geoemyda* Gray													
17 *Geoemyda spengleri* (Gmelin)									x	x		x	Southern China, Annam, the Malay Archipelago, Japan
18 *Geoemyda spinosa* Bell											x	x	Tenasserim, Peninsular Siam, the Malay Peninsula, Sumatra, Borneo, Natuna Islands
19 *Geoemyda silvatica* (Henderson)					x								Cochin
20 *Geoemyda depressa* (Anderson)									x				Arakan Hills
21. *Geoemyda tricarinata* (Blyth)							x	x					Chota Nagpur, North Bengal, North Assam

Families, Genera and Species	The Indian Subregion							The Indo Chinese Subregion					*Distribution and general remarks*
	1	2	3	4	5	6	7	8	9	10	11	12	
Family EMYDIDAE Gray—*contd*													
22 *Geoemyda trijuga* (Schweigger)					x	x	x	x	x				*trijuga* (forma typica) Bombay Madras Mysore State *trijuga edeniana* Burma *trijuga indopeninsularis* Chota Nagpur *trijuga coronata* Travancore and Cochin *trijuga thermalis* Ramnad and Ceylon
23 *Geoemyda grandis* Gray									x		x	x	Burma Siam Cambodia the Malay Peninsula
Genus *Damonia* Gray													
24 *Damonia subtrijuga* (Schlegel & Mull)											x	x	Siam Cochin China Northern Malay Peninsula Java
Genus *Hieremys* Boulenger													
25 *Hieremys annandalei* (Blgr)											x	x	Central Siam Cambodia N Malay Peninsula
Genus *Notochelys* Gray													
26 *Notochelys platynota* (Gray)											x	x	Cochin China the Malay Peninsula and Archipelago
Genus *Geoclemys* Gray													
27 *Geoclemys hamiltoni* (Gray)	x		x										Northern India from Sind to Bengal
Genus *Siebenrockiella* Lindholm													

Families, Genera and Species	The Indian Subregion							The Indo-Chinese Subregion					*Distribution and general remarks*
	1	2	3	4	5	6	7	8	9	10	11	12	
Family EMYDIDAE Gray—*contd*													
28 *Siebenrockiella crassicollis* (Gray)											x	x	Tenasserim, Siam, Cochin-China, the Malay Peninsula and Archipelago
Genus *Clemmys* Ritgen													
29 *Clemmys mutica* (Cantor)									x				Southern China, Formosa, Hainan
30 *Clemmys bealei* (Gray)									x	x			Southern China, Northern Annam, Hainan
Genus *Chinemys* Smith													
31 *Chinemys reevesi* (Gray)									x				Yunnan, S E China to Japan
Genus *Ocadia* Gray													
32 *Ocadia sinensis* (Gray)									x	x			Southern China, Formosa, Hainan, Annam
Genus *Morenia* Gray													
33 *Morenia ocellata* (Dum & Bib)									x			x	Southern Burma and Tenasserim
34 *Morenia petersi* Anderson			x										Bengal
Genus *Hardella* Gray													
35 *Hardella thurgi* (Gray)			x										The Gangetic and Brahmaputra River Systems
Genus *Kachuga* Gray													Fossils in the Pleistocene of the Siwalik Hills and the Narbada Valley
36 *Kachuga smithi* (Gray)	x		x										The Indus and the Ganges River Systems

Families, Genera and Species	The Indian Subregion							The Indo-Chinese Subregion				*Distribution and general remarks*
	1	2	3	4	5	6	7	8	9	10	11	12	
Family Emydidae Gray—*concld*													
37 *Kachuga tectum* (Gray)	x		x		x								*tectum* (forma typica), the Indus, Ganges and Brahmaputra River Systems
													tectum tentoria, the Mahanadi, Godavari and probably Kistna River Systems
38 *Kachuga sylhetensis* (Jerdon)								x					Garo, Khasi and Naga Hills, Assam
39 *Kachuga dhongoka* (Gray)			x					x					N E India, the Ganges as far west as Allahabad and north to Nepal Fossils in the Siwalik Hills
40 *Kachuga kachuga* (Gray)			x										The Gangetic River System
41 *Kachuga trivitta* (Dum & Bib)									x				Burma
Genus *Batagur* (Gray)													
42 *Batagur baska* (Gray)			x						x		x	x	Bengal, Burma to Cochin China and the Malay Peninsula, Sumatra
Family Testudinidae Gray													
Genus *Testudo* Linneaus													
43 *Testudo elegans* Schoepff	x			x	x	x							Throughout Central and Southern India, extending west as far as Sind and south to Ceylon
44 *Testudo platynota* Blyth									x				Burma as far south as Moulmein

Families, Genera and Species	The Indian Subregion							The Indo Chinese Subregion					*Distribution and general remarks*
	1	2	3	4	5	6	7	8	9	10	11	12	
Family TESTUDINIDAE Gray—*concld*													
45 *Testudo elongata* Blyth							x	x	x	x		x	North-eastern India to Tonkin and the Malay Peninsula as far south as Penang
46 *Testudo travancorica* Blgr						x							Travancore, Cochin and Coorg
47 *Testudo emys* Schleg & Mull.								x	x		x	x	Cachar and Naga Hills, Assam, Burma, Siam, the Malay Peninsula and the Archipelago
48 *Testudo impressa* (Gunther)									x	x	x	x	Burma, Siam, Annam, Tonkin, the Malay Peninsula
49 *Testudo horsfields* Gray	x												The Caspian and Aral Seas to the north western corner of British India
Family TRIONYCHIDAE Gray													Fossil *Trionyx*, *Chitra*, and *Lissemys* indistinguishable from present day forms have been found in the Pliocene and Pleistocene of the Siwalik Hills of India
Genus *Lissemys* Smith													
50 *Lissemys punctata* (Bonnaterre)	x		x	x	x	x			x				*punctata* (forma typica), the Indus and the Ganges river systems *punctata granosa*, the Indian Peninsula south of the Ganges, and Ceylon *punctata scutata*, the Irrawaddy and Salween rivers
Genus *Pelochelys* Gray													
51 *Pelochelys bibroni* (Owen)									x		x	?	? Bengal, the Indo-Chinese Peninsula and Southern China, Hainan, ? the Malay Peninsula, Sumatra, Borneo, the Philippine Islands, New Guinea

Families, Genera and Species	The Indian Subregion							The Indo Chinese Subregion				Distribution and general remarks
	1	2	3	4	5	6	7	8	9	10	11	12	
Family TRIONYCHIDAE Gray (*contd*)													
Genus *Chitra* Gray													
52 *Chitra indica* (Gray)	x	x	x					x	x		x	x	Northern India, Siam, the Malay Peninsula
Genus *Dogania* Gray													
53 *Dogania subplana* (Geoffrey)									x		x	x	Burma, Siam, the Malay Peninsula and Archipelago, the Philippine Islands, Asia, Africa and North America
Genus *Trionyx* Geoffrey													
54 *Trionyx gangeticus* Cuvier	x		x		x								The Indus, Ganges and Mahanadi River Systems
55 *Trionyx nigricans* Anderson									x				Chittagong
56 *Trionyx leithi* Gray			x		x								The Ganges and rivers of Peninsular India as far south as Madras
57 *Trionyx hurum* Gray			x										Lower reaches of the Ganges and Brahmaputra
58 *Trionyx formosus* Gray									x				Burma
59 *Trionyx cartilagineus* (Boddaert)											x	x	Southern Burma, Siam, French Indo China as far north as Tonkin, the Malay Peninsula and Archipelago
60 *Trionyx sinensis sinensis* Weigmann									x	x	x		*sinensis* (forma typica), Southern China, Indo-China, Annam, Hainan *sinensis tuberculatus*, Yunnan, Central China, Formosa
61 *Trionyx steindachneri* Siebenrock.									x	x			Southern China, Tonkin, Annam, Hainan

Of the 61 species of Crocodiles and Chelonians listed above, one species of Crocodile, *Crocodilus porosus*, is estuarine and rarely enters rivers above tidal limits, and four species of turtles, *Dermochelys coriacea*, *Eretmochelys imbricata*, *Chelonia mydas* and *Caretta caretta olivacea*, are marine These five species are not of any zoogeographical significance

Of the remaining 56 species, 8 are common to the Indian and Indo-Chinese Subregions, 14 are confined to the Indian Subregion and 34 to the Indo-Chinese Subregion I shall elucidate the significance of each of these groups separately

DISTRIBUTION OF THE SPECIES COMMON TO THE INDIAN AND INDO-CHINESE SUBREGIONS

Of the eight species common to the Indian and Indo-Chinese Subregions, the Gharial (*Gavialis gangeticus*) represents one of the most primitive of the living reptiles and is the sole survivor of a large number of species that once lived in South-eastern Asia Fossil forms of *Gavialis* have been described from the Pliocene deposits of the Siwalik Hills and the Narbada Valley and they are known to have attained a length of 50 to 60 feet The great antiquity of the crocodiles, as is well known to all students of Zoology, is reflected in their discontinuous distribution in the tropical Australian region, South-eastern Asia, Africa and tropical and subtropical America Thus they indicate the existence of the Gondwanaland which comprised the lands enumerated above and persisted as a large southern continent till the end of the Cretaceous or the beginning of the Tertiary period The point of interest is that *G gangeticus*, a survival of the Indobrahm, has persisted throughout the ages of the later Himalayan earth movements without undergoing any appreciable change in structure This is no doubt due to the fact that it lives in sluggish rivers and the ecological condition under which it lives have also persisted as such throughout all these ages

One species of freshwater Tortoise (Terrapins), *Geoemyda trijuga*, and one species of freshwater Turtle (Mud Turtle), *Lissemys punctata*, are of special interest from a zoogeographical point of view as both are found in Burma on the one hand and over a greater part of India on the other Both have budded out geographical races in different parts of their distribution (Text-fig 2), no doubt under the varying ecological conditions of their respective habitats

In the case of *G trijuga*, a chiefly aquatic and vegetarian species, five fairly well defined races are recognizable, dependent chiefly upon the colouration of the head and to a less extent that of the shell Variation in colouration is certainly associated with environmental factors and many similar cases are known from among fishes as well The typical form of the species is known from the Bombay and Madras Presidencies and the State of Mysore, the race *edeniana* from Burma, the race *indopeninsularis* from Chota Nagpur and North Bengal, the race *coronata* from Travancore and Cochin and the race *thermalis* from Ramnad District, Ceylon and the Maldive Islands[1] The distribution, though extensive, is discontinuous for the Chota Nagpur and North Bengal race is separated from the Bombay race in the west and the Burma race in the east by considerable distances Geographical isolation and habitudinal segregation may have led to the origin of the present races but there seems hardly any doubt that once *G trijuga* was uniformly distributed in the intervening portions of its range

The typical form of the Mud Turtle *Lissemys punctata* is found in the Indus and the Ganges and their tributaries, the race *granosa* is found in the Indian Peninsula south of the Ganges while its race *scutata* is found in the Irrawaddy and the Salween Rivers Its absence from Assam makes its distribution discontinuous but its occurrence in the Indus and the Ganges takes us back to the time of the Indobrahm or the Siwalik River Though more widely distributed than the Gharial, *Gavialis gangeticus*, it seems to be contemporaneous with it during the Siwalik period

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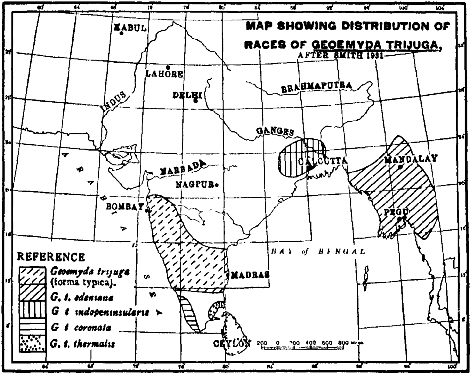
[1] Lt.-Col R B S Sewell has raised a point regarding the occurrence of this species in the Maldive Islands Either it is introduced there by human agency or it must have migrated there before the Laccadives and Maldives were separated from India Personally I favour the latter possibility and feel that such separation occurred not in a very remote geological period.



TEXT FIG 2 - Map showing the geographical raciation and present day discontinuous distribution of *Goemyda trijuga* (Schwel) *After Dr M A Smith*

The Terrapin, *Batagur baska*, and the Mud Turtle, *Chitra indica*, are essentially Indo Chinese forms but their range extends into India for a short distance The former, a herbivorous and entirely aquatic species, is found in Bengal and then its range extends from Burma to Cochin-China and the Malay Peninsula It is also found in Sumatra The animal is of exceedingly shy disposition and inhabits estuaries, deep slow flowing rivers and canals It is likely, therefore, that its distribution to Bengal may have been affected *via* the Bay of Bengal during the monsoons when the salinity near the head of the Bay falls very low

Chitra indica, on the other hand, feeds upon fish, molluscs and other animal food, and is a very pugnaceous animal It is found in Northern India, Siam and the Malay Peninsula It has been recorded from the Irrawaddy also, though this needs confirmation From its distribution, it appears to be a species of the Indobrahm

The Land Tortoise, *Testudo elongata*, is found in Chota Nagpur on the one hand and in north-eastern India to Tonkin and the Malay Peninsula on the other Dr Annandale had recognised the Chota Nagpur form as a separate species, which he called *T parallelus* and remarked on its close resemblance to *T elongatus*, an Indo-Chinese species

The Terrapin, *Geomyda tricarinata*, is found in the Chaibasa District of Chota Nagpur on the one hand and Northern Bengal (Jalpaiguri District) and Assam (Daflas Hills and Bisnath Plain) on the other This herbivorous, aquatic species must have once occurred over the intervening areas in the Vindhya and Satpura trend of mountains

The Terrapin, *Kachuga dhongoka*, at present occurs in North-east India and in the Ganges as far west as Allahabad and north to Nepal but its fossils are known from the Siwalik Hills It is, therefore, a species dating back to the Indobrahm when its range must have been more extensive

DISTRIBUTION OF THE SPECIES OF THE INDIAN SUBREGION

Of the 14 species of Crocodiles and Chelonians, 3 have a very restricted distribution The Terrapin, *Geoemyda silvatica*, is known only from the forests of the Cochin State and inhabits short burrows underground and does not show any partiality for water It can live entirely upon vegetable food The Land Tortoise, *Testudo travancorica*, is known from the hills of Travancore, Cochin and Coorg This species appears to have been derived from *T elongata*, the range of which extends from North-east India to Tonkin and the Malay Peninsula as far south as Penang The endemicity of a large number of species in the Malabar Tract of India and Ceylon and their close affinity with Malayan forms are well known facts and indicate their long isolation from the parent stock leading to the evolution of new species

The Terrapin *Morenia petersi*, is restricted to Eastern Bengal (Jessore, Dacca, Fatehgarh) but little is known about its habits or habitat The Terrapin, *Kachuga kachuga*, is also definitely known from the Gangetic River System

Another species of somewhat restricted distribution is the Mud Turtle, *Trionyx hurum*, definitely known so far from the lower reaches of the Ganges and the Brahmaputra The Mud Turtle, *Hardella thurgi*, is found in the Gangetic and Brahmaputra River Systems

The species that are found in the Indus and the Ganges River Systems are of special interest in so far as they support the view that before the Indus and the Ganges were evolved as present-day rivers, they formed the Indobrahm or the Siwalik River of the Pleistocene period (Pascoe 1919, Pilgrim 1919) The Terrapin, *Geoclemys hamiltoni*, which is represented by fossils in the Siwalik Hills, is at the present day found in Northern India from Sind to Bengal It is carnivorous in its habits Another Terrapin, *Kachuga smithi*, is found in the tributaries of the Indus and the Ganges though it is more common in the former river The Terrapin, *Kachuga tectum*, is represented in India by two geographical races, its typical form is found in the Indus, Ganges and the Brahmaputra River Systems while the race *tentoria* is found in the Mahanadi and the Godavari rivers Here again we find species in formation through isolation or some form of habitudinal segregation The Mud Turtle, *Trionyx gangeticus*, is also found in the Indus, Ganges and the Mahanadi River Systems It is interesting to note that Annandale had recognised the Mahanadi form as a separate race, *mahanaddicus*, though Smith did not agree with Annandale At any rate, there are some differences in colouration from the typical form and in the Mahanadi, therefore, we have an incipient new species of some future age

Another Mud Turtle which has spread southwards from the Ganges to the rivers of the Peninsula is *Trionyx leithi* at present known from the Ganges and the rivers of the Peninsula as far south as Madras This species is closely allied to *T gangeticus* and is co-extensive with it for a certain part of its range (Ganges and

Mahanadi River Systems) Here we have perhaps an instance of the budding of a new species from the older one of the Indobrahm

We have so far dealt with the forms associated in one way or the other with the Ganges System, but two species definitely exhibit Western Asiatic affinities The Starred Tortoise, *Testudo elegans*, is distributed throughout Central and Southern India, extending west as far as Sind and south to Ceylon Though a forest dweller, it is found in dry areas of the low country The genus *Testudo* is cosmopolitan in its distribution, except Australia and Papuasia, and it is likely that *T elegans* is a migrant or descendant of forms living north-west of Sind Another Land Tortoise, *Testudo horsfieldi*, shows definitely that it is a migrant into India from the Aral and Caspian Seas Regions It is only found in the north-western corner of India and has not spread further south

The most widely distributed species of the Indian Region is the Crocodile, *Crocodilus palustris* It is found in the whole of the Indian Peninsula and Ceylon, extending as far west as the Dasht River near the Persian Frontier, in Baluchistan, north to Nepal and east as far as the Darrang District on the Brahmaputra in Assam

GENERAL REMARKS CONCERNING THE INDIAN SUBREGION

Before taking up the analysis of the species only known from the Indo-Chinese Subregion, we may briefly summarise the trends of distribution of the forms known from the Indian Subregion The main points are —

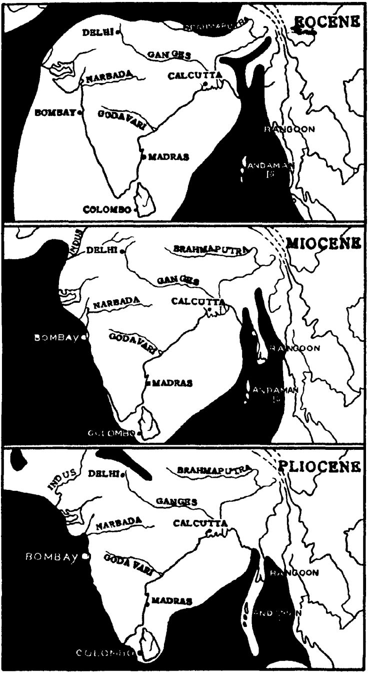
1 Certain species are still in the process of evolution and as a result of geographical isolation have developed certain racial features characteristic of well defined geographical areas There are various gradations in the process of species formation
2 Through long isolation or some form of habitudinal segregation, endemic species have established themselves in certain characteristic geographical areas
3 There is definite evidence of the 'Malayan' element in the fauna of India, particularly of Peninsular India
4 There is definite evidence of a once continuous Indus, Ganges and Brahmaputra river
5 There is definite evidence of a connection of the Ganges System with the River Systems of the Mahanadi and Godavari
6 There is evidence that certain desert-loving forms migrated into India from the north-west and spread over the Peninsula and Ceylon
7 There is some evidence that Chota Nagpur plateau formed a link in the route of migration of 'Malayan' forms to Peninsular India

An attempt will be made later to elucidate all these points by referring to the evolution of the present day physical features of India

DISTRIBUTION OF THE SPECIES OF INDO-CHINESE SUBREGION

Of the 34 species recorded from this subregion, as many as 9 (Nos. 14, 15, 20, 29, 31, 41, 44, 55 and 58 of the list) are endemic in the Trans-Himalayan Mountainous Area, one is endemic in Annam (No 12 of the list) and one in the Eastern Himalayas (No 38 of the list) Eight species (Nos 3, 13, 18, 24, 25, 26, 28, 33 and 59) are common to the Great Plains of Indo-China, Tenasserim and Peninsular Siam Four species (Nos 9, 23, 51 and 53) are found in the Trans-Himalayan Mountainous Area, the Great Plains of Indo-China, Tenasserim and Peninsular Siam Four species (Nos 16, 30, 32 and 61) are common to the Trans-Himalayan Mountainous Area and Annam One species (No. 11) is found all over the Indo-Chinese Subregion, one (No 10) is found in the Eastern Himalayan Area, Trans-Himalayan Mountainous Area and the Great Plains of Indo-China, one species

DISTRIBUTION OF LAND & SEA IN INDIA DURING EOCENE, MIOCENE & PLIOCENE PERIODS

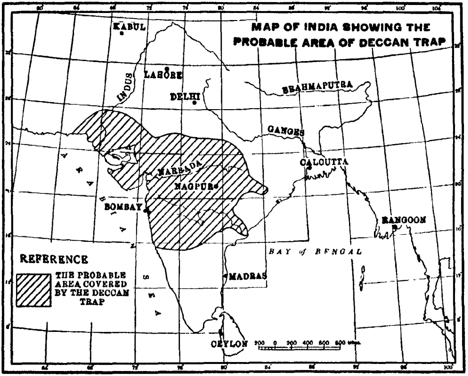


TEXT FIG 3 Maps of India showing the distribution of land and sea during the Eocene, Miocene and Pliocene periods
The figures reproduced here are parts of maps photographed from some work of which the reference is lost.

(No 33) is found in Trans-Himalayan Area and Tenasserim, one species (No 17) is found in the Trans-Himalayan Mountainous Area, Annam, Peninsular Siam and Tenasserim, one species (No 48) to the above three regions as in No 17 and also the Great Plains of Indo-China, one species (No 60) is found in the Trans-Himalayan Mountainous Area, Annam, and Great Plains of Indo-China and one species (No 47) to all the areas except Annam It will be noticed that the largest number of species is found in the Trans-Himalayan Mountainous Area and particularly in Southern China

PROBABLE CENTRE OF ORIGIN OF THE FAUNA

From the distributional records noted above, it would seem probable that Southern China formed the original home of these animals whence they radiated towards the south-west into Burma and India, into Burma, Siam and the Malay Peninsula and Archipelago and south-east to Indo China and Annam The directions of the mountain ranges and the rivers in the Indo Chinese Subregion support this view and I shall show later that geological evidence also favours such a hypothesis

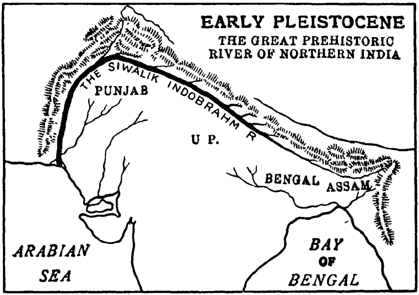

MAP OF INDIA SHOWING THE PROBABLE AREA OF DECCAN TRAP
KABUL
LAHORE
INDUS
DELHI
BRAHMAPUTRA
GANGES
CALCUTTA
NARBADA
NAGPUR
BOMBAY
RANGOON
BAY of BENGAL
MADRAS
CEYLON
200 0 200 400 600 800 MILES
REFERENCE
THE PROBABLE AREA COVERED BY THE DECCAN TRAP

TEXT FIG 4 Map of India showing the probable area covered by a succession of lava flows constituting the Deccan Trap

EVOLUTION OF THE GEOGRAPHY OF SOUTH-EASTERN ASIA

During the Gondwana period, when Australia, South-eastern Asia, East Africa and South America were parts of a single continent, Gondwanaland, the earth underwent a slow gentle buckling of the earth's crust which produced a series of extensive east-and-west valleys During such a period crocodiles must have extended all over Gondwanaland and this would explain their present-day discontinuous distribution

At the end of the Gondwana period, in late Cretaceous or early Tertiary, there was an intense folding of the crust accompanied by the upheaval of the Alpine-Himalayan System, which dismembered the Gondwana Continent and gave the Indian Peninsula more or less its present outline and caused the eruption of lava which formed the Deccan Trap The Himalayan upheaval, in its initial stage, probably confirmed some of the Gondwanaland valleys, but later the movements became more violent on lines tending north and south and disorganised the older drainage system, for with the upraised Central Asia as a vast block with long slopes towards the east and south the general drainage from Tibet was fundamentally altered The west-to east courses of the Upper Brahmaputra, the Hwang Ho, the Yangtze Kiang and the Si Kiang are probably the modern survivals of the ancient east to west rivers, but later arose the north-to-south rivers, such as the Dihang, the Irrawaddy, the Salween and the Mekong The latest east-to west big river was the Indobrahm or the Siwalik River along the southern base of the Himalayas in the Pleistocene period We have ample evidence of the existence of such a river in the distribution of the crocodiles and chelonians referred to above It must be noted that there was no present-day lower Ganges at that period and, therefore, probably there was no gap between the Garo Hills in the east and the Rajmahal Hills in the west



TEXT FIG 5 The Indobrahm or the Great Prehistoric River of Northern India *After Dr D N Wadia* (*Proc Nat Inst Sci Ind* 4, 389, 1938)
The Indobrahm was the successor of the Nummulitic Gulf which stretched from the head of the Sind Gulf to the Punjab and thence along the foot of the embryonic Himalayan chain through Simla and Naini Tal to Assam It carried the combined discharge of the Brahmaputra, the Ganges and the Indus rivers and seems to have existed all through the late Tertiary and early Pleistocene times

The Himalayan uplift was in several major and minor stages and from the violent earthquakes witnessed in this region, it is evident that they are still very unstable and are still rising. Usually three major stages are recognised During the first stage the central axis of the Himalayas was upheaved The second stage about Mid-Miocene upheaved the central part of the range and the third phase came at the end of the Tertiary period when the sub Himalayan zone was added to it The last phase is still continuing and was responsible for the uplift of the Siwalik Hills and the disturbance of the Karewas in Kashmir

The orogenic movements from the north that produced the Himalayas, met with strong resistance from the very old block of the Indian Peninsula, the middle portion of which, however, sagged and in course of time gave rise to the alluvial plain of the Ganges and a concave outline to the central part of the Himalayas The effect of the subsidence of the northern part of the Peninsula also affected the Vindhyas and the Satpuras which were at one time much loftier mountains and formed continuous ranges extending between the Assam Himalayas on the one hand and the Gujarat Western Ghats on the other The Himalayan movements met resistance in the east by the horn of the Peninsula presented by the hills of Assam and in the west by the hills round about Kohat At both these points the Himalayas were bent round and took a more or less north to south direction The direction and intensity of these movements determined the evolution of the zoogeographical features of India and in consequence the migration or dispersal of the animal life

Simultaneously with the rise of the Himalayas, lava flowed out from fissures in several parts of the Indian Peninsula and gave rise to the Deccan Traps (*vide supra*, text-fig 4) Like the Himalayan movements and probably contemporaneous with them, the outbursts of lava also occurred at varying intervals and during the quiescent period animals from neighbouring areas migrated to these lava rocks and some remains of them became entombed in the infra- and inter-trappean beds The last lava outburst may be contemporaneous with the formation of the Siwalik hills in the Pleistocene period or may be even younger than that as is evident from the distribution of the present day forms The lava flows completely annihilated the then existing fauna and recolonisation occurred during the dormant periods

In the Trans-Himalayan Area, particularly in Yunnan, there were corresponding earth movements To explain the origin of deep canyons of the rivers of Yunnan and Western China, it is believed by some that there was a regional uplift of some 6,000 to 10,000 feet in very recent geological times Gregory and Gregory (1923) have, however, found no evidence of any post-Pliocene high regional uplift of this area According to them, the physiography of central and south-western China could be explained by the subsidence of the surrounding country, which produced long slopes downward to the east and the south The geographical distribution of animals supports the latter view

One more palaeogeographical fact must also be borne in mind in connection with the origin and evolution of the Indian fauna In his study of the evolution of the river system of south-eastern Asia, Gregory (1925) found that in the Trans-Himalayan Area the rivers on the west generally beheaded the rivers on the east and thus diverted their waters, and consequently the aquatic fauna, westwards Probably this happened several times contemporaneous with the phases of the Himalayan uplift and enabled the fauna to spread westwards in a series of waves Evidence of such waves of migration is clear even among the Chelonians referred to above

AGE OF THE PRESENT-DAY CROCODILES AND CHELONIANS

Though the Crocodiles made their first appearance in the Upper Cretaceous of Europe and North America, the fossil forms of Gharials (*Gavialis*) are known from the Pliocene deposits of the Siwalik Hills and the Narbada Valley The Chelonians are a much older group, for they are found in the Triassic much as we see them now Their greatest development was towards the end of the Mesozoic and in the early Tertiary Period As shown above, some of the present-day Indian forms are known from the fossils of the Siwalik Hills and are, therefore, at least as old as the Pliocene

THE ORIGIN AND DISPERSAL OF THE FAUNA

As in the case of freshwater fishes, the distributional records of Indian Crocodiles and Chelonians show that they originated on the Yunnan tableland when the conditions were probably somewhat warmer than they are at present With the subsidence of the surrounding country, probably in a series of five or six phases, and the production of long slopes downward to the east and the south, the fauna became dispersed along these two routes in the first instance, the eastern branch colonised French Indo China and Annam, while the southern branch became dispersed into the hills of Burma, Siam and the Malay Peninsula The northern portion of this branch, through a series of river captures, was deflected westwards to the Eastern Himalayas and the Hills of Assam There is a barrier, yet undefined, in Central Nepal, which prevented the westward migration of the fauna along the Himalayas, but the Assam Hills were then continuous with the Rajmahal Hills and there stretched across India loftier ranges of the Vindhya and Satpura Mountains which captured the monsoons and produced ecological conditions similar to those of the Eastern Himalayas, Assam Hills and the Western Ghats In certain parts of the Chota Nagpur plateau similar conditions prevail even up to the present day Along this range the fauna was dispersed to the Western Ghats and thence along them to the south and Ceylon This was the first wave and could explain the occurrence of Malayan or Indo-Chinese element in the fauna of Ceylon The second wave came with a very rich new faunal element after Ceylon had been separated from the mainland through subsidence This would explain the number of characteristic species in the Malabar Tract (Cochin, Travancore) south of the Palghat Gap Through further subsidence or faulting the Palghat Gap made its appearance and prevented the third wave from reaching the extreme south of the Peninsula Through the latest eruption of the lava, a considerable north-western portion of the Deccan became denuded of animal life and when normal conditions returned, a fourth wave repopulated it There were then shorter waves which did not reach the Western Ghats but spread for varying distances along the Satpura and Vindhya Trend of Mountains It was at this stage, that the Indobrahm river of the Siwalik period became dismembered, the Garo-Rajmahal Gap formed, the present-day Ganges came into existence and flowed through the Garo-Rajmahal Gap to the Bay of Bengal Further changes in the migration of the mountainous forms between the Garo Hills and the Rajmahal Hills became interrupted but the Eastern Himalaya and the Assam Hills continued to receive eastern elements

The Western Ghats had another contact with the Himalayas through the Arravali Range and some Western Himalayan forms spread over this range southwards Similarly, the high country beyond Delhi to the Baluchistan and West Punjab Hills, which must have been more pronounced once, served to deflect some of the north-western forms to the south However, this element is represented only by a few forms

The most remarkable thing to note is that all these changes had occurred during the Pleistocene or later periods when the present-day fauna had established itself

CONCLUSIONS

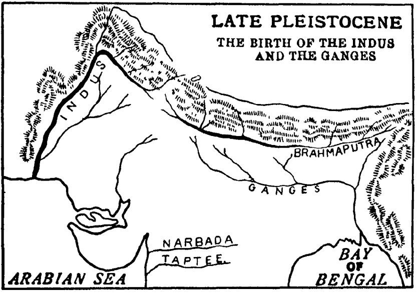
In his chapter on Zoogeography, Smith (1931, p 15) remarked

> 'The dispersal of most of the species that exist today took place at a time when the geographical configuration of the country was very different from what it is today The number of species that are common to the islands of the Malay Archipelago and the continent of Asia is one proof of this, the present distribution of Gharial (*Gavialis gangeticus*) and of the Freshwater Turtles, e g , *Chitra* and *Pelochelys*, in river systems that are not now connected together is another It seems equally certain that the Indo Chinese hill tortoises, *Testudo elongata* and *Geoemyda tricarinata* did not extend their range into the peninsula of India (Chota Nagpur) by crossing the Gangetic Plain True hill species for this reason are of greater interest and value in the study of zoological distribu-

> tion than lowland forms They are just as much isolated by the conditions under which they live as if they inhabited islands, and their occurrence on widely separated mountain ranges is good evidence that a more direct connection between these ranges existed in past times than is to be found today '

All this is very true, but Smith failed to indicate the connections which could account for the many anomalies in the distribution of the Crocodiles and Chelonians In fact, in his second volume in the *Fauna* (1935, p 15) on Sauria (Lizards), he is unable to explain these anomalies of distribution He states

> 'The close affinity which certain Indo Chinese and Malayan lizards have with others that inhabit Southern India—the northern part of the Indian Peninsula being without them—raises an interesting point in zoological distribution The resemblance which *Dasia olivacea* bears to *D subcaeruleum*, *Lygosoma maculatum* to *L dussumieri*, and *Riopa bowringi* to *R albopunctatus*, is so close that one feels convinced that if one has not been derived from the other they must surely have had a common ancestor The genus *Draco* has a similar distribution, *Varanus salvator* occurs in Ceylon and in Indo China, but is absent from the whole of the Indian Peninsula, and there are similar parallels in distribution among the mammals, birds, fishes, and insects Why are they absent from Northern India ? Have they died out in that area, or was there at one time a more southern route across the Indian Ocean by which they could travel ?'



TEXT FIG 6 Dismemberment of the Indobrahm in the late Pleistocene Period and the birth of the Indus and the Ganges as separate rivers *After Dr D N Wadia* (*Proc Nat Inst Sci India*, **4**, 389, 1938)

'At the end of the Siwalik epoch an uplift of the ground between Hardwar and Bikaner disconnected the Indus system from the Ganges portion of the Indobrahm, thus splitting up that river into two separate drainage basins' (Wadia)

I believe the route I have suggested above along the Vindhya-Satpura Trend of Mountains meets with all the requirements of zoogeography in India so far as the occurrence of Malayan element in its fauna is concerned The most interesting point is that all these changes were, comparatively speaking, recent and that they are more or less supported by the evidence that has accumulated in recent years concerning the palaeophysiography of south-eastern Asia For instance, Wadia (1943, p. 41) has pointed out that

> 'The period immediately succeeding the Tertiary was a period of intense orogenic activity in North-West India, it being the final phase of the uplift of the Himalayas,

> during which, to judge from various evidences observed in the Pir Panjal, the Kashmir Himalayas were lifted from 5,000 to 8,000 feet The tilting and folding of the river and lake formed Karewas with the fossil plants, fish, batrachians, elephas, rhinoceros, and a few human implements, and their extension to altitudes up to 11,500 feet, afford a rough estimate of the extent of the movements and of their time duration '

These intense movements in the North-West India must have had percussions over the entire Indo-Gangetic Basin and affected the Vindhya-Satpura trend of mountains It seems likely that orogenic movements of this period may have dismembered the Indobrahm river and produced the Garo-Rajmahal Gap which blocked the further migration of the so-called Malayan fauna to South India

ACKNOWLEDGMENTS

I am grateful to Dr Malcolm A Smith, Lt -Col R B S Sewell, Dr A T Hopwood and Professor H L Chhibber for kindly going through the typescript and favouring me with their comments and suggestions Dr Hopwood's note is reproduced here in its entirety under 'Discussion'

SUMMARY

The distribution of Crocodiles and Chelonians in Ceylon, India, Burma and Farther East is tabulated in accordance with the data given in Smith's revised edition in the *Fauna of British India* series Short descriptions of the zoogeographical areas recognised by Smith and of the ecological factors influencing animal life therein are given.

Of the 61 species of Crocodiles and Chelonians inhabiting the Indian Region, one species of crocodiles and four species of turtles are either estuarine or marine Of the remaining 56 species, 8 are common to the Indian and Indo Chinese Subregions, 14 are confined to the Indian Subregion and 34 to the Indo Chinese Subregion The significance and value of distribution of species of each of these groups is separately elucidated

The distribution of species of the Indian Subregion shows (i) that some of the species are still in the process of evolution and that in certain characteristic ecological complexes a number of endemic species have evolved, (ii) that there is definite evidence of the migration of Malayan forms to Peninsular India, (iii) that the Indus and the Ganges must have formed a continuous river system once and that at some stage the Mahanadi and Godavari had connections with the Ganges System, (iv) that there is evidence of the migration of certain North Western desert forms to Peninsular India and Ceylon, and (v) that the Chota Nagpur plateau must have been in the route of migration of the Malayan forms to Peninsular India

Southern China seems to be the probable centre of origin of the fauna and reference is made to the antiquity of the Crocodiles and Chelonians It is shown that the present-day Indian forms are at least as old as the Pliocene

A brief account of the evolution of the geography of South East Asia is given and the origin and dispersal of the fauna is discussed The most remarkable thing to note is that the various palaeogeographical changes responsible for the present day distribution of animals in India seem to be of comparatively recent origin and are probably associated with the intense orogenic activity immediately after the Tertiary when the Kashmir Himalayas were lifted from 5,000 to 8,000 feet

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DISCUSSION

Dr A Tindell Hopwood of the British Museum (Natural History) sent the following comments with regard to the evolution of the geography of South-Eastern Asia and distribution of certain selected species

'The E W valleys of Central Asia are connected with the Hercynian (i e Carboniferous) orogenesis Their relatively simple arrangement was strongly influenced by the much later Alpine Himalayan movements which began at the very end of the Cretaceous, continued at intervals during the Tertiary, gave a last heave at the beginning of the upper Pleistocene and have not yet entirely died away

The curious disposition of the mountains to the W N and E of India is a result of the squeezing out of the Tethyan geosyncline between Laurasia advancing from the north and the Indian block, or fragment of Gondwanaland from the south The opposing fronts squeezed the soft contents of the geosyncline into the E W Himalaya but, since the Indian block was much narrower than Laurasia its advance threw the sediments on either side into N S folds as in E Baluchistan and in Burma

Eventually Laurasia over rode the Indian block, or if you prefer it, India dived under Laurasia In any event the elevation of the Tibetan plateau was caused by this movement which also tilted the Indian block and carried down the Vindhya Satpura hills In the fore-deep so formed on the southern front of the Himalaya the Indo Gangetic plain was formed from the sediments washed down from the rising mountains

The Vindhya Satpura ranges, the Eastern Ghats and the Nilghiris are extremely ancient features representing early Palaeozoic (Cambrian ?) orogenesis

If the Brahmaputra is an old river which originally flowed in an E W valley, its mouth must have been somewhere on the China Sea, and one is tempted to speculate that its eastward continuation is now represented by the Yangtse Kiang If that were so the interruption would have been brought about by the elevation of the mountains which now fill the gap between the two rivers and the Brahmaputra would be a very old river indeed dating back to Permian or even Carboniferous times It seems more likely, however, that the river did not exist prior to the first elevation of the Himalaya and that it took its origin from the drainage of the northern slope of that chain accumulating in a parallel valley There it flowed eastward The southward bend in Tibet is conditioned by the general structure of that area, and the same factor governs the westward bend in northern Assam

The reversal of drainage which ended the Indo Brahm and gave rise to the Ganges and Indus is post Middle Pleistocene date The same earth movements were responsible for the formation of the Garo Rajmahal gap

Remarks on the Distribution of certain selected Species

a	*Gavialis gangeticus*	Occurs in regions	1, 3, 9
b	*Crocodilus palustris*	,,	1, 2, 3, 4, 5, 6, 7, 8
c	*Cyclemys mouhoti*	,,	8, 9, 11
d.	*Geoemyda tricarinata*	,,	7, 8
e.	*G tryuga*	,,	5, 6, 7, 8, 9.
f	*Kachuga dhongoka*	,,	3, 8
g	*Batagur baska*	,,	3, 9, 12
h	*Testudo elongata*	,,	7, 8, 9, 10, 12
i	*Lissemys punctata*	,,	1, 3, 4, 5, 6, 9
j	*Chitra indica*	,,	1, 2, 3, 8, 9, 11, 12

Of these *b*, *i*, *j* are old species in India and invaders in Indo China, *c* and *h* are old in Indo China and invaders in India The difficult genera are *Geoemyda*, *Kachuga*, and *Batagur* *Geoemyda* probably invaded India and *Kachuga* invaded Indo China *Batagur* might have done either, but I think it went from West to East '

The following discussion took place when the paper was communicated by Dr Hora at the Ordinary General Meeting held on May 4, 1948 —

In communicating his paper entitled 'The distribution of Crocodiles and Chelonians in Ceylon, India, Burma and Farther East', Dr S L Hora generally referred to the distribution of the Vertebrata in India, with particular reference to the so-called Malayan affinities of the vertebrate fauna of the Malabar Tract of the Western Ghats He invited attention to the fact that after the completion of the *Fauna of British India* volumes on Vertebrata, Blanford published an account of the distribution of vertebrate animals in the *Philosophical Transactions of the Royal Society of London* in 1901 Some of these *Fauna* volumes have now been revised and in the light of new knowledge concerning the systematics and geographical distribution of the various animals, it was now possible to comprehend more fully the zoogeography of the forms showing discontinuous distribution This article, Dr Hora pointed out was the first of a series that would be published on this subject

Dr Hora then described the probable routes of migrations of the Malayan forms to the Western Ghats and stated that as Bay of Bengal is a very old feature of the physiography of India, the migration of freshwater turtles and tortoises could not be across this region As most of these forms were not found in the Western Himalayas, they could not have been pushed down as a result of glaciation The resemblance of the Chelonian fauna of the Chota Nagpur Plateau with that of Assam, Burma and Farther East on the one hand and with that of the Malabar Tract on the other showed that Chota Nagpur must have formed a part of the route of this migration This, Dr Hora said, confirmed his earlier views based on the study of distribution of torrential fishes, and he was feeling more and more convinced that the Satpura Vindhya trend of mountains must have been continuous once with the Assam Hills and the Eastern Himalayas on the one hand and with the Gujerat section of the Western Ghats on the other

Dr Hora then referred to the necessary climatic conditions over the Vindhya Satpura trend for making possible the migration of the rain forest dwelling species common to the Malabar Tract and Assam Burma region He opined that when this migration took place the Satpuras and the Vindhyas must have been much higher ranges which could intercept monsoons more effectively so as to give an annual precipitation of 100" to 150" per annum on their slopes resulting in luxuriant vegetation He stated that Dr S K Banerji, Director General of Observatories, has computed that for this amount of precipitation, the Satpuras must be raised to 6,000 to 8,000 feet above sea level

Referring to the geological period when this migration took place, Dr Hora stated that the evidence so far available to him showed that active migration was facilitated by orogenic movements consequent upon the uplifts of the Himalayas during the later Tertiary or Quarternary Periods

Dr S K Banerji explained with reference to the monsoon currents in this region how the figure of 6,000 to 8,000 feet elevation of the Satpuras was computed so as to give Dr Hora climatic conditions, consequent on rainfall, equivalent to those prevailing in Assam and the Malabar Zone He stated that from the very nature of the proposition put to him, it was difficult to answer with any degree of precision but the values given could be taken as very good approximations

Dr S P Agharkar stated that the hill top floras of the Western Ghats and the Assam-Burma Hills showed resemblances almost identical with those described by Dr Hora among the faunas He also agreed with Dr Hora that the Satpura Vindhya trend of mountains could serve as the route of migration when the climatic conditions over the entire area were more or less uniform and conformed with the present day conditions in the Eastern Himalayas, Assam Hills and the Malabar Zone

Dr D N Wadia stated that as a geologist he welcomed the studies that were being conducted by Dr Hora, but that more detailed information and data were necessary to establish the probable route of migration of the Malayan forms to the Western Ghats

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PROCEEDINGS
OF THE
NATIONAL INSTITUTE OF SCIENCES OF INDIA

AUGUST, 1948

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A NOTE ON AN ANALOGY REGARDING OPERATORS IN DIRAC'S WAVE EQUATION FOR THE ELECTRON

By S Gupta

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(Communicated by Dr H J Bhabha, F R S)

(*Received March 15, read April 2, 1948*)

1 The Dirac equation for the electron in the absence of a field is

$$-\frac{\hbar}{i}\frac{\partial\psi}{\partial t} = H\psi, \tag{1}$$

where the Hamiltonian H is given by

$$H = c(\alpha_1 p_1+\alpha_2 p_2+\alpha_3 p_3+\alpha_4 m_0 c) \tag{1'}$$

With the four α operators in this Hamiltonian as pointed out by J v Neumann (1928), 16 operators can be formed (by multiplication) which are linearly independent and it is expected that some physical significance can be attributed to each of them Different authors associate these matrices with physical quantities in different manner Some have worked in terms of densities (in Dirac Darwin sense) and their operators are slightly different from those who have preferred to argue in terms of the operators themselves as representing the physical quantities Below is written the whole set of operators in a group S indicating the physical quantities with which they are associated and which are in common use *

S I $j_0 = 1$, charge operator,

$j_1, j_2, j_3 = \alpha_1, \alpha_2, \alpha_3$, current operators

II $P_1, P_2, P_3 = -i\alpha_4\alpha_1, -i\alpha_4\alpha_2, -i\alpha_4\alpha_3$ electric moment operators

$M_1, M_2, M_3 = -i\alpha_4\alpha_2\alpha_3, -i\alpha_4\alpha_3\alpha_1, -i\alpha_4\alpha_1\alpha_2$, magnetic moment operators

III $K_0, K_1, K_2, K_3 = -i\alpha_1\alpha_2\alpha_3, -i\alpha_2\alpha_3, -i\alpha_3\alpha_1, -i\alpha_1\alpha_2$ spin operators

IV $I, J = \alpha_4, \alpha_4\alpha_1\alpha_2\alpha_3$ not definitely identified

The densities (in Dirac Darwin sense) of I, II, III and IV are transformed as a four vector, an antisymmetrical tensor of the second rank, an antisymmetrical tensor of the third rank and invariants respectively , but these operators themselves have no such transformation properties

For the discussion of the Lorentz invariance the most useful form of (1) is

$$(\gamma^\mu p_\mu+m_0 c)\psi = 0, \tag{1''}$$

where

$$x_0 = ct,\ p_\mu = \frac{\hbar}{i}\frac{\partial}{\partial x^\mu},\ \gamma^0 = \alpha_4\ \ \gamma^k = \alpha_4\alpha_k \quad (k = 1, 2, 3)$$

---

* Many authors beginning from Dirac himself have interpreted many of these operators in the manner stated here We have left aside the constant factors which are associated with the operators

The Lorentz invariance of (1″) with *given* γ^μ demands th[illegible]or the orthogonal transformation defined by

$$r^{\mu} = L_\nu{}^\mu x^\nu$$

there shall exist a similarity transformation of γ^μ given by

$$\Lambda^{-1}\gamma^\mu\Lambda = L_\nu{}^\mu\gamma^\nu$$

If γ^μ, when transformed in this sense, is called a four vector, a set S′ of tensors can be defined as follows

S′ I′ γ^μ, a four vector

$$\jmath_0, \jmath_1, \jmath_2, \jmath_3 = \gamma^0, \gamma^1\ \gamma^2, \gamma^3 = \alpha_4\ \ \alpha_4\alpha_1, \alpha_4\alpha_2\ \ \alpha_4\alpha_3$$

II′ $i\gamma^\mu\gamma^\nu$, $(\mu \neq \nu)$, an antisymmetrical tensor of the second rank

$$P_1, P_2, P_3 = i\gamma^1\gamma^0, i\gamma^2\gamma^0, i\gamma^3\gamma^0 = -i\alpha_1, -i\alpha_2\ \ -i\alpha_3,$$

$$M_1, M_2, M_3 = i\gamma^2\gamma^3, i\gamma^3\gamma^1, i\gamma^1\gamma^2 = -i\alpha_2\alpha_3, -i\alpha_3\alpha_1, -i\alpha_1\alpha_2$$

III′ $i\gamma^\lambda\gamma^\mu\gamma^\nu$, $(\mu \neq \nu \neq \lambda)$, an antisymmetrical tensor of the third rank,

$$K_0, K_1, K_2, K_3 = i\gamma^1\gamma^2\gamma^3, i\gamma^2\gamma^3\gamma^0\ \ i\gamma^3\gamma^1\gamma^0, i\gamma^1\gamma^2\gamma^0$$
$$= -i\alpha_4\alpha_1\alpha_2\alpha_3, -i\alpha_4\alpha_2\alpha_3, -i\alpha_4\alpha_3\alpha_1\ \ -i\alpha_4\alpha_1\alpha_2$$

IV′ $\gamma^1\gamma^2\gamma^3\gamma^0 = \alpha_1\alpha_2\alpha_3 = J'$, an invariant

It is to be noted that the set S can be obtained from S′ simply by multiplication with α_4 from the left and vice versa In this note it will be shown that S and S′ are related to one another in a manner such that any operator of one set can be expressed in terms of the corresponding operators of the other set Moreover, this relation has a *formal analogy* with Lorentz transformation such that quantities in S′ correspond to a set of vectors in rest system and those in S to a set in a moving system

2 For a free electron H and p, the energy and linear momentum operators respectively are integrals of the equation and are constants, and further

$$H^2 = c^2p^2 + m_0{}^2c^4 \tag{2}$$

Using these properties of the free electron, S′ can be expressed in terms of S in the following manner

$$\left.\begin{aligned} \jmath_0 &= \frac{H}{m_0c^2}\left\{\jmath_0 - \frac{1}{c}\,(c^2\mathbf{p}H^{-1}, \mathbf{j})\right\}, \\ \jmath_k &= \left\{\frac{H}{m_0c^2} - \alpha_4\right\}\frac{[\mathbf{p}\times(\mathbf{p}\times\mathbf{j})]_k}{p^2} + \frac{H}{m_0c^2}\left\{\jmath_k - \frac{1}{c}\ \ c^2p_kH^{-1}\ \ \jmath_0\right\} \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} P_k &= \left\{\alpha_4 - \frac{H}{m_0c^2}\right\}\frac{(\mathbf{P}\ \ \mathbf{p})p_k}{p^2} + \frac{H}{m_0c^2}\left\{P_k - \frac{1}{c}[c^2\mathbf{p}H^{-1}, \mathbf{M}]_k\right\}, \\ M_k &= \left\{\alpha_4 - \frac{H}{m_0c^2}\right\}\frac{(\mathbf{M}\ \ \mathbf{p})p_k}{p^2} + \frac{H}{m_0c^2}\left\{M_k + \frac{1}{c}[c^2\mathbf{p}H^{-1}, \mathbf{P}]_k\right\} \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} K_0 &= \frac{H}{m_0c^2}\left\{K_0 - \frac{1}{c}\,(c^2\mathbf{p}H^{-1}, \mathbf{K})\right\}, \\ K_k &= \left\{\frac{H}{m_0c^2} - \alpha_4\right\}\frac{[\mathbf{p}\times(\mathbf{p}\times\mathbf{K})]_k}{p^2} + \frac{H}{m_0c^2}\left\{K_k - \frac{1}{c}\cdot c^2p_kH^{-1}\ \ K_0\right\}. \end{aligned}\right\} \tag{5}$$

The first equation of (3) is obvious from the Hamiltonian itself and the first equation of (5) can be obtained by multiplying from the right both sides of (1′) with $-i\alpha_1\alpha_2\alpha_3$ For the second equation of (4) we operate with M_k on (1′) from the right and have

$$M_k = \frac{H}{m_0c^2}\left\{M_k+\frac{1}{c}\,[c^2\mathbf{p}H^{-1},\,\mathbf{P}]_k\right\}+\frac{ip_k\tau}{m_0c}, \tag{6}$$

where $\tau = \alpha_1\alpha_2\alpha_3\alpha_4$ Again, since

$$(H-\alpha_4m_0c^2)\tau = ic(\mathbf{M}\ \mathbf{p}),$$

on operating with $H-\alpha_4m_0c^2$ from the left on both sides of this equation and taking equation (2) together with the relation

$$H\alpha_4+\alpha_4H = 2m_0c^2 \tag{7}$$

into consideration, we have

$$\tau = \frac{i}{cp^2}\,(H-\alpha_4mc^2)(\mathbf{M}\ \mathbf{p}).$$

When this expression for τ is substituted in (6), the second equation of (4) is obtained In a similar manner the first equation of (4) can also be deduced

For the second equation in (5) equation (1′) is multiplied from the right by K_k ($k = 1, 2, 3$) which gives

$$K'_k = \frac{H}{m_0c^2}\left\{K_k-\frac{1}{c}\ \ c^2p_kH^{-1}\ \ K_0\right\}+\frac{1}{m_0c}\,[\mathbf{p}\times i\boldsymbol{\alpha}]_k \tag{8}$$

Again,

$$\{H-\alpha_4m_0c^2\}i\alpha_k = icp_k+c\,[\mathbf{p}\times\mathbf{K}]_k$$

We operate with $H-\alpha_4m_0c^2$ from the left on both sides of this equation and taking (2) and (7) into consideration, we get

$$\frac{1}{m_0c}\ \ i\alpha_k = \left(\frac{H}{m_0c^2}-\alpha_4\right)\frac{[\mathbf{p}\times\mathbf{K}]_k}{p^2}+\frac{ip_k}{p^2}\left(\frac{H}{m_0c^2}-\alpha_4\right)$$

With this expression for $i\alpha_k$ equation (8) reduces to the second equation in (5) A similar process gives the second equation of (3)

3 In (3), (4) and (5) we have a set of equations in which the quantities of set S′ are expressed only in terms of the corresponding quantities in S and the elementary integrals H and p If now an interpretation of these equations be attempted by retaining the operator vectors on the right of these equations and replacing p, H by their expectation values, namely,

$$\frac{m_0v}{\sqrt{1-\beta^2}},\ \ \frac{m_0c^2}{\sqrt{1-\beta^2}}$$

respectively and α_4 by its eigenvalue +1 (positive energy) in the usual manner, we get to the following set of equations

$$\left.\begin{aligned} j'_0 &= \gamma\left\{j_0-\frac{1}{c}\,(\mathbf{v}\cdot\mathbf{j})\right\},\\ j'_k &= (\gamma-1)\,\frac{[\mathbf{v}\times(\mathbf{v}\times\mathbf{j})]_k}{v^2}+\gamma\left\{j_k-\frac{1}{c}\ \ v_k\ \ j_0\right\},\end{aligned}\right\} \tag{3′}$$

$$\left.\begin{aligned} P'_k &= (1-\gamma)\frac{(\mathbf{P}\;\mathbf{v})v_k}{v^2}+\gamma\left\{P_k-\frac{1}{c}[\mathbf{v}\times\mathbf{M}]_k\right\}, \\ M_k &= (1-\gamma)\frac{(\mathbf{M}\;\mathbf{v})v_k}{v^2}+\gamma\left\{M_k+\frac{1}{c}[\mathbf{v}\times\mathbf{P}]_k\right\}, \end{aligned}\right\} \tag{4'}$$

$$\left.\begin{aligned} K_0 &= \gamma\left\{K_0-\frac{1}{c}(\mathbf{v}\;\mathbf{K})\right\}, \\ K_k &= (\gamma-1)\frac{[\mathbf{v}\times(\mathbf{v}\times\mathbf{K})]_k}{v^2}+\gamma\left\{K_k-\frac{1}{c}\;v_k\;K_0\right\}, \end{aligned}\right\} \tag{5'}$$

where

$$\gamma = (1-\beta^2)^{-\frac{1}{2}},\ \beta = v/c$$

These relations are identical with the transformation formulae of a four vector, an antisymmetrical tensor of the second rank (electric and magnetic moment tensor) and an antisymmetrical tensor of the third rank (which is transformed like a vector) in the theory of relativity * (Frenkel, 1926) in which S and S′ behave as two co-ordinate systems, S′ moving relative to S with velocity v The reciprocal relations, namely, S set in terms of S′ are easily calculated and can be obtained from (3), (4) and (5) simply by changing the sign of p, just as in the Lorentz transformation formulae only *the sign of v has to be changed in such a case* This changing of the sign of p is the same as operating with α_4 on both sides of (3), (4) and (5) from the right The interpretation of (3), (4) and (5) by replacing only p, H by their expectation values, and α_4 by $+1$ † and retaining the other quantities in their apparent tensor form may not be quite legitimate so far as the behaviour of the Dirac electron is concerned but the resemblance of these equations with the Lorentz transformation of tensors and vectors is so striking that this identification may help to bring to light some inner relation between the two sets of operators for which physical interpretations relating to the elementary properties of the electron have been proposed It should be remembered that in S′ set the operators themselves behave as vectors and tensors in the sense defined before, while in S set only the densities, that is the operators associated with the wave functions, behave as such S′ set *behave* as quantities in their proper system and in this set only the charge ($\jmath_0$), magnetic moment (M'_1, M'_2, M'_3), and (K'_1, K'_2, K'_3) operators are hermitian (real) and the rest are non-hermitian, whereas in the S set all the quantities are hermitian (cf Frenkel, 1934, p 317) The operator α_4 which transforms one set into another *formally plays* the part of a Lorentz transformation These are the furthest limits to which probably the above analogy can be pushed Whether this analogy with Lorentz transformation is more real than it appears and represents some properties of the electron is more than we can say at present

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* Lehrbuch der Electrodynamik, J Frenkel, Erster Band, 1926 See page 288, Eqns (2, 2*a*) and page 294, Eqn (11)

† α_4 has the expectation value $+1$ in the rest system

STUDIES ON THE CYTOLOGY OF YEASTS

III THE TECHNIQUE OF HANDLING YEASTS FOR CYTOLOGICAL INVESTIGATIONS

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(Communicated by Sir J C Ghosh, Kt, D Sc, F N I)

(*Received September 20, 1947, read March 5, 1948*)

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INTRODUCTION

The recent demonstration of endopolyploidy in yeasts (Subramaniam, 1947*a*) makes it possible to give a rational explanation of the technique of handling yeasts for cytological investigations It would be admitted that cells should be in active vegetative division in order that mitotic stages may be studied by various methods of fixation and staining Therefore, a clear distinction should be made between (1) the handling of cultures of yeasts for cytological investigations and (2) the methods of fixation and staining employed to demonstrate the mitotic stages The importance of the above distinction can only be appreciated when it is realised that an aerobically growing culture would gradually change into a fermenting one with the gradual depletion of the dissolved oxygen in the medium Fermenting cells are like gland cells and if culture conditions are not standardised, preparations from such cultures would present a baffling array of pictures The use of a variety of methods of fixation and staining as done by Nagel (1946) cannot improve matters A careful perusal of the literature shows *that none of the previous investigators either tried to standardise the methods of handling yeasts for cytological investigations or attempted a correlation of the changes in the cytological behaviour of yeasts under varying physiological conditions*

It was emphasised in a provious publication (Subramaniam, 1946) that if cytological investigations are carried out on rapidly growing cultures, the nuclear phases in yeasts could be demonstrated with ordinary staining procedures From a preliminary investigation of the behaviour of the chromosomes in *Neurospora crassa*, McClintock (1945) remarks 'No distinctively unique features of chromosomal organisation were recognised'

Since yeasts are considered to be the products of degenerative evolution from higher Fungi (Guilliermond, 1940), there appears to be no valid reason why the cytology of yeasts should be different from that of higher Fungi

A CRITICAL EVALUATION OF SOME RECENT PUBLICATIONS

Nagel (1946) in a recent publication makes the amazing statement 'The body most frequently referred to as the nucleus (Guilliermond, 1910, Badian, 1937, Beams, Zell and Sulkin, 1940) is called the centriole by another school (Lindegren, 1945) and the nucleolus by a third (Wager and Peniston, 1910)', and arrives at the conclusion that 'after one hundred years of cytological work, the organisation of the yeast nucleus is still a matter for debate among authorities, even as to the elementary points' There appears to be very little justification for such pessimism, for identification of the organelles in a cell are based on established criteria A careful perusal of the Glossary in Darlington's Recent Advances in Cytology (1937) would show that there can be no confusion in the identification of a nucleus from a nucleolus and a centriole from both! Without trying to identify the various cell organelles in yeast based on the above established criteria, Nagel unnecessarily complicates issues by coining new terms

From the tabulated statement given by her (p 268), it would be evident that while Janssens and Leblanc (1898), Guilliermond (1903, 1910 and 1920), Kohl (1908), Henneberg (1916), Kater (1927), Badian (1937), Brandt (1941) and Subramaniam and Ranganathan (1945, 1946*a*, *b*) identify the same cell entity as the *nucleus*, it is only Wager and Peniston (1910) and Lindegren (1945) who try to identify the nucleus of other workers as the *nucleolus* and *centriole* respectively The 'vacuole' of other workers, on the other hand, is identified by Wager and Peniston and Lindegren as the *nucleus*

Nagel (1946) definitely states that the cell entity identified by the majority of workers as the nucleus is Feulgen positive, while the 'vacuole' is Feulgen negative Greenstein in his review on nucleoproteins (1944) states 'At the start of the mitotic cycle (prophase) there is an accumulation of nucleic acid in the chromosome which reaches a high value in the metaphase and largely disappears at the telophase This nucleic acid is of the desoxyribose type, for it yields a positive Feulgen reaction, in no other place, other than the chromosomes, is this type of nucleic acid found' (p 274) If this is the established location of desoxyribose nucleoproteins in organisms, the identification of a Feulgen positive body in the yeast as the 'nucleolus' or the 'centriole' is unwarranted The centrioles and nucleoli are Feulgen negative and Nagel admits this is so (p 266)

Lindegren tried to argue that 'the desoxyribose nucleoprotein nature (Feulgen positive) of nuclei of higher organisms does not necessarily indicate its universality in the chromatin of simpler organisms' (Nagel, 1946, p 266) There appears to be no justification for the above, since Avery, MacLeod and McCarty (1944) extracted not only a desoxyribose nucleic acid from Pneumococci but also showed that such an extract from cells of the encapsulated Type III induced the *R* variants of Type II to become encapsulated 'Inheritable morphological, serological and chemical alterations in a living organism are thus induced by a specific chemical substance, in this case a polymerised desoxypentose nucleic acid' (Greenstein, 1946, p. 280).

Realising probably the untenable position taken up by him, Lindegren (1947) suggests that in view of Rafalko's (1946) work, 'the yeast chromosomes contain desoxyribose nucleic acid and are apparently conventional in this regard' (p 63) The bodies identified by him as the 'chromosomes' in the 'vacuole' and claimed by him as composed of 'volutin' are now considered to be of the conventional type Nagel (1946) is sure that the 'vacuole' and its contents are Feulgen negative and that granules are lacking in the vacuole From her Table V (p 259) it is apparent that only 14% of the cells showed any granules in the vacuole and of these only 2%

showed any pairing It is from this negligible percentage (2%) that Lindegren identified these bodies as 'chromosomes' based on 'their structure and behaviour'. Nagel says 'If the magnicorp (vacuole) is nuclear, the variability of the number of particulate units is suspiciously great' (p 267) Lindegren does not offer any proof that the homologies of the structures described by him are identical with those described by Rafalko It appears that Rafalko himself is doubtful of his identifications of the stained bodies in Feulgen preparations as the centriole and the nucleolus, since these terms are used within inverted commas *The confusion is, therefore, not due to want of established criteria but appears merely to be a question of personal predilection* Thus having tried to identify a Feulgen positive body as a 'centriole' instead of a nucleus, Lindegren suggests in his recent contribution that 'the desoxyribose nucleoprotein in the centriole is the equivalent of the heterochromatin in cells of higher organisms' (1947, p 64) Caspersson and Brandt (1941) considered the volutin grains in the cytoplasm composed of ribonucleic acid as the equivalent of the heterochromatin and nucleolus of higher organisms The body identified by Lindegren in yeasts as the 'centriole is not only Feulgen positive but does not form a spindle during mitosis and even though lying outside the nucleus is yet considered to be heterochromatin Centrioles of the conventional type with centrospheres have already been demonstrated (Ranganathan and Subramaniam, 1947) Further, in Parts II and V of these studies (Subramaniam, 1947, Ranganathan and Subramaniam, 1948) criteria are discussed for the identification of heterochromatin and based on these certain parts of chromosomes have been *identified in yeasts as the heterochromatin*

In view of the above confusion a restatement of the criteria for the identification of chromosomes has become necessary

Criteria and Definitions

Investigators of the cytology of higher plants are never worried over criteria for the identification of chromosomes Those engaged in the study of the nuclear apparatus of yeasts and bacteria are forced to consider these criteria for a correct appreciation of the problems facing them To define criteria for the identification of a chromosome, one has to fall back on two different sources of evidence These are (1) physiological and (2) chemical To do this one has to start with the definition of the nucleus Darlington (1937) defines a nucleus as a 'cell body which arises or reproduces by mitosis' (p 51) Now, what is mitosis? Mitosis is the 'separation of identical halves of the split chromosomes into two identical groups from which two daughter nuclei are reconstituted' (p 22) This takes us to a definition of the chromosomes, for chromosomes are 'the bodies into which the nucleus resolves itself at the beginning of mitosis and from which it is derived at the end of mitosis' (p 574) The crucial test for a claim of mitosis is, therefore, the demonstration of the anaphase, which is defined as 'the stage at which daughter chromosomes move apart in a nuclear division' (p 572)

In the year 1879 Flemming introduced the word *chromatin* to refer to the 'deeply staining substance of the nuclear network and of the chromosomes, consisting of nuclein' (Wilson, 1904, p 439) Usages of particular terms have changed with the rapid advances in our knowledge, but even today, the term *chromatin* is used to refer to 'the part of the chromosome that stains deeply during mitosis as opposed to the achromatic part' (Darlington, 1937, p 574) Thus having obtained a clear idea of the physiological criteria for the identification of chromosomes let us turn to the chemist's idea of chromatin Mirsky and Pollister (1943) state 'From the above we may conclude tentatively that chromatin is almost entirely made up of a fibrous nucleoprotein, soluble in strong saline and precipitable by dilution This substance is a complex of desoxyribose nucleic acid with a protein It has long been generally accepted that nucleic acid bulks large in the composition of chromatin, and since the development and widespread application of the Feulgen nucleal reaction, it has become equally clear that the nucleic acid contains as its sugar component, the desoxypentose, d-ribo desose All this our analyses of chromatin confirm' (p 258) The chromatin isolated from leukaemic blood cells by Claude and Potter (1943) gave a positive Feulgen reaction and was shown to contain 40% desoxyribose nucleic acid

SPECIFICITY OF THE FEULGEN REACTION

The recent discovery of the acidic protein 'chromosomin' by Stedman and Stedman (1943) led to a controversy whether the Feulgen reaction stains the 'chromosomin' or the desoxyribose nucleic acid Stedman and Stedman were of the opinion that desoxyribose nucleic acid occurs not in the chromosomes but in the nuclear sap, and that it is the new compound formed as a result of the interaction of the above with the fuchsin-sulphurous acid that stains the chromosomes On the basis of the above argument, Choudhuri (1943) developed the colour in leuco-basic-fuchsin by addition of an aldehyde and using this claimed that it stained the chromosomes in an identical way as Feulgen's technique Carr (1945) claims (1) that by changing the pH, the colour of basic fuchsin could be changed, (2) that the specific staining of the nucleus is the result of the destruction of the cytoplasmic structures, and (3) that the staining of the chromosomes is a mere adsorption phenomenon However, the above arguments have been contested and proof has been adduced that desoxyribose nucleic acid occurs in the chromosomes and not in the nuclear sap Stowell and Albers (1943) showed by spectrophotometric analyses that basic fuchsin differed from leuco basic-fuchsin to which a trace of formaldehyde had been added, and stained nuclei in thymus gland from both The nucleic acid in the chromosomes were digested with the desoxyribose nuclease by Mazia and Jaeger (1939), Serra (1943), Dodson (1946), Stowell (1946) and Brachet (1946), and it was shown that the protoid cores of the chromosomes left after such digestion do not stain by the Feulgen technique It appears that after hydrolysis for one and a half hours (Stowell, 1946), when fuchsin sulphurous acid fails to stain any nuclei, the developed nucleal stain still stained the nuclei diffusely Sections stained by basic fuchsin, developed nucleal stain and by Feulgen technique also differed in the rate of fading when exposed to a carbon arc (Stowell, 1945) The Feulgen stained preparations were found to be more resistant than the others Though the colour of the structures stained by basic fuchsin could be altered by washing with hydrochloric acid, the resultant colour is different from that seen after Feulgen staining Hydrolysis may destroy the mitochondria but does not appreciably change the dry weight of the tissues (Dodson, 1946), but later staining of the various cytoplasmic structures depends on the type of fixative used But even after hydrolysis, some cytoplasmic structures and the nucleolus which do not stain with the Feulgen reagent could be stained by the routine stains (Stowell, 1945) Staining of the chromosomes has also been shown not to be a mere adsorption phenomenon, since attempts to induce nuclei, which were made Feulgen negative by prolonged hydrolysis, to adsorb thymonucleic acid, proved a failure (Stowell, 1946) It has thus been emphasised (Dodson, 1946, Brachet, 1946, Stowell, 1945, 1946) that in the present state of our knowledge, the Feulgen technique offers a specific test for the location of thymonucleic acid in tissues

TECHNIQUE

(a) *Preparation of Wort and Wort agar* —The usual method of preparation of wort in this laboratory differs in some details from the method suggested by Stelling-Dekker (Henrici, 1941) Fifty grams of barley malt with 250 c c of water and a layer of toluene in a 500 c c flask are incubated at 55°C overnight It is filtered through a Buchner funnel and while the filtrate is kept separate, the residue is mixed with 100 c c of water and cooked for 30 minutes at 10 lb pressure. After cooling, the filtrate from the first digestion is added and the flask and its contents again incubated overnight at the same temperature The contents are then filtered The specific gravity is adjusted to 1 020 and the *p*H to 4 5–5 0 Roughly every 50 gm of malt gives about 350 c c of wort This is sterilised at 10 lb pressure for 30 minutes, and if a precipitate appears on standing it is again filtered and sterilised This wort is used for experiments only after standing for a few days

For preparation of wort-agar, 2 gm of agar are added to every 100 c c of wort of specific gravity 1 020 and a *p*H of 6 8, autoclaved at the above pressure for the same duration, filtered through glass wool while hot, tubed and again sterilised before the preparation of slants

(*b*) *Methods of Seeding and Smearing* —Material from a 24-hour growth on an agar slant is inoculated into a test tube containing about 5 c c of wort After the lapse of 24 hours, the contents of the test tube are well shaken and discarded and an equal volume of wort added to the tube The tube is again well shaken and a loop from the above, which would contain some 30 to 50 cells, is inoculated into 100 c c conical flasks containing about 5 c c of wort forming a layer, a few millimetres in thickness, at the bottom The final inoculation is carried out inside a sterile chamber and the flasks are left inside the chamber for 24 hours Richards (1928) mentions that the same amount of seeding in an identical quantity of the medium produces almost the same amount of growth whether cultured in test tubes or in petri dishes, in spite of the differences in the area exposed to air Our observations indicate that growth is better when grown in thin layers of wort in conical flasks Growing in flasks simplifies handling and also shows less number of dead cells.

The few cells seeded utilise most of the sugar for growth and ferment the rest Hence, after the lapse of 24 hours, the flasks show a layer of resting cells with a very small percentage of budding ones The spent medium in the flasks is carefully poured out without disturbing the layer of yeast, and replaced with three times its volume of fresh wort and well shaken in order to ensure a uniform distribution of cells A stop watch is started at the time of changing the medium The cells begin to settle at the bottom of the flask 15 to 20 minutes after the addition of fresh medium and at the stipulated interval the wort is carefully poured out and the layer of cells left at the bottom utilised for making smears

It was found that to smear and transfer a slide to the fixing bath would take 30 seconds Thus, if the smearing of the contents of a flask is done in an orderly fashion, progressive stages separated by half-minute intervals could be obtained The contents of each flask are used for making only 10 slides For example, if the smearing is done of the contents of a flask containing a 30 minute growth, the first slide would give the picture at the 30th minute while the tenth would be that of 34 minutes and 30 seconds Thus, 10 slides each are made from the contents of flasks 30, 35, 40, 45, 50, 55 and 60 minutes after changing the medium These seventy slides give one a glimpse of the cytological changes taking place during these 35 minutes (30 to 65 minutes) at regular intervals of about 30 seconds

(*c*) *Methods of Fixation* —The yeast is removed from the flasks with a pippette, and a small quantity is placed on a slide coated lightly with Mayer's albumin It is smeared carefully to give a uniform layer, one cell thick The smear is then exposed to ammonia vapour for a few seconds and carefully transferred to troughs containing either Carnoy's alcohol acetic-chloroform mixture or Bouin's fluid If the cells show a tendency to get dislodged from the slides, which happens when the yeast emulsion is a bit thin, the slides are kept inverted over a trough of Carnoy's fluid for two minutes and then transferred to the fixatives

As a preliminary to staining by the Feulgen technique the smears without pretreatment with ammonia are fixed in osmic vapour for 30 minutes One c c of a 2% osmic acid solution is introduced into a staining trough with grooved sides intended to hold ten slides, and the smears are kept above the layer of osmic acid by placing two glass rods at the bottom It was found that to get a uniform fixation, the slides should be reversed at the end of 15 minutes.

From experiments, it appears that fixation for one hour in Carnoy's or Bouin's fluids gives the best pictures Bouin slides are washed in 70% alcohol and stored in the same, while Carnoy slides are first washed in rectified spirit and stored in 70% alcohol after being passed through 90% alcohol

Slides fixed in osmic vapour for 30 minutes are first dried and then transferred to 70% alcohol

(*d*) *Methods of Staining* —The slides are washed in water and mordanted for 24 hours in a 4% solution of iron alum They are then washed in running water for half an hour and stained in well-ripened 0 5% haematoxylin for 48 to 72 hours This long treatment with the mordant and the stain enables one to control the differentiation which is carried out with a 4% solution of iron alum Correct differentiation is a rather tricky affair and considerable experience is necessary to get beautifully stained preparations The differentiated cells are washed for 30 minutes in running water, exposed to ammonia vapour for a few seconds, taken through ascending grades of alcohol to xylol and mounted in Canada balsam Well-differentiated slides do not require any counterstaining

Staining by the Feulgen technique is comparatively easier If the smears are hydrolysed after a short wash in 70% alcohol, the staining is not very satisfactory The best preparations are obtained from slides kept for 24 hours in 70% alcohol The correct time of hydrolysis is found to be 7 minutes and the slides are kept overnight in leuco-basic fuchsin Gurr's basic fuchsin was employed throughout and the leuco basic fuchsin freshly prepared for each batch of experiments is shaken with Norite and filtered before use

The smears on removal from the fuchsin-sulphurous acid reagent are washed in three changes of SO_2 water, and then counterstained lightly with light green On comparison, light green is found to be more suitable than fast green After a wash in distilled water, the slides are quickly dehydrated and mounted in Canada balsam Even after the lapse of fifteen months the slides show no signs of fading

THE PRINCIPLES ON WHICH THE TECHNIQUE OF HANDLING YEASTS FOR CYTOLOGICAL INVESTIGATIONS WERE DEVISED

One of the stumbling blocks for advances in our knowledge of the cytology of yeasts was the non availability of an easily reproducible technique for handling yeasts for cytological investigations Kater (1940) remarks about his own previous work (1927) 'The value of the method rests upon the fact that in the presence of picric acid, the nucleus will stain before the cytoplasmic granules, but the required balance of dye and acid appears to be quite delicate so that duplication of results is difficult Although the clear cut cells on the slides were very convincing to an actual observer, it can hardly legitimately be the basis for a general acceptance of the conclusion by all workers in the field until others manage to duplicate the results '

Fixation and staining play only a minor rôle in the technique of handling yeasts for cytological investigations As pointed out by Subramaniam (1946) 'the demonstration of anaphase stages is the crucial test for any claim of mitosis in yeasts and the very fact that these could be demonstrated in material fixed and stained in the ordinary way shows that failure of the earlier workers to obtain the mitotic stages was more due to the inherent difficulties in the handling of material for cytological investigations than in the lack of availability of a suitable technique for demonstration' Thus the emphasis is shifted from the methods of fixation and staining to the handling of cultures in such a way that not only would it be possible to get at specified intervals larger percentages of cells at identical stages of division, but would also enable one to follow the course of cell division in an orderly manner

Since cells of no two yeast strains divide at identical intervals, any technique should be based on principles applying which one should be able to investigate the cytology of any strain Blind repetition of a technique without any grasp of the variable factors requiring control would, instead of clarifying issues, only enhance the confusion A clear logical presentation of the principles appears therefore necessary

In 1861 Pasteur demonstrated that in well aerated media the yeast is aerobic, completely decomposes sugar into carbon dioxide and water, and resembles other plants in its respiration as well as multiplication While during fermentation 100 gm of sugar is split up roughly into 51 parts of alcohol and 49 parts of carbon dioxide, the same quantity of sugar is used up during aerobic growth to form about 190 gm of yeast, since no alcohol is produced From a recent review (Neuberg, 1946), it appears that the biochemistry of respiration in yeasts yet awaits elucidation

Clark (1922) observed logarithmic growth for 15 hours and Richards (1928) unlimited growth, if the medium was changed once in three hours The increase in the number and volume of cells was considerably more when the spent medium was replaced with an equal volume of fresh medium than when added to the spent medium

While Clark (1922) found that the logarithmic phase ends at 15 hours, Richards (1932), in two separate studies, found it terminating at 30 and 35 hours after inoculation Such variations have been attributed to different concentrations of certain substances in the media which accelerate growth Though the age of the cells used for seeding was found by Richards to have 'no appreciable effect on the crop', he found that the quantity used for seeding 'determines the rate with which the events of the growth cycles are completed' An equilibrium in the number of cells persists for some hours after the logarithmic growth phase Thus, the variable factors like control of the quantity to be seeded and hence the time of termination of the logarithmic phase,the composition of wort, the age of material used for seeding, and temperature are possible within limits

It is true that even during the phase of equilibrium, a small percentage of cells would be budding In such a population, however, fermenting cells would be few, since the amount of sugar in the medium at this stage appears to be negligible, and because it is presumed (Richards, 1932) that it is the glycogen stored in the cells which is utilised for the second fermentation

Thus in the technique employed, the cells would be in a phase of equilibrium at the end of 24 hours and replacement of the spent wort with fresh medium induces multiplication of all the resting cells which grow at almost the same rate It was possible, therefore, to study the mitotic stages in a regular sequence from a series of preparations made at regular intervals without the necessity of trying to fit in the various stages in their proper sequence as in a jig-saw puzzle

SUMMARY

1 The need for a clear distinction between (a) the handling of cultures of yeasts for cytological investigations and (b) the methods of fixation and staining employed to demonstrate the mitotic stages is emphasised

2 A critical evaluation of some recent publications is made and it is shown that the confused state of our knowledge of the cytology of yeasts is not due to want of established criteria to identify the various cell organelles

3 Criteria for the identification of nuclei and chromosomes are restated and the question of the specificity of the Feulgen reaction is reviewed

4 Details are given of the preparation of wort and wort agar, the methods of seeding and smearing, and the methods of fixation and staining

5 A rational explanation is offered of the principles on which the technique of handling yeasts for cytological investigations were devised

6 It is pointed out that if material is handled in the manner indicated, the mitotic stages could be studied in a regular sequence without the necessity of trying to fit in the various stages in their proper sequence as in a jig saw puzzle.

ACKNOWLEDGMENTS

I am very grateful to Sir J C Ghosh, Kt , D Sc , F N I , for his active interest and encouragement and to the Council of the National Institute of Sciences for the award of an Imperial Chemical Industries Research Fellowship

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STUDIES ON THE CYTOLOGY OF YEASTS

IV ENDOPOLYPLOIDY IN YEASTS

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(Communicated by Sir J C Ghosh, Kt , D Sc , F N I)

(*Received September 20, 1947, read March 5 1948*)

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INTRODUCTION

It was realized early during our investigations on the cytology of yeasts (Subramaniam, 1946) that a clear distinction should be made between an aerobically growing culture and a fermenting one That the physiology of an aerobically growing yeast cell should be different from that of a fermenting one needs no reiteration (Ranganathan and Subramaniam, 1947) On the above basis it was suggested that division of the nucleus during the aerobic phase alone is comparable to the division of the nucleus in higher plants Further, it was emphasised that a knowledge of the nuclear behaviour during aerobic growth is an essential prerequisite for any attempt at correlation of the behaviour of the nucleus under varying physiological conditions

Logically, evaluation of results should be based on the right type of comparisons This necessitates a correct perspective, which is possible only if there is a clear appreciation of the variations in the cytological behaviour exhibited by different types of cells in response to their specialised rôle in the economy of an organism In higher animals, a clear distinction is made between the cytological behaviour of an embryonic cell from that of a glandular cell The behaviour of the nucleus in glandular cells is interpreted as changes from normal behaviour exhibited by embryonic cells in response to specialisation for particular functions, and no generalisation on nuclear behaviour in all types of cells is attempted from studies on glandular cells alone Yet, this is what has happened in yeasts Generalisations on the behaviour of the nucleus in yeasts were based on fermenting cells The fact that the yeast cell is unique and shows both oxidative and fermentative abilities was forgotten The difference between a growing cell and a fermenting one, as far as cytological investigations were concerned, was ignored Students of the histology of higher animals in particular would be aware of the marked difference in the cytological behaviour exhibited by glandular cells Taking their origin from what are called 'replacement' cells, the secretory cells may die after a single secretory cycle as in 'Holocrine' cells or may repeat the process as in 'Merocrine' cells before death supervenes (Bowen, 1929)

A fermenting yeast cell can legitimately be compared to an actively secreting glandular cell (Wager and Peniston, 1910) and until evidence is adduced to the contrary, there appears to be no reason why the cytological behaviour of a fermenting yeast cell should not be compared to that of a glandular cell When such a comparison is attempted, then some startling similarities become evident (Subramaniam, 1947*b*)

Since the mitotic cycle has already been described for the brewery strain Sc 9 (N C T C 3,007) (Subramaniam, 1946), it was thought desirable to investigate the behaviour of the nucleus during fermentation

ENDOPOLYPLOIDY

Though the earlier belief was, with a few exceptions, that the chromosome number was constant for all tissues of an organism, it has been conclusively proved in insects that this is not so and that the nuclei of the adult tissues are polyploid in varying degrees The irregularly shaped masses of chromatin, which represent the chromosomes in the resting stage, are in many cases sufficiently distinct, and since nuclei in different tissues show diploid, tetraploid and octoploid numbers, it has been surmised that these nuclei are polyploid That such a duplication in the number of chromosomes takes place by a process of endomitosis has been demonstrated in nurse cells of the ovary of *Drosophila* by Painter and Reindorp (1939) The duplication of the chromosomes takes place at periodic intervals during the early growth of the tissue and the splitting of the chromosomes is unaccompanied either by dissolution of the nuclear membrane or by the formation of asters or a spindle In the nurse cells of the ovary of *Drosophila* it has been shown that there is a progressive nucleination of the chromosomes during prophase succeeded by a splitting of these bodies and followed by denucleination until the characteristic original diffuse resting stage is reached As a result of these changes the nuclear volume becomes doubled It has recently been shown that not all nuclei need show endo-prophase, endo metaphase, endo-anaphase, and endo-telophase Witkus (1945) suggests that there are three methods by which a nucleus can become endopolyploid A double reduplication as in *Spinacia* (Witkus, 1945), or repeated duplication as in the ileum of *Culex* (Grell, 1946*a*, *b*) may take place during the resting stage According to her, only in the third type could typical endomitosis be observed

Heterochromatin and plasmosomes in resting nuclei also act as indices to the extent of polyploidy in various tissues In *Gerris lateralis* the odd chromosome in the complement of 21 is the X, which is heteropycnotic in the somatic nuclei In the diploid cells only one such heteropycnotic X occurs in the resting nucleus, and Geitler (1937) using these prominent indicators along with nuclear volume estimated that some of the giant nuclei in the salivary glands may even be 2048-ploid The same organ in different insects, and different tissues in the same insect, may show different endopolyploid constitution (White, 1945)

The remarkable advances in our knowledge of the cytogenetics of *Drosophila* dates from the rediscovery of the polytene chromosomes in the salivary gland cells (Heitz and Bauer, 1933, Painter, 1933) The salivary gland chromosomes occur only in Diptera, though they are not limited to the salivary glands alone Their structure is conceived to be similar to that of uncoiled prophase chromosomes which had repeatedly divided and in which there is no separation of the resulting strands The fact that only the haploid number occurs has led to the supposition that the products of division of similar pairs of chromosomes constitute the giant polytene salivaries The homologous threads actually fuse in some cases, while in others they remain in close apposition

The endopolyploidy in Diptera is of a special kind, for unlike in other cases there is no increase in the number of chromosomes by periodic splitting However, during the growth of the salivary gland cells, the chromosomes become thicker and thicker

Though increase in the size of the nuclei when taken along with the number of heterochromatic segments and plasmosomes may give one a fair idea of the degree of polyploidy, mere increase in size by itself offers no such evidence Different organs in the rat show different chromosomal and nuclear volumes It has been suggested (Biesele, 1944*b*) that though there appears to be some correlation between nuclear and chromosomal volumes, yet, as the changes in the rat from tissue to tissue are gradual and not discontinuous, they cannot be arranged in a polymeric series as conceived by Jacobj (1935) and others, since the nuclei of different volumes carry only the diploid number of plasmosomes, viz 6 in all the cases There appears to be a relation between the chromosome volume and total concentration of B vitamins—with the exception of inositol—since the adult liver which contains the maximum concentration in comparison with other tissues also shows the highest chromosome volume (Biesele, 1944*b*) The relative increase in the chromosome volume from the foetal to the adult liver is closely paralleled by the increase in the concentration of vitamins also, and it has been suggested that the former is responsible for the latter

It should be pointed out that, while, even doubling in volume of the nuclei of the liver cells of the rat from the foetal to the adult stage is not followed by a doubling of the plasmosome number (Biesele, 1944*a*, *b*), in mice similar changes are accompanied by a doubling of the plasmosomes indicative of polyploidy (Biesele, Poyner and Painter, 1942)

In the adult liver, the occasional metaphases seen may be in those cells which have retained their embryonic character, since mitotic divisions are common in foetal livers Is the failure of the cells of the adult liver to divide mitotically the result of their highly endopolyploid constitution? Polytene chromosomes become evident in cancer cells owing to the inherent impulse in such cells for rapid division Since partial removal of the liver results in rapid growth (Brues and Marble, 1936) and since such a phenomenon is reminiscent of the condition in malignant tumours, several investigators have been interested in the problem Thus, partial removal was expected to accelerate mitotic division and bring to light polytene chromosomes in the nuclei, if any While Beams and King (1942) thought that there was an increase in the number of polyploid nuclei in such regenerating livers as a result of fusion of mitotic figures in binucleate cells, Biesele (1944*a*) arrives at the conclusion that there is a very close agreement in the proportions of diploid, tetraploid and octoploid metaphases between the control and regenerating livers

As suggested before, if fermenting cells are compared to glandular cells, then amitosis appears merely to be a prelude to degeneration There appears to be serious possibilities that even in fermenting cultures some yeast cells may remain indifferent and divide by mitosis, while the rest of the actively fermenting cells may not again be able to revert to active vegetative reproduction.

TECHNIQUE

Under the circumstances the question arose whether fermenting cells become endopolyploid If so, it should be possible to demonstrate the same by experiments planned on the lines of Brues and Marble (1936) and Biesele (1944*a*) Just as surgical removal accelerates mitotic division in the liver, replacement of the spent wort with fresh medium in fermenting cultures produces the same effect.

Therefore, tubes of wort were inoculated with the brewery strain Sc 9, and after the lapse of five days the spent medium was poured out and replaced with the same quantity of fresh medium The contents of the tubes were centrifuged and smeared at five-minute intervals commencing from 40 minutes after the addition of fresh medium The descriptions are based on Feulgen preparations The technique of handling yeasts for cytological investigations has already been described elsewhere (Subramaniam, 1948)

OBSERVATIONS

Forty-five minutes after the addition of fresh wort to the five-day old culture, a variety of cytological pictures could be seen in Feulgen preparations The various pictures seen were first drawn and later arranged in a rational series Fig 1 is that of a cell showing no Feulgen positive structures and contains a vacuole This, however, does not appear to be the nuclear vacuole The nucleus could not be made out Two chromosomes are seen in the cell illustrated in Fig 2 and what appears to be the anaphase of the diploid in Fig 3 The absence of a vacuole in these cells is not surprising, since cells at identical stages in aerobic cultures do not have a vacuole at all A comparison of these stages with similar ones from actively dividing cultures shows that the shape and size of the chromosomes are slightly different The four chromosomes appear scattered in the cell in Fig 4, while in Fig 5 a reconstituted nucleus and a pair of chromosomes could be seen The bud and the mother cell each have two chromosomes in Fig 6 The pair in the bud appear in close apposition as a prelude probably to fusion What appears to be the next stage is illustrated in Fig 7 where in the bud the chromosomes appear to have fused together while those in the mother cell still remain discrete The above stages which occur only in very small percentages appear to be those of the 'replacement' cells It appears highly probable that it is the products of division of these cells which become endopolyploid

Since dying cells appear to be replaced from time to time, the same slide often shows an ascending series of endopolyploid constitution Fig 8 is that of a tetraploid as evidenced by the reconstituted nucleus and the four chromosomes Eight chromosomes are seen in the budding cell in Fig 9, one of which is bigger than the rest Whether the increased size is a mere abnormality or whether it is a compound chromosome is very difficult to judge In Fig 10, however, the two larger bodies ought to be compound ones if we conceive that duplication of chromosomes is by a regular division of all the chromosomes Since only six chromosomes are seen in the cell, the two larger ones should each be the product of fusion of two chromosomes Micronuclei formation is fairly common and two reconstituted nuclei and four chromosomes, one pair bigger than the other, are seen in Fig 11 Ten Feulgen positive bodies of different sizes occur in the cell in Fig 12 and it appears as if the compound chromosomes are separating into their component parts What appear in all probability to be octoploid cells are fairly common (Figs 13 and 14), if we concede that the odd numbers of Feulgen positive bodies seen are due to some of them being compound The anaphase stages of these octoploid nuclei are not regular and while in Fig 15 a reconstituted nucleus could be seen, in Fig 16 there are seven chromosomes apart from the reconstituted nucleus Apparently the chromosomes are not segregated into equal complements The cell in Fig 17 has a reconstituted nucleus and eight chromosomes, one of which is greater in size than the rest It appears likely that not all chromosomes, which are greater in size than the rest, need be compound ones Cells which are 16-ploid are common and the one shown in Fig 18 has about 19 chromosomes and a reconstituted nucleus

In tetraploids there is usually an attempt at segregation of the chromosomes into equal complements Fusion of four chromosomes may occur before (Fig 19) or after a bud begins to develop (Fig 21) and division is often normal (Fig. 20)

The picture seen in Fig 22 is slightly baffling There is a reconstituted nucleus and twelve chromosomes in the mother cell and two chromosomes in the bud Apparently it must be an octoploid All the chromosomes in the mother cell may fuse to form one or more nuclei (Fig 23) leaving only two chromosomes in the bud Is this a method by which a diploid cell can take its origin from an endopolyploid cell? Since there is 'somatic pairing' in the diploid, only an identical pair of chromosomes could have migrated to the bud There is as yet no method by which one could study the later behaviour of these buds with two chromosomes

Segregation of the chromosomes into unequal complements during the anaphase is quite general in highly endopolyploid cells (Figs 24, 25, 26, 27, 28 and 29) Micronuclei formation is fairly common in these cells (Figs 27, 29, 30 and 31) The larger complement of chromosomes usually remains in the mother cell (Figs 28 and 29) and gives rise to one (Fig 28) or two (Fig 30) large pycnotic nuclei and one or two micronuclei (Figs 29 and 30) The size of the complement of chromosomes passing on to the bud may be small (Fig 31) or fairly large (Fig 32) But the staining reactions of these cells (Figs 28, 29, 30, 31 and 32) give one the impression that they are dying

Multinucleate cells are of frequent occurrence (Figs 33, 34 and 35) and these have a single well defined vacuole (Figs 33, 34, 35, 45 and 46) The nuclei are of different sizes and should be the result of fusion of groups of chromosomes Amitotic stages are fairly common The nuclei stain brilliantly with the Feulgen stain and may or may not show (Figs 37, 38, 39, 40, 41 and 42) any chromatin grains inside The products of amitotic division may be equal (Figs 37, 41 and 42) or unequal (Figs 36 and 39) The resulting nuclei may bud smaller nuclei (Fig 38) and these usually separate and appear to move away (Fig 39) Figs 40, 43 and 44 give one the impression that the nucleus may break up into a number of pieces, one of which appears to migrate to the bud (Fig 44) This may even show deeply stained bodies (Fig 44)

It is likely that the above are highly endopolyploid cells whose nuclei are unable to resolve themselves into their component chromosomes in spite of the very favourable environmental conditions The stimulus afforded by the nutriment and the availability of dissolved oxygen leads to abortive attempts at division as evidenced by the phenomena of amitosis observed

In the final stages pycnotic nuclei may be observed to persist in the mother cells (Figs 45, 46 and 47) as well as in the buds (Fig 47)

DISCUSSION

(a) *The 'Nuclear Vacuole' of Wager and Peniston and its significance*

From the observations presented above it would be evident that with the progress of fermentation, the nucleus of the yeast cell becomes highly endopolyploid In view of the fact that some types of secretory cells show one or more vacuoles, a consideration of the significance of the 'nuclear vacuole' of Wager and Peniston (1910) is rendered necessary Guilliermond even in 1910 definitely disputed the identification on the basis that as the vacuole and its contents stain with neutral red, it can only be the secretory vacuole, since the nucleus in healthy cells never stains with vital dyes Wager and Peniston (1910) suggest a comparison of fermenting yeast cells with glandular cells in active secretion A comparison of the cytological behaviour of the fermenting yeast cell with that of the glandular cell offers no support for such an identification

The following cycle of changes were described by Wager and Peniston, basing their observations on the reactions for organic phosphorus exhibited by the yeast cell during different stages of fermentation In the early stages, when the 'nuclear vacuole' was small, the cytoplasm, the 'nucleolus' and the granules at its periphery show a reaction for phosphorus With the progress of fermentation, there is an increase in the size of the vacuole At the same time the cytoplasm loses its reaction, while, apart from the 'nucleolus' and the granules at its periphery, it becomes evident in the vacuole and the volutin granules also According to them, formation of organic phosphorus becomes evident some 14 hours after commencement of fermentation, reaches a peak at about 48 hours and is followed by a gradual fall

The changes in the volume of the 'nuclear vacuole' appears to follow a course similar to that of organic phosphorus The vacuole, which is small at the commencement of fermentation, increases in size and with the loss by the cytoplasm of its staining affinity shows a network of granules It fills almost the entire cell at the height of fermentation and decreasing slowly in size, occupies but a small space in the cell when these are slowly settling at the end of fermentation

Now, this synchronisation of the changes in the volume of the vacuole, the increase in the organic phosphorus and the stages of fermentation are all reminiscent of the usual rôle played by ribonucleic acid in active secretory synthesis It is true that in many gland cells the nuclei may become endopolyploid But the increase in volume in such cases has been shown to be rhythmic and discontinuous and never gradual, and a highly endopolyploid nucleus usually never reverts back to its original condition at the end of the secretory cycle Hence, even the possibility that the increase in size of the 'nuclear vacuole' may be due to endopolyploidy cannot be substantiated on the basis of evidence available in published literature

The 'nuclear vacuoles' described by Wager and Peniston (1910) and Janssens and Leblanc (1898) do not appear to be homologous On the basis of the recent careful investigations of Subramaniam (1946) it appears that the description of a nuclear vacuole with a nucleolus by Janssens (1902) is likely to refer to a prophase stage, when the chromatin mass inside the nucleus is capable of being confused with a nucleolus (see pictographic summary, Subramaniam, 1946)

(b) *Fermenting Cells and Glandular Cells*

Fermenting cultures contain very small percentages of cells showing regular mitotic phases, similar in essentials to those seen during the aerobic division, the significance of which has been lost sight of by previous investigators Richards' work (1932), though bearing on an entirely different problem, offers indirect evidence for the suggestion that as in glands, in fermenting cultures also there may be 'replacement' or embryonic cells which continue to divide normally, and that fermenting cells like other gland cells never regain their power of normal vegetative division, but that death and disintegration occur sooner or later He found that at the end of the logarithmic growth phase there was an increase in the percentage of dead cells as indicated by methylene blue staining His suggestion (1928) that the concentration of alcohol may be the inhibiting factor does not appear very convincing, for he found no such correlation during the second cycle of growth He says 'During the period of increased rate of growth, the production of alcohol also increases As there is no measurable amount of sugar in the medium this second period of increased fermentation probably comes from glycogen stored within the cells The greater amount of alcohol must increase the rate of killing of cells, *although there is no such direct correlation* between the two factors as was found for the first cycle of growth ' Judged on the basis of the cytology of glandular cells, increased production of alcohol means increased number of cells fermenting and hence the occurrence of increased percentages of killed cells

Proceeding on the above lines it is difficult to conceive that dying cells are merely the larger buds which had not become resistant to the injurious effects of the alcohol and other by-products of fermentation

It was observed that when fresh wort was added to a five-day-old culture, the stimulus afforded by the nutriment and the availability of dissolved oxygen leads to abortive attempts at division even by the highly endopolyploid cells In fact, in many cases the bud formation is completed, only the buds do not get detached from the mother Having an abnormal complement of chromosomes, it was also observed that both the products of division show pycnosis and hence eventual death It should be realised that even in fermenting cultures the nutriment available fluctuates from time to time Every batch of fermenting cells, when they die and disintegrate, should temporarily increase the available nutriment in the medium thus affording stimulus to the endopolyploid cells to bud But since there is no increase in the availability of dissolved oxygen, the budding is not of normal embryonic diploid cells It is perhaps owing to this reason that one observes an increased percentage of larger buds dying in later stages of fermentation

That even old cultures contain actively dividing cells, comparable to embryonic cells of glandular tissues, would be evident from Slator's (1919) observation that uncontaminated wort cultures show active cells even after several years Proceeding further, it would be admitted that in view of the above consideration, the 'durable' cells occurring after the lapse of 900 to 1,000 hours are not transformed fermenting cells but should have an entirely different origin That they may take their origin from the few actively dividing cells occurring during this period appears likely Richards states 'Budding continues throughout the period, becoming materially diminished only after most of the cells have changed into the resistant form The numbers and percentages of budding and of stained cells fluctuate in irregular cycles of small magnitude, but there is no general cycle other than the gradual change of the population into resting cells with a resulting decreased birth ,and death rate ' (Page 289)

At first sight it may appear that yeast is unique in that it becomes endopolyploid during fermentation Yet, this does not appear to be so In Ciliates, while the micronucleus is considered to be generative, the macronucleus has been supposed to subserve a purely physiological function In the same cell, therefore, different functions are controlled by different nuclei Though the structure and behaviour of the macronucleus has attracted considerable attention, protozoologists do not seem to have cared to consider whether it is endopolyploid From an analysis of the observations on the behaviour of the macronucleus recorded by other workers, Subramaniam (1947*c*) has suggested that it is in all probability endopolyploid and has offered a rational explanation as to why there is a need for endomixis Endopolyploidy in the yeast, therefore, does not appear to be an exception to the general rule among unicellular organisms

The usual fate of endopolyploid cells being death and disintegration after varying periods of activity, and since in the final stages of fermentation, excepting for a negligible percentage, most of the cells should be endopolyploid, the necessity for the rejuvenation of cultures after each fermentation would at once become apparent This may explain why continuous fermentation without rejuvenation is almost a virtual impossibility

SUMMARY

1 The physiology of an aerobically growing cell should be different from that of a fermenting one and hence a comparison of the cytological behaviour of fermenting cells with that of glandular cells is attempted

2 Since secretory cells usually become endopolyploid, a brief review of the literature on endopolyploidy is presented

3 Just as surgical removal of a part accelerates mitotic division in the liver, replacement of spent wort with fresh medium in fermenting cultures produces the same effect

4 Various cytological pictures are seen in preparations of five day old cultures in which the spent medium was replaced with fresh wort There are rare clusters of cells showing the typical stages seen in the aerobic phase The majority show varying degrees of endopolyploidy The segregation of chromosomes during anaphase is mostly irregular, and highly polyploid cells show amitosis like phenomena

5 A careful analysis of Wager and Peniston's observations, in the light of recent advances, indicates that their 'nuclear vacuole' is nothing but a secretory vacuole,

6 It is suggested that as in glands, in fermenting cultures also there may be 'replacement' or embryonic cells which continue to divide normally and that fermenting cells, like other glandular cells, never regain their power of normal vegetative division but that death and disintegration occur sooner or later

7 Endopolyploidy in the yeast does not appear to be an exception to the general rule among unicellular organisms since the macronucleus of Ciliates appears to be endopolyploid

8 The usual fate of endopolyploid cells being death and disintegration, may explain why continuous fermentation without rejuvenation is almost a virtual impossibility

ACKNOWLEDGMENTS

I am very grateful to Sir J C Ghosh, Kt , D Sc , F N I , for his active interest and encouragement and to the Council of the National Institute of Sciences for the award of an Imperial Chemical Industries Research Fellowship

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DESCRIPTION OF ILLUSTRATIONS

FIG 1 Cell showing a vacuole but no Feulgen positive structures
FIG 2 Metaphase of Diploid
FIG 3 Anaphase of Diploid
FIG 4 Late Anaphase
FIG 5 Cell showing a reconstituted nucleus and two chromosomes
FIG 6 Budding cell showing a pair of chromosomes in the mother and bud
FIG 7 Reconstituted nucleus in the bud and a pair of chromosomes in the mother cell
FIG 8 Tetraploid cell showing a reconstituted nucleus and four free chromosomes
FIG 9 Budding cell showing eight Feulgen positive bodies
FIG 10 Cell showing six Feulgen positive bodies, two of which are greater in size
FIG 11 A cell having two reconstituted nuclei, a pair of big and a pair of small Feulgen positive bodies
FIG 12 Feulgen positive bodies of differing sizes which seem to be breaking up into their component parts
FIGS 13 and 14 Cells showing 11 and 14 Feulgen positive bodies, some of which ought to be compound chromosomes
FIGS 15, 16, 17, 18 and 19 A reconstituted nucleus and differing numbers of chromosomes of varying sizes lying free in the cytoplasm
FIGS 20 and 21 Normal division of a tetraploid
FIGS 22, 23, 24, 25, 26, 27 and 28 Irregular segregation of chromosomes in cells of varying endopolyploid constitutions
FIGS 29 and 30 Formation of micronuclei
FIGS 31 and 32 Irregular segregation of chromosomes in highly endopolyploid cells
FIGS 33, 34 and 35 Multinuclear cells
FIGS 36, 37, 38, 39, 40, 41 and 42 Varying types of amitotic division
FIG 43 Breaking up of highly endopolyploid nuclei into smaller ones
FIG 44 Irregular distribution of nuclei between the mother and bud
FIGS 45, 46 and 47 Pycnotic nuclei persisting in cells with vacuoles

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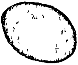
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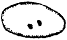
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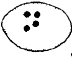
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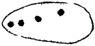
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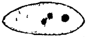
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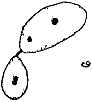
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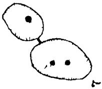
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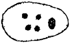
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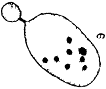
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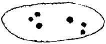
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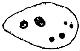
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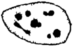
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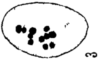
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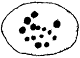
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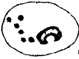
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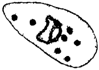
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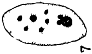
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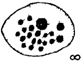
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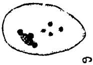
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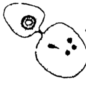
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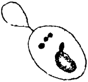
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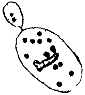
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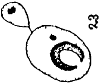
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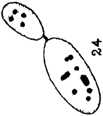
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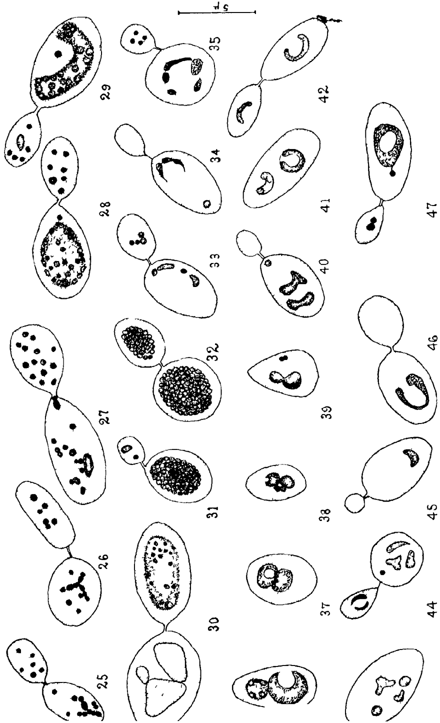
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ON $N_q(r)$ IN THE TARRY-ESCOTT PROBLEM

By HANSRAJ GUPTA, *Government College, Hoshiarpur*

(Communicated by Dr D S Kothari, F N I)

(*Received May 13, read August 8, 1948*)

The generalised[1] Tarry-Escott problem of degree r and order q is that of finding q different sets $A_1, A_2, A_3, \quad , A_q$ of s integers each—different in the sense that the members of one set are not merely permutations of those of another set—such that

$$(1) \qquad \sigma_k(A_1) = \sigma_k(A_2) = \sigma_k(A_3) = \quad = \sigma_k(A_q), \quad k = 1, 2, 3, \quad , r,$$

where $\sigma_k(A_m)$ denotes the sum of the kth powers of the members of A_m

The least value of s, for which sets A satisfying (1) exist, is denoted by $N_q(r)$

The object of this note is to show that

$$(2) \qquad N_q(r) < \frac{r(r+1)}{2} + 1$$

Write s for $\quad \dfrac{r(r+1)}{2} + 1$

Consider all the different sets A_m of s non-zero positive integers

$$(3) \qquad a_{m\ 1}, a_{m\ 2}, a_{m\ 3}, \quad , a_{m\ s}$$

whose sum is n The number of such sets is not less than[2]

$$\frac{1}{s!}\binom{n-1}{s-1},$$

because the sets A_m provide all the partitions of n into exactly s non-zero summands

Since

$$\sum_{t=1}^{s} a_{m\ t}^{k} < \left\{ \sum_{t=1}^{s} a_{m,t} \right\}^{k}, \quad k > 2,$$

we have

$$\sigma_k(A_m) \leq n^k, \quad k > 2$$

Hence, there are at most

$$\prod_{k=2}^{r} n^k = n^{2+3+4+ \quad +r} = n^{s-2}$$

different sequences

$$(4) \qquad \sigma_1(A_m), \sigma_2(A_m), \sigma_3(A_m), \quad , \sigma_r(A_m)$$

For a sufficiently large $n > n_0$,

$$\frac{1}{s!}\binom{n-1}{s-1} > (q-1)n^{s-2}.$$

Hence, there are at least q sets

$$A_{m_1}, A_{m_2}, A_{m_3}, \quad , A_{m_q}$$

which yield the same sequence (4) and the result follows readily

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PROCEEDINGS
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SEPTEMBER-OCTOBER, 1948

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SOME NON-RAMANUJAN CONGRUENCE PROPERTIES OF THE PARTITION FUNCTION

By D B LAHIRI, *Indian Statistical Institute, Calcutta*

(Communicated by Mr S N Roy, M Sc, F N I)

(*Received December 18, 1947, read January 1, 1948*)

While speaking about the arithmetical properties of the partition function, Hardy (1940) in his stimulating lectures on Ramanujan states

> 'Ramanujan was the first, and up to now the only, mathematician to discover any such properties,'

It would be highly interesting therefore for any one to come across any new properties I have found three new ones, namely,

$$p(49m+k) \equiv 0, \quad (\text{mod } 49), \quad k = 19,\ 33,\ 40$$

The case $k = 47$ was discovered by Ramanujan

What is remarkable about these three new congruences is that their possible existence, not to speak about their actual establishment, eluded us for such a long time, even though their genesis is to be found in one of the remarkable identities of Ramanujan This identity is

$$\sum_{0}^{\infty} p(7m+5)x^m = 7\,\frac{\{f(x^7)\}^3}{\{f(x)\}^4} + 49x\,\frac{\{f(x^7)\}^7}{\{f(x)\}^8},$$

where $\quad f(x) = \prod_{1}^{\infty}(1-x^m)$

Now,

$$\frac{\{f(x^7)\}^3}{\{f(x)\}^4} = \frac{\{f(x^7)\}^3}{\{f(x)\}^7}\ \{f(x)\}^3,$$

$$\equiv \{f(x^7)\}^2 \sum_{0}^{\infty}(-1)^n(2n+1)x^{\frac{1}{2}n(n+1)}, \quad (\text{mod } 7),$$

by making use of Jacobi's formula and the simple fact

$$\frac{1}{\{f(x)\}^7} \equiv \frac{1}{f(x^7)}, \quad (\text{mod } 7)$$

It follows therefore from Ramanujan's identity that

$$\sum_{0}^{\infty} p(7m+5)x^m \equiv 7\{f(x^7)\}^2 \sum_{0}^{\infty}(-1)^n(2n+1)x^{\frac{1}{2}n(n+1)}, \quad (\text{mod } 49)$$

Now, it is not difficult to see that powers of the form x^{7m+i}, $i = 2, 4, 5$ do not occur in

$$\sum_{0}^{\infty} (-1)^n (2n+1) x^{\frac{1}{2}n(n+1)}$$

This implies that powers of the same forms do not occur in the product

$$7\{f(x^7)\}^2 \sum_{0}^{\infty} (-1)^n (2n+1) x^{\frac{1}{2}n(n+1)}$$

This in turn finally leads us to the fact that the coefficients of powers of the same forms in

$$\sum_{0}^{\infty} p(7m+5) x^m$$

are divisible by 49 Thus

$$p(49m+k) \equiv 0, \quad (\text{mod } 49), \quad k = 19, 33, 40$$

Ramanujan's congruence, $p(49m+47) \equiv 0$, (mod 49), is also an immediate consequence of the fact that although powers of the form x^{7m+6} do occur in

$$\sum_{0}^{\infty} (-1)^n (2n+1) x^{\frac{1}{2}n(n+1)},$$

yet in every case the coefficient is a multiple of 7

An examination of a table of partitions shows that such congruences of non-Ramanujan type do not exist in respect of the moduli 25 and 121

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THE ANALOGUE OF BLASIUS' FORMULA IN SUBSONIC COMPRESSIBLE FLOW

By V R THIRUVENKATACHAR (*N I S Junior Research Fellow*), *Department of Mathematics, Central College, Bangalore*

(Communicated by Dr D S Kothari.)

(*Received April 3, read April 20, 1948*)

1 *Introduction* The object of this note is to derive a formula for the force in subsonic compressible flow, which is the analogue of the well-known Blasius formulae in the incompressible case The derivation is carried out on the basis of the hodograph method as recently developed by C C Lin[1] It is also shown that the familiar Prandtl Glauert rule[2] is derivable from the formula

2 *The equations of the hodograph method* In the hodograph method, the (p, ρ) relation is approximated by

$$p = A - \frac{B}{\rho} \tag{1}$$

If c is the local velocity of sound,

$$c^2 = dp/d\rho = B/\rho^2 = c_0^2\rho_0^2/\rho^2, \tag{2}$$

where c_0, ρ_0 are the values of c, ρ at the stagnation point of the flow The Bernoulli equation then gives

$$c^2 - c_0^2 = q^2 \tag{3}$$

$$c/c_0 = \rho_0/\rho = \sqrt{1+q^2/c_0^2} \tag{4}$$

Since an additive constant in p is immaterial, we choose $A = c_0^2\rho_0$, giving

$$p = -c_0^2\rho_0\left(\frac{\rho_0}{\rho}-1\right) = -c_0^2\rho_0\{\sqrt{1+q^2/c_0^2}-1\} \tag{5}$$

When $c_0 \to \infty$, we see from (3) and (4) that

$$c \to \infty,\ c/c_0 \to 1,\ \rho/\rho_0 \to 1,$$

i e $\rho = \rho_0$, so that the fluid is incompressible Also (5) gives in this limiting case

$$p = -\tfrac{1}{2}\rho_0 q^2$$

which is the usual equation for the pressure in the incompressible flow According to Lin, the compressible flow around a closed profile is constructed as follows Given the incompressible flow past a profile P_0 in the ζ-plane described by the complex potential $F(\zeta)$ and the complex velocity $w_0(\zeta) = -\dfrac{dF(\zeta)}{d\zeta}$ the compressible flow past a profile P in the z-plane is represented parametrically by the set of equations:

$$\text{Complex potential} = W = \phi + i\psi = F(\zeta) \tag{6}$$

$$w_0(\zeta) = -\frac{dF(\zeta)}{d\zeta} \tag{7}$$

$$\frac{2qe^{-i\theta}}{1+\sqrt{1+q^2/c_0^2}} = \frac{w_0(\zeta)}{k(\zeta)} \tag{8}$$

$$dz = k(\zeta)d\zeta - \frac{1}{4c_0^2}\overline{\frac{w_0^2(\zeta)}{k(\zeta)}}\,d\zeta \tag{9}$$

Here $qe^{-i\theta}$ is the complex velocity in the compressible flow and $k(\zeta)$ is to be chosen so that it is regular in the region R_0 outside P_0, has no root in R_0 and is such that

$$\left|\frac{1}{2c_0}w_0(\zeta)\right| < \left|k(\zeta)\right| < \infty \text{ on } P_0 \tag{10}$$

$$\oint k(\zeta)d\zeta = \frac{1}{4c_0^2}\overline{\oint \frac{w_0^2(\zeta)}{k(\zeta)}\,d\zeta}, \tag{11}$$

the integration being round any contour enclosing P_0

3 *The analogue of Blasius' formulae* The force and the moment in two-dimensional flow are given by [3]

$$\bar{\mathfrak{F}} = X - iY = i\oint pe^{-2i\theta}dz \tag{12}$$

$$M = \text{real part of } \oint pe^{-2i\theta}z\,dz = Re\oint pe^{-2i\theta}z\,dz \tag{13}$$

Since W is the complex potential in the compressible flow,

$$qe^{-i\theta} = -\frac{dW}{dz} = -\frac{dW}{d\zeta}\frac{d\zeta}{dz} = -\frac{dF(\zeta)}{d\zeta}\frac{d\zeta}{dz} = w_0(\zeta)\frac{d\zeta}{dz} \tag{14}$$

Substituting in (8) we get

$$1+\sqrt{1+q^2/c_0^2} = 2k(\zeta)\frac{d\zeta}{dz}$$

whence

$$\sqrt{1+q^2/c_0^2}-1 = 2\left(k\frac{d\zeta}{dz}-1\right) \tag{15}$$

and

$$q^2/c_0^2 = 4k\frac{d\zeta}{dz}\left(k\frac{d\zeta}{dz}-1\right) \tag{16}$$

From (5) and (15) we have

$$p = -2c_0^2\rho_0\left(k\frac{d\zeta}{dz}-1\right) \tag{17}$$

Using (14) and (17) in (12) we get

$$\bar{\mathfrak{F}} = \tfrac{1}{2}i\rho_0\oint\frac{w_0^2(\zeta)}{k(\zeta)}\,d\zeta \tag{18}$$

Similarly

$$M = Re\left\{-\tfrac{1}{2}\rho_0\oint\frac{w_0^2(\zeta)}{k(\zeta)}\,z\,d\zeta\right\}, \tag{19}$$

where

$$z = \int k(\zeta)d\zeta - \frac{1}{4c_0^2}\overline{\int\frac{w_0^2(\zeta)}{k(\zeta)}\,d\zeta}$$

The equations (18) and (19) are the analogues in compressible flow of the Blasius formulae for the incompressible case On account of the condition (11) we may also write

$$\mathfrak{F} = 2ic_0^2\rho_0 \overline{\oint k(\zeta)d\zeta}$$

or

$$\mathfrak{F} = -2\,c_0^2\rho_0 \oint k(\zeta)d\zeta \tag{20}$$

4 *Application to airfoil in uniform stream Prandtl Glauert formula* For a simple air wing in a uniform stream we take[4]

$$u_0(\zeta) = A + \frac{B}{\zeta} + \frac{C}{\zeta^2} + \tag{21}$$

where

$$A = -Ue^{i\alpha} \quad B = \Gamma/2\pi i$$

Assume

$$k(\zeta) = 1 + \frac{k_1}{\zeta} + \qquad k_1 = k_{11} + ik_{12} \tag{22}$$

Then

$$\frac{u_0^2(\zeta)}{k(\zeta)} = A^2 + (2AB - k_1A^2)/\zeta + O(\zeta^{-2})$$

The condition (11) gives

$$4c_0^2(k_{11} - ik_{12}) = -\{(i\Gamma U)/\pi\}e^{i\alpha} + (k_{11} + k_{12})U^2e^{2i\alpha}$$

Separating real and imaginary parts and solving for k_{11} k_{12} we find

$$k_{11} = -\frac{\Gamma U \sin\alpha}{\pi(U^2 + 4c_0^2)}$$

$$k_{12} = \frac{\Gamma U \cos\alpha}{\pi(U^2 + 4c_0^2)}$$

so that

$$k(\zeta) = 1 + \frac{i\Gamma U}{\pi(U^2 + 4c_0^2)} \frac{e^{i\alpha}}{\zeta} + O(\zeta^{-2}) \tag{23}$$

Then by (20)

$$\mathfrak{F} = \frac{4c_0^2\rho_0\Gamma U}{U^2 + 4c_0^2} ie^{i\alpha} \tag{24}$$

Hence the lift force L is given by

$$L = |\mathfrak{F}| = \frac{4c_0^2\rho_0\Gamma U}{U^2 + 4c_0^2} \tag{25}$$

From (8) we have for $|\zeta| \to \infty$

$$\frac{2q_\infty}{1 + \sqrt{1 + q_\infty^2/c_0^2}} = U$$

or, from (3)

$$q_\infty/(c_0 + c_\infty) = U/2c_0 \tag{26}$$

If $M = q_\infty/c_\infty$ is the free-stream Mach number of the compressible flow and if we set $M = \sin\beta$, then from (3) $c_0 = c_\infty \cos\beta$ The equation (26) then gives

$$\frac{U}{2c_0} = \frac{\sin\beta}{1+\cos\beta} = \tan\frac{\beta}{2}, \tag{27}$$

whence

$$4c_0^2/(U^2+4c_0^2) = \cos^2(\beta/2)$$

Substituting in (25) we find

$$\begin{aligned} L &= \rho_0 \Gamma U \cos^2(\beta/2) \\ &= \rho_0 c_0 \Gamma \sin\beta \quad \text{[using 27]} \\ &= \rho_\infty c_\infty \, \Gamma \, (q_\infty/c_\infty) \quad \text{[by (4)]} \end{aligned}$$

i e

$$L = \rho_\infty \Gamma q_\infty . \tag{28}$$

Also from (6) we have

$$\oint_P \frac{dW}{dz}\,dz = \oint_{P_0} \frac{dF}{d\zeta}\,d\zeta,$$

which shows that the circulation in the physical plane of the compressible flow is the same as that in the hodograph plane, i e Γ Thus writing $V = q_\infty$ for the velocity at infinity in the compressible flow, we obtain

$$L = \rho_\infty \Gamma V, \tag{29}$$

where Γ is the circulation in the compressible flow

But it is known[5] that the circulation in the compressible flow is connected with the corresponding value in the incompressible case by the relation

$$\Gamma = \Gamma_0/\surd(1-M^2)$$

Hence

$$L = \frac{\rho_\infty \Gamma_0 V}{\surd(1-M^2)} \tag{30}$$

which is the Prandtl-Glauert formula

I am grateful to Prof B S Madhava Rao for the interest he has taken in this work and for helpful remarks My thanks are also due to the National Institute of Sciences of India for the award of a Research Fellowship under which this work was carried out

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THE TEMPERATURE DEPENDENCE OF PARAMAGNETIC SUSCEPTIBILITY OF A RELATIVISTIC ELECTRON GAS

By K. S. Singwi,* *Delhi University, Delhi*

(Communicated by Dr. D. S. Kothari, F.N.I.)

(*Received February 21, read March 5, 1948.*)

Abstract

The expressions for the temperature dependence of the paramagnetic susceptibility of a relativistic electron gas are derived both for degeneracy and for non-degeneracy. The non-relativistic expressions, for both cases, are also given for the sake of completeness.

§1. Introduction

The paramagnetic susceptibility of a degenerate electron gas was first deduced by Pauli (1927) from energy considerations, using Fermi-Dirac statistics. For low temperatures he obtained the result which may be put in the form

$$\chi = \tfrac{3}{2}B^2 n/\xi_0, \tag{1}$$

where n is the electron concentration, ξ_0 the maximum electron energy in the completely degenerate state, and B the Bohr magneton. Bloch (1929) gave, as a higher approximation at low temperatures, the following expression

$$\chi = \tfrac{3}{2}B^2 \frac{n}{\xi_0}\left[1-\frac{\pi^2}{12}\left(\frac{kT}{\xi_0}\right)^2\right] \tag{2}$$

The problem of the temperature variation of free electron susceptibility, both at low and at high temperatures, has been re-examined by Stoner (1935). All these authors have considered a non-relativistic electron gas.

Recently the present author (1948), using the theory of perturbation, has given an expression for the paramagnetic susceptibility of a degenerate electron gas. The result is

$$\chi = \frac{e^2}{2\pi ch}\log\left(x+\sqrt{1+x^2}\right), \tag{3}$$

where the non-dimensional parameter x is given by

$$x = \frac{h}{mc}\left(\frac{3n}{8\pi}\right)^{\frac{1}{3}}, \tag{4}$$

n being the electron concentration and the other symbols have their usual meaning. In the non-relativistic case, i.e. $x \ll 1$, (3) reduces to the Pauli expression (1), whereas in the relativistic case, i.e. $x \gg 1$, (3) reduces to

$$\chi = \frac{e^2}{2\pi ch}\log 2x \tag{5}$$

---

* Research Fellow of the National Institute of Sciences, India.

In the completely degenerate case the maximum energy ξ_0 of the electron is related to x by

$$\xi_0 = mc^2\{(1+x^2)^{\frac{1}{2}}-1\}\,, \tag{6}$$

which reduces to

$$\xi_0 = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{\frac{2}{3}}, \tag{7}$$

in the non relativistic case, and to

$$\xi_0 = ch\left(\frac{3n}{8\pi}\right)^{\frac{1}{3}}, \tag{8}$$

in the relativistic case

So far we have not considered, in the relativistic case, the effect of temperature on the paramagnetic susceptibility The aim of the present paper is to derive expressions for the temperature variation, both at high and at low temperatures, of the paramagnetic susceptibility of a relativistic electron gas The high temperatures and densities necessary for the application of the relativistic formulae do not exist in the terrestrial laboratories However, the formulae may find some applications in astrophysics Moreover, for the sake of completeness it is worth while to derive them

§2 PARAMAGNETIC SUSCEPTIBILITY AT LOW TEMPERATURES

The paramagnetic susceptibility of a relativistic electron gas, as given by equation (28) of our previous paper (1948), when account is taken of temperature, becomes

$$\chi = \frac{e^2}{4}\frac{4\pi g}{(2\pi)^3}\int_0^\infty \frac{dk_0}{(k_0^2c^2\hbar^2+m^2c^4)^{\frac{1}{2}}}\,\frac{1}{1+e^{\epsilon-\xi/kT}}\,, \tag{9}$$

where $k_0\hbar$ is the momentum of the electron, ϵ its kinetic energy, g its statistical weight, and ξ is Gibb's free energy per particle Since

$$k_0^2\hbar^2c^2 = \epsilon^2+2\epsilon mc^2,$$

(9) reduces to

$$\begin{aligned}\chi &= \frac{e^2}{4c\hbar}\frac{4\pi g}{(2\pi)^3}\int_0^\infty \frac{(\epsilon+2\epsilon mc^2)^{-\frac{1}{2}}}{1+e^{\epsilon-\xi/kT}}\,d\epsilon\,,\\ &= \frac{e^2}{4c\hbar}\frac{4\pi g}{(2\pi)^3}\int_0^\infty \frac{d}{d\epsilon}\phi(\epsilon)\,\frac{d\epsilon}{1+e^{\epsilon-\xi/kT}}\,, \end{aligned} \tag{10}$$

where

$$\phi(\epsilon) = \log\left[(\epsilon+mc^2)+\{(\epsilon+mc^2)^2-mc^2\}^{\frac{1}{2}}\right]$$

An asymptotic series expansion of the integral in (10), where $\phi(\epsilon)$ is sufficiently regular and vanishes for $\epsilon = 0$, was first given by Sommerfeld for the case $\dfrac{\xi}{kT} \gg 1$, i e the degenerate case, subject to an error of the order of $e^{-\xi/kT}$ In the present case $\phi(0) = \log mc^2$ Applying Sommerfeld lemma we have

$$\int^\infty \frac{d}{d\epsilon}\phi(\epsilon)\,\frac{1}{1+e^{\epsilon-\xi/kT}}\,d\epsilon = \phi(\xi)-\phi(0)+\{2c_2(kT)^2\phi''(\epsilon)+\quad\}_{\epsilon=\xi}\,,$$

where $c_{2n} = (1-2^{1-2n})\zeta(2n)$, and $\zeta(2n)$ being the Riemann-Zeta function.

(10) now becomes

$$\chi = \frac{e^2}{4c\hbar}\frac{4\pi g}{(2\pi)^3}\left[\log\frac{(\xi+mc^2)+\sqrt{(\xi+mc^2)^2-(mc^2)^2}}{mc^2} - 2c_2(kT)^2(\xi+mc^2)(\xi^2+2\xi mc^2)^{\frac{3}{2}}\right], \tag{11}$$

where we retain terms up to $(kT)^2$

We shall now consider two different cases

Case 1 Non-relativistic degenerate, i e

$$\xi/mc^2 \ll 1,$$

and

$$\frac{\xi_0}{kT} \gg 1$$

To the order of approximation we desire (11) becomes

$$\chi = \frac{\rho^2}{4c\hbar}\frac{4\pi g}{(2\pi)^3}\left(\frac{2\xi}{mc^2}\right)^{\frac{1}{2}}\left\{1-c_2\left(\frac{kT}{\xi}\right)^2\frac{1}{2}\right\} \tag{12}$$

The series expansion of ξ in terms of x, as given by Kothari and Singh (1942), is

$$\xi = \xi_0\left\{1-\frac{\pi^2}{6}\left(\frac{kT}{\xi_0}\right)^2\frac{(1+2x^2)\{(1+x^2)^{\frac{1}{2}}-1\}}{x^2(1+x^2)^{\frac{1}{2}}} + \text{terms containing powers of } \left(\frac{kT}{\xi_0}\right)^2\right\} \tag{13}$$

Since $x \ll 1$, we have

$$\xi = \xi_0\left\{1-\frac{\pi^2}{12}\left(\frac{kT}{\xi_0}\right)^2\right\} \tag{14}$$

where ξ_0 is given by (7)

Substituting (14) in (12) and retaining terms up to $\left(\frac{kT}{\xi_0}\right)^2$ we have

$$\chi = \tfrac{3}{2}B^2 n\frac{1}{\xi_0}\left\{1-\frac{\pi^2}{12}\left(\frac{kT}{\xi_0}\right)^2\right\} \tag{15}$$

Case 2 Relativistic degenerate, i e

$$mc^2/\xi \ll 1,$$

and

$$\xi_0/kT \gg 1$$

To the order of approximation we desire (11) now becomes

$$\chi = \frac{e^2}{4c\hbar}\frac{4\pi g}{(2\pi)^3}\left\{\log\frac{2\xi}{mc^2} - 2c_2\left(\frac{kT}{\xi}\right)^2\right\} \tag{16}$$

Since $x \gg 1$, (13) becomes

$$\xi = \xi_0\left\{1-\frac{\pi^2}{3}\left(\frac{kT}{\xi_0}\right)^2\right\}, \tag{17}$$

where ξ_0 is given by (8).

Substituting for ξ in (16) from (17) and retaining terms up to $\left(\frac{kT}{\xi_0}\right)^2$ we have

$$\chi = \frac{e^2}{2\pi c\hbar}\left\{\log\frac{2\xi_0}{mc^2} - \frac{\pi^2}{2}\left(\frac{kT}{\xi_0}\right)^2\right\}, \tag{18}$$

which for low temperatures reduces to (5)

§3 PARAMAGNETIC SUSCEPTIBILITY AT HIGH TEMPERATURES

The classical case is characterised by $\frac{kT}{\xi_0} \gg 1$. We shall distinguish here two different cases

Case 1 Non-relativistic non degenerate, i e

$$\epsilon/mc^2 \ll 1,$$

and

$$\frac{\xi_0}{kT} \ll 1$$

The general expression (9) becomes

$$\begin{aligned}\chi &= \frac{e^2}{4c\hbar}\frac{4\pi g}{(2\pi)^3}\int_0^\infty (2\epsilon mc^2)^{-\frac{1}{2}}\frac{d\epsilon}{1+e^{\epsilon-\xi/kT}},\\ &= \frac{e^2}{4c\hbar}\frac{4\pi g}{(2\pi)^3}(2mc^2)^{-\frac{1}{2}}\sum_{n=1}^{\infty}(-1)^{n+1}e^{n\xi/kT}\int_0^\infty \epsilon^{-\frac{1}{2}}e^{-n\epsilon/kT}\,d\epsilon,\\ &= \frac{e^2}{4c\hbar}\frac{4\pi g}{(2\pi)^3}\left(\frac{kT}{2mc^2}\right)^{\frac{1}{2}}\sum_{n=1}^{\infty}(-1)^{n+1}e^{n\xi/kT}\frac{\Gamma(\frac{1}{2})}{n^{\frac{1}{2}}},\\ &= \frac{e^2}{4c\hbar}\frac{4\pi g}{(2\pi)^3}\left(\frac{\pi kT}{2mc^2}\right)^{\frac{1}{2}}e^{\xi/kT}\left(1-\frac{e^{\xi/kT}}{2^{\frac{1}{2}}}+\frac{e^{2\xi/kT}}{3^{\frac{1}{2}}}\qquad\right)\end{aligned} \tag{19}$$

Kothari and Singh (1942) have, for the non degenerate case, given the following expression for ξ

$$\xi = kT\,[\log A_0 + 2b_2A_0 + \text{terms containing higher powers of } A_0], \tag{20}$$

where, for the non-relativistic case,

$$A_0 = \frac{4}{3\sqrt{\pi}}\left(\frac{\xi_0}{kT}\right)^{\frac{3}{2}},\quad \xi_0 = \frac{mc^2x^2}{2},$$

and, for the relativistic case,

$$A_0 = \frac{1}{6}\left(\frac{\xi_0}{kT}\right)^3,\quad \xi_0 = mc^2x$$

The coefficient b_2 is

$$b_2 = \frac{1}{2^{\frac{5}{2}}} \quad \text{for non-relativistic case,}$$

and

$$b_2 = \frac{1}{2^4} \quad \text{for relativistic case.}$$

From (20) we have, for the case under consideration,

$$e^{\xi/kT} = \frac{4}{3\sqrt{\pi}}\left(\frac{\xi_0}{kT}\right)^{\frac{3}{2}} \left\{1+\frac{e^{\xi/kT}}{2^{\frac{3}{2}}}+\text{higher powers of } e^{\xi/kT}\right\} \tag{21}$$

From (19) and (21), and retaining terms containing $(kT)^2$ we have

$$\chi = \frac{e^2}{2\pi ch}\,\frac{4}{3}\,\frac{1}{(2mc^2)^{\frac{1}{2}}}\,\frac{\xi_0}{kT}\,\xi_0^{\frac{1}{2}}\left\{1-\frac{4}{3\sqrt{\pi}}\,\frac{1}{2^{\frac{3}{2}}}\left(\frac{\xi_0}{kT}\right)^{\frac{3}{2}}\right\}$$

Substituting for ξ_0 from (7) we have

$$\chi = B^2\,\frac{n}{kT}\left\{1-\frac{1}{3}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(\frac{\xi_0}{kT}\right)^{\frac{3}{2}}\right\} \tag{22}$$

Case 2 Relativistic non-degenerate, i e

$$\epsilon/mc^2 \gg 1,$$

and

$$\frac{kT}{\xi_0} \gg 1$$

The general expression (9) now becomes

$$\chi = \frac{e^2}{4c\hbar}\,\frac{4\pi g}{(2\pi)^3}\int_0^\infty \frac{1}{(\epsilon+mc^2)}\,\frac{d\epsilon}{1+e^{\epsilon-\xi/kT}},$$

$$= \frac{e^2}{4c\hbar}\,\frac{4\pi g}{(2\pi)^3}\sum_{n=1}^{\infty}(-1)^{n+1}\,e^{n\xi/kT}\int_{mc^2}^{\infty}\frac{e^{-\frac{n}{kT}(t-mc^2)}}{t}\,dt,$$

$$= \frac{e^2}{4c\hbar}\,\frac{4\pi g}{(2\pi)^3}\sum_{n=1}^{\infty}(-1)^{n+1}\,e^{(\xi+mc^2)\frac{n}{kT}}\int_{\frac{mnc^2}{kT}}^{\infty}\frac{e^{-y}}{y}\,dy,$$

$$= \frac{e^2}{4c\hbar}\,\frac{4\pi g}{(2\pi)^3}\sum_{n=1}^{\infty}(-1)^{n+1}\,e^{(\xi+mc^2)n/kT}$$

$$\times\left\{-\gamma-\log\frac{mc^2}{kT}\,n+n\,\frac{mc^2}{kT}-\frac{n^2}{2|\underline{2}}\left(\frac{mc^2}{kT}\right)^2+\quad\right\}, \tag{23}$$

where γ is Euler's constant = 56

For the case under consideration

$$e^{\xi/kT} = \frac{1}{6}\left(\frac{\xi_0}{kT}\right)^3\left\{1+\frac{b_2}{3}\left(\frac{\xi_0}{kT}\right)^3\right\}, \tag{24}$$

where ξ_0 is given by (8)

From (23) and (24) we obtain

$$\chi = \left(\frac{ech}{4\pi}\right)^2\frac{n}{2}\,\frac{1}{(kT)^3}\left[\left\{1+\frac{mc^2}{kT}+\frac{1}{|\underline{2}}\left(\frac{mc^2}{kT}\right)^2\right\}\log\frac{kT}{mc^2}-\gamma\left(1+\frac{mc^2}{kT}\right)+\frac{mc^2}{kT}\right.$$

$$\left.+\text{terms of the order }\left(\frac{mc^2}{kT}\right)^2\text{ and higher have been neglected}\right] \tag{25}$$

My thanks are due to Prof D S Kothari for his very kind interest in this work

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DIAMAGNETISM OF A RELATIVISTIC ELECTRON GAS

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(Communicated by Dr. D. S. Kothari, F.N.I.)

(*Received February 24, read March 5, 1948.*)

ABSTRACT

The diamagnetic susceptibility of a relativistic degenerate electron gas is deduced from energy considerations. The results of the present straightforward method agree with those deduced in a recent paper by the author from the perturbation theory in quantum mechanics.

INTRODUCTION

Bohr (1911) has shown that in classical mechanics a free electron gas will have no diamagnetic susceptibility. Landau (1930) gave the important result that in quantum mechanics a diamagnetic contribution to the susceptibility should arise because of the discreteness of the energy states in a magnetic field. Laudau's treatment of the diamagnetic susceptibility is based on energy considerations and holds for a non-relativistic electron gas. In an earlier paper (1947) the present author has given a relativistic generalisation of Klein's (1945) quantum mechanical theory of a free electron gas in a magnetic field, and using the theory of perturbation expressions for para- and dia-magnetic susceptibilities were derived. The present paper attempts to derive the diamagnetic susceptibility of a relativistic electron gas from purely energy considerations, as has been done previously, in the non-relativistic case, by Laudau. It is indeed very satisfactory that the perturbation method of the earlier paper (1948) and the one given here give the same results.

2. *Derivation of characteristic values*

It is unnecessary to derive the characteristic energy values for a free electron, in the non-relativistic case, in a magnetic field, as this has been done at length by several authors. We shall proceed to derive the characteristic values of the energy in the relativistic case.

The Dirac equation, as modified for the presence of a magnetic field, takes the form

$$E\psi = \left\{\left(\vec{\alpha}\ \vec{p} - e\vec{A}\right) + \beta\mu\right\}\psi. \tag{1}$$

where $\vec{\alpha}$ and β are the well-known Dirac matrices, $\vec{p}$ the momentum in energy units, $\vec{A}$ the vector potential of the field, E the total energy of the electron and $\mu = mc^2$ its rest energy.

A field H along the z axis may be derived from a vector potential of components

$$A_x = -\tfrac{1}{2}Hy,\quad A_y = A_z = 0 \tag{2}$$

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The equation then becomes

$$E\psi = \left\{\left(\vec{\alpha}\ \vec{p}+\beta\mu\right)+eH\alpha_x y\right\}\psi \tag{3}$$

Writing (3) in full we have

$$\left.\begin{aligned}(\mu-E)\psi_1+(p_x+ip_y+eHy)\psi_4+p_z\psi_3 &= 0\\ (\mu-E)\psi_2+(p_x-ip_y+eHy)\psi_3-p_z\psi_4 &= 0\\ -(\mu+E)\psi_3+(p_x+ip_y+eHy)\psi_2+p_z\psi_1 &= 0\\ -(\mu+E)\psi_4+(p_x-ip_y+eHy)\psi_1-p_z\psi_2 &= 0\end{aligned}\right\}, \tag{4}$$

where p_x, p_y and p_z are the momentum operators,

$$p_x = -i\hbar c\frac{\partial}{\partial x}, \text{ etc}$$

Let the solutions be

$$\psi_i = u_i(y)e^{i/\hbar(-p_x x+p_z z)}, \tag{5}$$

$$i = 1, 2, 3, 4$$

p_x and p_z in (5) and in what follows are ordinary numbers

From (4) and (5) we have

$$(\mu-E)u_1+cp_zu_3+D_1u_4 = 0, \tag{6a}$$

$$(\mu-E)u_2-cp_zu_4+D_2u_3 = 0, \tag{6b}$$

$$cp_zu_1-(\mu+E)u_3+D_1u_2 = 0, \tag{6c}$$

$$-cp_zu_2-(\mu+E)u_4+D_2u_1 = 0, \tag{6d}$$

where D_1 and D_2 are

$$D_1 = \left(-cp_x+c\hbar\frac{\partial}{\partial y}+eHy\right), \tag{7a}$$

$$D_2 = \left(-cp_x-c\hbar\frac{\partial}{\partial y}+eHy\right) \tag{7b}$$

Multiplying 6(b) by cp_z and 6(a) by $(\mu-E)$ and adding the two we have

$$-\{c^2p_z^2+(\mu^2-E^2)\}u_4+D_2\{cp_zu_3+(\mu-E)u_1\} = 0,$$

which on using 6(a) becomes

$$\{(cp_z)^2+(\mu^2-E^2)+D_2D_1\}u_4 = 0 \tag{8}$$

From (7) D_2D_1 is

$$D_2D_1 = (c^2p_x^2+e^2H^2y^2-2cHeyp_x)-eHc\hbar-c^2\hbar^2\frac{\partial^2}{\partial y^2}$$

(8) now becomes

$$\frac{\partial^2u_4}{\partial y^2}-\frac{1}{c^2\hbar^2}\left\{(cp_z)^2+(\mu^2-E^2)-eHc\hbar+c^2\left(\frac{eH}{c}y-p_x\right)^2\right\}u_4 = 0 \tag{9}$$

It can easily be shown that u_3 satisfies the equation

$$\frac{\partial^2u_3}{\partial y^2}-\frac{1}{c^2\hbar^2}\left\{(cp_z)^2+(\mu^2-E^2)+eHc\hbar+c^2\left(\frac{eH}{c}y-p_x\right)^2\right\}u_3 = 0 \tag{10}$$

u_2 and u_1, respectively, satisfy the same equations as do u_4 and u_3. There are thus only two independent solutions of (6). Eqns (9) and (10) differ only in the sign of the term $eHc\hbar$, which corresponds to the energy of the spin orientation in the magnetic field. Since we are interested only in the proper values of the energy of the translational motion, we drop the spin energy term in (9) and (10). The two equations then reduce to one single equation

$$\frac{\partial^2 u}{\partial y^2} - \frac{1}{c^2\hbar^2}\left\{(cp_z)^2+(\mu^2-E^2)+c^2\left(\frac{eH}{c}\,y-p_x\right)^2\right\}u = 0 \tag{11}$$

This corresponds to the equation for a harmonic oscillator with equilibrium position at

$$y = \frac{cp_x}{eH} \tag{12}$$

Putting

$$\frac{eH}{c}\,y-p_x = aq, \tag{13}$$

where

$$a = \hbar\left(\frac{eH}{c}\right),$$

(11) becomes

$$\frac{\partial^2 u}{\partial q^2}+\left\{\frac{1}{eHc\hbar}(E^2-\overline{c^2p_z^2+\mu^2})-q^2\right\}u = 0$$

or

$$\frac{\partial^2 u}{\partial q^2}+(\lambda-q^2)u = 0, \tag{14}$$

where

$$\lambda = \frac{1}{eHc\hbar}\left\{E^2-(c^2p_z^2+\mu^2)\right\} \tag{15}$$

The eigenvalues are given by $\lambda = (2n+1)$, where n is an integer.

Therefore

$$E^2 = (c^2p_z^2+\mu^2)+(2n+1)eHc\hbar \tag{16}$$

In the non-relativistic case (16) becomes

$$E = \frac{p_z^2}{2m}+\mu+(2n+1)\frac{eH}{4\pi mc} \tag{17}$$

We shall require the number of energy states for a given value of the quantum number n and for a range dp_z. If A, B, and C be the linear dimensions of the container, then $\nu(n)$, the number of states per unit range, is

$$\begin{aligned}\nu(n)dp_z &= \frac{AC}{\hbar^2}dp_z\int dp_x, \\ &= \frac{eH}{c\hbar^2}ACdp_z\int_0^B dy, \qquad \text{using (12)}, \\ &= V\frac{eH}{c\hbar^2}dp_z,\end{aligned} \tag{18}$$

V being the volume.

It is very important that y must be less than B, in order to avoid the distortion of the characteristic values by the boundary Out of all the electrons within the vessel a fraction of the order of $\frac{y}{B}$ have boundary orbits which classically hit the wall If B is sufficiently large, an overwhelming majority of the common stationary states will not have their characteristic values appreciably changed by the influence of the boundary The error involved then, as shown by Landau, becomes negligible

In relativistic mechanics

$$E^2-\mu^2=\epsilon^2+2\epsilon\mu,$$

where ϵ is the kinetic energy,

(16) then becomes

$$\epsilon=\left\{(c^2p_z^2+\mu^2)+(2n+1)eHc\hbar\right\}^{\frac{1}{2}}-\mu \tag{19}$$

3 *Diamagnetic susceptibility*

In this section we shall calculate the diamagnetic susceptibility of a degenerate electron gas The non relativistic case has already been treated in detail by Stoner (1935)

The magnetic moment is given by the general formula

$$M=-\left(\frac{\partial F}{\partial H}\right)_{T\ V}, \tag{20}$$

where F is the free energy With Fermi Dirac statistics

$$F=N\xi+\Omega, \tag{21}$$

$$\Omega=-kT\sum_s\log\left(1+\exp\left(\xi-\epsilon_s\right)/kT\right) \tag{22}$$

where ξ is the free energy per particle

Substituting (19) in (22) and using (18) we have

$$\Omega=-kT\frac{gVeH}{ch^2}\,2\sum_{n=0}^{\infty}\int_0^{\infty}\log\left(1+e^{A-\{\alpha+\beta(n+\frac{1}{2})\}^{\frac{1}{2}}}\right)dp_z, \tag{23}$$

g being the weight factor for the electron, and

$$\alpha=c^2p_z^2+\mu^2/(kT)^2, \tag{24}$$

$$\beta=2\,\frac{eHc\hbar}{(kT)^2}, \tag{25}$$

$$A=\xi+\mu/kT \tag{26}$$

The summation * is evaluated by using Euler's formula, which gives

$$\sum_{n=0}^{\infty}\log\left(1+e^{A-\{\alpha+\beta(n+\frac{1}{2})\}^{2}}\right)=\int_0^{\infty}\log\left(1+e^{A-(\alpha+\beta x)^{\frac{1}{2}}}\right)dx$$

$$-\frac{1}{24}\left|\frac{d}{dx}\log\left(1+e^{A-(\alpha+\beta x)^{\frac{1}{2}}}\right)\right|_0^{\infty}$$

$$=\frac{\beta}{2}\int_0^{\infty}\frac{x(\alpha+\beta x)^{-\frac{1}{2}}\,dx}{1+e^{(\alpha+\beta x)^{\frac{1}{2}}-A}}-\frac{\beta}{48}\,\frac{\alpha^{-\frac{1}{2}}}{1+e^{\alpha^{\frac{1}{2}}-A}} \tag{27}$$

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* The process of summation in (23) is performed before integrating over dp_z, and this is justified since the series is uniformly convergent

Denote the integral on the right by I, and let

$$\alpha+\beta x = t^2,$$

then

$$I = \frac{2}{\beta^2}\int_{\alpha^{\frac12}}^{\infty}\frac{(t^2-\alpha)dt}{1+e^{t-A}}$$

Putting $t = y+\alpha^{\frac12}$ we have

$$I = \frac{2}{\beta^2}\int_0^\infty \frac{y^2\,dy}{1+e^{y+\alpha^{\frac12}-A}}+\frac{4\alpha^{\frac12}}{\beta^2}\int_0^\infty \frac{y\,dy}{1+e^{y+\alpha^{\frac12}-A}} \tag{28}$$

Since in the relativistic case

$$\left.\begin{aligned}\alpha^{\frac12} &= cp_s/kT\\ \text{and}\quad A &= \xi/kT\end{aligned}\right\}, \tag{29}$$

I becomes

$$I = \frac{2}{\beta^2}\int_0^\infty \frac{y^2\,dy}{1+e^{y-(\eta-z)}}+\frac{4z}{\beta^2}\int_0^\infty \frac{y\,dy}{1+e^{y-(\eta-z)}}, \tag{30}$$

where we have put

$$\left.\begin{aligned}cp_s/kT &= z\\ \text{and}\quad \xi/kT &= \eta\end{aligned}\right\} \tag{31}$$

Let

$$F_n(\eta) = \int_0^\infty \frac{y^n\,dy}{1+e^{y-\eta}}, \tag{32}$$

we can then easily show that

$$\frac{\partial}{\partial\eta}F_n(\eta) = nF_{n-1}(\eta) \tag{33}$$

For $\eta \gg 1$, i e (degenerate case)

$$F_n(\eta) = \frac{1}{n+1}\eta^{n+1}+\frac{\pi^2 n}{6}\eta^{n-1}+ \qquad , \tag{34}$$

(see Stoner (1935))
(30) on using (32) becomes

$$I = \frac{2}{\beta^2}F_2(\eta-z)+\frac{4}{\beta^2}zF_1(\eta-z). \tag{35}$$

Let

$$\Omega = \Omega_1+\Omega_2,$$

where

$$\Omega_1 = -\frac{4kTVeH}{ch^2}\frac{kT}{c\beta}\left[\int_0^\infty F_2(\eta-z)dz+2\int_0^\infty zF_1(\eta-z)dz\right], \tag{36}$$

and

$$\Omega_2 = \frac{VeH}{ch^2}\frac{(kT)^2}{12}\beta\int_0^\infty (c^2p_s^2+\mu^2)^{-\frac12}\frac{dp_s}{1+e^{(c^2p_s^2+\mu^2)^{\frac12}-(\xi+\mu)/kT}} \tag{37}$$

In (37) we have not yet taken the relativistic approximation, we shall do so in the end

To evaluate the integrals in Ω_1, let

$$\eta - z = \eta',$$

$$\int_0^\infty F_2(\eta - z)\,dz = -\int_\eta^{-\infty} F_2(\eta')\,d\eta',$$

$$= -\frac{1}{3}\Big| F_3(\eta')\Big|_\eta^{-\infty} = \tfrac{1}{3}F_3(\eta) \tag{38}$$

Also

$$\int_0^\infty zF_1(\eta - z)\,dz = -\int_\eta^{-\infty} (\eta-\eta')F_1(\eta')\,d\eta',$$

$$= \tfrac{1}{6}F_3(\eta) \tag{39}$$

Substituting these values Ω_1 becomes

$$\Omega_1 = -\frac{8}{3}\left(\frac{kT}{ch}\right)^2 \frac{VeH}{\beta} F_3(\eta),$$

which, on substituting the value of β from (25), becomes

$$\Omega_1 = -\frac{8\pi}{3}(kT)^4 \frac{V}{c^3h^3} F_3(\eta) \tag{40}$$

Putting $(c^2p_z^2+\mu^2) = t^2$, and using (25) Ω_2 becomes

$$\Omega_2 = \frac{1}{6}\frac{Ve^2H^2}{2\pi ch}\int_\mu^\infty \frac{dt}{(t^2-\mu^2)^{\frac{1}{2}}}\,\frac{1}{1+e^{(t-\mu)-\xi/kT}}$$

$$= \frac{1}{6}\frac{Ve^2H^2}{2\pi ch}\int_\mu^\infty \frac{d}{dt}\phi(t)\,\frac{dt}{1+e^{(t-\mu)-\xi/kT}},$$

where $\phi(t) = \log\left(t+(t^2-\mu^2)^{\frac{1}{2}}\right)$

Put $t-\mu = z$, then

$$\Omega_2 = \frac{1}{6}\frac{Ve^2H^2}{2\pi ch}\int_0^\infty \frac{d}{dz}\phi(z)\,\frac{dz}{1+e^{z-\xi/kT}}, \tag{41}$$

where

$$\phi(z) = \log\left[(\mu+z)+\{(z+\mu)^2-\mu^2\}^{\frac{1}{2}}\right]$$

The integral in (41) can be evaluated by using Sommerfeld Lemma $\left(\frac{\xi}{kT} \gg 1\right)$ We then have

$$\Omega_2 = \frac{1}{6}\frac{Ve^2H^2}{2\pi ch}\left\{\phi(\xi)-\phi(0)+2c_2(kT)^2\phi''(\xi)+\quad\right\},$$

where $c_2 = \pi^2/12$

Or

$$\Omega_2 = \frac{1}{6}\frac{Ve^2H^2}{2\pi ch}\left\{\log\frac{(\xi+mc^2)+\{(\xi+mc^2)^2-(mc^2)^2\}^{\frac{1}{2}}}{mc^2} \right.$$

$$\left. -2c_2(kT)^2(\xi+mc^2)(\xi^2+2\xi mc^2)^{\frac{1}{2}}\right\} \tag{42}$$

For the relativistic case $\left(\frac{mc^2}{\xi} \ll 1\right)$ we have

$$\Omega_2 = \frac{1}{6}\frac{Ve^2H^2}{2\pi ch}\left\{\log\frac{2\xi}{mc^2} - 2c_2(kT)^2\frac{1}{\xi^2}\right\} \tag{43}$$

Hence we have

$$\Omega = \Omega_1 + \Omega_2$$

$$= -\frac{8\pi}{3}(kT)^4\frac{V}{c^3h^3}F_3(\eta) + \frac{1}{6}\frac{Ve^2H^2}{2\pi ch}\left\{\log\frac{2kT}{mc^2}\eta - 2c_2\frac{1}{\eta^2}\right\} \tag{44}$$

The magnetic moment is

$$M = -\frac{\partial\Omega}{\partial H},$$

$$= -\frac{1}{3}\frac{Ve^2H}{2\pi ch}\left\{\log\frac{2kT}{mc^2}\eta - 2c_2\frac{1}{\eta^2}\right\} \tag{45}$$

The number of particles N is given by

$$N = \frac{1}{kT}\frac{\partial\Omega}{\partial\eta}$$

$$= \frac{8\pi}{3}\left(\frac{kT}{ch}\right)^3 VF_3'(\eta) \qquad \text{+other terms which we may neglect,}$$

$$= 8\pi\left(\frac{kT}{ch}\right)^3 VF_2(\eta), \qquad \text{using (33),}$$

$$= \frac{8\pi}{3}V\left(\frac{kT}{ch}\right)^3\eta^3\left(1+\frac{\pi^2}{\eta^2}\right), \text{ using (34)} \tag{46}$$

The maximum energy ξ_0 of a particle in the completely degenerate state is given by

$$\xi_0 = mc^2\left[(1+x^2)^{\frac{1}{2}}-1\right], \tag{47}$$

(see Kothari and Singh (1942)),

where
$$x = \frac{h}{mc}\left(\frac{3N}{8\pi V}\right)^{\frac{1}{3}} \tag{48}$$

In the relativistic case $\xi_0/mc^2 \gg 1$, i e x very large,

$$N = \frac{8\pi V}{c^3h^3}\frac{\xi_0^3}{3},$$

$$= \frac{8\pi}{3}\frac{V}{c^3h^3}(kT)^3\eta_0^3 \tag{49}$$

From (46) and (49) we have

$$\begin{aligned} \eta &= \eta_0\left(1+\frac{\pi^2}{\eta^2}+\quad\right)^{-\frac{1}{3}} \\ &= \eta_0\left(1-\frac{1}{3}\frac{\pi^2}{\eta^2}\right) \qquad . \end{aligned} \tag{50}$$

The susceptibility is given by

$$\chi_D = \frac{M}{H} = -\frac{1}{3}\frac{e^2 V}{2\pi ch}\left(\log\frac{2kT}{mc^2}\eta-\frac{2c_2}{\eta^2}\right),$$

which on using (49) and (50) becomes

$$\begin{aligned} \chi_D &= -\left(\frac{ech}{4\pi}\right)^2\frac{N}{\xi_0^3}\left\{\log\frac{2kT}{mc^2}\eta_0\left(1-\frac{\pi^2}{3\eta_0^2}\right)-\frac{2c_2}{\eta_0^2}\right\}, \\ &= -\left(\frac{ech}{4\pi}\right)^2\frac{N}{\xi_0^3}\left\{\log\frac{2\xi_0}{mc^2}-\frac{\pi^2}{2}\left(\frac{kT}{\xi_0}\right)^2\right\} \end{aligned} \tag{51}$$

For $T=0$, (51) reduces to the expression* deduced earlier (1948) by the method of perturbation It is, therefore, very gratifying that the two methods which are so different give identical results

4 *Diamagnetic susceptibility (non-relativistic case)*

For the case under consideration, i e

$$\frac{mc^2}{\xi} \gg 1,$$

(42) becomes

$$\Omega_2 = \frac{1}{6}\frac{e^2H^2V}{2\pi ch}\frac{2^{\frac{1}{2}}\xi^{\frac{1}{2}}}{(mc^2)^{\frac{1}{2}}}\left\{1-c_2(kT)^2\frac{1}{2\xi^2}\right\} \tag{52}$$

The susceptibility is given by

$$\begin{aligned} \chi_D &= -\frac{1}{H}\frac{\partial\Omega}{\partial H}, \\ &= -\frac{1}{H}\frac{\partial\Omega_2}{\partial H}, \\ &= -\frac{1}{3}\frac{e^2V}{2\pi ch}\left(\frac{2\xi}{mc^2}\right)^{\frac{1}{2}}\left\{1-\frac{c_2}{2}\left(\frac{kT}{\xi}\right)^2\right\} \end{aligned} \tag{53}$$

Now

$$\xi = \xi_0\left\{1-\frac{\pi^2}{12}\left(\frac{kT}{\xi_0}\right)^2\right\}, \tag{54}$$

where

$$\xi_0 = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{\frac{2}{3}}, \tag{55}$$

(see Stoner (1935)).

---

* Actually formula (32) of the previous paper gives the paramagnetic susceptibility The diamagnetic susceptibility can at once be calculated from (24), and will come out to be $-\frac{1}{3}$ the paramagnetic susceptibility

Using (54) and (55), (53) becomes

$$\chi_D = -\frac{N}{2\xi_0} B^2 \left\{ 1 - \frac{\pi^2}{12} \left(\frac{kT}{\xi_0} \right)^2 \right\}, \tag{56}$$

where

$$B = \left(\frac{eh}{4\pi mc} \right)$$

I have much pleasure in expressing my thanks to Prof D S Kothari under whose supervision this work was carried out

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A CONTRIBUTION TO THE EMBRYOLOGY OF *WAHLENBERGIA GRACILIS* SCHRAD

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(Communicated by Prof P Maheshwari)

(*Received March 30, read April 20, 1948*)

INTRODUCTION

The genus *Wahlenbergia* is a member of the family Campanulaceae, placed by Engler and Prantl (1897) in the tribe Campanuloideae, subtribe Wahlenbergineae The only previous work on the embryology of this genus is by Rosen (1932) who has described the mode of endosperm formation in an unnamed species and reported it to be of the *Codonopsis* type

MATERIALS AND METHODS

The only species of this genus in South India is *Wahlenbergia gracilis* Schrad It is an erect perennial herb with linear leaves and blue flowers on long pedicels The fruit is a loculicidally dehiscing capsule with persistent calyx teeth The material was collected at Ootacamund at a height of about 6,800 ft It was fixed in formalin acetic-alcohol and at the 70% alcohol stage the mature ovaries were treated with Carnoy's fluid for half an hour Subsequent treatment was carried out according to customary methods and sections were cut at a thickness of 10–20μ Staining was done in Heidenhain's iron-alum haematoxylin with eosin as counter stain

MICROSPOROGENESIS AND MALE GAMETOPHYTE

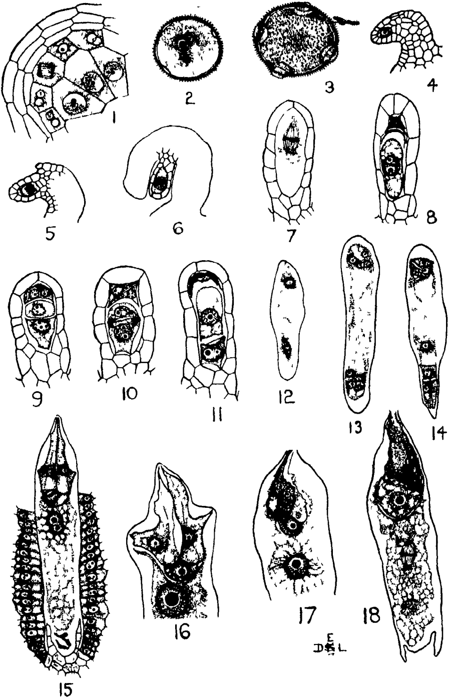
The wall of the young anther is made up of three layers in addition to the tapetum (Fig 1) Of these, the outermost is the epidermis, next we have the endothecium which acquires fibrous thickenings at a later stage, the third is the middle layer which remains undivided and disorganises when the anther is mature The tapetal cells are at first uninucleate but soon become binucleate and at the same time the cytoplasm becomes vacuolate

The microspore mother cells undergo the usual reduction divisions and form the microspores which are arranged tetrahedrally Quadripartition of the micro spore mother cell takes place by cleavage furrows

The mature pollen grain is trinucleate, with a prominent tube nucleus and two small male nuclei (Fig 2) The exine is in the form of a hard thick wall showing a reticulate pattern on the surface and beset with minute spinescent projections The intine appears as a thin delicate membrane There are four germ pores situated on slightly elevated portions of the exine (Fig 3)

MEGASPORANGIUM AND THE FEMALE GAMETOPHYTE

The tricarpellary, trilocular, inferior ovary has an indefinite number of ovules borne on axile placentae The ovules are anatropous and unitegmic A single hypodermal archesporial cell becomes differentiated in the nucellus (Fig 4) and is followed by the appearance of the integument (Fig 5) The archesporial cell is



TEXT-FIGS 1–18.

overarched by the nucellar epidermis and functions directly as the megaspore mother cell (Fig 6) After the usual reduction divisions a linear tetrad of megaspores is formed (Figs 7 and 8) Occasionally a T-shaped tetrad may be found (Fig 9) as in *Cephalostigma Schimperi* (Kausik and Subramanyam, 1947) and sometimes the upper dyad cell divides by an oblique wall (Fig 10) As a rule, the upper three megaspores degenerate and the lower enlarges further (Fig 8) It undergoes three divisions and produces the mature embryo sac (Figs 12 and 13) which is therefore of the monosporic eight nucleate type Rarely, the third megaspore may enlarge (Fig 11), a feature also recorded for *Lobelia trialata*, a member of the allied family Lobeliaceae (Kausik and Subramanyam, 1945*b*)

At about the two nucleate embryo sac stage the cells of the nucellar epidermis are destroyed, excepting a few towards the sides At the four nucleate stage even these are destroyed so that the embryo sac lies in direct contact with the inner epidermis of the integument which becomes differentiated as the integumentary tapetum The latter shows its maximum development when the embryo sac is fully formed and is ready for fertilisation (Fig 15)

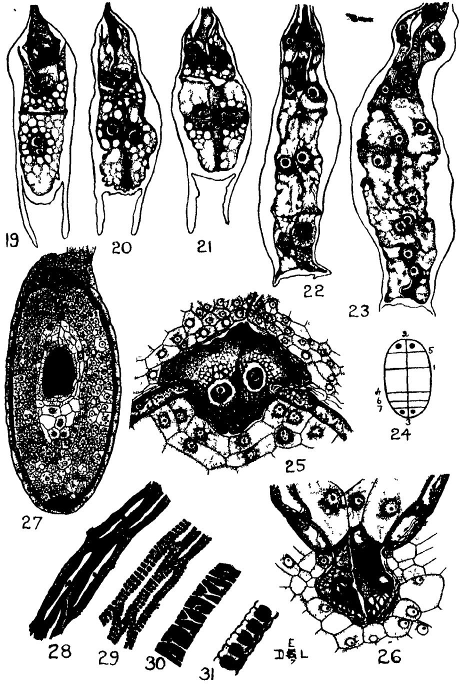
The fully organised embryo sac (Fig 15) is long and tapering at both ends The synergids are elongated and have pointed apices They also show the characteristic hooks (Fig 16) already reported for *Cephalostigma Schimperi* (Kausik and Subramanyam, 1947*a*) The pear-shaped egg is situated between the two synergids and shows a conspicuous nucleus at the base The two polar nuclei meet just above the centre of the embryo sac and fuse to form the secondary nucleus The antipodals are organised as definite cells Sometimes the antipodal cells are differentiated before the egg apparatus (Fig 14) Such a feature has also been reported in *Isotoma longiflora* (Kausik and Subramanyam, 1945*a*), a member of the Lobeliaceae The antipodals degenerate at the time of fertilisation and are then seen as darkly stained masses This is in general accordance with the condition in the Lobeliaceae and Campanulaceae (Kausik and Subramanyam, 1945*a*, *b* and 1946*a*, *b*) except that there is a slightly earlier degeneration of the antipodals in *Wahlenbergia*

The pollen tube destroys one of the synergids during its entry into the embryo sac, but sometimes both the synergids may remain intact Double fertilisation has been observed (Fig 17)

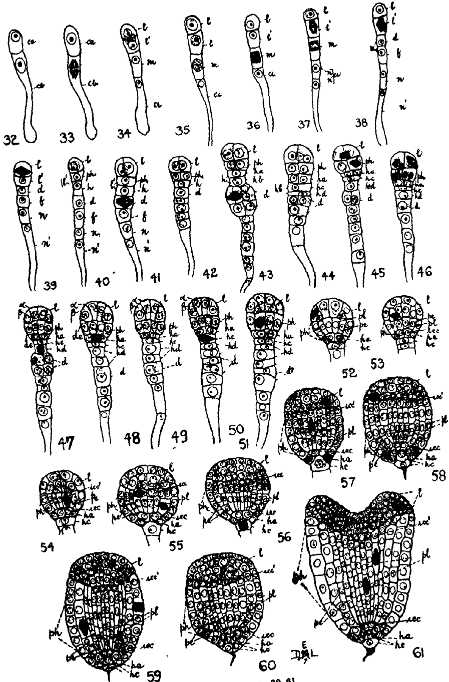
ENDOSPERM

Endosperm development is of the *ab initio* cellular type The first division of the primary endosperm nucleus is followed by the laying down of a transverse wall (Fig 18) to give rise to a small primary micropylar and a large primary chalazal chamber Next, a vertical wall is formed first in the micropylar (Fig 19) and then in the chalazal chamber (Fig 20), thus resulting in a four-celled stage A transverse division now follows in the lower pair of cells (Fig 21) and immediately afterwards in the upper pair (Fig 21), resulting in the formation of four tiers of paired cells The two cells of the first tier form the micropylar haustorium and those of the lower tier give rise to the chalazal haustorium after undergoing one (Fig 22) or perhaps two transverse divisions (Fig 23) The remaining tiers of cells which lie between the haustoria undergo further longitudinal and transverse divisions and give rise to the endosperm Thus the sequence of wall formation, schematically represented in Fig 24, closely corresponds with that in the *Codonopsis* type found in other members of Campanulaceae (Rosén, 1932, and Kausik and Subramanyam, 1947*a*)

The micropylar haustorium is two celled (Fig 25), each cell forming a prominent lateral hump or bulge and containing a conspicuous nucleus and a dense mass of cytoplasm Lying in the midst of a rich nutritive tissue belonging to the integument, this haustorium remains active for a long time The activity of the chalazal haustorium, which is also two-celled (Fig 26), stops at an earlier stage In a mature seed it is seen as a darkly stained compressed structure lying in a mass of collapsed



TEXT FIGS 19–31



TEXT-FIGS. 32–61.

cells The endosperm fills the entire seed cavity (Fig 27) and all its cells, except those in the neighbourhood of the developing embryo, contain large quantities of starch The embryo with its slender suspensor, which now appears to be quite shrivelled up, lies deeply buried in the mass of endosperm tissue

SEED-COAT

In the mature seed (Fig 27) the outer epidermis of the integument becomes thickened and forms a hard and rigid protective covering Its cells are elongated along the longitudinal axis of the seed (Fig 28) and owing to the conspicuous thickening of the inner tangential and the radial walls the cavity of each cell is reduced to a narrow space (cf *Lobelia trigona* studied by Kausik, 1935) Each cell is in communication with its neighbouring cells by means of canals which traverse the entire thickness of the cell wall (Figs 29 and 30) The canals are long and narrow and branch towards the outer as well as the inner sides of the cells In a transverse section at the region of these canals the cell cavity appears in the form of the letter 'T' (Fig 31)

EMBRYO

The development of the embryo closely follows that described by Souèges (1936, 1938) and Kausik and Subramanyam (1947*a*) for other members of Campanulaceae, and Crete (1938), Hewitt (1939) and Kausik and Subramanyam (1945 and 1947*b*) for some members of the Lobeliaceae Stages in development are presented in Figs 32–61 The embryogeny corresponds to the *Solanad* type of Johansen (1945) A case of polyembryony was met with in the present form and this has been separately described (Subramanyam, 1947)

SUMMARY

The wall of the anther is made up of three layers, external to the tapetum The tapetal cells become binucleate The endothecium is fibrous The pollen grains are trinucleate at the time of shedding The exine is very finely spinescent and there are four germ pores

The ovary is inferior and trilocular with an indefinite number of anatropous unitegmic ovules borne on an axile placenta The innermost layer of the integument forms an integumentary tapetum Megasporogenesis proceeds normally and the embryo sac is of the monosporic eight nucleate type The synergids are very long and show characteristic hook-like projections The antipodal cells are ephemeral Double fertilisation occurs

Endosperm is of the *ab initio* cellular type and follows the *Codonopsis* type (Rosén, 1932) The endosperm develops haustoria at the micropylar and chalazal ends The micropylar haustorium is made up of two uninucleate cells and appears to be more aggressive than the chalazal endosperm haustorium which is also two celled and uninucleate

Development of the embryo follows the *Solanad* type (Johansen, 1945) as seen in other members of Campanulaceae and Lobeliaceae

The mature seed contains a large mass of endosperm The seed-coat consists of a single layer of cells whose outer walls become thickened.

ACKNOWLEDGMENT

In conclusion I wish to thank Prof P Maheshwari of the University of Dacca for critically going through the manuscript and for valuable suggestions I am also grateful to him for permitting me to use some of his own preparations of this plant. Thanks are due to Prof L N Rao, Central College, Bangalore, for encouragement, Mr S N Chandrasekhara Iyer, Systematic Botanist, Coimbatore, for determining the species, and to Mr P A Nathan, Curator, Govt Botanic Gardens, Ootacamund, for collection of the material

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EXPLANATION TO FIGURES

Wahlenbergia gracilis Schrad (Figs 1 to 18)

1 Portion of a transverse section of a young anther showing wall layers, binucleate tapetum and microspore mother cells ×485
2 Three nucleate pollen grain ×485
3 Pollen grain in surface view showing four germ pores ×485
4 Young nucellar primordium and the archesporium ×291
5 Megaspore mother cell and the integument initials ×291
6. Anatropous ovule showing megaspore mother cell ×291
7 Megaspore mother cell in division ×679
8. Linear tetrad of megaspores, the chalazal cell enlarging ×679
9 T shaped tetrad of megaspores ×679
10 Megaspore tetrad with oblique wall in the upper dyad cell ×679
11 T shaped tetrad in which the third megaspore shows signs of enlargement ×679
12. Two nucleate embryo sac showing division of nuclei ×430
13 Formation of the eight-nucleate embryo sac ×485
14 An young eight nucleate embryo sac showing the early differentiation of the antipodals, before the organisation of the egg apparatus has been completed ×485
15 Mature embryo sac showing egg apparatus, the degenerating antipodal cells, and the two polars in close juxtaposition ×485
16 Upper part of the embryo sac showing the elongated hooked synergids and the egg cell ×679
17 A stage in double fertilisation, showing remnants of the pollen tube ×485
18 Primary endosperm nucleus in division ×485

Wahlenbergia gracilis Schrad (Figs 19 to 31)

19 to 23. Stages in the development of the endosperm and the differentiation of the micropylar and chalazal haustoria ×485 each
24 Diagram showing sequence of wall formation in endosperm
25 Two celled micropylar haustorium in advanced stage ×485
26. Two celled chalazal haustorium in advanced stage. ×485

27 Longitudinal section of a mature seed, showing the embryo with collapsed suspensor, the persisting micropylar haustorium, the almost collapsed chalazal haustorium, and the thick-walled seed coat ×291

28 Surface view of the epidermal cells of the seed coat showing thickenings ×485

29 The same under a separate focus ×485

30 The canals connecting the adjacent cells, ×970

31 Epidermal cells of the seed coat in transverse section. ×485

Wahlenbergia gracilis Schrad (Figs. 32 to 61)

32-61 Stages in the development of the embryo The primary segmentation walls are indicated by thicker lines ×291 each

ca—Apical cell of the two celled proembryo, *cb*—Basal cell of the two celled proembryo, *m* and *ci*—Cells derived from the basal cell *cb*, *d* and *f*—Cells derived from the cell *m*, *n* and *n'*—Cells derived from the cell *ci*, *l*—Superior cell derived from the apical cell *ca*, from which the cotyledons are later differentiated, *l'*—Inferior cell derived from the apical cell *ca*, α and β—Cells formed in the superior octant, *ph* and *h*—Cells derived from *l'*, from which part of the hypocotyl is formed, *ha* and *hb*—Cells produced from *h*, *iec*—Initials of radicle, *iec'*—Initials of the stele, *hc* and *hd*—Cells produced from *hb*, *d* or *d* ?—Cell or cells produced from one of the cells of the suspensor, *de*—Dermatogen, *pe*—Periblem, *pl*—Plerome

NOTES ON SOME ULOTRICHALES FROM NORTHERN INDIA

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(*Received January 26 , read March 5, 1948*)

Members of the order Ulotrichales have comparatively drawn very little attention from the algologists in Northern India In 1920 Ghose described a new species of **Uronema** from India, *U indicum* In 1926 Nellie Carter recorded three species of Ulothrix, *U zonata*, *U subtilis* and *U aequalis* from different localities in India In 1936 the present author described *Cylindrocapsa oedogonioides*, a new species from the Punjab, and in 1939 *Microspora indica* and *Cylindrocapsa scytonemoides*, both new species from Fyzabad In 1940 Iyengar and Kanthamma described *Hormidiella parvula* gen et sp nov , and in 1941 *Heterothrichopsis viridis* gen et sp nov

It is a remarkable fact that **Ulothrix** which is so common in the Punjab is practically absent from Oudh During the collection of algae in the districts of Fyzabad, Gonda, Bahraich, Lucknow, Partabgarh, Rai Bareli, and Allahabad the present author did not secure even a single sample, though these districts are very rich in Zygnemales, which flourish in the *jhils* and sluggish streams of Oudh The absence of **Ulothrix** from Oudh was also confirmed by making enquiries from Botany teachers of various colleges at Fyzabad and Allahabad Out of the six species of **Ulothrix** collected by the present writer, three were collected from the Punjab, one from a sulphur spring in Kashmir, one from a dripping rock near Almora in Kumaon Himalayas, and one from the river Jumna in Agra district From these data one may safely conclude that species of **Ulothrix** flourish in the comparatively colder regions of North India The related genus **Schizomeris**, which is recorded from India for the first time, shows a similar distribution, and its two samples were collected from Kashmir and the Punjab **Prasiola** with its single recorded species *P fluviatilis* flourishes only in cold alpine torrents of Kashmir and Kumaon and has not been collected from the plains

In addition to the above-mentioned members of Ulotrichales, two other remark able species were collected from the Himalayas *Binuclearia tatrana*, which, so far as the present author is aware, has not been reported from India up to now, was collected from a bog near Dhakuri, at an altitude of 8,500 feet in Kumaon

Only *Geminella* and *Microspora* flourish in the warm districts of the United Provinces *Geminella* is represented by a solitary species *G interrupta* Turp , which was collected from a fresh water stream near Meja in **Allahabad** district Microspora is represented by *M indica* Randh with its bright green flocculent masses with spots of red This is found in most of the *jhils* of Oudh, and was originally collected from a *jhil* in Fyzabad

Enteromorpha intestinalis was also collected from Agra district growing luxuriantly in Chambal river A similar form was also collected by R N Tandan in April 1937 from Jumna river near Allahabad

From the above account it is evident that members of the order Ulotrichales flourish in the cold fresh-water rivulets and torrential streams of the Himalayas, are comparatively well represented in the submontane districts of the Punjab, and are rather rare in the warmer districts of Oudh, whose sluggish streams and *jhils* are choked with various members of the Zygnemales, for whose growth and reproduction ideal conditions are found in Oudh

Systematic enumeration of the species observed

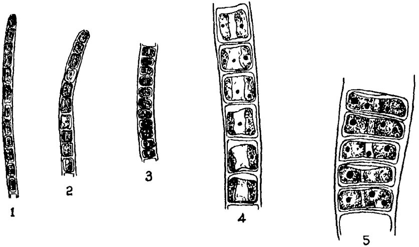
1 ULOTRICHALES

(1) ULOTRICHACEAE

Ulothrix Kutz

1 *Ulothrix subtilissima* Rabenhorst Krypt Flora V and Heering Susswasser flora 6, 1914, p 31

Cells 4 5μ broad and 8–12μ long Chloroplast with a single small pyrenoid in the middle



Figs 1–5 Species of *Ulothrix*

Fig 1 *U subtilissima* Rab
Fig 2 *U tennerima* Kutz
Fig 3 *U oscillarina* Kutz
Fig 4 *U tennuissima* Kutz
Fig 5 *U zonata* Kutz

Habit —Found epiphytic on a species of *Lyngbya*, growing at the sides of a water reservoir at Lahore in December 1929 Also collected from dripping rocks on Almora-Someshwar road on 5th August, 1939

2 *Ulothrix tennerima* Kutzing Heering Süsswasser-flora 6, 1914, p 32

Cells 7–10μ broad, 5–12μ long Chloroplast bears a single pyrenoid
Habit —Found attached to the sides of a water-trough at Lahore in December 1929 Also collected free-floating in Jumna river at Bateshwar, district Agra, in January 1941

3 *Ulothrix tennuissima* Kützing Heering Süsswasser-flora 6, 1914, p. 32.

Cells 15–18μ broad and 12–27μ long Chloroplast usually with two pyrenoids
Habit —Collected from a sulphur spring at Anantnag, Kashmir, on 23rd August, 1941.

4 *Ulothrix oscillarina* Kützing Heering Süsswasser-flora 6, 1914, p 32

Cells 8–10μ broad and 4–6μ long Chloroplast with 2-3 small pyrenoids Cell wall very thin

Habit —Found embedded in a mucilaginous stratum, forming a dark blue-green covering on brickwork at the sides of a water tap, mixed with desmids at Jullundur City railway station in August 1929

5 *Ulothrix zonata* Kutzing Heering Süsswasser flora 6, 1914, p 35

Cells 14–36μ broad and 8–14μ long Each chloroplast bearing 1–3 pyrenoids Cell wall thick, lamellated

Habit —Found attached to twigs in a dark green mass, in a puddle near a well at village Bodal, district Hoshiarpur, Punjab, in August 1929 Also collected from a sulphur spring at Anantnag, Kashmir, on 23rd August, 1941

Hormidium Klebs

1 *Hormidium flaccidum* A Br *forma* typica Heering Susswasser flora 6, p 45

Cells are rectangular or squarish, 6–9μ broad and 7–15μ long, and joined into stable filaments Each cell contains a single chloroplast in the form of a half girdle, each bearing a single conspicuous pyrenoid (Fig 6) The protoplast shows homogeneous contents Some of the filaments contain many empty cells, possibly on account of the escape of swarmers (Fig 7) The filaments do not have any specialised rhizoids

Filaments on dried soil become pale yellowish in colour, and cell walls become considerably thickened (Fig 7) A common mode of perennation in this alga is by means of akinetes, which are liberated by the decay of the outer wall (Fig 8)

Habit —Collected by the author from clayey soil at Binsar and Gananath, Almora, from an altitude of 6,000–7,000 feet above sea-level in September 1939, after the rains This alga has a great affinity for clayey soil and was especially abundant on the bridle path near Gananath forest bungalow, covering a big area When the soil dries up, the filaments become pale yellowish green Also collected from Bisaran, near Pahlgam, Kashmir, on 30th July, 1941, growing on clayey soil, and from village Nain in Rai Bareli district, U P, in January 1943

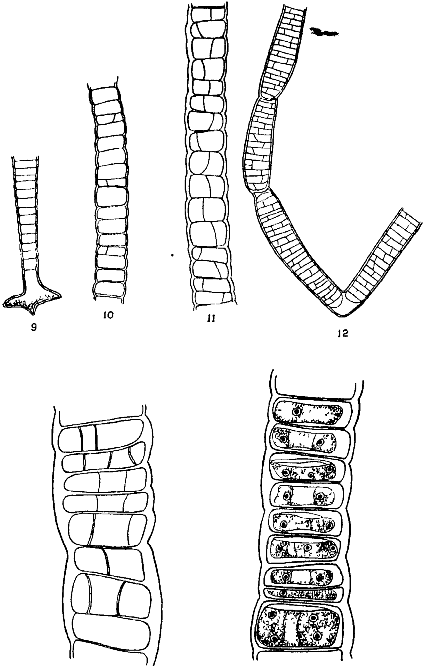
Schizomeris Kutzing

1 *Schizomeris Liebleinii* Fritsch and Rich *Trans Roy Soc S Africa*, Vol xi, p 317

Filaments 1-2 cm long, attached to the substratum by a basal hyaline cell (Fig 9) Filaments uniseriate in the lower end, resembling *Ulothrix zonata*, and higher up become separate and multiseriate on account of cell division Septation is irregular (Figs 11–13) Filaments are 30–60μ broad with a thick enclosing hyaline sheath, 3μ broad

Filaments are constricted at irregular intervals, as a prelude to fragmentation (Fig 12) which appears to be a common mode of vegetative propagation in this alga Chloroplasts are collar shaped bearing two or more pyrenoids as in *Ulothrix zonata*, in the lower parts of filaments

Apices of some of the thalli showed loose masses of cells with more or less complete breakdown of cross-septa and side walls (Fig 15) This stage possibly represents an early stage in zoospore formation, and the loose cells are very likely immature zoospores This mode of zoospore formation from the apex of the thallus is rather unique, and shows that *Schizomeris* is a distinct form resembling *Ulothrix* only in the structure of the chloroplasts, and differing widely from the latter in the mode of liberation of zoospores, which in *Ulothrix* escape laterally This apical



FIGS. 9–14. *Schizomeris Lieblienii.*

mode of zoospore-liberation is the necessary consequence of a solid multicellular thallus formation and thick enclosing cell wall

This form resembles *S irregularis* described by Fritsch and Rich from South Africa in the irregular septation of thalli In size it is much bigger than the African form, the maximum width of whose filaments is given as 37μ

Habit —This alga was collected in August 1929 growing on stems of water-plants in a tank at Bodal, district Hoshiarpur, Punjab A broader form of this alga was collected from a waterfall of Vernag Spring, Kashmir, the source of Jhelum river, on 22nd August, 1941

Enteromorpha (Link) Harvey

1 *Enteromorpha intestinalis* (L) Greville Heering Süsswasser-flora 6, p 27

Thallus tubular, sparsely branched, branches irregular, not constricted at the base Fronds intestine-like, inflated, free-floating, mature ones flattened at the top Younger thalli 180μ to 1 mm in diameter, thread-like in appearance, mature ones 1 2 cm broad and 12–15 cm long (Fig 16)

In younger thalli, cells are rectangular to polygonal in outline, arranged in regular rows $9–15\mu$ broad and $21–40\mu$ long Each cell contains a single chloroplast bearing a conspicuous pyrenoid In older thalli, cells are polygonal in outline and irregularly arranged and are $15-16\mu$ in diameter

Habit —Collected by the author from Chambal river below Pinahat in Agra district from January to June 1941 Young plants are found free floating in June, and those collected in January were mostly mature plants

Geminella Turpin

1 *Geminella interrupta* Turpin Heering Susswasser flora 6, 1914, p 41

Filaments enclosed by a gelatinous transparent sheath, which stains deep blue with Nile blue, and is only visible when stained Cells usually in pairs, oval, each containing a laminate chloroplast, with 1-2 pyrenoids (Fig 17)

Filaments inclusive of sheath $15–30\mu$ broad Cells $6\ 7\mu$ broad and $9–12\mu$ long

Habit —Collected from a sluggish fresh-water stream near Meja district, Allahabad, on 15th March, 1940

Binuclearia Wittrock

1 *Binuclearia tatrana* Wittrock Heering Süsswasser flora 6, 1914, p 39

Cells cylindrical, $6–10\mu$ broad, $15–30\mu$ long, oval in shape, sometimes appearing grouped in pairs End walls filled with mucilage which is deposited in layers There is a distinct bulging opposite the septa, which appear biconcave in shape (Fig 18)

The chloroplasts show a conspicuous pyrenoid Describing the chloroplast of this genus Smith writes, 'The protoplast of a *Binuclearia* cell has a single laminate chloroplast without a pyrenoid, that completely encircles the cell ' Fritsch also observes that 'a pyrenoid is not readily distinguishable' On the contrary, in this Himalayan form, the solitary pyrenoid in each protoplast is very conspicuous

Habit —Collected from a marshy piece of land near the D B bungalow at Dhakuri, on the Pindari glacier route in Almora district in the Himalayas, at an altitude of 8,500 feet on 16th September, 1939

Prasiola Meneghini

1 *Prasiola fluviatilis* (Sommerf) Areschoug Heering Susswasser-flora 6, 1914, p 59.

Thalli lanceolate, or irregular in outline, attached to stones by a thickened stipe, which may be slightly funnel-shaped at the base in some cases Thalli 3–16 cm long and 2-3 cm broad, and in some cases may be 8–10 cm broad (Fig 19)

Cells are grouped in quartettes, 5-6μ in diameter, each with a single, central, more or less stellate chloroplast, bearing a solitary pyrenoid (Fig 20)

This alga resembles the type in most respects, though some of the thalli are longer and broader than the biggest so far recorded

Habit —Collected by the author from a torrential stream above Diwali, on the Pindari glacier route in the Himalayas, Almora district, in September 1939 Also collected by the author from Liddar and Sheshnag rivers near Pahlgam, Kashmir, in August 1941, grows luxuriantly in ice cold water in Himalayan torrential streams attached to stones

Microspora Thuret

1 *Microspora indica* Randhawa

Vegetative cells 18–24μ broad and 21–36μ long Cell wall conspicuously lamellated, composed of H shaped overlapping halves (Fig 21) Chloroplast parietal with cushion like outgrowths at the sides Vegetative propagation by akinetes

Habit —Found free floating in a *jhil* near village Pachham Sareera, Tahsil Manjhanpore, district Allahabad, in February 1940

SUMMARY

The following twelve species of Ulotrichales have been collected from Northern India —

Ulothrix subtilissima Rabenhorst (Punjab, Kumaon), *U tennerima Kutzing* (Lahore, Agra), *U tennuissima* Kutzing (Kashmir), *U oscillarina* Kutzing (Punjab), *U zonata* Kutzing (Punjab, Kashmir), *Hormidium flaccidum* A Br (Kumaon, Kashmir), *Schizomeris Liebleinii* Fritsch and Rich (Punjab, Kashmir), *Enteromorpha intestinalis* (L) Creville (Agra), *Geminella interrupta* Turpin (Allahabad), *Binuclearia tatrana* Wittrock (Kumaon), *Prasiola fluviatilis* (Sommerf) Areschong (Kumaon, Kashmir), and *Microspora indica* Randhawa (Allahabad)

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EXPLANATION OF PLATES

Figs 6–8 *Hormidium flaccidum* A Br *forma* typica Heering (For explanation *see* text)

Fig 15 *Schizomeris Liebleinii* Fritsch and Rich For further explanation *see* text

Fig 16 *Enteromorpha intestinalis* (L) Greville shows different types of thalli

Fig 17 *Geminella interrupta* Turpin shows a filament

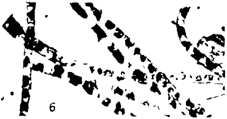
Fig 18 *Binuclearia tatrana* Wittrock

Figs 19 20 *Prasiola fluviatilis* (Somerf) Areschoug

Fig 19 shows different types of thalli

Fig 20 Arrangement of cells in the thallus

Fig 21 *Microspora indica* Randhawa shows structure of cell wall and chloroplasts



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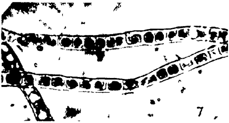

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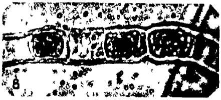

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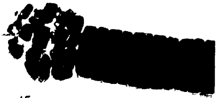

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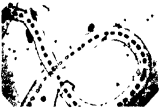

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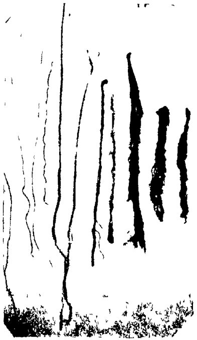

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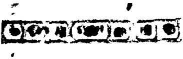
Randhawa Photo



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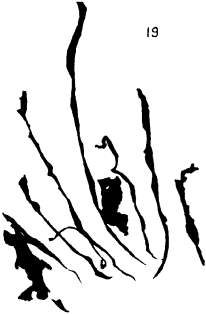

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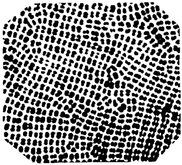

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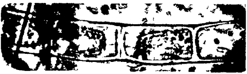

Randhawa – Photo

ON SOME ARCHIANNELIDS OF THE KRUSADAI ISLAND *

By K H Alikunhi, *M Sc*, *Freshwater Biological Research Station, Government Fisheries, Madras*

(Communicated by Prof R Gopala Aiyar, F N I)

(*Received November 15, 1947*, *read October 1, 1948*)

INTRODUCTION

Larval forms of *Polygordius* and *Saccocirrus* had been recorded from Indian waters (Goodrich, 1900, Aiyar, 1933 and 1935), but only recently have adult archiannelids been shown to occur in the intertidal zone of the Indian coast by Aiyar and Alikunhi (1944) who have described six new species of Archiannelids, viz *Polygordius madrasensis*, *P uroviridis*, *Protodrilus pierantonii*, *P indicus*, *Saccocirrus minor* and *S cirratus* from the Madras beach, two of these species—*Protodrilus pierantonii* and *Saccocirrus minor*—also from the Malabar coast (*loc cit*, p 126)

While on a visit to Krusadai in September 1940, I had an opportunity of examining the intertidal sand around the island, and an examination of samples of sand revealed the presence of *Polygordius madrasensis* Aiyar and Alikunhi, *Protodrilus pierantonii* Aiyar and Alikunhi and a new species of *Saccocirrus*, in fairly large numbers As this is the first record of archiannelids from this area, a brief account of these species is given in this paper

The intertidal sand around the island is considerably coarse, major portion being formed of large pieces of broken shells and corals, clean gravelly sand is rare and in most places the substratum at low-water level is formed of muddy sand or mud

Worms were collected by taking samples of sand in a glass trough and shaking them vigorously with sea water when the worms, disturbed and shaken off from their hiding places, could be seen swimming in water and were easily pipetted out

Polygordius madrasensis Aiyar and Alikunhi

Specimens of this common Madras species were obtained from 'Sandy Point', Krusadai, in coarse sand, a little above the low-water mark A dozen specimens, most of them mature females, filled with ova in the middle and posterior segments, were obtained in a single haul It is probable, therefore, that they occur in large numbers The worms were very active, and in the living condition each measured about 15 to 20 mm in length They are slightly longer than the Madras specimens which only rarely reach above 12 mm Examination under the microscope, however, revealed that they are identical with the Madras species in all essential features

Protodrilus pierantonii Aiyar and Alikunhi

This species, which is also one of the commonest archiannelids of the Madras beach, occurs in large numbers in clean sand in the intertidal zone near Pamban bridge The substratum here is different from that at 'Sandy Point' and consists of clean minute pebbles, with a considerable portion of fine silt. A similar substratum is present in the intertidal zone at Rameswaram also. Collections were made from this region in July 1943, and several specimens of these minute archiannelids obtained. Specimens from both these localities were fully ripe—males and females with mature gonads—and measured 2.5 to 5.0 mm in length.

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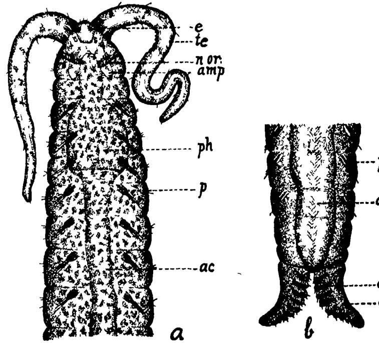
* Paper read before the 29th Annual Session of the Indian Science Congress Association, Baroda, 1942.

Saccocirrus krusadensis sp. nov.

A species of *Saccocirrus* was found to occur in considerable numbers in coarse sand at 'Sandy Point', Krusadai. Smaller specimens of the same species were also obtained from Pamban and Rameswaram, in the same samples of sand as contained specimens of *Protodrilus*. At 'Sandy Point', due to the coarse nature of the substratum and the constant disturbance of the upper layers of sand by the incessant waves, these worms are actually found two or three inches below the surface layer of sand, very near the low-water level.

A good number of specimens were collected and studied fresh at Krusadai itself, but some of them were brought alive to the Zoological Research Laboratory, Madras, and were examined for further details. It is found that the worm possesses important features in which it differs from all the known species of the genus, and hence in the following pages it is described as a new species under the name *Saccocirrus krusadensis*.



TEXT FIG. 1. *Saccocirrus krusadensis* sp. nov.

(*a*) Anterior end, dorsal aspect, drawn from life. × 90.
(*b*) Caudal end showing structure of pygidium, drawn from life. × 130.

ac—Alimentary canal, *al*—Anal lobe, *amp*—Ampulla, *ap*—Adhesive papilla, *e*—eye; *n or*—Nuchal organ, *p*—Parapodium, *ph*—Pharynx, *te*—Tentacle.

External Characters

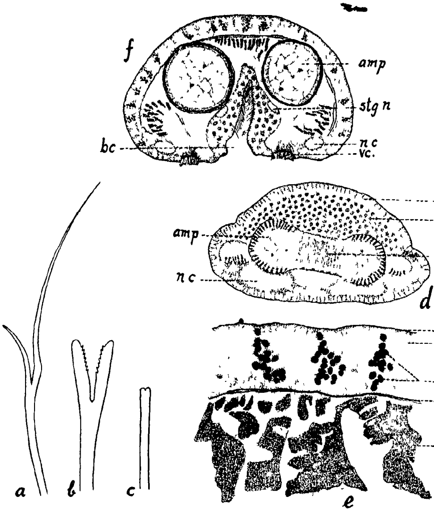
Worms are very active and creep about in a characteristic leech-like fashion when placed in a dish of clean sea water They measure 20 to 25 mm in length in the living condition when the body is normally extended The number of segments varies from 100 to 150 Colouration is pale white with a tinge of light green The body, as usual, is highly contractile and tapers to either extremity but more so to the posterior The prostomium is conical and carries the two characteristic long tentacles which as in *S minor* are not constricted into segments (Fig 1*a*) At the base of each tentacle is a group of actively vibratile cilia The eyes, situated immediately behind the anterior extremity of the prostomium, are in the form of a pair of dark, usually reniform pigment spots without lenses and with the convex sides facing each other The nuchal organs are situated behind the level of the tentacular bases The tentacles and the tip of the prostomium are provided with palpocils

The pygidium is provided with a pair of elongated anal lobes, slightly tapering to the tip (Fig 1*b*) Each lobe carries 6 to 9 glandular papillae situated on its ventro median aspect The anterior ones of these papillae are larger than the hinder ones On each papilla a large number of elongated adhesive glands open The secretion of these glands enables the worm to attach to the substratum by this end Palpocils are present on these papillae The structure of pygidium, as described above, differs from that in *S minor* and *S cirratus* in its typically bifurcated nature and in the absence of anal cirri In the number, relative size and disposition of these papillae the present form differs from the other three species of the genus also

There is a uniform development of epidermal glands on the body surface Groups of minute palpocils are present in every segment, on either side, at about the parapodial level

On the ventral surface, at the level of the buccal invagination on either side of the median line, there is a linear band of cilia, arising from a shallow groove In transverse sections these cilia are situated on either side between the wall of the buccal invagination and the region directly below the nerve cord (Fig 2*f*) The cells adjoining the cilia seem to be of a sensory nature

The parapodia are minute cylindrical structures which can be retracted into depressions on either side in the ventro lateral aspect of the segment The first segment behind the head and also the last two are apodous and achaetous, while the remaining segments possess parapodia with chaetae In all the other known species of the genus the last few segments varying from 5 to 12, or even more, are devoid of parapodia and chaetae Setae are all simple bristles and there are 8 or 9 of them in each foot These bristles are of three distinct types as follows (1) Extremely slender long bristles deeply bifid at the tip (*furcate seta*) the two prongs are markedly unequal, the longer one with a slight bend at the base, being about three or four times the length of the shorter prong (Fig 2*a*), in the anterior as well as the posterior segments there is only a single bristle of this kind in each foot, while in the middle segments two such setae may be present in each, these setae project beyond the others in the bundle (2) Comparatively stout bristles, also deeply bifid at the tip, but with equal prongs (Fig 2*b*), the inner aspect of each prong is delicately serrated, there are three such setae in each of the anterior and posterior parapodia, while there may be four in the middle ones (3) Simple slender capillary bristles with blunt tips which may be imperceptibly notched (Fig 2*c*), there are two or three such bristles in each foot It may be observed that in *S minor* and *S orientalis* the tips of setae are all blunt; in *S cirratus* all the setae, except one which is crutch-shaped, are similar to those in *S minor*, in *S papillocercus* one bristle in each foot is provided with three prongs at the tip and the rest with blunt tips; and in *S major* the setae tips have three short projections It is therefore clear



TEXT FIG 2 *Saccocirrus krusadensis* sp nov

(*a*) Slender furcate seta with unequal prongs × 1,750
(*b*) Seta with bifid serrated tip × 1,750
(*c*) Simple slender bristle × 1,750
(*d*) Transverse section through hind portion of head showing transverse communicating canal between ampullae of head cavity × 400
(*e*) Enlarged view of section showing structure of body wall × 1,300
(*f*) Transverse section showing commencement of buccal invagination × 400

bc—Buccal invagination, *c*—Cuticle, *cm*—Circular muscle, *epi*—Epithelium, *gc*—Ganglion cells, *lm*—Longitudinal muscle, *nc*—Nerve cord, *ng*—Nuclear granules, *stg n*—Stomatogastric nerve, *tc*—Transverse connective that the structure of the setae in *S. krusadensis* is different from that of the other species and is of taxonomic value.

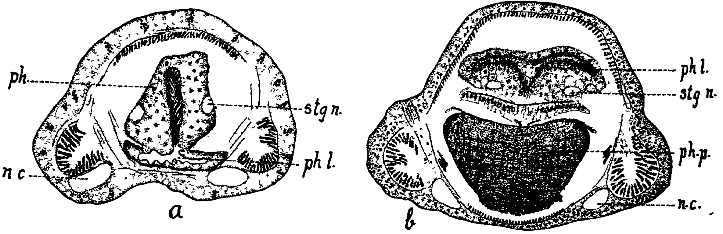
INTERNAL CHARACTERS

Head cavity.—In general structure the head cavity resembles that of the other species. The transverse communicating canal is situated at the posterior extremity of the brain. The ampullae are short and tapering, and, unlike the condition in *S. minor*, do not extend to the first setigerous segment (Fig. 1*a*). Circular muscles are well developed in the wall of the ampullae. A colourless fluid with nucleated corpuscles fills the head-cavity. In sections the lumen of the ampullae is filled with a reticulum of thick fibrils with nuclei at intervals (Fig. 2*d* and *f*). The passage of the fluid from one ampulla to the other is inhibited by a set of delicate muscle fibres that traverse the communicating canal (Fig. 2*d*).

Body wall.—The epidermal layer is considerably thick. Each nucleus is in the form of a deeply staining conspicuous body, usually broad at the inner and tapering towards the outer aspect of the cell, and under high magnification has a pronounced granular appearance (Figs. 2*e* and 3*a*). This peculiar structure of the nucleus seems to be characteristic of the genus and similar nuclear structure could be made out in *S. minor* and *S. cirratus* also. The circular and longitudinal muscle layers are well developed and an extremely thin coelomic epithelium lines the body cavity.

Alimentary Canal.—The structure of the anterior part of the alimentary canal, though greatly differing from that of both *S. minor* and *S. cirratus*, closely resembles the pharyngeal apparatus of *S. papillocercus*. The buccal invagination commences from behind the level of the transverse communicating canal of the head-cavity, and extends to the level of the hind end of the ampullae. The invaginated wall is enormously thickened, especially on the sides, and its roof or dorsal wall is powerfully ciliated. The ventral wall is lined by cuticle. Immediately behind, a muscular pad is developed on the ventral wall and this greatly obliterates the lumen. Further behind, this pad separates from the pharyngeal wall which is then completely devoid of muscles. The ventral wall is very thick and the lumen is ciliated on all sides. From the level of the posterior extremity of the muscular pad, that is, from the fourth setigerous segment, a layer of muscles makes its appearance, first on the ventral aspect and then gradually encroaching on to the dorsal wall. From the 4th to the 14th segment the alimentary canal is in the form of a narrow tube with thick walls, made up of a single layer of large secretory cells. They are invariably filled with secretory granules which stain deeply. It may be mentioned that in *S. papillocercus*, Marion and Bobretzky (1875), as well as Goodrich (1901) describe in the alimentary canal a glandular region representing the digestive stomach and, as in *S. krusadensis*, extending to the 14th segment. In the following segments the gut is expanded and saccular with intersegmental constrictions. In this region the cells are smaller and have rounded basal nuclei. Unlike the condition in *S. papillocercus* chloragogen cells are absent from the outer surface of the alimentary canal.

Nervous System.—In the brain the ganglion cells are crowded towards the dorsal aspect. The brain at its posterior extremity is continued as two ventro laterally directed, broad nerve trunks—the beginnings of the ventral nerve cords. In *S. papillocercus* (Goodrich, *loc. cit.*) the ventral nerve cords arise from the middle portion of the brain. The buccal invagination commences only after the nerve trunks have assumed their ventral position. In front of the buccal invagination each nerve trunk gets divided into two stout nerves, the one nearer to the median line getting itself associated with the wall of the buccal invagination, thereby constituting the stomatogastric system of nerves, while the outer nerves continue as the ventral nerve cords. In *S. papillocercus* Goodrich traces the origin of the



TEXT-FIG 3 *Saccocirrus krusadensis* sp nov

(*a*) Transverse section through anterior pharyngeal region Note the thick pharyngeal walls × 450
(*b*) Transverse section through hind region of pharynx Note muscular pad × 300

ph l —Pharyngeal lumen , *ph p* —Pharyngeal pad stomatogastric nerves up to the point of origin of the ventral nerve cords, which, as already mentioned, is situated more towards the middle portion of the brain The stomatogastric nerves running along the wall of the buccal invagination gradually get on to the ventral wall of the pharynx and, finally, near the posterior extremity of the muscular pad unite together into a small enlargement

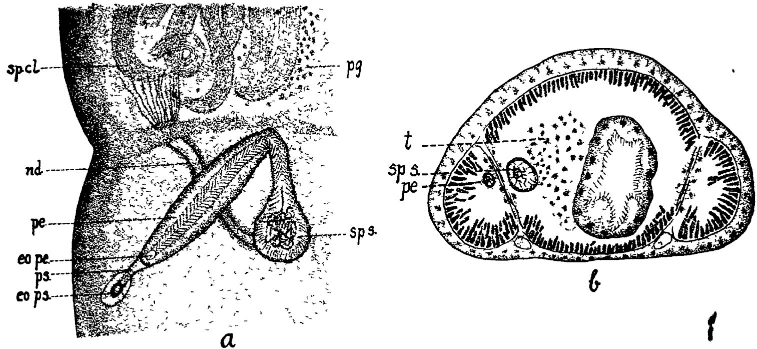
Reproductive Organs —Sexes are separate and almost all the specimens collected from Krusadai were fully mature, while those from Pamban and Rameswaram were small and immature

Male —The gonads are not developed in some of the anterior and a few extreme posterior segments In a specimen with 99 setigerous segments testes were developed in segments 27 to 90 It is interesting to observe that in all the specimens collected, both males and females, the genital elements were developed only on one side, usually the left Sections also show that the gonads are confined only to one side of the segmental chamber Further, the accessory reproductive structures like the sperm-sacs and penes in the male and the receptacula seminis in the female are all conspicuous by their unpaired nature and are developed only on the same side as the gonads Since this condition has been observed in all the mature specimens examined—more than 50 in number—it is likely that this may be the normal feature in this particular species However, the exact reason for this complete suppression of the genital elements from one side is not clear Whether the method of copulation, which unfortunately has not been observed so far, would possibly explain this very peculiar one-sided condition of the gonads is not now known

The testis is diffuse In living specimens a number of reddish-brown pigment granules surround each testis mass In sections they take up a dark stain The sperms are elongated, whip-like and extremely slender When pressed out they show activity by movements of their long flagella from which the head is hardly differentiated

Paired nephridia in the form of simple straight tubes are present from the 2nd segment backwards In the genital segments, a group of long cilia is developed surrounding the nephrostome, and the nephridial duct also gets slightly enlarged behind the septum It then enters a thin-walled circular ciliated chamber—the sperm-sac (Fig 4*a*) The sperm sac is situated in the median chamber of the body cavity (Fig 4*b*) In this feature it markedly differs from the other species of the genus which have the sperm sacs situated in the lateral chamber of the body cavity In living specimens the wall of the sperm sac has a shrivelled up or fringed appearance It is formed of a single layer of cells with rather prominent nuclei The cavity of the sperm sac is usually filled with sperms which are carried down into it from the coelom by the nephridial duct The sperm sac is continued as a comparatively broad ciliated duct, which entering the lateral chamber of the body cavity turns up to the dorsal aspect and slightly enlarges to form the penis (Fig 4*a*) The penis is in the form of an elongated cylindrical organ, hardly tapering to the tip It is internally ciliated and fairly thick walled, the cells being granular in appearance Unlike the condition in *S papillocercus* and *S minor* cuticular rods supporting the penis are extremely slender There is a short penis sheath formed by the invagination of the body-wall The tip of the penis, in the normal retracted condition, reaches the base of this sheath The penis is easily protrusible The external aperture of the penis is oval and the penis sheath is situated just above the parapodium of each segment, in the dorso-lateral aspect of the body

Female —Mature females are usually of a slightly deeper colour than the males As in the males, the genital elements are absent from some of the anterior and few of the hindmost segments In a specimen with 107 setigerous segments ova were developed in segments 26 to 94, but only on one side as in the males They are comparatively large and are of a greyish colour In ripe individuals they are closely packed and in the normal condition they do not show any tendency to pass over to the other side



Text fig 4 *Saccocirrus krusadensis* sp nov

(*a*) Position and arrangement of reproductive organs of male, drawn from life × 470
(*b*) Transverse section through a male genital segment Note sperm sac in median chamber of body cavity and testis only on one side × 280

eo pe—External opening of penis *eo ps*—External opening of penis sheath, *nd*—Nephridial duct, *pe*—Penis, *pg*—Pigment granules, *ps*—Penis sheath, *sp cl*—Sperm clusters, *sp s*—Sperm sac, *t*—Testis

Receptacula seminis are developed in the ovigerous segments and are in communication with the nephridial duct (Fig 5a) They are not paired, each segment having only one receptaculum seminis At its proximal end it is a thin-walled spacious sac, invariably containing spermatozoa It is situated, like the sperm-sac, in the median chamber of the body cavity, but is not ciliated internally The saccular portion is continued into a broad duct which suddenly develops a considerably thick wall of granular cells and a powerful internal lining of cilia The duct gradually narrows and finally opens to the exterior on the ventral side along with the nephridial duct (Fig 5b) Sections reveal that between the proximal portion where the sperms are stored and the duct of the receptaculum seminis there is a second saccular portion which markedly differs from the proximal part in the nature of the lining epithelium It takes lighter stain than the proximal part while the duct itself stains deeply

In these segments the nephridia on the same side as the receptacula seminis are slightly more enlarged than usual and the cilia near the nephrostome cover a larger portion of the septal surface The nephridial duct running between the lateral longitudinal and circular muscles finally opens to the exterior along with the receptaculum seminis by a common aperture (Fig 5b)

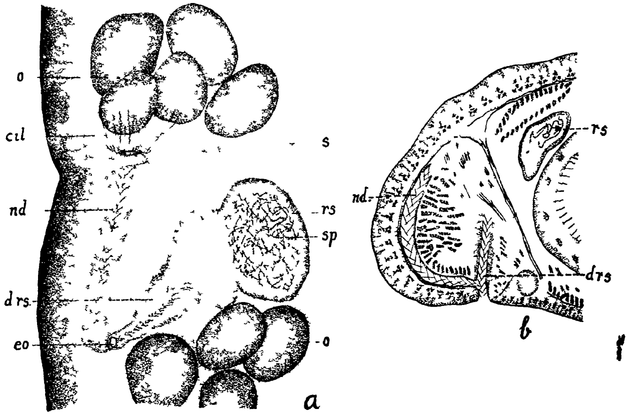
The presence of sperms in the receptaculum seminis can only be explained, as in other species, by assuming that copulation has previously taken place, though this has not been observed The relation between the receptaculum seminis and the nephridial duct suggests that the eggs when fully mature might be carried down the nephridial duct and get fertilised at the point of extrusion, by the sperms sent down from the receptaculum seminis Bobretzky (*loc cit*) has seen the nephridial duct dilated with eggs in ripe specimens In the present instance also transverse sections have been obtained of a ripe female in which the ovum is actually half way down the nephridial duct, in its way to the exterior Sperms have never been observed in the coelomic cavity of the female but are always found stored within the receptaculum seminis

Remarks—Including the present form the genus *Saccocirrus* at present consists of six species, viz *S papillocercus* and *S major* from Europe and Japan, *S minor*, *S. cirratus* and *S orientalis* from Madras, and the present form, *S krusadensis*, from the Gulf of Manaar In the structure of the setae and pygidium *S krusadensis* markedly differs from the Madras species In the musculature of the pharynx it resembles *S papillocercus* but the nature of the setae and pygidium, shape of the penis, the position of the sperm sacs and the one-sided nature of the gonads clearly mark it out from both *S papillocercus* and *S major*

DIAGNOSTIC FEATURES

Slender worms, 20 to 25 mm long, with 100 to 150 segments, parapodia on all segments excepting the first and the two hindmost ones, setae simple, deeply forked and of three types, anal lobes with 6 to 9 adhesive papillae, the distal ones much smaller than the proximal ones, ampullae of the head-cavity confined to the head segment, pharyngeal apparatus well developed and muscular, reproductive organs in both sexes developed only on one side, sperm-sac situated in the median chamber of body cavity, penis in the form of an elongated, cylindrical, easily protrusible structure, with inconspicuous cuticular rods, and receptaculum seminis with ciliated external duct

Locality—Sandy beach, Krusadai, Pamban and Rameswaram in the Gulf of Manaar



TEXT FIG 5 *Saccocirrus krusadensis* sp nov

(*a*) Relative position and arrangement of reproductive structures of female drawn from life × 500
(*b*) Transverse section showing common external aperture of nephridium and receptaculum seminis × 650

cil.—Cilia, *d rs* —Duct of receptaculum seminis, *eo* —External opening *o* —Ovum *rs* —Receptaculum seminis, *s* —Septum, *sp* —Sperms

KEY TO SPECIES

The following key has been prepared in the hope that it will facilitate easy diagnosis of the six species now included in the genus *Saccocirrus*, external characters alone have been taken into consideration —

1	Pygidium bifurcated, with or without adhesive papillae	2
	Pygidium not bifurcated, with two adhesive pads	*Saccocirrus minor*
2	Without adhesive papillae, with two long anal cirri	*Saccocirrus cirratus*
	With adhesive papillae, without anal cirri	3
3	Tips of setae smooth, without prongs	*Saccocirrus orientalis*
	Setae with two or three prongs at the tip	4
4	With *two* prongs at the tip	5
	With *three* short equal prongs at the tip	*Saccocirrus major*
5	With the prongs unequal in few and equal in others	*Saccocirrus krusadensis*
	Prongs short and equal in all setae (three prongs in one seta only)	*Saccocirrus papillocercus*

ACKNOWLEDGMENT.

The present work was carried out at the University Zoological Research Laboratory, Madras, during 1940-41, under the guidance of Prof R Gopala Aiyar, then Director of that laboratory My grateful thanks are due to him for his kind interest and the suggestive criticisms offered during the course of the work I am also indebted to the University of Madras for affording me facilities to carry out this investigation

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